Information Technologies, Economic Growth and Productivity Shocks

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Abstract

This paper develops a multi-sectoral endogenous growth model in order to reproduce some of the essential characteristics of the so-called "ICT Revolution". The economy consists of four sectors and the most important features are the embodied nature of technological progress, the horizontal differentiation and the "lab-equipment" specification of R&D. After the description of the different sectors, the equilibrium conditions are obtained, the balanced growth path is characterized analytically and the corresponding steady state system is derived. From this system some analytical results can be obtained, in particular it turns out that shocks on the productivity of the different sectors have permanent effects on long-term growth (contrary to the version of the model without the "lab-equipment" assumption, where only a shock on the productivity of the R&D sector influences long-term growth). These results are confirmed, in the last part of the paper, by the numerical simulation of the model, that allows also to analyse the short-run response of the system to the different shocks that can hit the economy and to study the robustness of the model.

Keywords: Information Technology, Endogenous Growth.

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1 Introduction

The sector of Information and Communication Technologies (ICT) has been recently considered of fundamental importance in the explanation of the economic performance of several countries. In particular, the strong productivity growth registered in the computer sector (i.e. in the production of hardware) has led some analysts to conclude that the era of a "New Economy" has begun, an era that can be considered a sort of "Third Industrial Revolution" in which information and communication technologies can be compared with the great inventions of the past that characterized the traditional Industrial Revolution.

For all these reasons, a great attention has been devoted to the study of what has been called the "ICT Revolution" and of its effects on the economy, and the debate is largely open, both from an empirical and from a theoretical point of view.

On the empirical side, the main studies (Gordon (1999, 2000), Jorgenson and Stiroh (2000), Oliner and Sichel (2000), Whelan (2000)) outline the strong productivity growth in the computer sector (particularly in the years 1995-1999), but at the same time evidence also problems of measurement of the real contribution of ICT to the growth and productivity of the economy, together with the fact that the productivity growth in the computer sector has not been accompanied by spillovers from this sector to the rest of the economy.

On the theoretical side (the most important contributions are Greenwood and Yorukoglu (1997), Greenwood and Jovanovic (1998, 1999), Hobijn and Jovanovic (1999), Jovanovic and Rousseau (2000)) it has been underlined the importance of embodiment of technological progress (i.e. the fact that only the new machines incorporate the latest technological advances), and at the same time the fact that the "ICT Revolution" has been accompanied by some "puzzling phenomena".

In fact, it is possible to observe that this "revolution" has been characterized by effects both on the real and on the financial side of the economy, with the emergence of some "puzzling" aspects. In particular, on the real side there has been an initial strong decrease in the productivity of the whole economy (the so-called "productivity slowdown") immediately after the beginning of the "ICT Revolution" (in the early '70s - the microprocessor, that can be considered the "starting point" of this revolution, was invented in 1971 -), followed only recently by a rise (as outlined above, in the late '90s the rise of productivity in the computer sector has been very strong, more than 40% in the USA). At the same time, on the financial side there has been an initial strong decrease in the ratio between stock market capitalization and GDP (from about 1 at the beginning of the '70s to 0.45 in 1974 for the leading OECD countries), followed only recently by a rise (today this ratio is close to 2).

The main explanations that have been proposed to this situation are based on the idea that the initial drop in productivity (on the real side of the economy) is due to an adoption period of the new technologies (because the pre-existing firms are not able to use immediately these new technologies at their full potential); this period is characterized by learning costs and slow diffusion (and it is precisely in this phase that the "productivity slowdown" takes place), and it is followed by an age of maturity during which the ICT sector starts driving the whole economy. Furthermore, according to the contributions proposed, the initial drop in the stock market capitalization/GDP ratio (on the financial side of the economy) should be due to the fact that a major technological innovation determines a temporary undervaluation of the stock market. In fact, again, old firms are not able to implement the new technologies and new firms are created to use these technologies; initially these firms do not "appear" in the stock market (for instance it took 10 years to Microsoft to go public), and this determines the drop in the ratio considered, while when these new firms are IPO'd the claims to their future dividends enter the stock market, and this determines a rise in the same ratio.

The model presented here takes a different view and tries to explain some of the essential features of the "ICT Revolution" considering the framework of endogenous growth theories. In particular, it is a Romer-like model (1990) in order to capture the R&D effort of the firms operating in the ICT sector, and in addition it considers embodied technological progress. More precisely, it is based on the original contribution developed by Boucekkine and de la Croix (2001), with the main difference represented by the "lab-equipment" specification for the R&D sector (see Rivera-Batiz and Romer (1991)), and it is a multi-sectoral model of endogenous growth that reproduces some of the essential characteristics of the ICT-based economy, in particular the embodied nature of technological progress (since the technological innovations that characterize the ICT sector are typically embodied in the new capital goods), the preminent role of the R&D sector (since the amount of resources devoted to research is particularly high, especially in the USA), and the link between innovation and market power (since ICT markets are typically non-competitive).

The paper is organized as follows. Section 2 provides the illustration of the model with the description of the different sectors that characterize the economy. Section 3 describes the equilibrium conditions, characterizes analytically the balanced growth path and derives the corresponding steady state system. From this system it is possible to find some analytical results concerning the effects on growth of different shocks that can interest the economy. In particular, it turns out that shocks on the productivity of the final good sector, of the equipment sector and of the intermediate good sector have permanent effects on long-term growth. Section 4 considers the numerical simulation of a calibrated version of the model, confirming the analytical findings and giving some further insights, concerning the short-run response of the system to the shocks and the robustness of the model. Section 5 concludes.

2 The model

The model developed is based on Boucekkine and de la Croix (2001) and Romer (1990), and is a multi-sectoral model written in discrete time with infinite horizon (time goes from 0 to ∞), endogenous growth and horizontal differentiation. The economy is characterized by 4 sectors: the final good sector, the equipment sector, the intermediate good sector and the R&D sector. In particular, the final good sector produces a composite good (used to consume or to invest in physical capital) using efficient capital (bought from the equipment sector) and labor; the equipment sector produces efficient capital (sold to the final good sector) using physical capital (that can be interpreted as hardware) bought from the final good producers and immaterial capital (that can be interpreted as software) bought from the intermediate good producers; the intermediate good sector produces the immaterial capital (sold to the equipment sector) using only labor; the R&D sector researches for new varieties of immaterial capital and in this way increases the range of softwares (horizontal differentiation).

In this model technological progress is mainly embodied (the idea is that the new softwares can only be run on the most recent hardware) and the innovators have a market power represented by copyrights, in order to stimulate innovation and growth (in particular, innovation corresponds to an expansion in the varieties of softwares that are available). All these elements are important to reproduce the essential characteristics of the ICT sector.

2.1 The final good sector

The final good sector produces a composite good (used to consume or to invest in physical capital) using efficient capital (bought from the equipment sector) and labor. Production is obtained through the following Cobb-Douglas technology (as in Solow (1960)):

$$Y_t = z_t K_t^{\alpha} L_t^{1-\alpha} \qquad \alpha \in [0;1] \tag{1}$$

where z_t represents total factor productivity (disembodied technological progress). The stock of capital is defined as:

$$K_t = \sum_{s=-1}^{t} E_s (1-\delta)^{t-s} \tag{2}$$

where E_s represents the efficient capital bought from the equipment sector at time s and δ is the physical depreciation rate (constant).

The discounted profits of investing E_t in efficient capital are given by:

$$\pi_t = \sum_{s=t}^{\infty} [Y_s - w_s L_s] R_t^s - d_t E_t$$

where:

$$R_t^t = 1 \qquad R_t^s = \prod_{\tau=t+1}^{s} \left(\frac{1}{1+r_\tau} \right)$$

represent the discount factors at time t and at time s respectively, r_τ is the interest rate at time τ, w_s is the wage for labor input at time s and d_t is the price of efficient capital at time t. The representative firm chooses efficient capital and labor input in order to maximize its discounted profits taking prices as given and subject to its technological constraints:

$$\max_{E_t; \{L_s\}_{s=t}^{\infty}} \pi_t$$
$$s.t.: \quad (1); (2)$$

The first order conditions characterizing an interior maximum for π_t are:

$$\alpha E_t^{\alpha-1} \sum_{s=t} R_t^s z_s (1-\delta)^{\alpha(s-t)} L_s^{1-\alpha} = d_t \tag{3}$$

$$(1-\alpha) z_s K_s^{\alpha} L_s^{-\alpha} = w_s \qquad \forall s \geq t \tag{4}$$

and from (4) we obtain:

$$L_t = \left(\frac{(1-\alpha) z_t}{w_t} \right)^{\frac{1}{\alpha}} K_t \tag{5}$$

that is the demand for labor by the final good sector.

2.2 The equipment sector

The equipment sector produces efficient capital (sold to the final good sector) using physical capital (hardware) bought from the final good producers and immaterial capital (software) bought from the intermediate good producers. Efficient capital is produced with a constant return to scale technology:

$$E_t = e_t Q_t^{\lambda} I_t^{1-\lambda} \qquad \lambda \in (0;1) \tag{6}$$

where e_t is a productivity variable, I_t represents physical capital (hardware) and Q_t represents immaterial capital (software). The immaterial capital is built from a series of specialized intermediate goods, according to a Dixit-Stiglitz CES function:

$$Q_t = \left(\int_0^{n_t} x_{i;t}^{\frac{\sigma-1}{\sigma}} di \right)^{\frac{\sigma}{\sigma-1}} \tag{7}$$

where n_t is the number of varieties of intermediate input available in t, $x_{i;t}$ is the quantity of intermediate input of variety i used in t and $\sigma > 1$ is the elasticity of substitution between two varieties.

