Technological Progress, Organizational Change and the Size of the Human Resources Department

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Technological Progress, Organizational Change and the Size of the Human Resources Department*

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Abstract

Innovative workplace practices based on multi-tasking and ICT that have been diffusing in most OECD countries since the 1990s have strong consequences on working conditions. Available data show together with the emergence of new organizational forms like multi-tasking, the increase in the proportion of workers employed in managerial occupation and the increase in skill requirements. This paper proposes a theoretical model to analyze the optimal number of tasks per worker when switching to multi-tasking raises coordination costs between workers and between tasks. Firms can reduce coordination costs by assigning more workers to human resources management. Human capital is endogenously accumulated by workers. The model reproduces pretty well the regularities observed in the data. In particular, exogenous technological accelerations tend to increase both the number of tasks performed and the skill requirements, and to raise the fraction of workers devoted to management.

Keywords: Information Technology, Organizational Change, Human Capital, Multi-Tasking.

Journal of Economic Literature: J22, J24, L23, O33, C62.

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1 Introduction

Admittedly the recent Information and Communication Technologies (ICT) revolution has favored the development of numerous innovative workplace practices in most OECD countries (see for example Osterman, 2004, for the US case, and Boucekkine and Crifo, 2007, for more recent empirical evidence for OECD countries). A view emerges as the productivity gains from investing in ICT cannot be significant without an appropriate evolution in the workplace organization towards more flexibility, which in turn is likely to rise the demand for skilled labor. This view is advocated and tested for example in the well-known paper of Bresnahan *et al.* (2002).

Table 1 illustrates this rising organizational flexibility following the ICT revolution. Work organization inside firms evolved from specialization to multi-tasking, and flexible forms of workplace organization have largely diffused in most OECD economies. For example, the two thirds of American firms and 57% of German establishments now rely on multi-tasking, and the proportion of workers involved in organizational changes increased continuously (and even doubled) in Great Britain and France.

Table 1: Tasks and computer use in the EU (% of workers), 1991-2000

	1991	1996	2000
Job involving complex tasks		57	56
Job involving repetitive tasks	23.3	16	15
Working with computers	13.9	38	41

Source: European surveys on working conditions, European foundation for the improvement of living and working conditions.

Parallel to this trend, we can also observe an increasing employment share of skilled workers in major OECD countries during the 1990s along with the dissemination of ICT, as shown in Table 2 (additional evidence confirming this trend in other OECD countries can be found in Boucekkine and Crifo, 2007).

Table 2: High-skilled (HS) ICT workers in the European Union and the United States, Average annual employment growth (1995-01) (* = in 2001)

	HS workers	HS ICT-related workers	Share of HS ICT workers in total occupations*
United States	2.79	5.29	2.63
France	1.67	7.11	2.05
Italy	5.99	8.58	1.30
Belgium	2.13	8.91	2.01
Germany	1.66	9.41	1.90
Denmark	3.08	9.49	2.58
EU	2.79	10.11	2.01
United Kingdom	1.37	12.63	2.60

Source: OECD (July 2004) based on the Eurostat Labour Force Survey and the US Current Population Survey, May 2003.

A definitely much less stressed aspect of organizational change, which is central in this paper, is the impact on human resources departments. However, one would expect that the role of such department will be significantly altered in a situation where flexible forms of work organization are so massively adopted. Indeed, a quick look at the data confirms this intuition. In particular, the management ratio increased in many OECD countries since the early 1980s. In France for instance, the percentage of workers employed in managerial and professional specialty occupations rose from 4% in 1980 to 6% in 1990 and 14% in 2000. Similarly, this ratio ranges from 12% in 1980 to 15% in 1990 and 22% in 2000 in Great Britain and from 22% in 1980 to 26% in 1990 and 30% in 2000 in the United States. Additional evidence for other OECD countries are reproduced in Table 3.

Table 3: Employment in administrative, legislative and managerial occupations (in percentage of total employment) in Canada, Denmark, Norway, UK and US, 1981-2000

	1981	1985	1991	1995	1997	2000
Canada	8.1	11.5	13.4	10.9	10.8	9.7
Denmark	4.06	3.4	4.4	6.5	7.019	7.12
Norway	5.3	6.5	6.6	7.02	7.8	8.02
UK			14.1	16.05	15.9	16.1
US	11.5	11.4	12.6	13.7	14.2	14.6

Source: International Labour Organization, Labour Statistics Database. The employment levels in administrative, legislative and managerial occupations corresponds to the employment levels of legislative officials and government administrators and managers.

In sum, the evolution of skills, job content and work organization observed in many OECD countries over the past decades can be summarized in the following three main characteristics:

- increase in the proportion of workers employed in managerial occupations
- diffusion of innovative workplace practices based on multi-tasking and computer use
- increase in skills requirements

The fact that the development of multi-tasking seems to be accompanied by an increase in the management ratios could suggest that coordinating workers and tasks should become more complex in the new economy. Becker and Murphy (1992) already argued that unlike Adam Smith's argument, specialization and the optimal division of labor would be determined more by the cost of combining specialized workers - i.e. coordination costs - than by the extent of the market. In particular, two types of coordination costs matter: the costs incurred by firms when coordinating workers' effort across various tasks and the costs of versatility incurred by workers subject to increased pressure and work rhythms. This contribution is only concerned with the former type of costs.

This paper studies the determination of the optimal number of tasks performed per worker in an economy where individuals devote time to production and human capital accumulation, and where multi-tasking both increases production and gives rise to coordination costs. Coordination costs could take several forms, which are surveyed in detail in the next section. In particular, we shall distinguish between horizontal coordination costs, which involve the costs of coordinating the tasks accomplished by each production worker, and vertical coordination costs, which reflect coordinating different workers. In our model, the first can reduce the coordination costs by assigning more workers to pure coordination non-productive tasks. We then examine how the economy reacts to permanent exogenous technological accelerations. The model is able to reproduce the three stylized facts outlined above. Importantly enough, the model delivers a permanent trend towards multi-tasking and human capital accumulation following permanent technological accelerations, while the size of the human resource department, that is the fraction of workers devoted to reduce coordination costs, is also significantly raised.

The paper produces two important contributions to the literature. In first place, it brings out a novel modelling of coordination costs and the associated human resources management workers. The framework is additionally found to replicate the recent evidence on the evolution of the size of human resources departments in the OECD countries following the ICT revolution. Second, it generalizes the findings of Boucekkine and Crifo (2007) in many respects. These authors have produced a simple OLG model with a fixed given number of tasks with human capital accumulation but without coordination costs. In this paper, the number of tasks is endogenous in the presence of well-motivated coordination costs. In this sense, it has a much broader scope than Boucekkine and Crifo's 2007 model.

The paper is organized as follows: Section 2 defines precisely coordination costs. Section 3 presents the setup of the general equilibrium model and derives the optimality and equilibrium conditions. Section 4 derives the steady-state values and the associated comparative statics. Section 5 reports some simulations exercises to analyze the impact of permanent exogenous technological accelerations on the economy. Section 6 concludes, while the main computations are reported in the Appendix.

2 Defining and accounting for coordination costs

As we analyze within-firm work organization, the evidence we report here is focused on "internal" coordination costs, i.e. intra (or within) firm coordination costs[1].

[1] External coordination costs correspond for instance to the costs of finding suppliers, negotiating contracts and paying bills to them and such issues are outside the scope of our model (external coordination costs are examined for instance in the transaction costs literature or in the industrial organization literature).

An important and complex problem is to estimate empirically the costs resulting from work organization and occupational conditions and changes. Injuries are the extreme outcome of poor working conditions, excessive and unmanageable pressures but they imply direct and obvious costs for workers and firms like absenteeism, sick pay, turnover costs, compensation and litigation costs, damage to equipment and production or reduced performance. However, the costs of coordinating activities and tasks between workers are not limited to safety and health conditions in the workplace. Precise empirical evidence on the extent of coordination costs is hard to find in the economics literature (unlike psychology, sociology, management and computer science where coordination costs are subject to a wider attention)[2] but we can give a simplified typology of available empirical measures by distinguishing three types of coordination costs.

The first category of coordination costs are *switch costs*, that denote the fact that people can flexibly switch back and forth between tasks but they cannot do it without costs. Switch costs hence measure the individual costs - mental or physical - of switching from one task to another, they correspond to "inter-tasks learning costs". Empirically, they are essentially measured by psychologists in terms of response time and acuracy[3]. Interestingly, switch costs occur even when people know in advance the identity of the new task and have ample time to prepare for the switch. Hence, if multi-tasking involves complicated and multi-faceted sub-activities, switching costs will be far from negligible.

The second category of individual costs of task switching is easier to account for in an economic perspective because it is measured by the direct *financial losses* of tasks coordination. These losses are evaluated in terms of human resources management. The number of workers employed in human resources services has grown in major OECD countries between 1980 and 2005 and more significantly, US managers reported a waste of 34 days per year and senior executives 7 weeks a year - an hour per day - in managing employees underperformance. Overall "poor hiring and people management practices" would have induced a shortfall worth 1% of US GDP in 2004 (SHL, 2004). But these financial losses also include the loss of wages or income and the additional expenditures (health care and medical treatment) associated with failures of tasks coordination. In the US, health care costs and individuals' health insurance have been rising by approximately 50% over the past two decades

[2]A notable exception comes from the literature on transaction costs economics. For Wallis and North (1986) transactions costs can be decomposed into motivation costs (agency costs and conflict of interests among managers, owners and debt holders as well as costs of cheating and opportunistic behaviours) and coordination costs (costs of obtaining information, coordinating input in production and measurement costs). Both marketed and non-marketed transaction costs (e.g. resources spent in waiting, getting permits to do business, cutting through red tapes etc.) are concerned. In developed economies, the transaction sector would represent 60% of GNP and non-marketed transactions costs 11.3% of GDP per capita (Wang, 2003).

[3]For Lien *et al.* (2006), when a typical response to a single stimulus takes 300 milliseconds, adding a second task increases the response to about 800 milliseconds. Extending the difference to a car driving 60 miles an hour, the response rate more than doubles.

and part of this rise could be imputed to organizational changes (Cartwright and Cooper, 1997).

The third category of tasks coordination costs are measured in terms of *human costs*, and they embed the pain, fear and general reduction in the quality of life associated with stress and poor working conditions. Such costs are difficult to evaluate but they are often embedded into indexes of well-being. In the 2000 European working condition survey, work-related stress was the second most common work-related health problem across European economies. Workers involved in innovative work organizational practices also tend to be subject to greater psychological discomfort and to face more mental strain than their non innovative counterparts (Askenazy *et al.*, 2002). Moreover, as far as employers are concerned, the effects of work related stress and pressure on the organizations and the firms' costs can be high when they result in greater sickness absenteeism, impaired performance and productivity and higher turnover rates. In the British industry for instance, according to a survey conducted in 2000, almost 40% of all absenteeism could be attributed to stress at the workplace at a cost of £4.2 billion a year (Hoel *et al.*, 2001). Adding the replacement costs and the loss of productivity in connection with stress, firms' total cost from stressful (changing and/or multi-tasking) work environments is important.

The literature on coordination problems mostly focuses on the division of labor and the returns to workers' specialization (Yang and Borland, 1991, Tamura, 1992). Some papers examine the impact of ICT on coordination (Brynjolfsson *et al.*, 1994), but very few papers analyze the role of coordination costs per se (see e.g. Becker and Murphy, 1992, or Hobijn, 1999). As coordination costs are multi-faceted, the theoretical literature has focused on various types of costs borne by the firm and the employees, among which:

- Switching costs between activities in terms of time, response delay or reduced goods quality (Lien *et al.* 2003, 2006).
- Communication costs and knowledge acquisition costs between workers (see Dessein and Santos, 2003, Bolton and Dewatripont, 1994, Radner, 1993, Garicano, 2000, Borghans and Ter Weel, 2006). In Garicano's model, for instance, higher layers of problem solvers in the firm increases the utilization rate of knowledge, thus economizing on knowledge acquisition, at the cost of increasing the communication required.
- Administrative coordination costs (assigning tasks, allocating resources and integrating outputs) and expertise coordination costs (managing knowledge and skill dependencies for complex, nonroutine intellectual tasks) (see Faraj and Sproull, 2000).
- Contracts incompleteness (see Acemoglu, Antras and Helpman, 2005). The basic idea here is that a firm decides a range of tasks to be performed and the division of labor among its workers. Coordination costs come from the fact that only a fraction of the activities that workers have to realize are contractible, the others are nonverifiable and noncontractible (and lead to under-investment).

In our model, focusing on internal coordination costs requires to account for both employee-level and firm-level coordination costs. In particular, it is important to explicitely incorporate human resources management as an activity necessary to coordinate the tasks performed by production workers. We therefore propose a typology that distinguishes two categories of internal coordination costs: horizontal and vertical coordination costs.

Horizontal coordination costs are the costs (at the firm level) of coordinating the tasks realized by each production worker. They embed switch costs between tasks, inter-tasks learning and information sharing costs, but also the strain associated with versatility and stressful working conditions. Such costs mainly concern production workers.

Vertical coordination costs are the costs of coordinating workers, that is the "costs of managers and others who decide when, where and how to produce" (Brynjolfsson *et al.*, 1994). Coordinating workers is a costly activity which induces expenses in addition to the direct wage costs of human resources managers and specialists. Vertical coordination costs mainly concern workers employed in human resources services.

3 The model

The model considers an economy in discrete time (from 0 to ∞) with an active population of size L[4]. The firm occupational structure is composed of two types of jobs: human resources jobs (in fraction ρ of the workforce employed) and production jobs (in fraction $1-\rho$ of the workforce employed). Workers devote time to production (either in the human resources service or in the production service) and to human capital accumulation.

3.1 Technology and coordination costs

The economy is characterized by a representative firm that produces a homogeneous (numeraire) good according to the following technology:

$$y_t = A_t \cdot \int_0^{n_t} \left[(1-\rho_t) \cdot h_t \cdot x_t(i) \cdot L_t\right]^{1-\alpha} di \qquad 0 < \alpha < 1 \tag{1}$$

where A_t is a productivity parameter, L_t is the volume of hours worked with human capital h_t, $x_t(i)$ is the time devoted to task i, n_t is the number of tasks performed per worker and ρ_t is the fraction of the workforce in the personnel (human resources) service.

[4]Note that when labor is divisible L measures either the number of workers for a fixed working time, or the volume of hours worked when working time can vary (given a fixed upper bound). Here we consider the second interpretation.

