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Origami and the Emergence of Hybrid Diagrams

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Abstract. The paper discusses the emergence of *hybrid diagrams* in the context of origami practice with respect to the study of the crease pattern, a particular diagram that can be associated with any origami model. We introduce the expression "hybrid diagram" to refer to a 2D diagram that embeds physical parts of the origami model and information about transformations occurred in space, or to an origami model on which attempts to grasp parts of the crease pattern appear. We focus on some university students working with the crease pattern for a given origami model. A first analysis of the work of these students allows for a preliminary characterization of hybrid diagrams: they encapsulate relations between the 3D model and the crease pattern and reveal the entanglement of diagrammatic activity with the gestural and the material. Drawing on the cognitive perspective of semiotic representations by R. Duval and diagrammatic thinking by C. Peirce, we interpret the emergence of hybrid diagrams as relevant to the conversion between different (mathematical) registers.

Keywords: Origami, Crease Pattern, Hybrid Diagram, Mathematical Thinking.

1 Origami and Diagrams

In this paper we use the idea of "hybrid diagram" drawing on observations made in the context of a teaching experiment that involved a group of university students in activities with origami models and the crease pattern, a diagram that consists of all or most of the creases that are folded in the final origami model.

The making of an origami consists in repeatedly folding one or more squared sheets of paper to obtain other (three-dimensional) shapes which can resemble animals or flowers, as well as recall geometric shapes or patterns. Far from being just a recreational activity, in recent years it has had important applications in many fields, like the aerospatial and medical field. From an educational perspective, a major interest in paper folding lies in the possibility of exploring geometric properties through material activity. The geometry of origami has its mathematical formalization in a set of seven axioms, which identify the ways in which it is possible to create a fold. These axioms have become famous as Huzita-Justin or Huzita-Hatori Axioms [9]. The list is also complete [2]. Most of the movements that contribute to the creation of an origami model are based on axioms, making the underlying mathematical theory particularly rich and interesting from the didactic point of view for they allow the discovery and study of mathematical relations in a concrete context (e.g., [6] and [8]).

Most of the available books on origami illustrate the process of making a model through instruction diagrams. In such diagrams, the sheet of paper is shown generally as a square, and each step of the construction is accompanied by arrows that indicate the direction of the movements to be performed and by marks that capture the position and type of fold (valley or mountain creases). The instruction diagrams provide an iconic representation of the steps in the construction, while the final model incorporates all the transformations made by paper folding. The relationships between an origami model and the set of transformations undergone by the sheet of paper through the activity of folding is captured by another diagram: the *crease pattern*. Some beautiful examples of crease patterns are available on the site of the origamist Robert J. Lang [12]. In the initial page of the website, Lang points out how in a crease pattern, one can see everything that is hidden in the folded work.

Intuitively, we can revisit the definition of crease pattern given by Hull [7], introducing it as the plane diagram that consists of the lines representing the fundamental valley and mountain folds, i.e., all the folds that are folded in the origami in its final form. An example of an origami model and the relative crease pattern is given in Fig. 1.

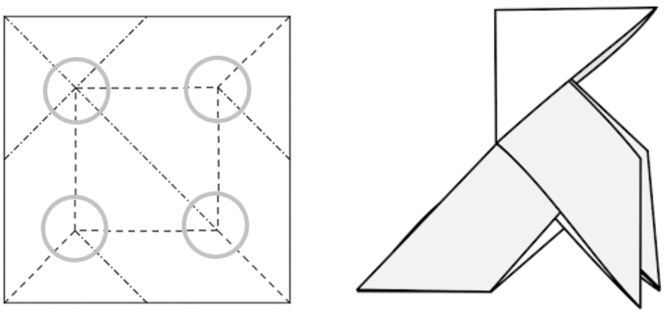


Fig. 1. The crease pattern and the model of the Pajarita, a classical origami. The vertexes of the crease pattern are circled in the first diagram.

The crease pattern is therefore a plane diagram that contains important information about the nature of the folds composing the final model, but only the expert eye can "reconstruct" (or imagine) a model starting from its crease pattern. Even the reverse process (i.e., building the crease pattern starting from a folded origami model) is not obvious, because it requires a considerable effort of three-dimensional visualization. It is not sufficient, indeed, to reopen the origami model and highlight the traces of the folds. Two distinctions must be made: (1) fundamental folds must be recognized and distinguished from those that are folded in the construction but no longer in the final model; (2) these fundamental folds can be *valley* or *mountain* creases. Following Hull [7], in a crease pattern generally valley folds are represented with dashed lines and mountain folds with dash-dot lines. Moreover, when drawing a crease pattern while looking at the relative origami model, it is necessary to "always look at the sheet of paper from the same side", since when the sheet is turned upside down, mountain folds become valley folds and vice versa.