The profits of the equipment sector at time t are:

$$\pi_t^0 = d_t E_t - I_t - \int_0^{n_t} p_{i;t} x_{i;t} di$$

where $p_{i,t}$ is the price of software of variety i at time t. The representative firm chooses the investment in physical capital and in immaterial capital in order to maximize profits taking prices as given and subject to its technological constraints:

$$\max_{I_t, x_{i,t}} \pi'_t$$
$$s.t. \quad (6); (7)$$

The first order conditions for this problem are:

$$(1-\lambda) d_t e_t q_t^{\lambda} = 1 \tag{8}$$

$$\lambda d_t e_t q_t^{\lambda-1} \left(\frac{Q_t}{x_{i,t}}\right)^{\frac{1}{\sigma}} = p_{i,t} \qquad \forall i \in [0; n_t] \tag{9}$$

where $q_t = \frac{Q_t}{I_t}$ is the software-hardware ratio, and from (8) and (9) we obtain:

$$x_{i,t} = \left(\frac{\phi}{q_t}\right)^{\sigma} Q_t p_{i,t}^{-\sigma} \tag{10}$$

that is the demand for intermediate input i by the firms of the equipment sector at time t (here we have defined $\phi = \frac{\lambda}{1-\lambda}$).

2.3 The intermediate good sector

The intermediate good sector produces immaterial capital (software, sold to the equipment sector) and it researches for new varieties, in order to expand the range of software.

The variety i of software is produced according to a linear technology that uses labor as the only input:

$$x_{i,t} = \tau_t L_{i,t} \tag{11}$$

where $L_{i,t}$ is the labor employed in the intermediate good sector and τ_t represents labor productivity. The producer behaves monopolistically (since market power is given by the presence of copyrights which have an infinite lifetime - i.e. the inventor of a new variety of software obtains these copyrights forever -) and its profit is:

$$\pi^{\infty}_{i,t} = p_{i,t}x_{i,t} - w_t L_{i,t} = \left(p_{i,t} - \frac{w_t}{\tau_t} \right) x_{i,t}$$

The price of output is chosen so as to maximize this profit subject to the demand formulated by the equipment sector, hence the problem solved by the firm is:

$$\max_{p_{i,t}} \pi^{\infty}_{i,t}$$

$$s.t. \qquad (10)$$

The first order condition for this problem is:

$$\left(\frac{\phi}{q_t} \right)^{\sigma} Q_t p_{i,t}^{-\sigma} = \sigma \left(p_{i,t} - \frac{w_t}{\tau_t} \right) \left(\frac{\phi}{q_t} \right)^{\sigma} Q_t p_{i,t}^{-\sigma-1}$$

from which we get:

$$p_{i,t} = \left(\frac{\sigma}{\sigma - 1} \right) \frac{w_t}{\tau_t} \qquad \forall i \in [0; n_t] \qquad (12)$$

i.e. the output price is a mark-up over unit labor cost.

2.4 The R&D sector

Besides producing softwares, the intermediate good sector researches for new varieties of immaterial capital, in order to expand their range. In this version of the model we assume the so-called "lab equipment" specification of R&D (see Rivera-Batiz and Romer (1991)), according to which the cost to create a new type of product (i.e. a new variety of software) is fixed at η units of Y. There will be entry of new firms in the economy until this cost is equal to the discounted flow of profits linked to one invention, and this equilibrium condition can be written as:

$$\eta = \sum_{z=t} R_t^z \pi^{\infty}_{i,z}$$

and since:

$$\pi^{\infty}_{i;t} = \left(p_{i;t} - \frac{w_t}{\tau_t} \right) x_{i;t} = \frac{1}{\sigma - 1} \frac{w_t}{\tau_t} x_{i;t}$$

the free-entry condition is:

$$\eta = \sum_{z=t}^{\infty} R_t^z \frac{1}{\sigma - 1} \frac{w_z}{\tau_t} x_{i;z}$$

2.5 Household behavior

After the 4 sectors that characterize the economy it is also possible to consider the household present in this economy. With reference to this aspect, the representative household consumes, saves for future consumption and supplies labor. The utility of the representative household at time 0 is:

$$u_0 = \sum_{t=0}^{\infty} \rho^t \ln C_t$$

i.e. it is the discounted sum of instantaneous utilities from 0 to ∞, where ρ is the psychological discount factor and the instantaneous utility function is assumed logarithmic. The corresponding budget constraint is:

$$A_{t+1} = (1 + r_{t+1}) A_t + w_t L - C_t \tag{13}$$

where A_t represents the assets detained by the household at time t.

The representative household chooses the assets detained in order to maximize its discounted utility subject to the budget constraint:

$$\max_{\{A_{t+1}\}_{t=0}^{\infty}} u_0 \quad \text{s.t.} \quad (13)$$

and the first order condition for this problem leads to:

$$\frac{C_{t+1}}{C_t} = (1 + r_{t+1}) \rho \tag{14}$$

that, together with the usual transversality condition, is sufficient for an optimum.

3 The equilibrium

It is now possible to characterize the equilibrium of the economy in the model considered, that is determined by the equilibrium on the labor market and on the final good market.

First of all, equilibrium on the labor market implies that the labor force is employed either in the final good sector or in the intermediate good sector:

$$L = L_t + \int_0^{n_t} L_{i,t}\, di \tag{15}$$

where the supply of labor can be normalized to 1 (i.e. $L = 1$).

Equilibrium on the final good market, then, implies:

$$Y_t = C_t + I_t + \eta \Delta n_t \tag{16}$$

where $\eta \Delta n_t$ is the cost of research for new varieties.

3.1 The equilibrium conditions

We can now derive the different equilibrium conditions, that summarize the first order optimality conditions and the market equilibrium relationships derived above.

First of all, the demand for labor by the final good sector is given by equation (5), while the demand for labor by the intermediate good sector can be obtained from equations (10), (11) and (12) and is given by:

$$\int_0^{n_t} L_{i,t}\, di = n_t \left(\frac{\sigma - 1}{\sigma}\right)^{\sigma} \left(\frac{\chi}{w_t}\right)^{\sigma} q_t^{1-\sigma} I_t \tau_t^{\sigma-1}$$

As a consequence, the equilibrium on the labor market, given by equation (15), is:

$$\left(\frac{(1-\alpha) z_t}{w_t}\right)^{\frac{1}{\alpha}} K_t + n_t \left(\frac{\sigma - 1}{\sigma}\right)^{\sigma} \left(\frac{\chi}{w_t}\right)^{\sigma} q_t^{1-\sigma} I_t \tau_t^{\sigma-1} = 1 \tag{17}$$

The equilibrium on the final good market can be obtained from equations (1), (5) and (16) and is:

$$z_t^{\frac{1}{\alpha}} K_t \left(\frac{1-\alpha}{w_t}\right)^{\frac{1-\alpha}{\alpha}} = C_t + I_t + \eta \Delta n_t \tag{18}$$

The law of motion for q_t can be obtained substituting the expression (5) into (3), using the definition of K_s and replacing the value of d_t from (8), so that we get:

$$\alpha z_t^{\frac{1}{\alpha}}(1-\lambda)e_t q_t^{\lambda}\left(\frac{1-\alpha}{w_t}\right)^{\frac{1-\alpha}{\alpha}} = 1-\left(\frac{1-\delta}{1+r_{t+1}}\right)\left(\frac{e_t}{e_{t+1}}\right)\left(\frac{q_t}{q_{t+1}}\right)^{\lambda} \tag{19}$$

The optimal consumption is then given by equation (14):

$$\frac{C_{t+1}}{C_t} = (1+r_{t+1})^{\rho} \tag{20}$$

while the accumulation rule of capital is obtained from equations (2) and (6) and is:

$$K_t = (1-\delta)K_{t-1} + e_t q_t^{\lambda} I_t \tag{21}$$

where $e_t q_t^{\lambda}$ can be seen as a measure of embodied technological progress (in contrast to the variable z_t that appears in the final good sector and that measures disembodied or neutral technological progress).

The value of Q_t can be determined using equations (7), (10) and (12) that lead to:

$$\frac{w_t q_t}{\tau_t \phi} = n_t^{\frac{1}{\sigma-1}}\left(\frac{\sigma-1}{\sigma}\right) \tag{22}$$

that links the embodied technological progress to the expansion in the varieties of intermediate products.