The worker's productive time is equal to T_t, hence we also have the constraint:

$$\int_0^{n_t} x_t(i)di = T_t \tag{2}$$

Tasks are symmetric, i.e. $x_t(i) = x_t$, and from (2) we then get:

$$x_t = \frac{T_t}{n_t}$$

and substituting this expression in (1) we obtain the production function:

$$y_t = A_t \cdot [(1 - \rho_t) \cdot h_t \cdot T_t \cdot L_t]^{1-\alpha} n_t^{\alpha} \tag{3}$$

Producing the good implies two types of costs: production costs and coordination costs (a similar element can be found in Brynjolfsson *et al.*, 1994). Since production requires physical resources and knowledge about how to combine them, production costs correspond to traditional costs of transforming inputs into output (physical - productive - resources expenses) whereas coordination costs correspond to the costs of combining and managing interactions and dependencies between resources (tasks and/or workers). In our model, labor is the sole input, therefore production costs equal the total wage bill and coordination costs depend on the number of tasks realized per worker (n) and on the fraction of workers in the human resources service (ρ).

The firm's profits (given that output is the numeraire) then write:

$$\pi_t = A_t \cdot [(1 - \rho_t) \cdot h_t \cdot T_t \cdot L_t]^{1-\alpha} n_t^{\alpha} - C(n, \rho) - w_t \cdot h_t \cdot T_t \cdot L_t$$

where w_t is the wage rate per efficiency unit of labor and $C(n, \rho)$ represents coordination costs measured as pure output loss.

The coordination costs function depends on horizontal and vertical coordination costs as follows:

$$C(n, \rho) = \frac{h(n, \rho) \cdot v(\rho)}{d}$$

where $h(n, \rho)$ denotes horizontal coordination costs and $v(\rho)$ denotes vertical coordination costs, while d reflects the extent of coordination costs (a higher d reduces the importance or magnitude of coordination costs).

Horizontal coordination costs are the firm-level costs of coordinating the tasks realized by each production worker. They embed switch costs between tasks, inter-tasks learning and information sharing costs, but also the strain associated with versatility and stressful working conditions. The higher the number of tasks per worker, the higher the switch costs, the higher the inter-tasks learning and information sharing costs and the higher the stress costs. Since production workers are in fraction $1 - \rho$, we assume that the costs of coordinating n tasks for each worker writes:

$$h(n, \rho) = n^{\xi} \cdot (1 - \rho)^{\theta}$$

where $\xi, \theta > 0$ (we do not impose $\xi \neq \theta$).

In other words, the costs of horizontal tasks coordination are based upon the idea that allocating workers' attention over various activities is likely to raise the occurrence of mistakes on the job. This increasing risk of production failure reduces the value of output and profits by an amount that depends both on the number of tasks per capita n and on the size of the production service $(1-\rho)$, given elasticities parameters ξ and θ that will be discussed below.

Vertical coordination costs are the costs of coordinating workers. Consistently with Section 2, they reflect the administrative cost of running a human resource department in a firm. In our set-up, such a cost involves of course a direct labor cost, which is already accounted for by the last term of the profit expression given above, but also an extra-cost depending on the size of the human resources department to be operated. To this end, we assume that vertical coordination costs is an increasing function of the share of workers in the human resources service, that is:

$$v(\rho) = \rho^{\eta}$$

where $\eta > 0$.

The fact that vertical coordination costs are linked to the size of human resources service and add to the wage bill can be related to the literature on vertical integration. Indeed, since Williamson's theory of transaction costs, it is well known that a major limit to the growth of firms lies in the costs of internal organization. As explained by Joskow (2003), the volume of auditing information that must be processed by management grows non-linearly with the size and scope of the firm and becomes more difficult to use to control costs and quality effectively and to adapt to changing market conditions. Moreover, monitoring becomes also more difficult in large organizations and there are therefore potential shirking problems resulting from low power internal compensation incentives. In sum, the decision whether or not to vertically integrate comes from a tradeoff between the costs of market-based arrangements and the costs of internal organization described by the relatively inferior adaptive properties of bureaucratic hierarchies to rapidly changing outside opportunities over the longer term and the difficulty of designing compensation mechanisms to give managers and employees appropriate incentives to control costs and product quality[5]. Here, we rely on this debate on the "make or buy" decision by accounting for the fact that firms relying on large human resources departments are characterized by specific management costs (bureaucratic and incentives costs) labelled as vertical coordination costs.

We now make clear two working assumptions for the optimization problem of the firm to make sense. We first need to ensure that function $C(n, \rho)$ is increasing in n

[5]However, as highlighted by Joskow (2003), this literature has focused much more on the inefficiencies of market transactions than it has on the strengths and weaknesses of internal organization. Our focus on vertical and horizontal coordination costs can therefore provide an interesting complementary contribution to this debate.

and decreasing in ρ. We need the latter condition on ρ to have an interior solution to the optimization problem tackled. Indeed, an increase in ρ decreases production labor, and therefore production. To balance this negative impact on profits, we need the increase in ρ lowers the coordination costs. With analytical forms postulated, we need the decrease in horizontal coordination costs due to an increment in ρ more than compensates the induced rising vertical coordination costs. This property will be put in more formal terms in Proposition 2.

The convexity of the cost function can be easily investigated. We have the following result:

Proposition 1 *The cost function* $C(n_t, \rho_t) = \frac{n_t^{\xi} \cdot (1-\rho_t)^{\theta} \cdot \rho_t^{\eta}}{d_t}$ *is convex when* $\xi > 1$, $\theta < 1$ *and* η *is small enough.*

Proof. The first-order partial derivatives of the cost function are given by:

$$\frac{\partial C}{\partial n_t} = \frac{\xi n_t^{\xi-1} (1-\rho_t)^{\theta} \rho_t^{\eta}}{d_t} \qquad \frac{\partial C}{\partial \rho_t} = \frac{n_t^{\xi}}{d_t} (1-\rho_t)^{\theta-1} \rho_t^{\eta-1} [\eta - (\eta+\theta) \rho_t]$$

while the second-order partial derivatives are:

$$\begin{aligned}
\frac{\partial^2 C}{\partial n_t^2} &= \frac{\xi (\xi-1) n_t^{\xi-2} (1-\rho_t)^{\theta} \rho_t^{\eta}}{d_t} \\
\frac{\partial^2 C}{\partial \rho_t \partial n_t} &= \frac{\partial^2 C}{\partial n_t \partial \rho_t} = \frac{\xi n_t^{\xi-1}}{d_t} (1-\rho_t)^{\theta-1} \rho_t^{\eta-1} [\eta - (\eta+\theta) \rho_t] \\
\frac{\partial^2 C}{\partial \rho_t^2} &= \frac{n_t^{\xi}}{d_t} (1-\rho_t)^{\theta-2} \rho_t^{\eta-2} [\eta (\eta-1)(1-\rho_t) - \rho_t \eta (\theta-1) + \\
& \quad -\rho_t \eta (\eta+\theta)(1-\rho_t) - \rho_t^2 (\eta+\theta)(\theta-1)]
\end{aligned}$$

and the hessian matrix is:

$$H = \begin{bmatrix} \frac{\partial^2 C}{\partial n_t^2} & \frac{\partial^2 C}{\partial \rho_t \partial n_t} \\ \frac{\partial^2 C}{\partial n_t \partial \rho_t} & \frac{\partial^2 C}{\partial \rho_t^2} \end{bmatrix}$$

and for the cost function to be convex this matrix must be positive definite, i.e. we must have:

$$H_1 > 0 \qquad H_2 > 0$$

With reference to the first condition we have:

$$H_1 > 0 \Rightarrow \frac{\partial^2 C}{\partial n_t^2} > 0 \Rightarrow \frac{\xi (\xi-1) n_t^{\xi-2} (1-\rho_t)^{\theta} \rho_t^{\eta}}{d_t} > 0 \Rightarrow \xi > 1$$

i.e. it is satisfied if $\xi > 1$, while with reference to the second condition we have:

$$H_2 > 0 \Rightarrow \det H > 0 \Rightarrow \frac{\partial^2 C}{\partial n_t^2} \cdot \frac{\partial^2 C}{\partial \rho_t^2} - \left(\frac{\partial^2 C}{\partial \rho_t \partial n_t} \right)^2 > 0$$

that is:

$$\frac{\xi(\xi-1)n_t^{\xi-2}(1-\rho_t)^{\theta}\rho_t^{\eta}}{d_t}\cdot\frac{n_t^{\xi}}{d_t}(1-\rho_t)^{\theta-2}\rho_t^{\eta-2}\left[\eta(\eta-1)(1-\rho_t)-\rho_t\eta(\theta-1)+\right.$$
$$\left.-\rho_t\eta(\eta+\theta)(1-\rho_t)-\rho_t^2(\eta+\theta)(\theta-1)\right]-\left[\frac{\xi n_t^{\xi-1}}{d_t}(1-\rho_t)^{\theta-1}\rho_t^{\eta-1}\left[\eta-(\eta+\theta)\rho_t\right]\right]^2>0$$

that leads to:

$$\frac{\xi n_t^{2\xi-2}(1-\rho_t)^{2\theta-2}\rho_t^{2\eta-2}}{d_t^2}\left[(\xi-1)\eta(\eta-1)(1-\rho_t)-\rho_t\eta(\xi-1)(\theta-1)+\right.$$
$$-\rho_t\eta(\xi-1)(\eta+\theta)(1-\rho_t)-\rho_t^2(\xi-1)(\eta+\theta)(\theta-1)+$$
$$\left.-\xi\eta^2+2\xi\eta\rho_t(\eta+\theta)+\xi\left(\eta^2+2\eta\theta+\theta^2\right)\rho_t^2\right]>0$$

The fraction outside the square bracket is positive, while considering the expression inside the square bracket and letting η tend to 0 we get:

$$-\rho_t^2(\xi-1)\theta(\theta-1)+\xi\theta^2\rho_t^2>0,$$

which holds when $\xi>1$ and $\theta<1$, and hence the cost function is convex under the conditions of the proposition. ■

At this point the profit function writes:

$$\pi_t=A_t\cdot\left[(1-\rho_t)\cdot h_t\cdot T_t^d\cdot L_t\right]^{1-\alpha}n_t^{\alpha}-\frac{n_t^{\xi}\cdot(1-\rho_t)^{\theta}\cdot\rho_t^{\eta}}{d_t}-w_t\cdot h_t\cdot T_t^d\cdot L_t$$

where $d_t>0$ and where T_t^d denotes now the working time demanded by the firm. In the decentralized economy the firm's intertemporal optimization program is then given by[6]:

$$\max_{\left\{n_t,T_t^d,\rho_t\right\}_{t=0}^{\infty}}\sum_{t=0}^{\infty}\frac{1}{(1+r_1)(1+r_2)...(1+r_t)}\left[A_t\left[(1-\rho_t)h_tT_t^dL_t\right]^{1-\alpha}n_t^{\alpha}+\right.$$
$$\left.-\frac{n_t^{\xi}(1-\rho_t)^{\theta}\rho_t^{\eta}}{d_t}-w_th_tT_t^dL_t\right]$$

The first-order conditions of this program are:

$$\frac{\partial f}{\partial n_t}=\frac{1}{(1+r_1)(1+r_2)...(1+r_t)}\left[\alpha A_t\left[(1-\rho_t)h_tT_t^dL_t\right]^{1-\alpha}n_t^{\alpha-1}-\frac{\xi n_t^{\xi-1}(1-\rho_t)^{\theta}\rho_t^{\eta}}{d_t}\right]=0$$
$$\frac{\partial f}{\partial T_t^d}=\frac{1}{(1+r_1)(1+r_2)...(1+r_t)}\left[(1-\alpha)A_t\left[(1-\rho_t)h_tT_t^dL_t\right]^{-\alpha}(1-\rho_t)h_tL_tn_t^{\alpha}-w_th_tL_t\right]=0$$
$$\frac{\partial f}{\partial \rho_t}=\frac{1}{(1+r_1)(1+r_2)...(1+r_t)}\left[-(1-\alpha)A_t\left[(1-\rho_t)h_tT_t^dL_t\right]^{-\alpha}h_tT_t^dL_tn_t^{\alpha}+\right.$$
$$\left.-\frac{n_t^{\xi}}{d_t}\left[\eta(1-\rho_t)^{\theta}\rho_t^{\eta-1}-\theta(1-\rho_t)^{\theta-1}\rho_t^{\eta}\right]\right]=0$$

[6] plus the standard transversality conditions.

from which we obtain:

$$\alpha A_t (1-\rho_t)^{1-\alpha} \left(h_t T_t^d L_t\right)^{1-\alpha} n_t^{\alpha} = \frac{\xi n_t^{\xi} (1-\rho_t)^{\theta} \rho_t^{\eta}}{d_t} \tag{4}$$

$$(1-\alpha) A_t (1-\rho_t)^{1-\alpha} \left(h_t T_t^d L_t\right)^{-\alpha} n_t^{\alpha} = w_t \tag{5}$$

$$(1-\alpha) A_t (1-\rho_t)^{-\alpha} \left(h_t T_t^d L_t\right)^{1-\alpha} n_t^{\alpha} = \frac{n_t^{\xi}}{d_t} \left[\theta (1-\rho_t)^{\theta-1} \rho_t^{\eta} - \eta (1-\rho_t)^{\theta} \rho_t^{\eta-1}\right] \tag{6}$$

The second-order conditions of the problem of the firm, that guarantee the presence of a maximum, are checked in Appendix 7.1.

Equation (4) gives the optimality condition for the number of tasks by equalizing the marginal productivity of a task (that is, the increase in output due to an additional task) with the marginal cost of a task (that is, the marginal increase in horizontal coordination costs). Similarly, the optimal demand for productive time T determined in equation (5) equalizes the marginal product of productive time with its marginal remuneration.

Equation (6) is the most important condition of the firm's block, as it provides the optimality condition for the fraction of labor devoted to coordination tasks. The left-hand side clearly reflects the loss in production induced by diverting workers from production. The right-hand side reflects the marginal impact of a larger share of human resources specialists on the costs of internal coordination. By definition, this impact is twofold. On the one hand, more workers in the human resources department implies less workers in production, and less productive workers means a lower exposure by the firm to the risk of productive mistakes, thereby a lower level of horizontal costs of tasks coordination. On the other hand however, more workers in the human resources department drives the vertical component of coordination costs in the opposite direction by construction. More human resources specialists means indeed a higher bureaucratic and incentives burden, thereby driving vertical coordination costs upward. For condition (6) to make sense, it is of course necessary to ensure that the right-hand side is positive at least locally, which is done in Proposition 2. Note that this needed property amounts to guarantee that the coordination costs function $C(n, \rho)$ is a decreasing function of ρ, a point made before.

3.2 Household and human capital accumulation

The household in the economy has a utility function given by:

$$u(c_t) = \frac{c_t^{1-\tau} - 1}{1-\tau} \qquad \tau > 0$$

where c_t is consumption, and to simplify the analysis we then assume $\tau = 1$, that is:

$$u(c_t) = \ln c_t \tag{7}$$

The household is then endowed with one unit of time supplied each period, that is spent on working (the fraction T_t^s) or on human capital accumulation (the fraction $1-T_t^s$), and the accumulation of human capital is described by the following equation:

$$h_{t+1} = E_t \cdot h_t^{\delta} \cdot \left(1 - T_t^s\right)^{1-\delta} \qquad 0 < \delta < 1 \tag{8}$$

where E_t is an efficiency parameter.