In a crease pattern, which is a square-shaped diagram within which folds are represented through segments, we call a *vertex* each point inside the square where at least two distinct lines concur (see Fig. 1 again). In our work we have focused our attention on the characteristics of particular origami, called flat origami. Intuitively, a flat origami can be closed in a book without creating further folds and without removing any fundamental fold: it is therefore an object that, despite being three-dimensional, since it is made up of multiple layers folded over each other, can be treated as two-dimensional.

Studying the crease pattern is of interest for many reasons. We select here two of them, which are relevant to this paper. First, the crease pattern *shows* what is hidden in the model once folded, therefore it opens a different window on the creation process of an origami and the relations among folds in the origami. Secondly, properties of a flat origami can be illustrated and expressed through the crease pattern, so this is a space for rich mathematical explorations.

In this paper, we will focus on some students working on the task of drawing the crease pattern of an origami model, and we will describe the emergence of types of diagrams in their activity, which we call *hybrid diagrams*. We will present a qualitative analysis of the students' activity that shows how such diagrams emerge and sustain the mathematical exploration. In the next section we will frame these ideas drawing on research in mathematics education on semiotic representation.

2 Semiotic Representation in Mathematical Thinking

Duval [5] stresses the importance of *semiotic representation* for any mathematical activity. He introduces semiotic representation in relation to the attempt of better understanding the difficulties that students have with comprehension of mathematics, and their nature. One specificity of mathematical thinking exactly is the cognitive activity required by mathematics, which makes use of semiotic systems of representation. Signs, or semiotic systems of representation, play a role not only to designate mathematical objects or to communicate but also to work on, and with, mathematical objects. For Duval, no kind of mathematical process is performed without using a semiotic system of representation: mathematical processes always involve "*substituting some semiotic representation for another*" (p. 107, *emphasis in the original*). Therefore, in mathematical activity what matters is not representations but the transformation of representations. Semiotic activity is so relevant to mathematics (and mathematics education) because signs and semiotic representations allow access to mathematical objects. Ambiguity can emerge when learners must distinguish objects and their representations. According to Duval, the ability to change from one representation system to another is critical to progress and problem solving. Mathematical activity has different semiotic representation systems, called *registers*: the verbal, the numerical, the graphical, the symbolic, each providing specific possibilities for performing mathematical processes. There are two different types of transformations of semiotic representations: treatments and conversions. Treatments occur within the same register and can be carried out depending on the possibilities of semiotic transformation which are specific to the register used. Conversions instead are transformations of representation that consist of changing a register without changing the objects, like when we pass from the algebraic notation for a function to its graph. Briefly speaking, these transformations capture changes in or of register.

Today, semiotic activity in mathematics is regarded as more complex than just implying treatments of and conversions between the semiotic registers à la Duval and has been expanded to incorporating bodily-based signs, like gesture, gazes, tones of voice, sketches, tool usages, and so on, so that we speak of *multimodal* or *sensuous* mathematical cognition [11], meaning that mathematical cognition involves multiple modalities and senses, besides registers. Thus, we refer to *semiotic sets* instead of registers. Arzarello [1], for example, has introduced the notion of *semiotic bundle* to capture the relationships in and within different semiotic sets. In this paper, we consider *diagrams* as one possible semiotic resource that is activated in mathematical thinking. In so doing, we must refer to Peirce's theory of cognitive activity and his attempt to rescue the import of perception [10]. Peirce considers diagrammatic thinking as central to discovery of new conceptual relations, which remained hidden before or beyond the realm of our attention and are instead made apparent by perceptual inspection.

What matters to us in respect to Peirce's consideration of diagrams is therefore the role that they can play in reasoning about mathematical relations. We are not interested in the appearance of diagrams but more in their nature (how they emerge) and function (why they emerge), because this helps us to better investigate cognitive activity in mathematics. In addition, the history of mathematics shows that relevant mathematical ideas were discovered or advanced with a productive semiotic activity involving an interplay of gestures and diagrams [3]. Borrowing from these ideas, we see diagrams as a semiotic set consisting of graphs, sketches, figures, and any form of visual thinking expressed in the written. Focus is put on the emergence of kinds of diagrams in mathematical activity, which we call *hybrid diagrams*.