Finally, the free-entry condition can be written as:

$$\eta = \frac{\phi^{\sigma}}{\tau_t^{1-\sigma}(\sigma-1)^{1-\sigma}\sigma^{\sigma}}\sum_{z=t}^{\infty} R_t^z w_z^{1-\sigma} q_z^{1-\sigma} I_z$$

and then:

$$\eta R_t^{t+1} = \frac{\phi^{\sigma}}{\tau_t^{1-\sigma}(\sigma-1)^{1-\sigma}\sigma^{\sigma}}\sum_{z=t+1}^{\infty} R_t^z w_z^{1-\sigma} q_z^{1-\sigma} I_z$$

The two expressions for t and $t+1$ are therefore:

$$\frac{\tau_t^{1-\sigma}(\sigma-1)^{1-\sigma}\sigma^{\sigma}}{\phi^{\sigma}}\eta = \sum_{z=t}^{\infty} R_t^z w_z^{1-\sigma} q_z^{1-\sigma} I_z$$

$$\frac{\tau_t^{1-\sigma}(\sigma-1)^{1-\sigma}\sigma^{\sigma}}{\phi^{\sigma}}\eta R_t^{t+1} = \sum_{z=t+1} R_t^z w_z^{1-\sigma} q_z^{1-\sigma} I_z$$

and subtracting the second from the first:

$$\frac{\tau_t^{1-\sigma}(\sigma-1)^{1-\sigma}\sigma^{\sigma}}{\phi^{\sigma}}\frac{\eta r_{t+1}}{1+r_{t+1}} = w_t^{1-\sigma} q_t^{1-\sigma} I_t \tag{23}$$

These results can be summarized in the following Proposition:

Proposition 1 *Given the initial conditions K_{-1} and n_{-1} an equilibrium is a path:*

$$\{w_t; q_t; C_t; I_t; K_t; n_t; r_{t+1}\}_{t\geq 0}$$

that satisfies the equations (17) – (23) illustrated above.

3.2 The balanced growth path

After the characterization of the equilibrium, the next step is the analysis of the balanced growth path of the model. In this case we assume that the exogenous productivity variables z_t, e_t and τ_t and the interest rate r_t are constant in the long term, while each endogenous variable grows at a constant rate along a balanced growth path. In general, if g_x is the growth factor of the variable x and $\overline{x}$ is the initial level of the variable, we have:

$$x_t = \overline{x} g_x^t \tag{24}$$

Since a balanced growth path must satisfy the equations (17) – (23), we have seven restrictions among the various growth factors, that are:

$$g_w; g_q; g_C; g_I; g_K; g_n$$

In particular, it is possible to rewrite each of the equations (17) – (23) substituting each variable with the corresponding expression similar to (24). In this way, from equation (17) we get:

$$g_K (g_w)^{-\frac{1}{\alpha}} = 1 = \frac{g_n}{(g_w)^{\sigma}}(g_q)^{1-\sigma} g_I \tag{25}$$

From equation (18) we obtain:

$$g_C = g_I = g_n = g_K (g_w)^{-\frac{1-\alpha}{\alpha}} \tag{26}$$

(and from the good market equilibrium also $g_Y = g_C = g_I = g_n$). From equation (19) we get:

$$(g_q)^{\lambda} (g_w)^{-\frac{1-\alpha}{\alpha}} = 1 \tag{27}$$

From equation (20) we have:

$$g_C = (1 + r)\rho \tag{28}$$

From equation (21) we obtain:

$$g_K = (g_q)^{\lambda} g_I \tag{29}$$

From equation (22) we get:

$$g_w g_q = g_n^{\frac{1}{\sigma-1}} \tag{30}$$

Finally, from equation (23) we have:

$$(g_w)^{1-\sigma} (g_q)^{1-\sigma} g_I = 1 \tag{31}$$

In correspondence of a balanced growth path, the various growth rates must therefore satisfy the restrictions expressed by equations (25) – (31), and using these restrictions it is possible to determine the relations among the different growth rates.

First of all, from (27) we have:

$$g_w = (g_q)^{\frac{\lambda\alpha}{1-\alpha}}$$

From (25) we get:

$$g_K = (g_q)^{\frac{\lambda}{1-\alpha}}$$

From (26) we then have:

$$g_C = g_I = g_n = (g_q)^{\frac{\lambda\alpha}{1-\alpha}}$$

The same result can be obtained from (29), that is therefore redundant. From (30) we have:

$$g_n = (g_q)^{(\sigma - 1)\frac{1-\alpha+\lambda\alpha}{1-\alpha}}$$

and since we also have $g_n = (g_q)^{\frac{\alpha}{1-\alpha}}$ it must be $\sigma = \frac{1-\alpha+2\lambda\alpha}{1-\alpha+\lambda\alpha}$ (this restriction is a consequence of the "lab-equipment" specification for R&D). We then have that (31) is redundant, finally from (28) we get:

$$r = \frac{(g_q)^{\frac{\alpha}{1-\alpha}}}{\rho} - 1$$

In conclusion, the five unknowns of the problem (g_w, g_q, g_Y, g_K, r) are related by a system of four equations (the equations (25) $-$ (28), while (29) and (31) are redundant and (30) is used to obtain the expression for σ derived above), and for given g_q all the other unknowns can be found, and therefore they are parameterized by g_q. The results are expressed in the following Proposition:

Proposition 2 *If q_t grows at a factor $g_q > 1$, then all the other variables grow at strictly positive rates with:*

$$g_Y = g_C = g_I = g_n = g_w = (g_q)^{\frac{\alpha}{1-\alpha}}$$
$$g_K = (g_q)^{\frac{1}{1-\alpha}}$$

Hence, along a balanced growth path output, consumption, investment, number of varieties and wages grow at the same rate, while the stock of capital grows faster (since it includes improvements in the embodied productivity). The system is therefore able to display growth of the economy.

3.3 The stationarized dynamic system and the steady state system

The next step of the analysis is the study of the restrictions on the long-run levels, that give the additional information necessary to determine g_q. Computing these restrictions from the dynamic system expressed by equations (17) $-$ (23) (i.e. rewriting the equations substituting each variable with the corresponding expression similar to (24)), we end with 7 equations for

8 unknowns ($\bar w$, $\bar q$, $\bar C$, $\bar I$, $\bar K$, $\bar n$, $\bar r$ and g_q - since all the other growth rates can be expressed in terms of g_q -). The system in terms of levels is therefore undetermined (this is a usual property of endogenous growth models), but it is possible to rewrite it in such a way that we get rid of this indeterminacy. This is done by "stationarizing" the equations by means of some auxiliary variables, that is by rewriting the system in terms of variables that are stationary (i.e. constant) in the steady state. More precisely, the dynamic system (17) $-$ (23) can be rewritten as a function of the following seven stationary variables:

$$\hat q_t = \frac{q_t}{w_t^{\frac{1-\alpha}{\lambda\alpha}}} \qquad \hat C_t = \frac{C_t}{w_t} \qquad \hat I_t = \frac{I_t}{w_t} \qquad \hat K_t = \frac{K_t}{n_t^{\frac{1}{\alpha}}}$$

$$\hat n_t = \frac{n_t}{w_t} \qquad g_t = \frac{n_t}{n_{t-1}} \qquad r_t$$

These variables are such that they are constant in the steady state, for instance for $\hat q_t$ we have:

$$\hat q_t = \frac{q_t}{w_t^{\frac{1-\alpha}{\lambda\alpha}}} = \frac{\bar q g_q^t}{(\bar w g_w^t)^{\frac{1-\alpha}{\lambda\alpha}}} = \frac{\bar q g_q^t}{\bar w^{\frac{1-\alpha}{\lambda\alpha}} (g_q)^{\frac{\lambda\alpha}{1-\alpha} t \frac{1-\alpha}{\lambda\alpha}}} = \frac{\bar q}{\bar w^{\frac{1-\alpha}{\lambda\alpha}}} = \text{constant}$$

and therefore they are stationary variables. As a consequence, it is possible to obtain the stationarized dynamic system corresponding to the equations (17) $-$ (23). In particular, from equation (17) we obtain:

$$((1-\alpha) z_t)^{\frac{1}{\alpha}} \hat K_t \hat n_t^{\frac{1}{\alpha}} + \left(\frac{\sigma - 1}{\sigma}\right)^{\sigma} \phi^{\sigma} \tau_t^{\sigma-1} \hat n_t \hat q_t^{1-\sigma} \hat I_t = 1 \tag{32}$$

From (18) we have:

$$z_t^{\frac{1}{\alpha}} \hat K_t \hat n_t^{\frac{1}{\alpha}} (1-\alpha)^{\frac{1-\alpha}{\alpha}} = \hat C_t + \hat I_t + \eta \hat n_t \left(1 - \frac{1}{g_t}\right) \tag{33}$$

From (19) we get:

$$\alpha z_t^{\frac{1}{\alpha}} (1-\lambda) e_t \hat q_t (1-\alpha)^{\frac{1-\alpha}{\alpha}} + \frac{1-\delta}{1+r_{t+1}} \left(\frac{e_t}{e_{t+1}}\right) \left(\frac{\hat q_t}{\hat q_{t+1}}\right)^{\lambda} \left(\frac{\hat n_{t+1}}{\hat n_t g_{t+1}}\right)^{\frac{1-\alpha}{\alpha}} = 1 \tag{34}$$

From (20) we obtain:

$$\frac{\hat C_{t+1}}{\hat C_t} \frac{\hat n_t}{\hat n_{t+1}} g_{t+1} = (1 + r_{t+1}) \rho \tag{35}$$

From (21) we then have:

$$\hat{K}_t - (1-\delta)\hat{K}_{t-1} g_t^{-\frac{1}{\alpha}} = e_t \hat{q}_t \hat{I}_t \hat{n}_t^{-\frac{1}{\alpha}} \tag{36}$$

From (22) we get:

$$\frac{\hat{q}_t}{\tau_t \phi} = \hat{n}_t^{\frac{1}{\sigma-1}} \left(\frac{\sigma-1}{\sigma}\right) \tag{37}$$

Finally, from (23) we have:

$$\frac{\tau_t^{1-\sigma}(\sigma-1)^{1-\sigma}\sigma^{\sigma}}{\phi^{\sigma}} \frac{\eta r_{t+1}}{1+r_{t+1}} = \hat{q}_t^{1-\sigma}\hat{I}_t \tag{38}$$

The equations (32) − (38) represent therefore the stationarized dynamic system corresponding to the original system formed by equations (17) − (23). As for the original system, also in the stationarized one there are two pre-determined variables, $\hat{K}_t$ and g_t, therefore the stationarization doesn't alter the dynamic order of the original system.