The household's intertemporal optimization program in the decentralized economy is then given by[7]:

$$\max_{\{c_t, T_t^s, a_{t+1}, h_{t+1}\}_{t=0}^{\infty}} \quad \sum_{t=0}^{\infty} \beta^t \ln c_t$$

$$s.t. \quad \begin{aligned} a_{t+1} &= \left(1 + r_t\right) a_t + w_t h_t T_t^s - c_t \\ h_{t+1} &= E_t h_t^{\delta} \left(1 - T_t^s\right)^{1-\delta} \end{aligned}$$

where β is the discount factor (with $0 < \beta < 1$) and a_t represents the assets held at time t.

The Lagrangian for this problem is:

$$\begin{aligned} \mathcal{L} \;=\; & \sum_{t=0}^{\infty} \left\{ \beta^t \ln c_t + \beta^t \mu_t \left[\left(1 + r_t\right) a_t + w_t h_t T_t^s - c_t - a_{t+1} \right] + \right. \\ & \left. + \beta^t \lambda_t \left[E_t h_t^{\delta} \left(1 - T_t^s\right)^{1-\delta} - h_{t+1} \right] \right\} \end{aligned}$$

and the first-order conditions are given by:

$$\begin{aligned} \frac{\partial \mathcal{L}}{\partial c_t} &= \frac{\beta^t}{c_t} - \beta^t \mu_t = 0 \\ \frac{\partial \mathcal{L}}{\partial T_t^s} &= \beta^t \mu_t w_t h_t - \beta^t \lambda_t \left(1 - \delta\right) E_t h_t^{\delta} \left(1 - T_t^s\right)^{-\delta} = 0 \\ \frac{\partial \mathcal{L}}{\partial a_{t+1}} &= \beta^{t+1} \mu_{t+1} \left(1 + r_{t+1}\right) - \beta^t \mu_t = 0 \\ \frac{\partial \mathcal{L}}{\partial h_{t+1}} &= \beta^{t+1} \mu_{t+1} w_{t+1} T_{t+1}^s + \beta^{t+1} \lambda_{t+1} \delta E_{t+1} h_{t+1}^{\delta-1} \left(1 - T_{t+1}^s\right)^{1-\delta} - \beta^t \lambda_t = 0 \\ \frac{\partial \mathcal{L}}{\partial \mu_t} &= \beta^t \left[\left(1 + r_t\right) a_t + w_t h_t T_t^s - c_t - a_{t+1} \right] = 0 \\ \frac{\partial \mathcal{L}}{\partial \lambda_t} &= \beta^t \left[E_t h_t^{\delta} \left(1 - T_t^s\right)^{1-\delta} - h_{t+1} \right] = 0 \end{aligned}$$

from which, rearranging and substituting, we get the following relevant equations:

$$\frac{1}{c_t} = \frac{\beta}{c_{t+1}} \left(1 + r_{t+1}\right) \tag{9}$$

[7] plus the standard the transversality conditions.

$$\frac{w_t h_t}{c_t (1-\delta) E_t h_t^{\delta} (1-T_t^s)^{-\delta}} = \frac{\beta \delta w_{t+1} \left(1-T_{t+1}^s\right)}{c_{t+1}(1-\delta)} + \frac{\beta w_{t+1} T_{t+1}^s}{c_{t+1}} \tag{10}$$

$$a_{t+1} = (1+r_t) a_t + w_t h_t T_t^s - c_t \tag{11}$$

$$h_{t+1} = E_t h_t^{\delta} (1-T_t^s)^{1-\delta} \tag{12}$$

Equation (9) is the typical Euler equation for optimal consumption over time. Equation (10) is the optimality condition for human capital accumulation after substitution of the Lagrange multipliers λ_t and μ_t using the first-order conditions with respect to consumption and production time supply notably. The left-hand side measures the marginal cost of human capital accumulation, which is simply reflected in the wage forgone in period t due to education. The right-hand side is the sum of two marginal benefit terms: the increase in the marginal productivity of education time in $t+1$ and the increasing labor remuneration in the same period. Equations (11) and (12) are just the law of evolution of consumer's wealth and the education technology respectively.

3.3 Market equilibrium conditions

Together with the solution of the problem of the firm and of the household, the stationary equilibrium of the decentralized economy is characterized also by the market equilibrium condition:

$$y_t = c_t + \frac{n_t^{\xi} (1-\rho_t)^{\theta} \rho_t^{\eta}}{d_t}$$

from which:

$$A_t \left[(1-\rho_t) h_t T_t^s L_t\right]^{1-\alpha} n_t^{\alpha} = c_t + \frac{n_t^{\xi} (1-\rho_t)^{\theta} \rho_t^{\eta}}{d_t} \tag{13}$$

and by the labor market equilibrium condition:

$$T_t^d = T_t^s = T_t \tag{14}$$

We are now able to set the following definition of equilibrium for the economy under investigation:

Definition *Given the initial condition h_0, an equilibrium is a path:*

$$\{r_t, T_t, h_t, \rho_t, n_t, w_t, c_t, a_t\}_{t \geq 0}$$

that satisfies the equations (4)-(6) and (9)-(13) derived above and the corresponding standard transversality conditions.

4 Steady-state and comparative statics of the model

Given the decentralized economy considered in the previous Section, it is now possible to obtain the steady-state values of the different variables (for the computations see Appendix 7.2), that are given by:

$$r=\frac{1-\beta}{\beta} \tag{15}$$

$$T=\frac{1-\beta\delta}{1+\beta-2\beta\delta} \tag{16}$$

$$h=E^{\frac{1}{1-\delta}}\frac{\beta\left(1-\delta\right)}{1+\beta-2\beta\delta} \tag{17}$$

$$\rho=\frac{\alpha\eta}{\alpha\left(\xi+\eta+\theta\right)-\xi} \tag{18}$$

$$\begin{aligned} n &= \left(\frac{\alpha Ad}{\xi}\right)^{\frac{1}{\xi-\alpha}} E^{\frac{1-\alpha}{(1-\delta)(\xi-\alpha)}} L^{\frac{1-\alpha}{\xi-\alpha}} \left(\frac{\beta\left(1-\delta\right)\left(1-\beta\delta\right)}{\left(1+\beta-2\beta\delta\right)^{2}}\right)^{\frac{1-\alpha}{\xi-\alpha}} \cdot \\ &\cdot\left(\alpha\left(\xi+\theta\right)-\xi\right)^{\frac{1-\alpha-\theta}{\xi-\alpha}}\left(\alpha\eta\right)^{\frac{\eta}{\alpha-\xi}}\left(\alpha\left(\xi+\eta+\theta\right)-\xi\right)^{\frac{1-\alpha-\theta-\eta}{\alpha-\xi}} \end{aligned} \tag{19}$$

$$\begin{aligned} w &= (1-\alpha)\left(\frac{\alpha d}{\xi}\right)^{\frac{\alpha}{\xi-\alpha}} A^{\frac{\xi}{\xi-\alpha}} E^{\frac{\alpha(1-\xi)}{(1-\delta)(\xi-\alpha)}} L^{\frac{\alpha(1-\xi)}{\xi-\alpha}} \left(\frac{\beta\left(1-\delta\right)\left(1-\beta\delta\right)}{\left(1+\beta-2\beta\delta\right)^{2}}\right)^{\frac{\alpha(1-\xi)}{\xi-\alpha}} \cdot \\ &\cdot\left(\alpha\left(\xi+\theta\right)-\xi\right)^{\frac{\xi-\alpha(\xi+\theta)}{\xi-\alpha}}\left(\alpha\eta\right)^{\frac{\alpha\eta}{\alpha-\xi}}\left(\alpha\left(\xi+\eta+\theta\right)-\xi\right)^{\frac{\alpha(\xi+\eta+\theta)-\xi}{\xi-\alpha}} \end{aligned} \tag{20}$$

$$\begin{aligned} c &= \frac{\xi-\alpha}{\xi}\left(\frac{\alpha d}{\xi}\right)^{\frac{\alpha}{\xi-\alpha}} A^{\frac{\xi}{\xi-\alpha}} E^{\frac{\xi(1-\alpha)}{(1-\delta)(\xi-\alpha)}} L^{\frac{\xi(1-\alpha)}{\xi-\alpha}} \left(\frac{\beta\left(1-\delta\right)\left(1-\beta\delta\right)}{\left(1+\beta-2\beta\delta\right)^{2}}\right)^{\frac{\xi(1-\alpha)}{\xi-\alpha}} \cdot \\ &\cdot\left(\alpha\left(\xi+\theta\right)-\xi\right)^{\frac{\alpha(\xi+\theta)-\xi}{\alpha-\xi}}\left(\alpha\eta\right)^{\frac{\alpha\eta}{\alpha-\xi}}\left(\alpha\left(\xi+\eta+\theta\right)-\xi\right)^{\frac{\alpha(\xi+\eta+\theta)-\xi}{\xi-\alpha}} \end{aligned} \tag{21}$$

$$\begin{aligned} a &= \frac{\beta}{1-\beta}\left(\frac{\xi-\alpha}{\xi}L-1+\alpha\right)\left(\frac{\alpha d}{\xi}\right)^{\frac{\alpha}{\xi-\alpha}} A^{\frac{\xi}{\xi-\alpha}} E^{\frac{\xi(1-\alpha)}{(1-\delta)(\xi-\alpha)}} L^{\frac{\alpha(1-\xi)}{\xi-\alpha}} \left(\frac{\beta\left(1-\delta\right)\left(1-\beta\delta\right)}{\left(1+\beta-2\beta\delta\right)^{2}}\right)^{\frac{\xi(1-\alpha)}{\xi-\alpha}} \cdot \\ &\cdot\left(\alpha\left(\xi+\theta\right)-\xi\right)^{\frac{\xi-\alpha(\xi+\theta)}{\xi-\alpha}}\left(\alpha\eta\right)^{\frac{\alpha\eta}{\alpha-\xi}}\left(\alpha\left(\xi+\eta+\theta\right)-\xi\right)^{\frac{\alpha(\xi+\eta+\theta)-\xi}{\xi-\alpha}} \end{aligned} \tag{22}$$

The following result can then be stated:

Proposition 2 *Provided the following restrictions on the parameters hold:*

$$\delta<\frac{1+\beta}{2\beta} \qquad \text{and} \qquad \frac{\xi}{\xi+\theta}<\alpha<\xi$$

there exists a unique steady state of the model with $0<T<1$ *and* $0<\rho<1$ *where the values of the different variables are given by the expressions (15)-(22).*

Proof. The steady-state values of the variables are obtained in Appendix 7.2. Concerning the restrictions on the parameters, given the expression obtained for T:

$$T = \frac{1-\beta\delta}{1+\beta-2\beta\delta}$$

the fact that $0 < T < 1$ implies $0 < \frac{1-\beta\delta}{1+\beta-2\beta\delta} < 1$, and since the numerator is positive (because $0 < \beta < 1$ and $0 < \delta < 1$) we must have (for the first inequality to hold):

$$1+\beta-2\beta\delta > 0 \Rightarrow \delta < \frac{1+\beta}{2\beta}$$

while the second inequality is always verified (being $\delta < 1$). Given the expression obtained for ρ:

$$\rho = \frac{\alpha\eta}{\alpha(\xi+\eta+\theta)-\xi}$$

then, the fact that $0 < \rho < 1$ implies $0 < \frac{\alpha\eta}{\alpha(\xi+\eta+\theta)-\xi} < 1$, from which:

$$\alpha(\xi+\eta+\theta)-\xi > 0 \Rightarrow \alpha > \frac{\xi}{\xi+\eta+\theta}$$

and also:

$$\alpha\eta < \alpha(\xi+\eta+\theta)-\xi \Rightarrow \alpha > \frac{\xi}{\xi+\theta}$$

Since we also have $\alpha < \xi$ (because the latter must be larger than 1 for the cost function to be convex), this implies the following restrictions:

$$\frac{\xi}{\xi+\theta} < \alpha < \xi$$

■

The steady-state values obtained above can then be used for the comparative statics analysis of the decentralized equilibrium. The results (see computations in Appendix 7.3) can be summarized in the following Table (for the missing elements, it is not possible to derive analytically the sign of the relationship between the variable and the parameter that affects it, and it is necessary to resort to simulations to have this indication):

Table 4: Comparative statics of the model

	r	T	h	ρ	n	w	c	a
α	0	0	0	$-$				
β	$-$	$-$	$+$	0				
δ	0	$+$	$+$	0				
ξ	0	0	0	$+$				
θ	0	0	0	$-$				
η	0	0	0	$+$				
A	0	0	0	0	$+$	$+$	$+$	$+$
E	0	0	$+$	0	$+$	$-$	$+$	$+$
L	0	0	0	0	$+$	$-$	$+$	$-$
d	0	0	0	0	$+$	$+$	$+$	$+$

These results can be interpreted as follows.

The interest rate r depends negatively on the discount factor (β), and the same factor (that represents also the degree of impatience) has a negative impact on the time spent working T and a positive impact on human capital h. A higher productivity of human capital (through the elasticity parameter δ) has a positive influence on the time spent working T and (either through the efficiency parameter E or through the elasticity parameter δ - in this latter case if E is high enough -) on human capital h. In other words, when households are less impatient or when human capital is more productive, the level of human capital improves.

The fraction of the workforce in the human resources service ρ depends positively on the weight (elasticity) of tasks in horizontal coordination costs (ξ) and on the weight of human resources in vertical coordination costs (η) and it depends negatively on the weight of production workers in horizontal coordination costs (θ) and on the weight of tasks in total output (α). In other words, more workers are allocated to human resources management when coordination costs are more resources intensive or consuming, or when the production failure risk (due to switch costs between tasks) is higher. On the contrary, more workers are allocated to production activities when task specialization is more productive or when bureaucratic coordination is more costly.

The intensity of multi-tasking n (the number of tasks per individual) as well as the level of consumption c increase with the efficiency of human capital E, the size of the workforce L and the technological efficiency of production A; and they decrease with the extent of coordination costs (recall that a higher d means a lower importance of coordination costs).

The wage rate w decreases with the efficiency of human capital E, the size of the workforce L and the extent of coordination costs, and increases with the technological efficiency of production A.

The level of assets a decreases with the size of the workforce L and the extent of coordination costs and increases with the efficiency of human capital E and the technological efficiency of production A.

5 Short-term dynamics of the model

The model considered can then be simulated in order to study the effects of different types of technological shocks that can hit the economy. Hereafter we present some representative simulation experiments on a calibrated model. Since the model is in many aspects highly stylized (no physical capital for example), calibrating it on a real economy is a daunting task. Below we consider a benchmark calibration and the corresponding short-term dynamics. The equilibrium paths generated by this calibration have been checked to be qualitatively robust to numerous parameters' changes. While the quantitative results displayed below are only indicative and should not be taken too seriously, the qualitative dynamics are found to be strongly robust.