3 The Emergence of Hybrid Diagrams

3.1 The Mathematical Activity

For this paper, whose purpose is to present and discuss the emergence of hybrid diagrams in the context of mathematical paper folding, we centre our attention on a specific task. Some university students were asked to draw the crease pattern corresponding to each step of the construction process of an origami model. This task is relevant to the issue of conversion between different registers in mathematics, considering the origami model and the crease pattern as two different registers for the same object. The teaching experiment was aimed at creating the opportunity for university students to engage with origami and their representations and explore the features of flat origami regarding the mathematical properties of their crease pattern. The experiment was designed by the authors and carried out during the first semester of the academic year

2020/21, when university courses were held online because of the Covid pandemic. It engaged 29 master's degree students in mathematical explorations of origami models using 7 worksheets and the Google Meet platform. The conditions of distance teaching and learning are relevant to our research study. Initially, main interest was in the creation and study of mathematical activities involving origami to make the students explore non-elementary properties of flat origami. Additional interest arose concerning the understanding of the way in which the online environment could trigger new strategies for mathematical exploration and communication. Data for the analysis mostly consists of the video recordings of the Google Meet rooms in which the students worked in groups to face the tasks of the worksheets. Also, the written materials produced by the groups were uploaded to online shared folders. Our qualitative analysis employed techniques from micro-ethnography [13] to understand how the students make sense of the paper folding activities.

The first two worksheets focused on the creation and analysis of the crease patterns of two simple origami: the triangle base and the square base, which generally are the basis of folds for more complex origami constructions. In the third worksheet, the focus was on the analysis of the crease pattern created by another group and on the concept of vertex in the crease pattern. Worksheet 4 was divided into two parts (a and b) and centred on the request to create the sequence of crease patterns corresponding to the various steps of the construction of the "crane". The tasks of worksheets 5 to 7 finally guided the investigation of flat origami and the exploration and discovery of the theorems of Maekawa and Kawasaki, which advance peculiar properties of the flat origami's crease pattern. In this paper, we draw attention to the request given by the first part of Worksheet 4. The students were given the instruction diagrams for the origami model and a sequence of squares, which each group was asked to fill in with the crease pattern at each construction step. The first step was the crease pattern of the square base, which the students had already encountered. The last step was the complete crease pattern of the crane (Fig. 2). The students were also asked to assign a different role to different members of the group, as a folder or sketcher.

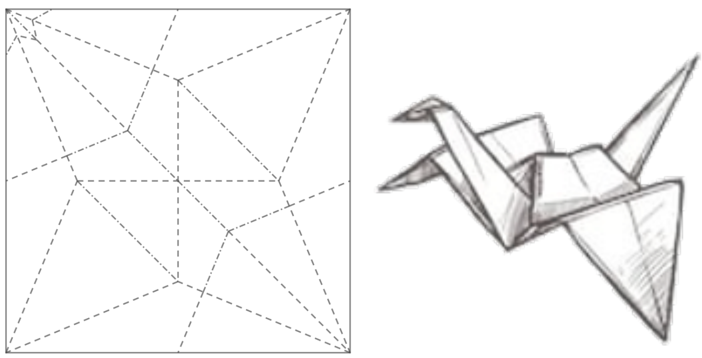


Fig. 2. The crease pattern (left) of the crane (right).

3.2 The Work of Two Groups

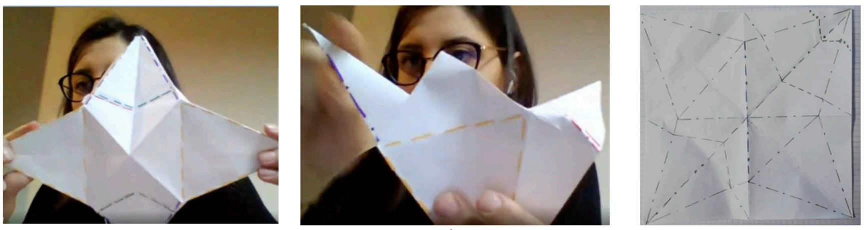
In this section we analyse the work of two groups (1 and 2). Group 1 is made up of three female students (S, G, H) and one male student (A). In solving the worksheet, S and A create the origami, while G and H create the crease pattern at each step (Fig. 3c).

Interestingly, S, in addition to building the model, draws the crease pattern directly on the model step by step, re-opening it and tracing the basic lines on paper, where it is possible to see the trace of the crease and therefore detect both the position and the nature of the fold (Fig. 3a).

To check the correspondence with what the groupmates do on paper, the model is often opened and closed again, but only halfway (Fig. 3b), as the model is substantially symmetric, for almost the whole process, with respect to the diagonals of the square.

We see that the group creates a type of hybrid diagram, given by the origami with folds added and marked with the same notation used in the crease pattern. We consider it *hybrid* because we recognize that the characteristics of origami are crucially merged with those of the crease pattern, and the model then is manipulated with different interest and in new ways (for example, just half-opened). The model thus modified can be conceived as a diagram, since the set of relations it contains becomes predominant perceptually other than semiotically, and such information is conveyed through appropriate conventions. The diagram is hybrid also in that it combines the material nature of the model with the usual way of representing the nature of the folds in a plane drawing.