At this point we can consider the steady state system corresponding to the stationarized system, to this end it is possible to define the following stationary variables (corresponding to those introduced above):

$$\hat{q} = \frac{\bar{q}}{\bar{w}^{\frac{1-\alpha}{\alpha}}} \qquad \hat{C} = \frac{\bar{C}}{\bar{w}} \qquad \hat{I} = \frac{\bar{I}}{\bar{w}} \qquad \hat{K} = \frac{\bar{K}}{\bar{n}^{\frac{1}{\alpha}}}$$
$$\hat{n} = \frac{\bar{n}}{\bar{w}} \qquad g = g_n \qquad r$$

and to observe that we can write:

$$\lim_{t\to+\infty} z_t = z \qquad \lim_{t\to+\infty} e_t = e \qquad \lim_{t\to+\infty} \tau_t = \tau$$

The stationarized steady state system is now given by the following equations:

$$((1-\alpha)z)^{\frac{1}{\alpha}} \hat{K}\hat{n}^{\frac{1}{\alpha}} + \left(\frac{\sigma-1}{\sigma}\right)^{\sigma} \phi^{\sigma}\tau^{\sigma-1}\hat{n}\hat{q}^{1-\sigma}\hat{I} = 1 \tag{39}$$

$$z^{\frac{1}{\alpha}}\hat{K}\hat{n}^{\frac{1}{\alpha}}(1-\alpha)^{\frac{1-\alpha}{\alpha}} = \hat{C} + \hat{I} + \eta\hat{n}\left(1-\frac{1}{g}\right) \tag{40}$$

$$\alpha z^{\frac{1}{\alpha}}(1-\lambda)e\hat{c}\,(1-\alpha)^{\frac{1-\alpha}{\alpha}} + \frac{1-\delta}{1+r}g^{-\frac{1-\alpha}{\alpha}} = 1 \tag{41}$$

$$g = (1+r)\rho \tag{42}$$

$$\hat{K}\left[1-(1-\delta)g^{-\frac{1}{\alpha}}\right]\hat{n}^{\frac{1}{\alpha}} = e\hat{c}\hat{I} \tag{43}$$

$$\tau\phi\left(\frac{\sigma-1}{\sigma}\right)\hat{n}^{\frac{1}{\sigma-1}} = \hat{c} \tag{44}$$

$$\frac{\tau^{1-\sigma}(\sigma-1)^{1-\sigma}\sigma^{\sigma}}{\phi^{\sigma}}\frac{\eta r}{1+r} = \hat{c}^{1-\sigma}\hat{I} \tag{45}$$

We therefore have a system of 7 equations with 7 unknowns ($\hat{c}$, $\hat{C}$, $\hat{I}$, $\hat{K}$, $\hat{n}$, g, r) that can be solved (at least from a theoretical point of view). In reality, given the complexity of the long-run steady state, it is impossible to derive an analytical solution. Nevertheless it is possible to obtain some interesting intermediate results, in particular it is possible to express each of the other unknowns as a function of the growth factor g, because there are explicit functions expressing the long-run levels ($\hat{c}$, $\hat{C}$, $\hat{I}$, $\hat{K}$, $\hat{n}$, r) exclusively in terms of g.

First of all, from equation (42) we have:

$$r = \Psi_r(g) = \frac{g}{\rho} - 1$$

From (41) we then have:

$$\hat{c} = \Psi_{\hat{c}}(g) = \left(\frac{1-\frac{1-\delta}{1+\Psi_r(g)}g^{-\frac{1-\alpha}{\alpha}}}{\alpha z^{\frac{1}{\alpha}}(1-\lambda)e\,(1-\alpha)^{\frac{1-\alpha}{\alpha}}}\right)^{\frac{1}{\lambda}}$$

From (44) we obtain:

$$\hat{n} = \Psi_{\hat{n}}(g) = \left(\frac{\tau\phi}{\Psi_{\hat{c}}(g)}\right)^{1-\sigma}\left(\frac{\sigma-1}{\sigma}\right)^{1-\sigma}$$

From (44) we also have:

$$\hat{n}\hat{q}^{1-\sigma} = \tau^{1-\sigma}\phi^{1-\sigma}\left(\frac{\sigma-1}{\sigma}\right)^{1-\sigma} \qquad (*)$$

while from (43) we have:

$$\hat{k}\hat{n}^{\frac{1}{\alpha}} = \frac{e\,(\Upsilon_{\hat{q}}(g))^{\lambda}\,\hat{l}}{1-(1-\delta)g^{-\frac{1}{\alpha}}} \qquad (**)$$

and putting (*) and (**) into (39) we get:

$$\hat{l} = \Upsilon_{\hat{l}}(g) = \frac{1}{((1-\alpha)z)^{\frac{1}{\alpha}}\frac{e}{1-(1-\delta)g^{-\frac{1}{\alpha}}}(\Upsilon_{\hat{q}}(g))^{\lambda} + \frac{\sigma-1}{\sigma}\phi}$$

From (40) we now have:

$$\hat{c} = \Upsilon_{\hat{c}}(g) = z^{\frac{1}{\alpha}}\frac{e}{1-(1-\delta)g^{-\frac{1}{\alpha}}}(\Upsilon_{\hat{q}}(g))^{\lambda}\,\Upsilon_{\hat{l}}(g)\,(1-\alpha)^{\frac{1-\alpha}{\alpha}} - \Upsilon_{\hat{l}}(g) - \eta\left(1-\frac{1}{g}\right)\Upsilon_{\hat{n}}(g)$$

and finally from (**) we have:

$$\hat{k} = \Upsilon_{\hat{k}}(g) = \frac{e}{1-(1-\delta)g^{-\frac{1}{\alpha}}}(\Upsilon_{\hat{q}}(g))^{\lambda}\frac{\Upsilon_{\hat{l}}(g)}{(\Upsilon_{\hat{n}}(g))^{\frac{1}{\alpha}}}$$

In conclusion, the following result holds:

Proposition 3 At any growth factor g, there exist explicit functions expressing the long-run levels $\hat{q}$, $\hat{c}$, $\hat{l}$, $\hat{k}$, $\hat{n}$, r exclusively in terms of g:

$$\hat{q} = \Upsilon_{\hat{q}}(g) \qquad \hat{c} = \Upsilon_{\hat{c}}(g) \qquad \hat{l} = \Upsilon_{\hat{l}}(g)$$
$$\hat{k} = \Upsilon_{\hat{k}}(g) \qquad \hat{n} = \Upsilon_{\hat{n}}(g) \qquad r = \Upsilon_{r}(g)$$

From these expressions, some other interesting results can be deduced. In particular, it can be observed that $\Upsilon_{\hat{l}}(g)$ doesn't depend neither on z nor on e nor on τ (the variables that measure the productivity, respectively, in the final good sector, in the efficient capital sector and in the intermediate good sector) - in fact, substituting in $\Upsilon_{\hat{l}}(g)$ the expression corresponding to $\Upsilon_{\hat{q}}(g)$ that appears in the denominator, the terms in z and e cancel out

-. Furthermore, the function $\Phi_{\hat{\phi}}(g)$ depends (negatively) on z and e and the functions $\Phi_{\hat{r}}(g)$, $\Phi_{\hat{k}}(g)$ and $\Phi_{\hat{c}}(g)$ depend on z, e and τ (in particular the first function is positively related to these variables, while the other two functions are negatively related to them). In addition, if we consider equation (45) and we substitute $\hat{l}$ and $\hat{\phi}^{1-\sigma}$ by their respective g-functional expressions, we obtain an equation involving only g that depends on z, e and τ, and this means that the long term growth factor is affected by the productivity variables z, e and τ.