5.1 Calibration

As usual, the simulation of the model requires a calibration, and the values chosen for the different parameters are reported in the following table:

Table 5: Calibration of the model, benchmark case

Parameter	*Symbol*	*Value*
Parameter α in the production function	α	0.65
Psychological discount factor	β	0.99
Productivity of human capital	δ	0.9
Parameter ξ in the coordination costs function	ξ	1.2
Parameter θ in the coordination costs function	θ	0.7
Parameter η in the coordination costs function	η	0.01
Productivity of output	A	1
Efficiency of human capital	E	2
Size of the workforce	L	1
Parameter d in the coordination costs function	d	0.5

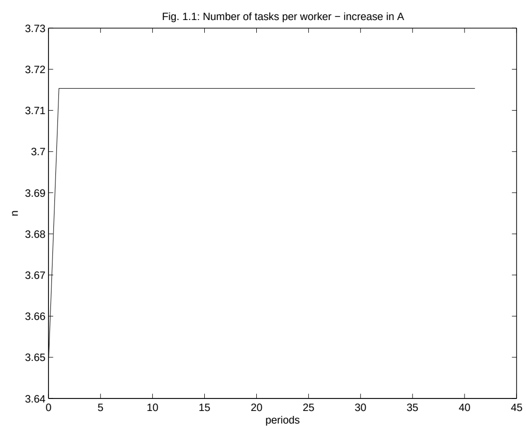
In particular, on the one hand these parameters are such that they satisfy the restrictions obtained above (in order to have convexity of the cost function and existence of the steady-state). On the other hand, the same parameters allow to obtain steady-state values of some relevant quantities that are reasonable on the basis on the empirical evidence that is available. For instance, with this parameterization in the steady state the fraction of time devoted to production is slightly more than 0.5, the ratio consumption/output is close to 0.95 and the fraction of the workforce in the human resources services is larger than 0.10.

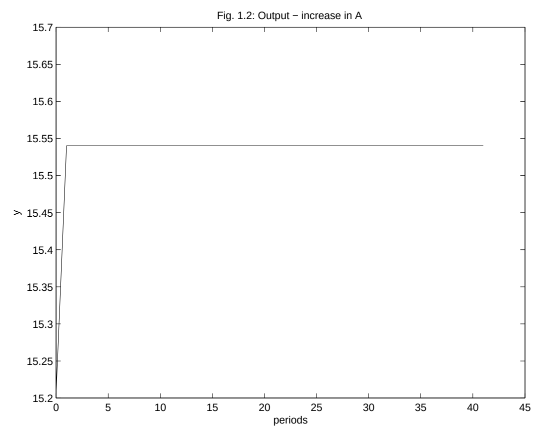
5.2 Simulation

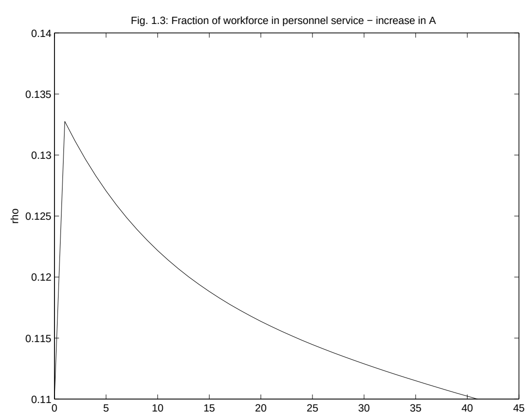
Starting from the values chosen, different types of shocks are considered, and the consequences on the economy are analyzed. In particular, it is possible to consider a shock on the productivity of output A, a shock on the efficiency of human capital E and a shock on the coordination costs through the parameter d. All these shocks are permanent.

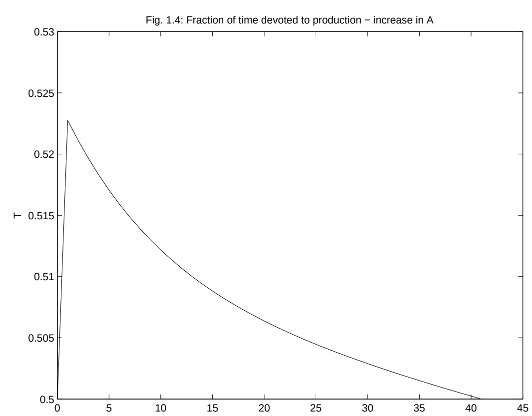
The first situation considered is an increase in A (the productivity of output). From the analytical results derived previously we know that this increase has a long run effect on the number of tasks performed per worker and on output (that both increase). The simulation allows studying the behaviour of all quantities also in the short run (Figures 1.1 – 1.4).

In particular, the simulations show that the number of tasks per worker n increases also in the short run, immediately after the increase in A, and remains at this level in the long run. The output level behaves in the same way, since it increases immediately after the increase in A, and then remains at this value in the long run. Also the fraction of the workforce in the human resources service ρ increases immediately after the increase in A, but then it returns to its initial level, since the long run value of this variable is not affected by A. The same holds for the level of human capital, that is characterized by an increase (of small amount) followed by a decrease, and for the allocation of time between production and human capital accumulation (with an initial increase of the time devoted to productive activity immediately after the shock on A, that in the end returns to its initial level).









If we consider an increase in the parameter d that characterizes the extent of coordination costs (a rise in d reduces the importance of coordination costs), we get the same results as for the increase in A.

More precisely, the increase in d increases the intensity of multi-tasking and the output level immediately after the shock, and this effect persists also in the long run. The fraction of the workforce in the human resources service, the level of human capital and the allocation of time in favour of production, on the other hand, increase immediately after the shock on d, but then return to their initial levels since this shock does not affect their long run values. Intuitively, an increase in d corresponds to a reduction in coordination costs and represents therefore the same kind of qualitative productivity shock as a rise in the technological parameter A.

The last situation analyzed is an increase in the efficiency of human capital E. In this case, we have shown analytically that a rise in E has a long run effect on the number of tasks performed per worker, on the level of output and on the level of human capital (that all increase).

In the short run (Figures $2.1 - 2.4$), the timing of the effects of the shock on the different variables is different from the previous cases. In effect, the number of tasks per worker n gradually increases after the rise in E and reaches, in the long run, a level higher than the initial one (while in the other cases examined there is immediately the jump in the variable, that then remains at the new value). Following the rise in E, output also increases gradually reaching the new long run level, and the same happens for the level of human capital. On the contrary, the fraction of the workforce in the human resources service and the allocation of time in favour of productive activity initially increase but then return to their initial levels, since their long run values are not affected by the shock on E.

Fig. 2.1: Number of tasks per worker − increase in E

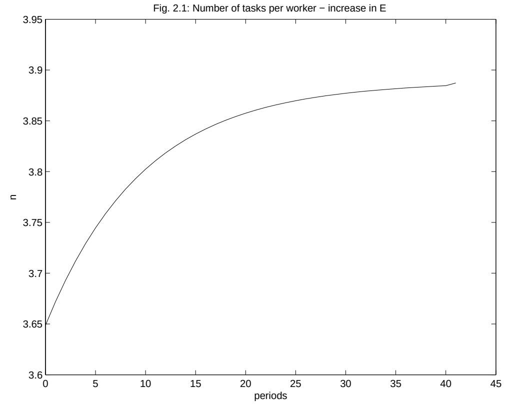


Fig. 2.2: Output − increase in E

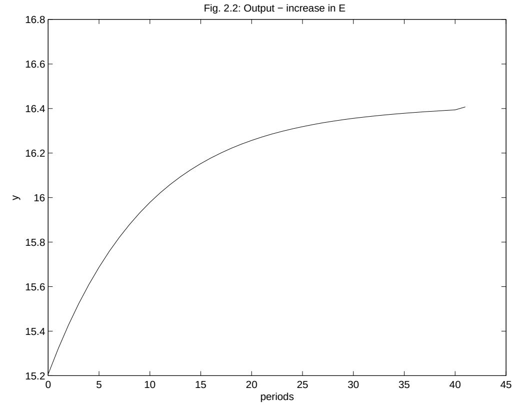


Fig. 2.3: Fraction of workforce in personnel service − increase in E

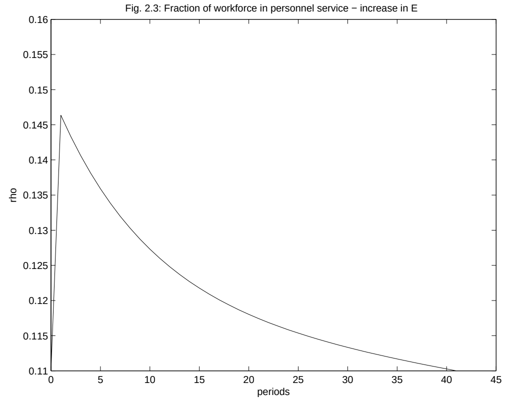
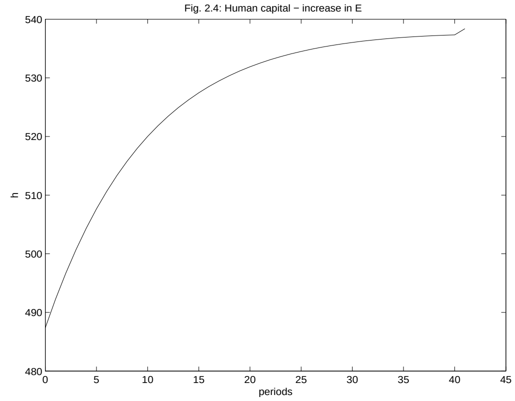


Fig. 2.4: Human capital − increase in E



It is important to observe that the results of these simulations (in particular, as outlined above, from the qualitative point of view) are consistent with the available data, concerning for instance the intensity of multi-tasking and other flexible organizational forms, and the behaviour of the workforce employed in the human resources service (see Tables 2 and 3) following the ICT Revolution of the 90s. This fraction of the workforce has increased in major economies. This is the kind of behaviour that also emerges from the simulation exercises of the present model, which therefore behaves extremely well in replicating the organizational features of the ICT revolution. Interestingly enough, our model predicts a transitory increase in the size of the human resources department in response to all the performed permanent technological shocks, which certainly needs to be corroborated on updated data.

6 Conclusion

This paper develops a model to analyze the intensity of multi-tasking under various exogenous technological accelerations. The model has two original characteristics: it includes endogenous coordination costs, and it introduces the size of the human resources department as a key variable for the firms to control their coordination costs. In our modelling, and building on recent economic and management literature, we distinguish between vertical and horizontal coordination costs, which proves crucial in the equilibrium properties of the model. The model also includes endogenous human capital accumulation, and therefore brings together enough ingredients to study some highly relevant stylized facts identified in the introduction section in OECD data. Although technological progress is exogenous in our set-up, and no technology adoption decision is to be taken, we believe that the model offers a useful shortcut to analyze the consequences of technological accelerations on workplace organization. The fact that all performed numerical simulations corroborate the ability of the model to replicate the observed stylized facts is a good indication of that.

Two extensions are currently in our agenda. One concerns the incorporation of an explicit technology adoption decision where the firms have to decide whether they should buy a more efficient technology also involving costly organizational restructuring. This sensitive issue is left in the dark in our framework. A second issue concerns a component of coordination costs not treated in this paper, but already explicitly mentioned in the introduction, that's the negative impact of flexible organizational forms (like multi-tasking) on the health of workers by inducing more stress, pressure and so on, as documented for example by Askenazy *et al.* (2002). We are currently studying the normative implications of such a situation (Boucekkine *et al.*, 2008).

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7 Appendix

7.1 Second-order conditions at the steady-state in the decentralized economy

To check, at the steady state, the second-order conditions of the problem of the firm that guarantee the presence of a maximum it is possible to observe that we have (at the steady-state):

$$
\begin{aligned}
\frac{\partial^2 f}{\partial n^2} &= \alpha(\alpha-1) A\left[(1-\rho) hT^d L\right]^{1-\alpha} n^{\alpha-2} - \frac{\xi(\xi-1)(1-\rho)^\theta \rho^\eta}{d} n^{\xi-2} \\
\frac{\partial^2 f}{\partial (T^d)^2} &= -\alpha(1-\alpha) A\left[(1-\rho) hT^d L\right]^{-\alpha-1} (1-\rho)^2 h^2 L^2 n^\alpha \\
\frac{\partial^2 f}{\partial \rho^2} &= -\alpha(1-\alpha) A\left[(1-\rho) hT^d L\right]^{-\alpha-1} h^2 \left(T^d\right)^2 L^2 n^\alpha + \\
& -\frac{n^\xi}{d}\left[\eta\left[(\eta-1)(1-\rho)^\theta \rho^{\eta-2} - \theta(1-\rho)^{\theta-1}\rho^{\eta-1}\right] + \right. \\
& \left. -\theta\left[\eta(1-\rho)^{\theta-1}\rho^{\eta-1} - (\theta-1)(1-\rho)^{\theta-2}\rho^\eta\right]\right] \\
\frac{\partial^2 f}{\partial T^d \partial n} &= \frac{\partial^2 f}{\partial n \partial T^d} = \alpha(1-\alpha) A\left[(1-\rho) hT^d L\right]^{-\alpha} (1-\rho) hLn^{\alpha-1} \\
\frac{\partial^2 f}{\partial \rho \partial n} &= \frac{\partial^2 f}{\partial n \partial \rho} = -\alpha(1-\alpha) A\left[(1-\rho) hT^d L\right]^{-\alpha} hT^d Ln^{\alpha-1} + \\
& -\frac{\xi n^{\xi-1}}{d}\left[\eta(1-\rho)^\theta \rho^{\eta-1} - \theta(1-\rho)^{\theta-1}\rho^\eta\right] \\
\frac{\partial^2 f}{\partial \rho \partial T^d} &= \frac{\partial^2 f}{\partial T^d \partial \rho} = -(1-\alpha)^2 A\left[(1-\rho) hL\right]^{-\alpha} h\left(T^d\right)^{-\alpha} Ln^\alpha
\end{aligned}
$$