We observe that the diagram is used by the students to operate a conversion between the register of the origami model and that of the crease pattern, which entails to check relations and modifications in space and in the plane and to discern the fundamental folds and their nature.

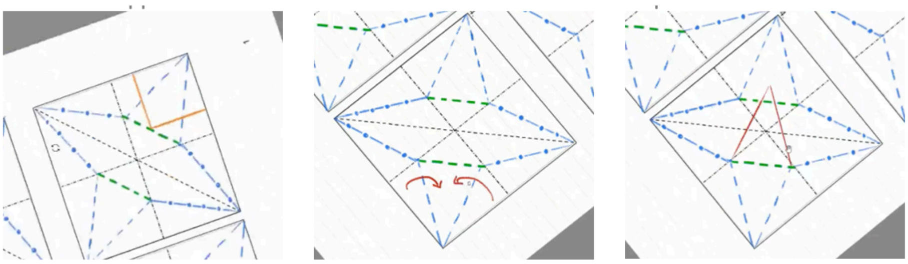


a b c

Fig. 3. (a) - (b) The hybrid diagram of group 1, then folded in half; (c) the crease pattern of the crane created by group 1.

Group 2 works in a different manner: a student (M) shares his tablet screen, in particular the window of a graphic editor software through which he modifies the assigned worksheet drawing the crease pattern; the rest of the group work on the origami model. The group is convinced that they are not allowed to reopen the model and observe the position of the folds with respect to the initial square. Therefore, they all proceed by imagining the changes occurred in the ongoing crease pattern, without comparing this directly with the folds traced on the paper sheet.

Each time the group works on a new crease pattern, M copies and pastes the crease pattern created in the previous step and then adds the changes directly on that diagram. The new added lines are of a different colour (Fig. 4a; as already done by group 1 in the hybrid diagram) and the online worksheet is rotated several times through the editor to show the crease pattern in the same position in which the other members of the group hold the origami. New folds are often first drawn as segments and, only later, the nature of the fold is captured by means of the appropriate marks.



a b c

Fig. 4. (a) - (b) - (c) Lines and arrows added by group 2 on the crease pattern.

Other signs are also drawn on the crease pattern to support the students' conjectures: in particular, arrows refer to the folding movement (Fig. 4b) or materialize parts of the origami in that passage, as they look like in the 3D space (Fig. 4c).

We observe the emergence of a "hybrid" diagram also in the case of group 2: the crease pattern, phase by phase, incorporates folding movements or captures representations of elements of the three-dimensional model.

3.3 Conclusions

Although at the very end the crease pattern of the crane produced by the groups is not entirely correct, we observe that the emergence of hybrid diagrams fosters the students' mathematical reasoning on the conversion between the origami and the crease pattern. In this paper, we analyse these diagrams focusing on the work of two groups. The ways in which we talk about the hybrid nature of the diagrams for the two groups are dual of each other. In the case of the first group, the 3D model incorporates qualities of the plane representation. In the case of the second group, during the process of diagramming, the crease pattern is transiently inhabited by arrows that literally bring in folding movements or new elements that mirror actual parts of the 3D origami. This seems to be an important characteristic of a hybrid diagram, which is provisionally arranged to incorporate aspects that usually belong to different registers and do not appear together.

In this sense, we see hybrid diagrams as semiotic and cognitive tools to operate a conversion, borrowing from Duval's language, between the register of the origami model and that of the crease pattern. The crease pattern crystallizes the process of folding, which essentially is a movement that happens in space but leaves a material trace, a material modification on the piece of paper. This is probably the reason why hybrid diagrams either capture movements (group 2) or are manipulated to perform such movements while controlling the nature and position of the folds (group 1). This way of addressing students' diagramming and gesturing aligns with de Freitas and Sinclair's [4] vision of them "as inventive and creative acts by which "immovable mathematics" can come to be seen as a deeply material enterprise" (p. 134).

Moreover, a hybrid diagram is nonstandard (does not belong entirely to one system of representation or another) and open to new modification and configuration. These features fundamentally evoke the dynamic character that Châtelet [3] sees as constitutive of diagrams. Tracing the emergence of hybrid diagrams allows us to better illuminate the semiosis that is at play in the process of conversion in mathematics.

Despite the huge interest in the field of origami practice and its relationship with mathematics, research that focuses on the cognitive side of this relationship is missing. Other studies, even in other contexts, might enhance the characterization of hybrid diagrams and help elucidate their role in mathematical thinking. Further qualitative research is needed to enlarge understanding of hybrid diagrams and their cognitive and didactical relevance. Wider implications could build on these first observations to better characterize hybrid diagrams and their cognitive value in mathematical activity.

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