All these results can be summarized in the following Proposition:

Proposition 4 *Assuming that a solution for the steady state system exists, the long run values of z and e affect the stationary values $\hat{\phi}$, $\hat{r}$, $\hat{k}$ and $\hat{c}$ and the long run value of τ affects the stationary values $\hat{r}$, $\hat{k}$ and $\hat{c}$. Furthermore z, e and τ have an impact on the long term growth factor g.*

According to these results, permanent changes in z_t (the productivity in the final good sector), in e_t (the productivity in the efficient capital sector) and in τ_t (the labor productivity in the intermediate good sector) will affect the long run growth rate of the economy. This is the main difference of this version of the model (based on the "lab-equipment" specification of R&D) with respect to the version without "lab-equipment" specification of R&D, in which long term growth turns out to be insensitive to changes in z_t and e_t (in that case only if the productivity of R&D is boosted, stimulating the creation of softwares, there is long term growth of the economy). Considering for instance the effects of changes in z_t, the difference between the version of the model without "lab-equipment" and the version with "lab-equipment" is based on the fact that in both versions long term growth relies on horizontal differentiation of R&D, but in the "lab-equipment" version the production function in the R&D sector is implicitly the same as in the final good sector, while in the other version the production function in the R&D sector is more labor intensive. The result is that a shock on the total factor productivity of the final good sector has an effect on long term growth in the "lab-equipment" model (because it corresponds to a shock on the productivity of the R&D sector, that is the engine of growth in this kind of model), while it doesn't have this effect in the other model.

These are the results that can be obtained analytically; in order to get further insights it is necessary to resort to numerical simulation, in fact from Proposition 3 we can derive the following Corollary:

Corollary 5 *There exists an explicit function $\Psi(g)$ such that the long run equilibrium growth factor solves the equation $\Psi(g) = 0$.*

This means that by using the g-functional expressions of the long run levels (those of Proposition 3) in any equation of the steady state system it is possible to obtain an explicit equation involving only g, in this way the system can be reduced to an explicit scalar equation involving the growth factor g, and once this equation is solved the remaining long run levels can be determined using the explicit g-functions. The problem is that the equation $\Psi(g) = 0$ (that gives the eventual steady state growth factor) is very complicated (given the complexity of the long run steady state), and it is impossible to derive an analytical solution. For this reason, to obtain further results it is necessary to resort to the numerical simulation of the model.

4 Simulation of the model

The model described above can be simulated numerically in order to verify the analytical findings and to get some other insights, concerning in particular the behavior of the economy as a consequence of shocks that can hit the system and the robustness of the model. This requires a calibration, that is chosen in such a way that it reproduces the essential features of the "new economy" on the basis of the empirical evidence available.

4.1 Calibration

The calibration of the model is tailored on the data of the US economy, therefore the different parameters are fixed to values that can be considered reasonable on the basis of the empirical data, and they are also chosen in order to match a series of moments of the steady state of the model. In particular, in the benchmark case the labor share in the final good sector $1 - \alpha$ is equal to 0.65 (hence $\alpha = 0.35$), while the share of software in the production of efficient capital λ is equal to 0.85 (this parameter, together with the labor productivity in the intermediate good sector τ, is used to calibrate the size of the new economy in terms of the labor force employed in the intermediate good and in the R&D sector). As a consequence, the elasticity of substitution between varieties of softwares σ is equal to 1.31[1].

[1] These values imply a mark-up rate of about 4, which can be considered very high and therefore not realistic. The reason is that, as a consequence of the "lab-equipment"

The rate of depreciation of physical capital δ is 10%, and the psychological discount factor ρ is 97%. The productivity in the final good sector z is then equal to 3, while the productivity in the equipment sector e is fixed to 12 (in this way we have a ratio capital to output of 2) and the productivity in the intermediate good sector τ is equal to 0.25 (this, together with the value chosen for the parameter λ, implies that about 8% of the labor force is employed in the intermediate good and in the R&D sector). Finally, the cost of a new variety of software expressed in units of output η (that derives from the "lab-equipment" specification for R&D) is used to calibrate the size of the R&D sector and is equal to 20 (in this way the R&D expenditure is approximately equal to 3.5% of GDP). We have therefore:

Parameter	Symbol	Value
Labor share in the final good sector	$1-\alpha$	0.65
Share of software in the production of efficient capital	λ	0.85
Elasticity of substitution between varieties of sofwares	σ	1.31
Rate of depreciation of physical capital	δ	0.10
Psychological discount factor	ρ	0.97
Total factor productivity in the final good sector	z	3
Total factor productivity in the equipment sector	e	12
Labor productivity in the intermediate good sector	τ	0.25
Cost of a new variety of software in units of output	η	20

Table 1: Values of the parameters, benchmark case

and the corresponding relevant moments of the steady state that are reproduced are:

Labor share in intermediate and research sector	8%
Capital/output ratio	2
R&D expenditure in terms of GDP	3.5%
Interest rate	3.9%

Table 2: Relevant moments of the steady state, benchmark case

assumption introduced in this model, the elasticity of substitution σ (and hence the mark-up, equal to $\frac{\sigma}{\sigma-1}$) is strictly linked to the parameters α and λ. When we choose for these parameters values that allow to match the moments of the steady state we are interested in, this high mark-up rate appears.

With these values, the model leads to a growth rate of output equal to 0.8% per year, that can be interpreted as the part of output growth generated by embodied technical progress, and is in line with the available data (see for instance Greenwood, Hercowitz and Krusell (1997)). Furthermore, for this parameterization the model has a unique steady state equilibrium with positive growth, and this equilibrium is locally a saddle point (for simulations and stability assesments the "Dynare" package - see Juillard (1996) - has been used).

At this point, the benchmark case can be used first of all to verify the effects on the calibrated version of the model of different types of shocks that can interest the economy. More precisely, after that the steady state for the initial calibration of the model has been computed, a shock on a particular variable is considered (here all the shocks have an intensity equal to 1%), then the new steady state is obtained and the dynamics of transition from the old steady state to the new one is determined. In this way it is also possible to determine the magnitude of the effects of the shocks on the relevant variables in the short and in the long run, and this is an interesting contribution of the analysis developed.

The benchmark case, then, can be used also to verify the robustness of the model. With reference to this aspect, starting from the benchmark it is possible to consider significant changes in the relevant parameters and to simulate again the model (in order to verify the response of the economy to the different shocks when the new values of the parameters are taken into account). The results obtained show that the model is sufficiently robust, and represent therefore another important contribution of the present study.

4.2 The benchmark case: productivity shocks

The first result that can be obtained with the calibrated version of the model described above is represented by the analysis of the effects of shocks that can hit the economy. The shocks considered are productivity shocks, more precisely it is possible to consider a shock on z (the productivity in the final good sector), a shock on e (the productivity in the equipment - efficient capital - sector) and a shock on τ (the productivity in the intermediate good - software - sector); it is also possible to analyse how the economy reacts to a reduction in η (the cost of a new variety of software in units of output). All the shocks considered are permanent (from $t=0$) and have an intensity equal to 1%.

The first simulation concerns a shock on the final good sector productivity parameter z, that is increased permanently by 1%. From the analytical results (Proposition 4) we know that this should have an impact on the long term growth rate (this is a central difference of the version of the model with "lab-equipment" with respect to the version without "lab-equipment"), and the simulation confirms this result. In effect, both the growth rate of efficient capital and the growth rate of production (that, in this version of the model, is also equal to the growth rate of the number of patents, i.e. of softwares) increase in the long run (Figures 11 and 12 - the values reported are those of the growth factors, from which those of the growth rates can be easily deduced -). This is due to the fact that the "lab-equipment" specification implies that the production function in the R&D sector is the same as in the final good sector. As a consequence, an increase in the productivity of the final good sector is equivalent to an increase in the productivity of the R&D sector, and since this sector is the engine of growth in the model (through the expansion in the varieties of softwares) this determines an effect on long term growth.

In particular, the increase in the productivity of the final good sector reduces the cost of production of the final good and initially determines a reallocation of the labor force favorable to the final good sector and at the expenses of the intermediate good sector. As a consequence, in the first period immediately after the shock there is a very strong increase in the growth rates (almost 8%), due to the contemporaneous increase in the productivity and in the labor force of the final good sector. At the same time, the labor force employed in the intermediate good sector (i.e. in the production of software) is characterised by an important reduction (about 1.3%). As time passes, then, the labor force employed in the final good sector reduces (because productive capacity has reached its maximum) and there is again a reallocation of this labor force in favor of the intermediate good sector. As a consequence, the growth rates partially reduce, and the long run effect is an increase in both the growth rate of efficient capital and the growth rate of production (of more than 4.5%) with respect to the initial steady state values. For the same reason the initial reduction in the labor force employed in the intermediate good sector is almost completely recovered in the following two periods, and from $t = 4$ there is a small increase (with respect to the initial steady state value). The long run result is that, as a consequence of an increase in the productivity of the final good sector, there