The hessian matrix is then:

$$
H = \begin{bmatrix}
\frac{\partial^2 f}{\partial n^2} & \frac{\partial^2 f}{\partial T^d \partial n} & \frac{\partial^2 f}{\partial \rho \partial n} \\
\frac{\partial^2 f}{\partial n \partial T^d} & \frac{\partial^2 f}{\partial (T^d)^2} & \frac{\partial^2 f}{\partial \rho \partial T^d} \\
\frac{\partial^2 f}{\partial n \partial \rho} & \frac{\partial^2 f}{\partial T^d \partial \rho} & \frac{\partial^2 f}{\partial \rho^2}
\end{bmatrix}
$$

and in order to have a maximum for the problem of the firm the sequence of the signs of the north-west principal minors of this matrix must be:

$$H_1 < 0 \qquad H_2 > 0 \qquad H_3 < 0$$

With reference to this aspect we have:

$$H_1 < 0 \Rightarrow \frac{\partial^2 f}{\partial n^2} < 0 \Rightarrow \alpha(\alpha-1) A\left[(1-\rho) hT^d L\right]^{1-\alpha} n^{\alpha-2} - \frac{\xi(\xi-1)(1-\rho)^\theta \rho^\eta}{d} n^{\xi-2} < 0$$

that is always true (since it must be $\xi > 1$). We then have:

$$H_2 > 0 \Rightarrow \frac{\partial^2 f}{\partial n^2} \cdot \frac{\partial^2 f}{\partial \left(T^d\right)^2} - \left(\frac{\partial^2 f}{\partial T^d \partial n}\right)^2 > 0$$

that implies:

$$\left[\alpha\left(\alpha-1\right)A\left(1-\rho\right)^{1-\alpha}\left(hL\right)^{1-\alpha}\left(T^d\right)^{1-\alpha}n^{\alpha-2} - \frac{\xi\left(\xi-1\right)\left(1-\rho\right)^{\theta}\rho^{\eta}}{d}n^{\xi-2}\right] \cdot$$
$$\cdot\left[-\alpha\left(1-\alpha\right)A\left(1-\rho\right)^{1-\alpha}\left(hL\right)^{1-\alpha}\left(T^d\right)^{-\alpha-1}n^{\alpha}\right] +$$
$$-\left[\alpha\left(1-\alpha\right)A\left(1-\rho\right)^{1-\alpha}\left(hL\right)^{1-\alpha}\left(T^d\right)^{-\alpha}n^{\alpha-1}\right]^2 > 0$$

that leads to:

$$\frac{\alpha}{d}\left(1-\alpha\right)\xi\left(\xi-1\right)A\left(1-\rho\right)^{1-\alpha+\theta}\rho^{\eta}\left(hL\right)^{1-\alpha}\left(T^d\right)^{-\alpha-1}n^{\alpha+\xi-2} > 0$$

that is always true (since $\xi > 1$). We finally have:

$$H_3 < 0 \Rightarrow \det H < 0$$

that can be checked during the simulations (because it is not possible to obtain an analytic solution for this inequality). In this case the second-order conditions of the problem of the firm are satisfied and the value found is effectively a maximum.

7.2 Steady-state in the decentralized economy

The steady-state of the decentralized economy is obtained considering all the quantities constant in the first-order conditions of the problem of the firm and of the household and in the market equilibrium conditions, that is:

$$\alpha A\left(hT^dL\right)^{1-\alpha}\left(1-\rho\right)^{1-\alpha}n^{\alpha} = \frac{\xi}{d}n^{\xi}\left(1-\rho\right)^{\theta}\rho^{\eta} \tag{23}$$

$$\left(1-\alpha\right)A\left(hT^dL\right)^{-\alpha}\left(1-\rho\right)^{1-\alpha}n^{\alpha} = w \tag{24}$$

$$\left(1-\alpha\right)A\left(hT^dL\right)^{1-\alpha}\left(1-\rho\right)^{1-\alpha}n^{\alpha} = \frac{n^{\xi}}{d}\left(1-\rho\right)^{\theta}\rho^{\eta}\left[\theta - \frac{\eta\left(1-\rho\right)}{\rho}\right] \tag{25}$$

$$\frac{1}{c} = \frac{\beta}{c}\left(1+r\right) \tag{26}$$

$$\frac{h}{\left(1-\delta\right)Eh^{\delta}\left(1-T^s\right)^{-\delta}} = \frac{\beta\delta\left(1-T^s\right)}{1-\delta} + \beta T^s \tag{27}$$

$$c = whT^s + ra \tag{28}$$

$$h = Eh^{\delta}\left(1-T^s\right)^{1-\delta} \tag{29}$$

$$A\left(hT^{s}L\right)^{1-\alpha}(1-\rho)^{1-\alpha}n^{\alpha}=c+\frac{n^{\xi}(1-\rho)^{\theta}\rho^{\eta}}{d} \tag{30}$$

$$T^{d}=T^{s}=T \tag{31}$$

From these equations it is possible to get the steady-state values of the different variables. In particular, from (26) we have:

$$\beta(1+r)=1\Rightarrow 1+r=\frac{1}{\beta}\Rightarrow r=\frac{1-\beta}{\beta}$$

Using from now on (31), so that we only write T, from (27) and (29) we obtain:

$$\begin{aligned}
\frac{Eh^{\delta}(1-T)^{1-\delta}}{(1-\delta)Eh^{\delta}(1-T)^{-\delta}} &= \frac{\beta\delta(1-T)}{1-\delta}+\beta T\Rightarrow\frac{1-T}{1-\delta}=\frac{\beta\delta(1-T)}{1-\delta}+\beta T\\
&\Rightarrow\ 1-T=\beta\delta(1-T)+\beta T(1-\delta)\\
&\Rightarrow\ 1-\beta\delta=T(1+\beta-2\beta\delta)\\
&\Rightarrow\ T=\frac{1-\beta\delta}{1+\beta-2\beta\delta}
\end{aligned}$$

From (29) (using the expression just found for T) we then get:

$$\begin{aligned}
h &= Eh^{\delta}\left(1-\frac{1-\beta\delta}{1+\beta-2\beta\delta}\right)^{1-\delta}\Rightarrow h^{1-\delta}=E\left(\frac{\beta(1-\delta)}{1+\beta-2\beta\delta}\right)^{1-\delta}\\
&\Rightarrow\ h=E^{\frac{1}{1-\delta}}\frac{\beta(1-\delta)}{1+\beta-2\beta\delta}
\end{aligned}$$

From (23) and (25) we then have:

$$\begin{aligned}
\frac{1-\alpha}{\alpha} &= \frac{1}{\xi}\left[\theta-\frac{\eta(1-\rho)}{\rho}\right]\\
&\Rightarrow\ \frac{1-\alpha}{\alpha}\xi=\frac{\theta\rho-\eta+\eta\rho}{\rho}\Rightarrow\frac{1-\alpha}{\alpha}\xi\rho=(\eta+\theta)\rho-\eta\\
&\Rightarrow\ \left(\eta+\theta-\frac{1-\alpha}{\alpha}\xi\right)\rho=\eta\Rightarrow\frac{\alpha\eta+\alpha\theta-\xi+\alpha\xi}{\alpha}\rho=\eta\\
&\Rightarrow\ \rho=\frac{\alpha\eta}{\alpha(\xi+\eta+\theta)-\xi}
\end{aligned}$$

From (23) (using the expressions found above for the different variables) we obtain:

$$\begin{aligned}
\alpha Ad(hTL)^{1-\alpha}(1-\rho)^{1-\alpha} &= \xi n^{\xi-\alpha}(1-\rho)^{\theta}\rho^{\eta}\\
&\Rightarrow\ n^{\xi-\alpha}=\frac{\alpha Ad(hTL)^{1-\alpha}(1-\rho)^{1-\alpha}}{\xi(1-\rho)^{\theta}\rho^{\eta}}\\
&\Rightarrow\ n=\left(\frac{\alpha Ad}{\xi}\right)^{\frac{1}{\xi-\alpha}}(hTL)^{\frac{1-\alpha}{\xi-\alpha}}(1-\rho)^{\frac{1-\alpha-\theta}{\xi-\alpha}}\rho^{\frac{\eta}{\alpha-\xi}}
\end{aligned}$$

$$\Rightarrow\;\; n=\left(\frac{\alpha Ad}{\xi}\right)^{\frac{1}{\xi-\alpha}}\left(E^{\frac{1}{1-\delta}}\cdot\frac{\beta\left(1-\delta\right)}{1+\beta-2\beta\delta}\cdot\frac{1-\beta\delta}{1+\beta-2\beta\delta}\cdot L\right)^{\frac{1-\alpha}{\xi-\alpha}}\cdot$$

$$\cdot\left(1-\frac{\alpha\eta}{\alpha\left(\xi+\eta+\theta\right)-\xi}\right)^{\frac{1-\alpha-\theta}{\xi-\alpha}}\left(\frac{\alpha\eta}{\alpha\left(\xi+\eta+\theta\right)-\xi}\right)^{\frac{\eta}{\alpha-\xi}}$$

$$\Rightarrow\;\; n=\left(\frac{\alpha Ad}{\xi}\right)^{\frac{1}{\xi-\alpha}}E^{\frac{1-\alpha}{(1-\delta)(\xi-\alpha)}}L^{\frac{1-\alpha}{\xi-\alpha}}\left(\frac{\beta\left(1-\delta\right)\left(1-\beta\delta\right)}{\left(1+\beta-2\beta\delta\right)^{2}}\right)^{\frac{1-\alpha}{\xi-\alpha}}\cdot$$

$$\cdot\left(\alpha\left(\xi+\theta\right)-\xi\right)^{\frac{1-\alpha-\theta}{\xi-\alpha}}\left(\alpha\eta\right)^{\frac{\eta}{\alpha-\xi}}\left(\alpha\left(\xi+\eta+\theta\right)-\xi\right)^{\frac{1-\alpha-\theta-\eta}{\alpha-\xi}}$$

From (24) we then have:

$$\begin{aligned}
w &= (1-\alpha)\,A\left(hTL\right)^{-\alpha}\left(1-\rho\right)^{1-\alpha}n^{\alpha}\\
&\Rightarrow w=(1-\alpha)\,A\left(E^{\frac{1}{1-\delta}}\cdot\frac{\beta\left(1-\delta\right)}{1+\beta-2\beta\delta}\cdot\frac{1-\beta\delta}{1+\beta-2\beta\delta}\cdot L\right)^{-\alpha}\left(1-\frac{\alpha\eta}{\alpha\left(\xi+\eta+\theta\right)-\xi}\right)^{1-\alpha}\cdot\\
&\cdot\left(\frac{\alpha Ad}{\xi}\right)^{\frac{\alpha}{\xi-\alpha}}E^{\frac{\alpha(1-\alpha)}{(1-\delta)(\xi-\alpha)}}L^{\frac{\alpha(1-\alpha)}{\xi-\alpha}}\left(\frac{\beta\left(1-\delta\right)\left(1-\beta\delta\right)}{\left(1+\beta-2\beta\delta\right)^{2}}\right)^{\frac{\alpha(1-\alpha)}{\xi-\alpha}}\cdot\\
&\cdot\left(\alpha\left(\xi+\theta\right)-\xi\right)^{\frac{\alpha(1-\alpha-\theta)}{\xi-\alpha}}\left(\alpha\eta\right)^{\frac{\alpha\eta}{\alpha-\xi}}\left(\alpha\left(\xi+\eta+\theta\right)-\xi\right)^{\frac{\alpha(1-\alpha-\theta-\eta)}{\alpha-\xi}}\\
&\Rightarrow w=(1-\alpha)\left(\frac{\alpha d}{\xi}\right)^{\frac{\alpha}{\xi-\alpha}}A^{\frac{\xi}{\xi-\alpha}}E^{\frac{\alpha(1-\xi)}{(1-\delta)(\xi-\alpha)}}L^{\frac{\alpha(1-\xi)}{\xi-\alpha}}\left(\frac{\beta\left(1-\delta\right)\left(1-\beta\delta\right)}{\left(1+\beta-2\beta\delta\right)^{2}}\right)^{\frac{\alpha(1-\xi)}{\xi-\alpha}}\cdot\\
&\cdot\left(\alpha\left(\xi+\theta\right)-\xi\right)^{\frac{\xi-\alpha(\xi+\theta)}{\xi-\alpha}}\left(\alpha\eta\right)^{\frac{\alpha\eta}{\alpha-\xi}}\left(\alpha\left(\xi+\eta+\theta\right)-\xi\right)^{\frac{\alpha(\xi+\eta+\theta)-\xi}{\xi-\alpha}}
\end{aligned}$$

From (30) we have:

$$\begin{aligned}
c &= A\left(hTL\right)^{1-\alpha}\left(1-\rho\right)^{1-\alpha}n^{\alpha}-\frac{n^{\xi}\left(1-\rho\right)^{\theta}\rho^{\eta}}{d}\\
&\Rightarrow c=A\left(E^{\frac{1}{1-\delta}}\cdot\frac{\beta\left(1-\delta\right)}{1+\beta-2\beta\delta}\cdot\frac{1-\beta\delta}{1+\beta-2\beta\delta}\cdot L\right)^{1-\alpha}\left(1-\frac{\alpha\eta}{\alpha\left(\xi+\eta+\theta\right)-\xi}\right)^{1-\alpha}\cdot\\
&\cdot\left(\frac{\alpha Ad}{\xi}\right)^{\frac{\alpha}{\xi-\alpha}}E^{\frac{\alpha(1-\alpha)}{(1-\delta)(\xi-\alpha)}}L^{\frac{\alpha(1-\alpha)}{\xi-\alpha}}\left(\frac{\beta\left(1-\delta\right)\left(1-\beta\delta\right)}{\left(1+\beta-2\beta\delta\right)^{2}}\right)^{\frac{\alpha(1-\alpha)}{\xi-\alpha}}\cdot\\
&\cdot\left(\alpha\left(\xi+\theta\right)-\xi\right)^{\frac{\alpha(1-\alpha-\theta)}{\xi-\alpha}}\left(\alpha\eta\right)^{\frac{\alpha\eta}{\alpha-\xi}}\left(\alpha\left(\xi+\eta+\theta\right)-\xi\right)^{\frac{\alpha(1-\alpha-\theta-\eta)}{\alpha-\xi}}+\\
&-\frac{1}{d}\left(\frac{\alpha Ad}{\xi}\right)^{\frac{\xi}{\xi-\alpha}}E^{\frac{\xi(1-\alpha)}{(1-\delta)(\xi-\alpha)}}L^{\frac{\xi(1-\alpha)}{\xi-\alpha}}\left(\frac{\beta\left(1-\delta\right)\left(1-\beta\delta\right)}{\left(1+\beta-2\beta\delta\right)^{2}}\right)^{\frac{\xi(1-\alpha)}{\xi-\alpha}}\cdot\\
&\cdot\left(\alpha\left(\xi+\theta\right)-\xi\right)^{\frac{\xi(1-\alpha-\theta)}{\xi-\alpha}}\left(\alpha\eta\right)^{\frac{\xi\eta}{\alpha-\xi}}\left(\alpha\left(\xi+\eta+\theta\right)-\xi\right)^{\frac{\xi(1-\alpha-\theta-\eta)}{\alpha-\xi}}\cdot\\
&\cdot\left(1-\frac{\alpha\eta}{\alpha\left(\xi+\eta+\theta\right)-\xi}\right)^{\theta}\left(\frac{\alpha\eta}{\alpha\left(\xi+\eta+\theta\right)-\xi}\right)^{\eta}
\end{aligned}$$

$$
\begin{aligned}
\Rightarrow \quad c = \frac{\xi-\alpha}{\xi}\left(\frac{\alpha d}{\xi}\right)^{\frac{\alpha}{\xi-\alpha}} A^{\frac{\xi}{\xi-\alpha}} E^{\frac{\xi(1-\alpha)}{(1-\delta)(\xi-\alpha)}} L^{\frac{\xi(1-\alpha)}{\xi-\alpha}} \left(\frac{\beta\left(1-\delta\right)\left(1-\beta\delta\right)}{\left(1+\beta-2\beta\delta\right)^{2}}\right)^{\frac{\xi(1-\alpha)}{\xi-\alpha}} \cdot \\
\cdot \left(\alpha\left(\xi+\theta\right)-\xi\right)^{\frac{\alpha(\xi+\theta)-\xi}{\alpha-\xi}} \left(\alpha\eta\right)^{\frac{\alpha\eta}{\alpha-\xi}} \left(\alpha\left(\xi+\eta+\theta\right)-\xi\right)^{\frac{\alpha(\xi+\eta+\theta)-\xi}{\xi-\alpha}}
\end{aligned}
$$