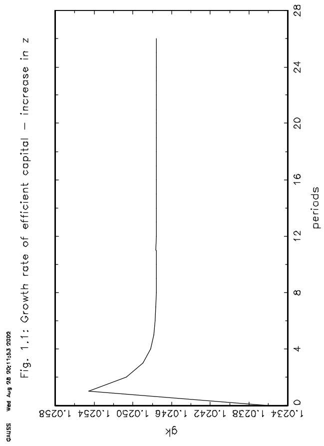

Fig. 1.1: Growth rate of efficient capital - increase in z

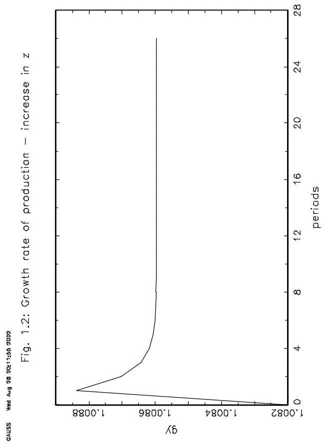

Fig. 1.2: Growth rate of production - increase in z

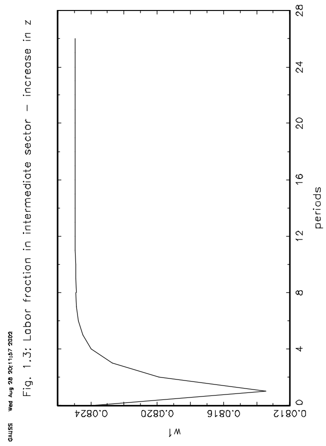

Fig. 1.3: Labor fraction in intermediate sector - increase in z

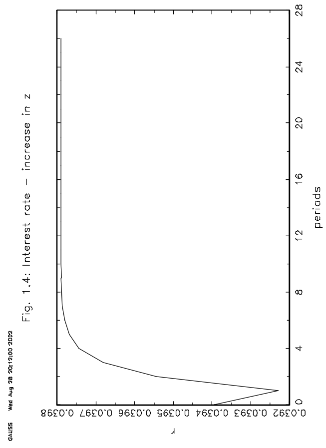

Fig. 1.4: Interest rate - increase in z

is a small reallocation (about 0.1%) of workers from the final good sector to the intermediate good sector (Figure 1.3). The last result concerns the interest rate (Figure 1.4), that decreases immediately after the shock (of about 0.5%), then recovers and in the long run has an increase of about 1% with respect to the initial value.

The second simulation considered concerns a shock on the equipment sector productivity parameter e, that is increased permanently by 1%. Also in this case we know from the analytical results that this should have an impact on the long run growth rate (contrary to what happens in the version of the model without "lab-equipment"), and the simulation again confirms this result. As before, this long run effect is due to the fact that the "lab-equipment" specification implies the same production function in the final good sector and in the R&D sector. In this case an increase in the productivity of the equipment sector increases the production of efficient capital and therefore of the final good, this corresponds to an expansion in the R&D sector and since this sector is the engine of growth in the model the result is an effect on long term growth. Nevertheless, the short run behavior of the variables is very different with respect to the situation that we have in the case of a shock on the productivity of the final good sector (in fact it is the opposite).

In effect, the increase of productivity in the equipment sector increases the profitability of producing efficient capital and increases the marginal return to both softwares and hardware, stimulating the demand for these inputs. This in turn stimulates the creation and production of softwares and determines an initial strong increase (about 2%) in the labor fraction employed in the intermediate good sector (while in the case of a shock on the productivity of the final good sector in the short run there is a reallocation of workers in favor of the final good sector), launching the growth of the economy. Since this growth is based on the expansion of the intermediate good sector (and not directly of the final good sector), nevertheless, the increase of the growth rates is less strong than in the first simulation considered, and it requires more time. In fact, initially there is a reduction of both the growth rate of efficient capital and the growth rate of production (due to the reduction of the labor force in the final good sector, that affects growth directly, while in the first simulation considered initially there is a strong increase of these growth rates) - the decrease with respect to the initial steady state values is of the order of 3% -, then from $t = 2$ these growth rates recover and in the

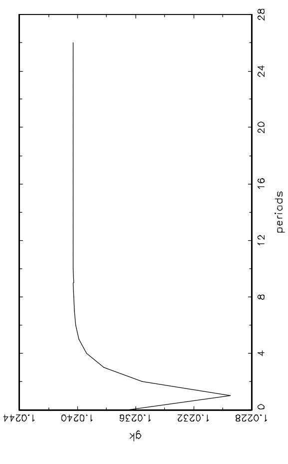

Fig. 2.1: Growth rate of efficient capital - increase in e

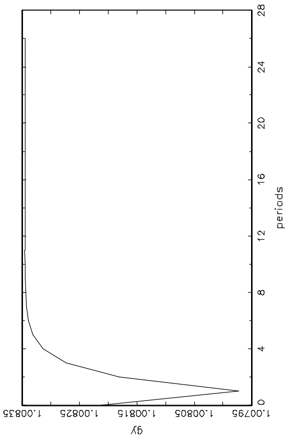

Fig. 2.2: Growth rate of production - increase in e

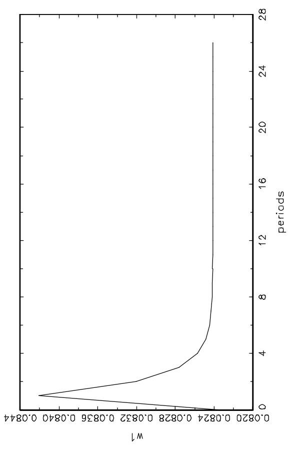

Fig. 2.3: Labor fraction in intermediate sector - increase in e

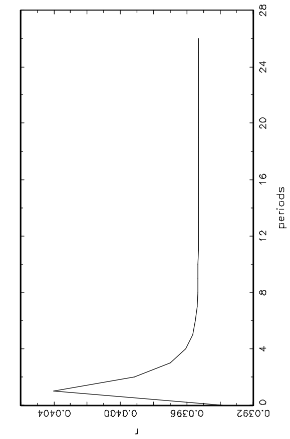

Fig. 2.4: Interest rate - increase in e

long run they increase (of about 1.5%) with respect to their initial steady state values (Figures 2.1 and 2.2). Furthermore, the initial strong reallocation of workers in favor of the intermediate good sector is not long lasting, in fact in a few periods the labor fraction in the software sector returns to a level that is only slightly higher (about 0.05%) than the initial steady state value (since labor force reallocates in favor of the final good sector, contributing to the increase in the growth rates from $t = 2$), and remains to this level in the long run (Figure 2.3). A similar behavior is that of the interest rate, that immediately after the shock increases of 2.5%, then returns to a level only slightly higher (about 0.3%) than the initial one (Figure 2.4).

The same kind of results obtained considering a shock on the productivity of the equipment sector hold when we consider a shock on the intermediate good sector productivity parameter ¿, increased permanently by 1%. Also in this case, in fact, there is a permanent effect on growth (always due to the fact that the "lab-equipment" specification implies the same production function in the final good sector and in the R&D sector). In this situation the increase in the productivity of the intermediate good sector reduces the cost of production of softwares and hence determines an initial strong reallocation of the labor force favorable to this sector. In fact, in the first period after the shock there is an increase (of more than 1.5%) in the labor fraction that is employed in the intermediate good sector, at the expenses of the labor force employed in the final good sector. As before, since in this situation the growth of the economy is based on the expansion of the intermediate good sector (and not directly of the final good sector), it is less strong than in the first simulation considered and it requires more time. In fact, initially there is a reduction of both the growth rate of efficient capital and the growth rate of production (due to the reduction of the labor force in the final good sector, that affects growth directly) - the decrease with respect to the initial steady state values is of the order of 2.5% -, then from $t = 2$ these growth rates start to rise again and in the long run they increase (of about 1.5%) with respect to their initial steady state values (Figures 3.1 and 3.2). We also have that the increase in the labor force employed in the intermediate good sector is not long lasting, in fact after a few periods the labor fraction in the software sector returns to a level only slightly higher (about 0.05%) than the initial value (since labor force reallocates in favor of the final good sector, determining the increase in the growth rates from $t = 2$) and remains to this level in the long run (Figure 3.3). The same kind of behavior, finally, GAUSS Wed Aug 28 20:31:13 2002

Fig. 3.1: Growth rate of efficient capital - increase in t

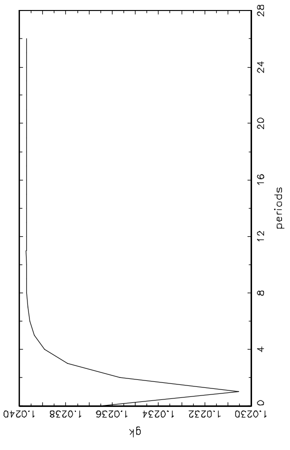

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Fig. 3.2: Growth rate of production - increase in t

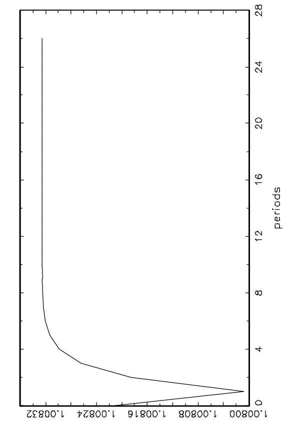

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Fig. 3.3: Labor fraction in intermediate sector - increase in t

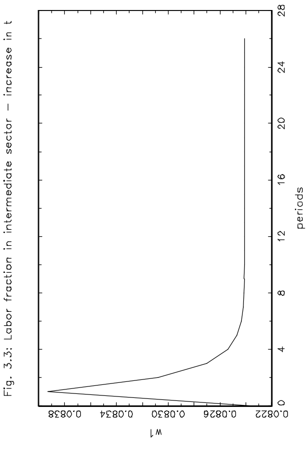

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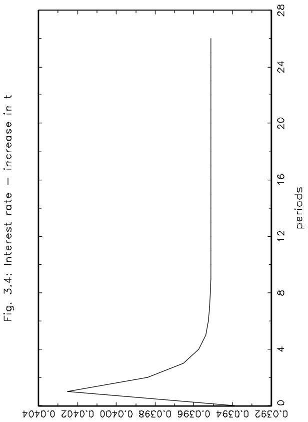
Fig. 3.4: Interest rate - increase in t



can be observed for the interest rate, that increases (of about 2%) immediately after the shock, then decreases and in the long run reaches a level slightly higher (about 0.3%) than the initial one (Figure 3.4).

After the shocks on the productivity variables described above, it is also possible to analyse how the economy reacts to a decrease in ´, the cost to create a new variety of software in units of output (as a consequence of the "lab-equipment" specification adopted for R&D), equal to 1%. The results turn out to be very close to those obtained in the first simulation, where an increase in the productivity of the final good sector was considered. In particular, the effect on the long run growth rate of a reduction in ´ is permanent (due to the fact that, in this model, the R&D sector is the engine of growth), and it is an increase in both the growth rate of efficient capital and the growth rate of production (Figures 4.1 and 4.2). In particular, the reduction in ´ determines a temporary reallocation of the labor force favorable to the final good sector and at the expenses of the intermediate good sector. As a consequence, in the first period after the decrease of ´ there is a very strong increase in the growth rates (about 8%), due to the increase in the labor force of the final good sector. At the same time, the labor force employed in the intermediate good sector is characterised by an important reduction (of the order of 1.5%). As time passes, then, the labor force employed in the final good sector reduces and there is again a reallocation of this labor force in favor of the intermediate good sector. As a consequence, the growth rates partially reduce, and the long run effect is an increase in both the growth rate of efficient capital and the growth rate of production of about 4.5% with respect to the initial steady state values. For the same reason, the initial reduction in the labor force employed in the intermediate good sector is almost completely recovered in the following two periods, and the long run effect is a small increase (about 0.1%) with respect to the initial steady state value (Figure 4.3). The last result concerns the interest rate (Figure 4.4), that decreases immediately after the decrease in ´ (of about 1%), then recovers and in the long run has an increase (of about 1%) with respect to the initial steady state level.

The results of the simulations illustrated above can be used also to compare the different shocks. First of all, it is possible to observe that the various shocks considered affect in a different measure the growth rates and the allocation of the labor force between final good sector and intermediate

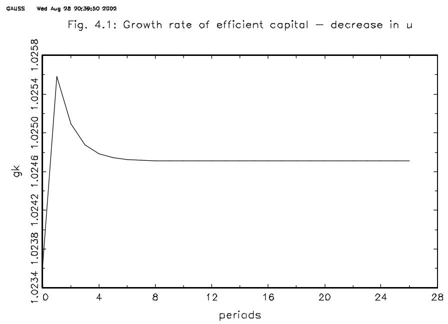


Fig. 4.1: Growth rate of efficient capital – decrease in u

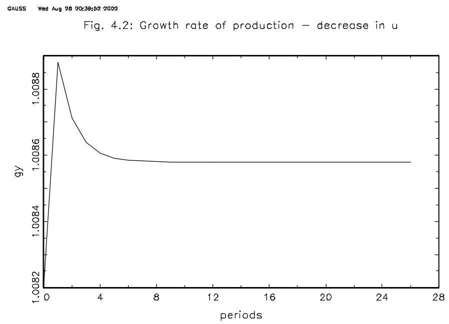


Fig. 4.2: Growth rate of production – decrease in u

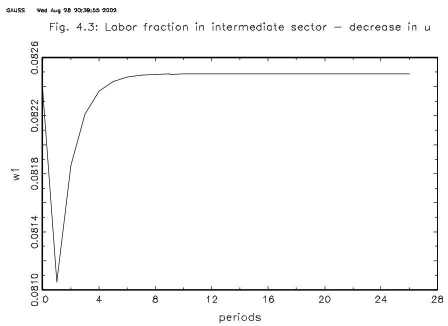


Fig. 4.3: Labor fraction in intermediate sector – decrease in u

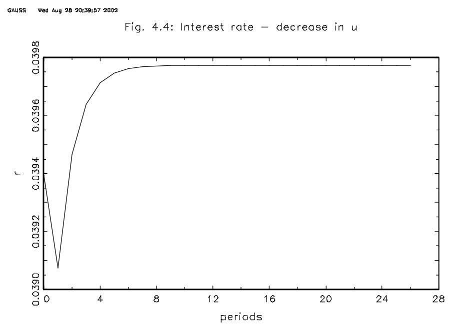


Fig. 4.4: Interest rate – decrease in u

good sector (it also turns out that the shock on the productivity of the final good sector and that on the cost of the R&D sector produce very similar consequences, and the same is true for the shock on the productivity of the equipment sector and for that on the productivity of the intermediate good sector). In particular, the increase in the productivity of the final good sector z and the decrease in the cost to create a new variety of software ´ both determine a strong increase in the growth rate of efficient capital and in the growth rate of production (that then partially reduce), while the increase in the productivity of the equipment sector e and the increase in the productivity of the intermediate good sector ¿ have a less strong effect on these growth rates (that initially decrease, then increase). With reference to the allocation of the labor force, the increase in z and the decrease in ´ initially determine a reallocation of the labor force favorable to the final good sector and at the expenses of the intermediate good sector (then the situation reverses), while the increase in e and the increase in ¿ have the opposite effect (initially the labor force employed in the intermediate good sector increases, then decreases). Finally, with reference to the interest rate, the increase in z and the decrease in ´ initially determine a decrease in the interest rate (that then increases), while the increase in e and the increase in ¿ have the opposite effect (initally the interest rate increases, then decreases).

All these results can be summarized in the following Table:

	increase in z decrease in ´	increase in e increase in ¿
growth rate of efficient capital	first $\uparrow$ then $\downarrow$ (partially)	first $\downarrow$ then $\uparrow$
growth rate of production	first $\uparrow$ then $\downarrow$ (partially)	first $\downarrow$ then $\uparrow$
labor force in intermediate sector	first $\downarrow$ then $\uparrow$	first $\uparrow$ then $\downarrow$
interest rate	first $\downarrow$ then $\uparrow$	first $\uparrow$ then $\downarrow$

Table 3: Qualitative effects of different types of shocks, benchmark case

The last aspect concerns the quantitative effects determined by the different types of shocks on the relevant variables in the long run. The conclusion is that the increase in z and the decrease in ´ have the strongest effect on growth (since, on the one hand, the final good sector affects growth directly, and on the other hand the R&D sector represents the engine of growth in this model) - the long run effect of the two types of shocks is an increase in the growth rate of efficient capital and in the growth rate of production of about 4.5% with respect to the initial steady state values -. On the contrary, the increase in e and the increase in ¿ have a less strong effect on growth (since these sectors affect growth indirectly) - the long run effect of these two other types of shocks is an increase in the growth rates of about 1.5% with respect to the initial steady state values -. Furthermore, the shocks on z and on ´ have a stronger effect also on the labor force employed in the intermediate good sector (that, in the long run, increases of about 0.1% with respect to the initial steady state value, while the increase is of the order of 0.05% when the shocks on e and on ¿ are considered) and on the interest rate (the increase in the long run is 1% with respect to the initial steady state value, while it is only 0.3% when the shocks on e and on ¿ are analysed).

All these results are reported in the following Table:

	shock on z shock on ´	shock on e shock on ¿
growth rate of efficient capital	$+4.5\%$	$+1.5\%$
growth rate of production	$+4.5\%$	$+1.5\%$
labor force in intermediate sector	$+0.1\%$	$+0.05\%$
interest rate	$+1\%$	$+0.3\%$

Table 4: Long run quantitative effects (with respect to the initial steady state levels) of different types of shocks, benchmark case

4.3 The benchmark case: robustness analysis

The benchmark case presented above can be used also to verify the robustness of the model. The idea in this case is to start from the benchmark and to modify significantly some parameter. With this new value of the parameter that has been changed the model is then simulated again, and the effects of the different shocks are studied (exactly as in the benchmark case). If the results are sufficiently close to those obtained for the initial parameterization, it is possible to conclude that the model is robust, and this represents a good property of the model itself.

In particular, three types of changes in the parameters are considered. The first is a change in λ (the share of software in the production of efficient capital), that is used to study the effect of a variation in the level of competition in the economy (since this parameter influences the elasticity of substitution σ and hence the mark-up rate). The second is a change in z (the productivity in the final good sector), that is used to analyse the effect of a change in the technology. The third is a change in η (the cost of a new variety of software in units of output), that is used to study the consequences of a variation in the cost of $R\&D$.

The most interesting aspect for this kind of analysis is represented by the effects of the different shocks on the relevant variables in the long run. These effects have been discussed, for the benchmark case, in the previous subsection (and are summarized in Table 4); they can now be determined for the different cases that depart from the benchmark, in order to verify the robustness of the model.