Finally from (28) we obtain:

$$
\begin{aligned}
a \quad &= \quad \frac{1}{r}\left(c-whT\right) \\
\Rightarrow \quad a &= \frac{\beta}{1-\beta}\left[\frac{\xi-\alpha}{\xi}\left(\frac{\alpha d}{\xi}\right)^{\frac{\alpha}{\xi-\alpha}} A^{\frac{\xi}{\xi-\alpha}} E^{\frac{\xi(1-\alpha)}{(1-\delta)(\xi-\alpha)}} L^{\frac{\xi(1-\alpha)}{\xi-\alpha}} \left(\frac{\beta\left(1-\delta\right)\left(1-\beta\delta\right)}{\left(1+\beta-2\beta\delta\right)^{2}}\right)^{\frac{\xi(1-\alpha)}{\xi-\alpha}} \cdot\right. \\
&\cdot \left(\alpha\left(\xi+\theta\right)-\xi\right)^{\frac{\alpha(\xi+\theta)-\xi}{\alpha-\xi}} \left(\alpha\eta\right)^{\frac{\alpha\eta}{\alpha-\xi}} \left(\alpha\left(\xi+\eta+\theta\right)-\xi\right)^{\frac{\alpha(\xi+\eta+\theta)-\xi}{\xi-\alpha}} + \\
&-\left(1-\alpha\right)\left(\frac{\alpha d}{\xi}\right)^{\frac{\alpha}{\xi-\alpha}} A^{\frac{\xi}{\xi-\alpha}} E^{\frac{\alpha(1-\xi)}{(1-\delta)(\xi-\alpha)}} L^{\frac{\alpha(1-\xi)}{\xi-\alpha}} \left(\frac{\beta\left(1-\delta\right)\left(1-\beta\delta\right)}{\left(1+\beta-2\beta\delta\right)^{2}}\right)^{\frac{\alpha(1-\xi)}{\xi-\alpha}} \cdot \\
&\cdot \left(\alpha\left(\xi+\theta\right)-\xi\right)^{\frac{\xi-\alpha(\xi+\theta)}{\xi-\alpha}} \left(\alpha\eta\right)^{\frac{\alpha\eta}{\alpha-\xi}} \left(\alpha\left(\xi+\eta+\theta\right)-\xi\right)^{\frac{\alpha(\xi+\eta+\theta)-\xi}{\xi-\alpha}} \cdot \\
&\left.\cdot E^{\frac{1}{1-\delta}} \cdot \frac{\beta\left(1-\delta\right)}{1+\beta-2\beta\delta} \cdot \frac{1-\beta\delta}{1+\beta-2\beta\delta}\right] \\
\Rightarrow \quad a &= \frac{\beta}{1-\beta}\left(\frac{\xi-\alpha}{\xi}L-1+\alpha\right)\left(\frac{\alpha d}{\xi}\right)^{\frac{\alpha}{\xi-\alpha}} A^{\frac{\xi}{\xi-\alpha}} E^{\frac{\xi(1-\alpha)}{(1-\delta)(\xi-\alpha)}} L^{\frac{\alpha(1-\xi)}{\xi-\alpha}} \left(\frac{\beta\left(1-\delta\right)\left(1-\beta\delta\right)}{\left(1+\beta-2\beta\delta\right)^{2}}\right)^{\frac{\xi(1-\alpha)}{\xi-\alpha}} \cdot \\
&\cdot \left(\alpha\left(\xi+\theta\right)-\xi\right)^{\frac{\xi-\alpha(\xi+\theta)}{\xi-\alpha}} \left(\alpha\eta\right)^{\frac{\alpha\eta}{\alpha-\xi}} \left(\alpha\left(\xi+\eta+\theta\right)-\xi\right)^{\frac{\alpha(\xi+\eta+\theta)-\xi}{\xi-\alpha}}
\end{aligned}
$$

7.3 Comparative statics in the decentralized economy

The comparative statics of the model in the decentralized economy can be obtained from equations (15)-(22). In particular, from (15) we have:

$$
\frac{dr}{d\beta} = \frac{-\beta-1+\beta}{\beta^{2}} = -\frac{1}{\beta^{2}}
$$

From (16) we then have:

$$
\begin{aligned}
\frac{\partial T}{\partial \beta} \quad &= \quad \frac{-\delta\left(1+\beta-2\beta\delta\right)-\left(1-\beta\delta\right)\left(1-2\delta\right)}{\left(1+\beta-2\beta\delta\right)^{2}} = \frac{-\delta-\beta\delta+2\beta\delta^{2}-1+2\delta+\beta\delta-2\beta\delta^{2}}{\left(1+\beta-2\beta\delta\right)^{2}} = \\
&= \quad \frac{\delta-1}{\left(1+\beta-2\beta\delta\right)^{2}}
\end{aligned}
$$

and also:

$$
\begin{aligned}
\frac{\partial T}{\partial \delta} \quad &= \quad \frac{-\beta\left(1+\beta-2\beta\delta\right)+2\beta\left(1-\beta\delta\right)}{\left(1+\beta-2\beta\delta\right)^{2}} = \frac{-\beta-\beta^{2}+2\beta^{2}\delta+2\beta-2\beta^{2}\delta}{\left(1+\beta-2\beta\delta\right)^{2}} = \\
&= \quad \frac{\beta-\beta^{2}}{\left(1+\beta-2\beta\delta\right)^{2}} = \frac{\beta\left(1-\beta\right)}{\left(1+\beta-2\beta\delta\right)^{2}}
\end{aligned}
$$

From (17) we get:

$$\frac{\partial h}{\partial E}=\frac{1}{1-\delta}E^{\frac{1}{1-\delta}-1}\frac{\beta\left(1-\delta\right)}{1+\beta-2\beta\delta}=E^{\frac{\delta}{1-\delta}}\frac{\beta}{1+\beta-2\beta\delta}$$

and also:

$$\begin{aligned}
\frac{\partial h}{\partial \beta} &= E^{\frac{1}{1-\delta}}\frac{\left(1-\delta\right)\left(1+\beta-2\beta\delta\right)-\beta\left(1-\delta\right)\left(1-2\delta\right)}{\left(1+\beta-2\beta\delta\right)^2}= \\
&= E^{\frac{1}{1-\delta}}\frac{1+\beta-2\beta\delta-\delta-\beta\delta+2\beta\delta^2-\beta+2\beta\delta+\beta\delta-2\beta\delta^2}{\left(1+\beta-2\beta\delta\right)^2}= \\
&= E^{\frac{1}{1-\delta}}\frac{1-\delta}{\left(1+\beta-2\beta\delta\right)^2}
\end{aligned}$$

and then:

$$\begin{aligned}
\frac{\partial h}{\partial \delta} &= E^{\frac{1}{1-\delta}}\frac{-\beta\left(1+\beta-2\beta\delta\right)+2\beta^2\left(1-\delta\right)}{\left(1+\beta-2\beta\delta\right)^2}+\frac{\beta\left(1-\delta\right)}{1+\beta-2\beta\delta}E^{\frac{1}{1-\delta}}\frac{1}{\left(1-\delta\right)^2}\log E= \\
&= E^{\frac{1}{1-\delta}}\frac{-\beta-\beta^2+2\beta^2\delta+2\beta^2-2\beta^2\delta}{\left(1+\beta-2\beta\delta\right)^2}+E^{\frac{1}{1-\delta}}\frac{\beta}{\left(1-\delta\right)\left(1+\beta-2\beta\delta\right)}\log E= \\
&= E^{\frac{1}{1-\delta}}\frac{\beta^2-\beta}{\left(1+\beta-2\beta\delta\right)^2}+E^{\frac{1}{1-\delta}}\frac{\beta}{\left(1-\delta\right)\left(1+\beta-2\beta\delta\right)}\log E= \\
&= \frac{E^{\frac{1}{1-\delta}}}{1+\beta-2\beta\delta}\left[\frac{\beta\left(\beta-1\right)}{1+\beta-2\beta\delta}+\frac{\beta}{1-\delta}\log E\right]
\end{aligned}$$

From (18) we have:

$$\begin{aligned}
\frac{\partial \rho}{\partial \alpha} &= \frac{\left(\alpha\left(\xi+\eta+\theta\right)-\xi\right)\eta-\alpha\eta\left(\xi+\eta+\theta\right)}{\left(\alpha\left(\xi+\eta+\theta\right)-\xi\right)^2}= \\
&= \frac{\alpha\xi\eta+\alpha\eta^2+\alpha\theta\eta-\xi\eta-\alpha\xi\eta-\alpha\eta^2-\alpha\theta\eta}{\left(\alpha\left(\xi+\eta+\theta\right)-\xi\right)^2}=-\frac{\xi\eta}{\left(\alpha\left(\xi+\eta+\theta\right)-\xi\right)^2}
\end{aligned}$$

and then:

$$\frac{\partial \rho}{\partial \theta}=\frac{-\alpha\eta\cdot\alpha}{\left(\alpha\left(\xi+\eta+\theta\right)-\xi\right)^2}=-\frac{\alpha^2\eta}{\left(\alpha\left(\xi+\eta+\theta\right)-\xi\right)^2}$$

and also:

$$\frac{\partial \rho}{\partial \xi}=\frac{-\alpha\eta\left(\alpha-1\right)}{\left(\alpha\left(\xi+\eta+\theta\right)-\xi\right)^2}=\frac{\alpha\eta\left(1-\alpha\right)}{\left(\alpha\left(\xi+\eta+\theta\right)-\xi\right)^2}$$

and finally:

$$\begin{aligned}
\frac{\partial \rho}{\partial \eta} &= \frac{\left(\alpha\left(\xi+\eta+\theta\right)-\xi\right)\alpha-\alpha\eta\cdot\alpha}{\left(\alpha\left(\xi+\eta+\theta\right)-\xi\right)^2}= \\
&= \frac{\alpha^2\xi+\alpha^2\eta+\alpha^2\theta-\alpha\xi-\alpha^2\eta}{\left(\alpha\left(\xi+\eta+\theta\right)-\xi\right)^2}=\frac{\alpha\left[\alpha\left(\xi+\theta\right)-\xi\right]}{\left(\alpha\left(\xi+\eta+\theta\right)-\xi\right)^2}
\end{aligned}$$

From (19) we then have:

$$\begin{aligned}
\frac{\partial n}{\partial E} &= \frac{1-\alpha}{(1-\delta)(\xi-\alpha)}\left(\frac{\alpha A d}{\xi}\right)^{\frac{1}{\xi-\alpha}} E^{\frac{1-\alpha}{(1-\delta)(\xi-\alpha)}-1} L^{\frac{1-\alpha}{\xi-\alpha}}\left(\frac{\beta(1-\delta)(1-\beta\delta)}{(1+\beta-2\beta\delta)^2}\right)^{\frac{1-\alpha}{\xi-\alpha}} \cdot \\
&\cdot\left(\alpha(\xi+\theta)-\xi\right)^{\frac{1-\alpha-\theta}{\xi-\alpha}}(\alpha\eta)^{\frac{\eta}{\alpha-\xi}}\left(\alpha(\xi+\eta+\theta)-\xi\right)^{\frac{1-\alpha-\theta-\eta}{\alpha-\xi}} = \\
&= \frac{1-\alpha}{(1-\delta)(\xi-\alpha)}\left(\frac{\alpha A d}{\xi}\right)^{\frac{1}{\xi-\alpha}} E^{\frac{1-\xi(1-\delta)-\alpha\delta}{(1-\delta)(\xi-\alpha)}} L^{\frac{1-\alpha}{\xi-\alpha}}\left(\frac{\beta(1-\delta)(1-\beta\delta)}{(1+\beta-2\beta\delta)^2}\right)^{\frac{1-\alpha}{\xi-\alpha}} \cdot \\
&\cdot\left(\alpha(\xi+\theta)-\xi\right)^{\frac{1-\alpha-\theta}{\xi-\alpha}}(\alpha\eta)^{\frac{\eta}{\alpha-\xi}}\left(\alpha(\xi+\eta+\theta)-\xi\right)^{\frac{1-\alpha-\theta-\eta}{\alpha-\xi}}
\end{aligned}$$

and then:

$$\begin{aligned}
\frac{\partial n}{\partial L} &= \frac{1-\alpha}{\xi-\alpha}\left(\frac{\alpha A d}{\xi}\right)^{\frac{1}{\xi-\alpha}} E^{\frac{1-\alpha}{(1-\delta)(\xi-\alpha)}} L^{\frac{1-\alpha}{\xi-\alpha}-1}\left(\frac{\beta(1-\delta)(1-\beta\delta)}{(1+\beta-2\beta\delta)^2}\right)^{\frac{1-\alpha}{\xi-\alpha}} \cdot \\
&\cdot\left(\alpha(\xi+\theta)-\xi\right)^{\frac{1-\alpha-\theta}{\xi-\alpha}}(\alpha\eta)^{\frac{\eta}{\alpha-\xi}}\left(\alpha(\xi+\eta+\theta)-\xi\right)^{\frac{1-\alpha-\theta-\eta}{\alpha-\xi}} = \\
&= \frac{1-\alpha}{\xi-\alpha}\left(\frac{\alpha A d}{\xi}\right)^{\frac{1}{\xi-\alpha}} E^{\frac{1-\alpha}{(1-\delta)(\xi-\alpha)}} L^{\frac{1-\xi}{\xi-\alpha}}\left(\frac{\beta(1-\delta)(1-\beta\delta)}{(1+\beta-2\beta\delta)^2}\right)^{\frac{1-\alpha}{\xi-\alpha}} \cdot \\
&\cdot\left(\alpha(\xi+\theta)-\xi\right)^{\frac{1-\alpha-\theta}{\xi-\alpha}}(\alpha\eta)^{\frac{\eta}{\alpha-\xi}}\left(\alpha(\xi+\eta+\theta)-\xi\right)^{\frac{1-\alpha-\theta-\eta}{\alpha-\xi}}
\end{aligned}$$