The first variation that is considered with respect to the benchmark is a change in the parameter λ (that affects the competition in the economy). In particular, the value of this parameter changes from 0.85 (as in the benchmark) to 0.95 (hence the change is larger than 10%), while the values of all the other parameters remain unchanged with respect to the benchmark. At this point, the new steady state of the model is computed, then the simulations are performed (exactly as in the initial version of the model) and the different types of shocks considered in the benchmark case are introduced. First of all, with the new value of λ the growth rate in the steady state is equal to 0.96%, we then have that the behavior of the economy, as a consequence of the shocks, is very close to the one obtained in the initial calibration of the model. In particular, both the shocks on z and on η determine an initial very strong increase in the growth rates (than then partially reduce) and an initial reduction in the labor fraction of the intermediate sector and in the interest rate (that then recover and in the long run increase with respect to the initial steady state values). On the contrary, the shocks on e and on τ produce an initial reduction in the growth rates (that then recover and in the long run increase with respect to the initial values) and an increase in the labor force of the intermediate sector and in the interest rate (that then decrease and reach a level, in the long run, only slightly higher than the initial one). It is then important to evaluate the magnitude of the effects of the shocks on the different variables in the long run, in order to make a comparison with the benchmark case; the values obtained are the following:

	shock on z	shock on e	shock on τ	shock on η
growth rate of efficient capital	$+4\%$	$+1.4\%$	$+1.4\%$	$+4\%$
growth rate of production	$+4\%$	$+1.4\%$	$+1.3\%$	$+4\%$
labor force in intermediate sector	$+0.2\%$	$+0.1\%$	$+0.1\%$	$+0.2\%$
interest rate	$+1\%$	$+0.5\%$	$+0.5\%$	$+1\%$

Table 5: Long-run quantitative effects (with respect to the initial steady state levels) of different types of shocks, case 1 (higher λ with respect to the benchmark)

From their analysis it turns out that they are very close to those obtained in the benchmark case, hence the model proves to be robust with respect to the change introduced in the parameter λ. With reference to this first case considered in the robustness study of the model, first of all it is possible to observe that the increase in λ determines an increase in σ (the elasticity of substitution between varieties of softwares) and a decrease in the mark-up rate, that corresponds to an increase in the level of competition in the economy. It is known that in the variety expanding growth models (like the one considered here) the relationship between mark-up and growth is not necessarily positive (as a consequence of the presence of the so-called "resource competition effect"), and in fact this is what happens in the present model, where a decrease in the mark-up is accompanied by an increase (and not a decrease) in the growth rate.

When, in this context, we consider the different types of shocks introduced in the previous subsection, then, we get that the long run effects on the growth rates are lower than in the benchmark (this is true especially for the shocks on z and on η, whose effects on the growth rates are 0.5% lower than in the benchmark). This is due to the fact that a higher value of λ determines not only an increase in the elasticity of substitution σ, but also in the demand for the intermediate good (software). As a consequence, a shock on z or on η determines (as in the benchmark case) an initial reallocation of workers favorable to the final good sector and at the expenses of the intermediate good sector, nevertheless this reallocation is less strong than in the benchmark case (because higher labor force in the intermediate good sector is now necessary to face the increased demand determined by the increase in $¸$). The dynamics is then of the same type analysed in the previous subsection, and the final result is an increase in the growth rates, but this increase is less strong than in the benchmark case. For the same reason (i.e. the increased demand of the intermediate good) the shocks on z and on $´$ have a higher long run effect (with respect to the benchmark) on the labor force employed in the intermediate good sector. When a shock on e or on $¿$ is considered the situation is similar (even if, as in the benchmark case, the short run dynamics are different with respect to the case of a shock on z or on $´$) and the long run effects are less strong than those obtained when the shocks on z or on $´$ are considered (since the shocks on e and on $¿$ affect growth only indirectly). Also in this case the fact that higher labor force in the intermediate sector is needed to satisfy the increased demand determines, in the long run, an increase in the growth rates lower than in the benchmark case, and an increase in the labor force of the intermediate sector higher than in the benchmark.

The second variation considered concerns the parameter z (hence the technology of the economic system), that changes from 3 to 3.5 (i. e. of more than 15%), while again the values of all the other parameters remain unchanged with respect to the benchmark. Also in this case the new steady state of the model is computed and the simulations are performed. The new growth rate in the steady state turns out to be 1.45%, and again the economy reacts qualitatively to the different shocks as in the benchmark case; in particular, then, the quantitative effects of the shocks on the different variables are the following:

	shock on z	shock on e	shock on $¿$	shock on $´$
growth rate of efficient capital	+3.4%	+1.2%	+1%	+3%
growth rate of production	+3.4%	+1.4%	+1.4%	+3%
labor force in intermediate sector	+0.1%	+0.1%	+0.1%	+0.1%
interest rate	+1%	+0.4%	+0.4%	+1%

Table 6: Long-run quantitative effects (with respect to the initial steady state levels) of different types of shocks, case 2 (higher z with respect to the benchmark)

Again it is possible to observe that they are sufficiently close to the values obtained in the benchmark case, hence also when a change in z is introduced the model proves to be robust. Considering the different shocks, then, in this situation we have that the long run effects on the growth rates are lower than in the benchmark case (and also lower than in the situation examined before, where an increase in ¸ was assumed). This is true especially for the shocks on z and on ´ (whose long run effects on the growth rates are about 1% lower than in the benchmark case), and can be explained observing that, with the new value of z, the initial growth rates in the steady state are quite large. As a consequence, shocks on the different parameters still determine increases in these growth rates (following the same dynamics of the benchmark model), but these increases are proportionally less strong than in the benchmark case, where the initial level of the growth rates was lower.

The last variation considered is that of the parameter ´ (that is a measure of the cost of R&D), that changes from 2.0 to 1.7 (hence exactly of 15%), while the other parameters remain at the values of the benchmark. The new growth rate in the steady state in this case is 1.45%; the model is then simulated and the different types of shocks are introduced, and also in this situation the qualitative behavior of the economy in response to these shocks is the same of the benchmark case, while the quantitative effects are the following:

	shock on z	shock on e	shock on ¿	shock on ´
growth rate of efficient capital	+ 3%	+ 1%	+ 1%	+ 3%
growth rate of production	+ 3.5%	+ 1%	+ 1.4%	+ 3%
labor force in intermediate sector	+ 0.1%	+ 0.1%	+ 0.1%	+ 0.1%
interest rate	+ 1%	+ 0.4%	+ 0.2%	+ 1%

Table 7: Long-run quantitative effects (with respect to the initial steady state levels) of different types of shocks, case 3 (lower ´ with respect to the benchmark)

Also in this situation they are close to the values obtained for the original parameterization of the model, that therefore is sufficienlty robust also with respect to changes in ´. In particular, it is possible to observe that, as in the previous situation, the long run effects of the different shocks on the growth rates are lower than in the benchmark case (and also lower than in the case in which a higher λ was assumed). This is true also for the shocks on e and on τ (whose long run effects on the growth rates are 0.5% lower than in the benchmark, while in the other two cases considered in the robustness study the difference with respect to the benchmark was smaller). The explanation that can be given is that, first of all, with the new value of δ the initial growth rates in the steady state are quite large, hence the shocks on the different parameters determine increases in these growth rates, but these increases are proportionally lower than in the benchmark case. Furthermore, in this situation the importance of the $R\&D$ sector in stimulating growth is even larger than in the benchmark, because the cost of $R\&D$ is lower. As a consequence, when the shocks on e and on τ are considered, since these shocks determine growth only indirectly through the expansion of the intermediate good sector, their effects on the long run growth rates are less strong than in the benchmark case (where the cost of $R\&D$ is higher, hence the importance of the $R\&D$ sector in promoting growth is lower).

In conclusion, from the discussion presented we can deduce that the model considered is quite robust, since the results obtained for the initial calibration remain valid when some parameter is significantly altered. This is a good property of the model, and represents an important achievement of the analysis developed.

5 Conclusion

This paper has been devoted to the presentation of a model that tries to explain some of the characteristics of the recent "ICT Revolution" using the framework of endogenous growth theory. More precisely, it is a multi-sectoral growth model with embodied technological progress, horizontal differentiation and "lab-equipment" specification of $R\&D$. As a consequence of this latter assumption, in particular, it is possible to show analytically that an increase in the productivity of the different sectors (final good sector, equipment sector, intermediate good sector) has an everlasting effect on growth (contrary to what happens in the version of the model without "lab-equipment").

This result is then confirmed resorting to numerical simulation, that allows also to get some further insights. In order to do this, a calibrated version of the model has been considered, where the values of the different parameters have been chosen so as to reproduce the empirical evidence that is available concerning in particular the US economy. In this simulation, then, the different types of productivity shocks have been analysed, and also the shock represented by a decrease in the cost of R&D in terms of output (strictly linked to the peculiarity of the "lab-equipment" specification adopted in this sector) has been studied.

The main conclusion obtained from the simulation is that all these shocks have permanent effects on long term growth (and this is the central difference with respect to the version of the model without the "lab-equipment" assumption, where only a shock on the productivity of the R&D sector influences the growth of the economy in the long run). In addition, the shocks on the productivity of the final good sector and on the cost of R&D on the one hand, and the shocks on the productivity of the equipment sector and of the intermediate good sector on the other hand, affect differently, in the short run, the economy, and influence the growth with different intensity. Finally, the model turns out to be sufficiently robust; when some parameter is significantly modified with respect to the benchmark case, both the qualitative and the quantitative implications of the model remain valid.

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