and also:

$$\begin{aligned}
\frac{\partial n}{\partial A} &= \frac{1}{\xi-\alpha}\left(\frac{\alpha A d}{\xi}\right)^{\frac{1}{\xi-\alpha}-1} \frac{\alpha d}{\xi} E^{\frac{1-\alpha}{(1-\delta)(\xi-\alpha)}} L^{\frac{1-\alpha}{\xi-\alpha}}\left(\frac{\beta(1-\delta)(1-\beta\delta)}{(1+\beta-2\beta\delta)^2}\right)^{\frac{1-\alpha}{\xi-\alpha}} \cdot \\
&\cdot\left(\alpha(\xi+\theta)-\xi\right)^{\frac{1-\alpha-\theta}{\xi-\alpha}}(\alpha\eta)^{\frac{\eta}{\alpha-\xi}}\left(\alpha(\xi+\eta+\theta)-\xi\right)^{\frac{1-\alpha-\theta-\eta}{\alpha-\xi}} = \\
&= \frac{1}{\xi-\alpha}\left(\frac{\alpha A d}{\xi}\right)^{\frac{1-\xi+\alpha}{\xi-\alpha}} \frac{\alpha d}{\xi} E^{\frac{1-\alpha}{(1-\delta)(\xi-\alpha)}} L^{\frac{1-\alpha}{\xi-\alpha}}\left(\frac{\beta(1-\delta)(1-\beta\delta)}{(1+\beta-2\beta\delta)^2}\right)^{\frac{1-\alpha}{\xi-\alpha}} \cdot \\
&\cdot\left(\alpha(\xi+\theta)-\xi\right)^{\frac{1-\alpha-\theta}{\xi-\alpha}}(\alpha\eta)^{\frac{\eta}{\alpha-\xi}}\left(\alpha(\xi+\eta+\theta)-\xi\right)^{\frac{1-\alpha-\theta-\eta}{\alpha-\xi}}
\end{aligned}$$

and finally:

$$\begin{aligned}
\frac{\partial n}{\partial d} &= \frac{1}{\xi-\alpha}\left(\frac{\alpha A d}{\xi}\right)^{\frac{1}{\xi-\alpha}-1} \frac{\alpha A}{\xi} E^{\frac{1-\alpha}{(1-\delta)(\xi-\alpha)}} L^{\frac{1-\alpha}{\xi-\alpha}}\left(\frac{\beta(1-\delta)(1-\beta\delta)}{(1+\beta-2\beta\delta)^2}\right)^{\frac{1-\alpha}{\xi-\alpha}} \cdot \\
&\cdot\left(\alpha(\xi+\theta)-\xi\right)^{\frac{1-\alpha-\theta}{\xi-\alpha}}(\alpha\eta)^{\frac{\eta}{\alpha-\xi}}\left(\alpha(\xi+\eta+\theta)-\xi\right)^{\frac{1-\alpha-\theta-\eta}{\alpha-\xi}} = \\
&= \frac{1}{\xi-\alpha}\left(\frac{\alpha A d}{\xi}\right)^{\frac{1-\xi+\alpha}{\xi-\alpha}} \frac{\alpha A}{\xi} E^{\frac{1-\alpha}{(1-\delta)(\xi-\alpha)}} L^{\frac{1-\alpha}{\xi-\alpha}}\left(\frac{\beta(1-\delta)(1-\beta\delta)}{(1+\beta-2\beta\delta)^2}\right)^{\frac{1-\alpha}{\xi-\alpha}} \cdot \\
&\cdot\left(\alpha(\xi+\theta)-\xi\right)^{\frac{1-\alpha-\theta}{\xi-\alpha}}(\alpha\eta)^{\frac{\eta}{\alpha-\xi}}\left(\alpha(\xi+\eta+\theta)-\xi\right)^{\frac{1-\alpha-\theta-\eta}{\alpha-\xi}}
\end{aligned}$$

From (20) we get:

$$\begin{aligned}
\frac{\partial w}{\partial E} &= \frac{\alpha\left(1-\xi\right)}{\left(1-\delta\right)\left(\xi-\alpha\right)}\left(1-\alpha\right)\left(\frac{\alpha d}{\xi}\right)^{\frac{\alpha}{\xi-\alpha}} A^{\frac{\xi}{\xi-\alpha}} E^{\frac{\alpha(1-\xi)}{(1-\delta)(\xi-\alpha)}-1} L^{\frac{\alpha(1-\xi)}{\xi-\alpha}} \left(\frac{\beta\left(1-\delta\right)\left(1-\beta\delta\right)}{\left(1+\beta-2\beta\delta\right)^2}\right)^{\frac{\alpha(1-\xi)}{\xi-\alpha}} \cdot \\
&\quad \cdot \left(\alpha\left(\xi+\theta\right)-\xi\right)^{\frac{\xi-\alpha(\xi+\theta)}{\xi-\alpha}} \left(\alpha\eta\right)^{\frac{\alpha\eta}{\alpha-\xi}} \left(\alpha\left(\xi+\eta+\theta\right)-\xi\right)^{\frac{\alpha(\xi+\eta+\theta)-\xi}{\xi-\alpha}} = \\
&= \frac{\alpha\left(1-\xi\right)}{\left(1-\delta\right)\left(\xi-\alpha\right)}\left(1-\alpha\right)\left(\frac{\alpha d}{\xi}\right)^{\frac{\alpha}{\xi-\alpha}} A^{\frac{\xi}{\xi-\alpha}} E^{\frac{\alpha(2-\xi-\delta)-\xi(1-\delta)}{(1-\delta)(\xi-\alpha)}} L^{\frac{\alpha(1-\xi)}{\xi-\alpha}} \left(\frac{\beta\left(1-\delta\right)\left(1-\beta\delta\right)}{\left(1+\beta-2\beta\delta\right)^2}\right)^{\frac{\alpha(1-\xi)}{\xi-\alpha}} \cdot \\
&\quad \cdot \left(\alpha\left(\xi+\theta\right)-\xi\right)^{\frac{\xi-\alpha(\xi+\theta)}{\xi-\alpha}} \left(\alpha\eta\right)^{\frac{\alpha\eta}{\alpha-\xi}} \left(\alpha\left(\xi+\eta+\theta\right)-\xi\right)^{\frac{\alpha(\xi+\eta+\theta)-\xi}{\xi-\alpha}}
\end{aligned}$$

and then:

$$\begin{aligned}
\frac{\partial w}{\partial L} &= \frac{\alpha\left(1-\xi\right)}{\xi-\alpha}\left(1-\alpha\right)\left(\frac{\alpha d}{\xi}\right)^{\frac{\alpha}{\xi-\alpha}} A^{\frac{\xi}{\xi-\alpha}} E^{\frac{\alpha(1-\xi)}{(1-\delta)(\xi-\alpha)}} L^{\frac{\alpha(1-\xi)}{\xi-\alpha}-1} \left(\frac{\beta\left(1-\delta\right)\left(1-\beta\delta\right)}{\left(1+\beta-2\beta\delta\right)^2}\right)^{\frac{\alpha(1-\xi)}{\xi-\alpha}} \cdot \\
&\quad \cdot \left(\alpha\left(\xi+\theta\right)-\xi\right)^{\frac{\xi-\alpha(\xi+\theta)}{\xi-\alpha}} \left(\alpha\eta\right)^{\frac{\alpha\eta}{\alpha-\xi}} \left(\alpha\left(\xi+\eta+\theta\right)-\xi\right)^{\frac{\alpha(\xi+\eta+\theta)-\xi}{\xi-\alpha}} = \\
&= \frac{\alpha\left(1-\xi\right)}{\xi-\alpha}\left(1-\alpha\right)\left(\frac{\alpha d}{\xi}\right)^{\frac{\alpha}{\xi-\alpha}} A^{\frac{\xi}{\xi-\alpha}} E^{\frac{\alpha(1-\xi)}{(1-\delta)(\xi-\alpha)}} L^{\frac{\alpha(2-\xi)-\xi}{\xi-\alpha}} \left(\frac{\beta\left(1-\delta\right)\left(1-\beta\delta\right)}{\left(1+\beta-2\beta\delta\right)^2}\right)^{\frac{\alpha(1-\xi)}{\xi-\alpha}} \cdot \\
&\quad \cdot \left(\alpha\left(\xi+\theta\right)-\xi\right)^{\frac{\xi-\alpha(\xi+\theta)}{\xi-\alpha}} \left(\alpha\eta\right)^{\frac{\alpha\eta}{\alpha-\xi}} \left(\alpha\left(\xi+\eta+\theta\right)-\xi\right)^{\frac{\alpha(\xi+\eta+\theta)-\xi}{\xi-\alpha}}
\end{aligned}$$

and also:

$$\begin{aligned}
\frac{\partial w}{\partial A} &= \frac{\xi}{\xi-\alpha}\left(1-\alpha\right)\left(\frac{\alpha d}{\xi}\right)^{\frac{\alpha}{\xi-\alpha}} A^{\frac{\xi}{\xi-\alpha}-1} E^{\frac{\alpha(1-\xi)}{(1-\delta)(\xi-\alpha)}} L^{\frac{\alpha(1-\xi)}{\xi-\alpha}} \left(\frac{\beta\left(1-\delta\right)\left(1-\beta\delta\right)}{\left(1+\beta-2\beta\delta\right)^2}\right)^{\frac{\alpha(1-\xi)}{\xi-\alpha}} \cdot \\
&\quad \cdot \left(\alpha\left(\xi+\theta\right)-\xi\right)^{\frac{\xi-\alpha(\xi+\theta)}{\xi-\alpha}} \left(\alpha\eta\right)^{\frac{\alpha\eta}{\alpha-\xi}} \left(\alpha\left(\xi+\eta+\theta\right)-\xi\right)^{\frac{\alpha(\xi+\eta+\theta)-\xi}{\xi-\alpha}} = \\
&= \frac{\xi}{\xi-\alpha}\left(1-\alpha\right)\left(\frac{\alpha d}{\xi}\right)^{\frac{\alpha}{\xi-\alpha}} A^{\frac{\alpha}{\xi-\alpha}} E^{\frac{\alpha(1-\xi)}{(1-\delta)(\xi-\alpha)}} L^{\frac{\alpha(1-\xi)}{\xi-\alpha}} \left(\frac{\beta\left(1-\delta\right)\left(1-\beta\delta\right)}{\left(1+\beta-2\beta\delta\right)^2}\right)^{\frac{\alpha(1-\xi)}{\xi-\alpha}} \cdot \\
&\quad \cdot \left(\alpha\left(\xi+\theta\right)-\xi\right)^{\frac{\xi-\alpha(\xi+\theta)}{\xi-\alpha}} \left(\alpha\eta\right)^{\frac{\alpha\eta}{\alpha-\xi}} \left(\alpha\left(\xi+\eta+\theta\right)-\xi\right)^{\frac{\alpha(\xi+\eta+\theta)-\xi}{\xi-\alpha}}
\end{aligned}$$

and finally:

$$\begin{aligned}
\frac{\partial w}{\partial d} &= \frac{\alpha}{\xi-\alpha}\left(1-\alpha\right)\left(\frac{\alpha d}{\xi}\right)^{\frac{\alpha}{\xi-\alpha}-1} \frac{\alpha}{\xi} A^{\frac{\xi}{\xi-\alpha}} E^{\frac{\alpha(1-\xi)}{(1-\delta)(\xi-\alpha)}} L^{\frac{\alpha(1-\xi)}{\xi-\alpha}} \left(\frac{\beta\left(1-\delta\right)\left(1-\beta\delta\right)}{\left(1+\beta-2\beta\delta\right)^2}\right)^{\frac{\alpha(1-\xi)}{\xi-\alpha}} \cdot \\
&\quad \cdot \left(\alpha\left(\xi+\theta\right)-\xi\right)^{\frac{\xi-\alpha(\xi+\theta)}{\xi-\alpha}} \left(\alpha\eta\right)^{\frac{\alpha\eta}{\alpha-\xi}} \left(\alpha\left(\xi+\eta+\theta\right)-\xi\right)^{\frac{\alpha(\xi+\eta+\theta)-\xi}{\xi-\alpha}} = \\
&= \frac{1-\alpha}{\xi\left(\xi-\alpha\right)} \alpha^{\frac{\xi}{\xi-\alpha}} \left(\frac{d}{\xi}\right)^{\frac{2\alpha-\xi}{\xi-\alpha}} A^{\frac{\xi}{\xi-\alpha}} E^{\frac{\alpha(1-\xi)}{(1-\delta)(\xi-\alpha)}} L^{\frac{\alpha(1-\xi)}{\xi-\alpha}} \left(\frac{\beta\left(1-\delta\right)\left(1-\beta\delta\right)}{\left(1+\beta-2\beta\delta\right)^2}\right)^{\frac{\alpha(1-\xi)}{\xi-\alpha}} \cdot \\
&\quad \cdot \left(\alpha\left(\xi+\theta\right)-\xi\right)^{\frac{\xi-\alpha(\xi+\theta)}{\xi-\alpha}} \left(\alpha\eta\right)^{\frac{\alpha\eta}{\alpha-\xi}} \left(\alpha\left(\xi+\eta+\theta\right)-\xi\right)^{\frac{\alpha(\xi+\eta+\theta)-\xi}{\xi-\alpha}}
\end{aligned}$$

From (21) we have:

$$\begin{aligned}\frac{\partial c}{\partial E} &= \frac{\xi\left(1-\alpha\right)}{\left(1-\delta\right)\left(\xi-\alpha\right)}\cdot\frac{\xi-\alpha}{\xi}\left(\frac{\alpha d}{\xi}\right)^{\frac{\alpha}{\xi-\alpha}}A^{\frac{\xi}{\xi-\alpha}}E^{\frac{\xi(1-\alpha)}{(1-\delta)(\xi-\alpha)}-1}L^{\frac{\xi(1-\alpha)}{\xi-\alpha}}\left(\frac{\beta\left(1-\delta\right)\left(1-\beta\delta\right)}{\left(1+\beta-2\beta\delta\right)^{2}}\right)^{\frac{\xi(1-\alpha)}{\xi-\alpha}}\cdot\\&\cdot\left(\alpha\left(\xi+\theta\right)-\xi\right)^{\frac{\alpha(\xi+\theta)-\xi}{\alpha-\xi}}\left(\alpha\eta\right)^{\frac{\alpha\eta}{\alpha-\xi}}\left(\alpha\left(\xi+\eta+\theta\right)-\xi\right)^{\frac{\alpha(\xi+\eta+\theta)-\xi}{\xi-\alpha}}=\\&= \frac{1-\alpha}{1-\delta}\left(\frac{\alpha d}{\xi}\right)^{\frac{\alpha}{\xi-\alpha}}A^{\frac{\xi}{\xi-\alpha}}E^{\frac{\alpha(1-\xi)+\delta(\xi-\alpha)}{(1-\delta)(\xi-\alpha)}}L^{\frac{\xi(1-\alpha)}{\xi-\alpha}}\left(\frac{\beta\left(1-\delta\right)\left(1-\beta\delta\right)}{\left(1+\beta-2\beta\delta\right)^{2}}\right)^{\frac{\xi(1-\alpha)}{\xi-\alpha}}\cdot\\&\cdot\left(\alpha\left(\xi+\theta\right)-\xi\right)^{\frac{\alpha(\xi+\theta)-\xi}{\alpha-\xi}}\left(\alpha\eta\right)^{\frac{\alpha\eta}{\alpha-\xi}}\left(\alpha\left(\xi+\eta+\theta\right)-\xi\right)^{\frac{\alpha(\xi+\eta+\theta)-\xi}{\xi-\alpha}}\end{aligned}$$

and then:

$$\begin{aligned}\frac{\partial c}{\partial L} &= \frac{\xi\left(1-\alpha\right)}{\xi-\alpha}\cdot\frac{\xi-\alpha}{\xi}\left(\frac{\alpha d}{\xi}\right)^{\frac{\alpha}{\xi-\alpha}}A^{\frac{\xi}{\xi-\alpha}}E^{\frac{\xi(1-\alpha)}{(1-\delta)(\xi-\alpha)}}L^{\frac{\xi(1-\alpha)}{\xi-\alpha}-1}\left(\frac{\beta\left(1-\delta\right)\left(1-\beta\delta\right)}{\left(1+\beta-2\beta\delta\right)^{2}}\right)^{\frac{\xi(1-\alpha)}{\xi-\alpha}}\cdot\\&\cdot\left(\alpha\left(\xi+\theta\right)-\xi\right)^{\frac{\alpha(\xi+\theta)-\xi}{\alpha-\xi}}\left(\alpha\eta\right)^{\frac{\alpha\eta}{\alpha-\xi}}\left(\alpha\left(\xi+\eta+\theta\right)-\xi\right)^{\frac{\alpha(\xi+\eta+\theta)-\xi}{\xi-\alpha}}=\\&= \left(1-\alpha\right)\left(\frac{\alpha d}{\xi}\right)^{\frac{\alpha}{\xi-\alpha}}A^{\frac{\xi}{\xi-\alpha}}E^{\frac{\xi(1-\alpha)}{(1-\delta)(\xi-\alpha)}}L^{\frac{\alpha(1-\xi)}{\xi-\alpha}}\left(\frac{\beta\left(1-\delta\right)\left(1-\beta\delta\right)}{\left(1+\beta-2\beta\delta\right)^{2}}\right)^{\frac{\xi(1-\alpha)}{\xi-\alpha}}\cdot\\&\cdot\left(\alpha\left(\xi+\theta\right)-\xi\right)^{\frac{\alpha(\xi+\theta)-\xi}{\alpha-\xi}}\left(\alpha\eta\right)^{\frac{\alpha\eta}{\alpha-\xi}}\left(\alpha\left(\xi+\eta+\theta\right)-\xi\right)^{\frac{\alpha(\xi+\eta+\theta)-\xi}{\xi-\alpha}}\end{aligned}$$

and also:

$$\begin{aligned}\frac{\partial c}{\partial A} &= \frac{\xi}{\xi-\alpha}\cdot\frac{\xi-\alpha}{\xi}\left(\frac{\alpha d}{\xi}\right)^{\frac{\alpha}{\xi-\alpha}}A^{\frac{\xi}{\xi-\alpha}-1}E^{\frac{\xi(1-\alpha)}{(1-\delta)(\xi-\alpha)}}L^{\frac{\xi(1-\alpha)}{\xi-\alpha}}\left(\frac{\beta\left(1-\delta\right)\left(1-\beta\delta\right)}{\left(1+\beta-2\beta\delta\right)^{2}}\right)^{\frac{\xi(1-\alpha)}{\xi-\alpha}}\cdot\\&\cdot\left(\alpha\left(\xi+\theta\right)-\xi\right)^{\frac{\alpha(\xi+\theta)-\xi}{\alpha-\xi}}\left(\alpha\eta\right)^{\frac{\alpha\eta}{\alpha-\xi}}\left(\alpha\left(\xi+\eta+\theta\right)-\xi\right)^{\frac{\alpha(\xi+\eta+\theta)-\xi}{\xi-\alpha}}=\\&= \left(\frac{\alpha d}{\xi}\right)^{\frac{\alpha}{\xi-\alpha}}A^{\frac{\alpha}{\xi-\alpha}}E^{\frac{\xi(1-\alpha)}{(1-\delta)(\xi-\alpha)}}L^{\frac{\xi(1-\alpha)}{\xi-\alpha}}\left(\frac{\beta\left(1-\delta\right)\left(1-\beta\delta\right)}{\left(1+\beta-2\beta\delta\right)^{2}}\right)^{\frac{\xi(1-\alpha)}{\xi-\alpha}}\cdot\\&\cdot\left(\alpha\left(\xi+\theta\right)-\xi\right)^{\frac{\alpha(\xi+\theta)-\xi}{\alpha-\xi}}\left(\alpha\eta\right)^{\frac{\alpha\eta}{\alpha-\xi}}\left(\alpha\left(\xi+\eta+\theta\right)-\xi\right)^{\frac{\alpha(\xi+\eta+\theta)-\xi}{\xi-\alpha}}\end{aligned}$$

and finally:

$$\begin{aligned}\frac{\partial c}{\partial d} &= \frac{\alpha}{\xi-\alpha}\cdot\frac{\xi-\alpha}{\xi}\left(\frac{\alpha d}{\xi}\right)^{\frac{\alpha}{\xi-\alpha}-1}\frac{\alpha}{\xi}A^{\frac{\xi}{\xi-\alpha}}E^{\frac{\xi(1-\alpha)}{(1-\delta)(\xi-\alpha)}}L^{\frac{\xi(1-\alpha)}{\xi-\alpha}}\left(\frac{\beta\left(1-\delta\right)\left(1-\beta\delta\right)}{\left(1+\beta-2\beta\delta\right)^{2}}\right)^{\frac{\xi(1-\alpha)}{\xi-\alpha}}\cdot\\&\cdot\left(\alpha\left(\xi+\theta\right)-\xi\right)^{\frac{\alpha(\xi+\theta)-\xi}{\alpha-\xi}}\left(\alpha\eta\right)^{\frac{\alpha\eta}{\alpha-\xi}}\left(\alpha\left(\xi+\eta+\theta\right)-\xi\right)^{\frac{\alpha(\xi+\eta+\theta)-\xi}{\xi-\alpha}}=\\&= \frac{\alpha^{2}}{\xi^{2}}\left(\frac{\alpha d}{\xi}\right)^{\frac{2\alpha-\xi}{\xi-\alpha}}A^{\frac{\xi}{\xi-\alpha}}E^{\frac{\xi(1-\alpha)}{(1-\delta)(\xi-\alpha)}}L^{\frac{\xi(1-\alpha)}{\xi-\alpha}}\left(\frac{\beta\left(1-\delta\right)\left(1-\beta\delta\right)}{\left(1+\beta-2\beta\delta\right)^{2}}\right)^{\frac{\xi(1-\alpha)}{\xi-\alpha}}\cdot\\&\cdot\left(\alpha\left(\xi+\theta\right)-\xi\right)^{\frac{\alpha(\xi+\theta)-\xi}{\alpha-\xi}}\left(\alpha\eta\right)^{\frac{\alpha\eta}{\alpha-\xi}}\left(\alpha\left(\xi+\eta+\theta\right)-\xi\right)^{\frac{\alpha(\xi+\eta+\theta)-\xi}{\xi-\alpha}}\end{aligned}$$

Finally, from (22) we get:

$$\begin{aligned}
\frac{\partial a}{\partial E} &= \frac{\xi\left(1-\alpha\right)}{\left(1-\delta\right)\left(\xi-\alpha\right)}\cdot\frac{\beta}{1-\beta}\left(\frac{\xi-\alpha}{\xi}L-1+\alpha\right)\left(\frac{\alpha d}{\xi}\right)^{\frac{\alpha}{\xi-\alpha}}A^{\frac{\xi}{\xi-\alpha}}E^{\frac{\xi(1-\alpha)}{(1-\delta)(\xi-\alpha)}-1}L^{\frac{\alpha(1-\xi)}{\xi-\alpha}}\cdot\\
&\quad\cdot\left(\frac{\beta\left(1-\delta\right)\left(1-\beta\delta\right)}{\left(1+\beta-2\beta\delta\right)^{2}}\right)^{\frac{\xi(1-\alpha)}{\xi-\alpha}}\left(\alpha\left(\xi+\theta\right)-\xi\right)^{\frac{\xi-\alpha(\xi+\theta)}{\xi-\alpha}}\left(\alpha\eta\right)^{\frac{\alpha\eta}{\alpha-\xi}}\left(\alpha\left(\xi+\eta+\theta\right)-\xi\right)^{\frac{\alpha(\xi+\eta+\theta)-\xi}{\xi-\alpha}}=\\
&= \frac{\xi\left(1-\alpha\right)}{\left(1-\delta\right)\left(\xi-\alpha\right)}\cdot\frac{\beta}{1-\beta}\left(\frac{\xi-\alpha}{\xi}L-1+\alpha\right)\left(\frac{\alpha d}{\xi}\right)^{\frac{\alpha}{\xi-\alpha}}A^{\frac{\xi}{\xi-\alpha}}E^{\frac{\alpha(1-\xi)+\delta(\xi-\alpha)}{(1-\delta)(\xi-\alpha)}}L^{\frac{\alpha(1-\xi)}{\xi-\alpha}}\cdot\\
&\quad\cdot\left(\frac{\beta\left(1-\delta\right)\left(1-\beta\delta\right)}{\left(1+\beta-2\beta\delta\right)^{2}}\right)^{\frac{\xi(1-\alpha)}{\xi-\alpha}}\left(\alpha\left(\xi+\theta\right)-\xi\right)^{\frac{\xi-\alpha(\xi+\theta)}{\xi-\alpha}}\left(\alpha\eta\right)^{\frac{\alpha\eta}{\alpha-\xi}}\left(\alpha\left(\xi+\eta+\theta\right)-\xi\right)^{\frac{\alpha(\xi+\eta+\theta)-\xi}{\xi-\alpha}}
\end{aligned}$$

and then:

$$\begin{aligned}
\frac{\partial a}{\partial L} &= \frac{\alpha\left(1-\xi\right)}{\xi-\alpha}\cdot\frac{\beta}{1-\beta}\left(\frac{\xi-\alpha}{\xi}L-1+\alpha\right)\left(\frac{\alpha d}{\xi}\right)^{\frac{\alpha}{\xi-\alpha}}A^{\frac{\xi}{\xi-\alpha}}E^{\frac{\xi(1-\alpha)}{(1-\delta)(\xi-\alpha)}}L^{\frac{\alpha(1-\xi)}{\xi-\alpha}-1}\cdot\\
&\quad\cdot\left(\frac{\beta\left(1-\delta\right)\left(1-\beta\delta\right)}{\left(1+\beta-2\beta\delta\right)^{2}}\right)^{\frac{\xi(1-\alpha)}{\xi-\alpha}}\left(\alpha\left(\xi+\theta\right)-\xi\right)^{\frac{\xi-\alpha(\xi+\theta)}{\xi-\alpha}}\left(\alpha\eta\right)^{\frac{\alpha\eta}{\alpha-\xi}}\left(\alpha\left(\xi+\eta+\theta\right)-\xi\right)^{\frac{\alpha(\xi+\eta+\theta)-\xi}{\xi-\alpha}}=\\
&= \frac{\alpha\left(1-\xi\right)}{\xi-\alpha}\cdot\frac{\beta}{1-\beta}\left(\frac{\xi-\alpha}{\xi}L-1+\alpha\right)\left(\frac{\alpha d}{\xi}\right)^{\frac{\alpha}{\xi-\alpha}}A^{\frac{\xi}{\xi-\alpha}}E^{\frac{\xi(1-\alpha)}{(1-\delta)(\xi-\alpha)}}L^{\frac{2\alpha-\xi(1+\alpha)}{\xi-\alpha}}\cdot\\
&\quad\cdot\left(\frac{\beta\left(1-\delta\right)\left(1-\beta\delta\right)}{\left(1+\beta-2\beta\delta\right)^{2}}\right)^{\frac{\xi(1-\alpha)}{\xi-\alpha}}\left(\alpha\left(\xi+\theta\right)-\xi\right)^{\frac{\xi-\alpha(\xi+\theta)}{\xi-\alpha}}\left(\alpha\eta\right)^{\frac{\alpha\eta}{\alpha-\xi}}\left(\alpha\left(\xi+\eta+\theta\right)-\xi\right)^{\frac{\alpha(\xi+\eta+\theta)-\xi}{\xi-\alpha}}
\end{aligned}$$

and also:

$$\begin{aligned}
\frac{\partial a}{\partial A} &= \frac{\xi}{\xi-\alpha}\cdot\frac{\beta}{1-\beta}\left(\frac{\xi-\alpha}{\xi}L-1+\alpha\right)\left(\frac{\alpha d}{\xi}\right)^{\frac{\alpha}{\xi-\alpha}}A^{\frac{\xi}{\xi-\alpha}-1}E^{\frac{\xi(1-\alpha)}{(1-\delta)(\xi-\alpha)}}L^{\frac{\alpha(1-\xi)}{\xi-\alpha}}\cdot\\
&\quad\cdot\left(\frac{\beta\left(1-\delta\right)\left(1-\beta\delta\right)}{\left(1+\beta-2\beta\delta\right)^{2}}\right)^{\frac{\xi(1-\alpha)}{\xi-\alpha}}\left(\alpha\left(\xi+\theta\right)-\xi\right)^{\frac{\xi-\alpha(\xi+\theta)}{\xi-\alpha}}\left(\alpha\eta\right)^{\frac{\alpha\eta}{\alpha-\xi}}\left(\alpha\left(\xi+\eta+\theta\right)-\xi\right)^{\frac{\alpha(\xi+\eta+\theta)-\xi}{\xi-\alpha}}=\\
&= \frac{\xi}{\xi-\alpha}\cdot\frac{\beta}{1-\beta}\left(\frac{\xi-\alpha}{\xi}L-1+\alpha\right)\left(\frac{\alpha d}{\xi}\right)^{\frac{\alpha}{\xi-\alpha}}A^{\frac{\alpha}{\xi-\alpha}}E^{\frac{\xi(1-\alpha)}{(1-\delta)(\xi-\alpha)}}L^{\frac{\alpha(1-\xi)}{\xi-\alpha}}\cdot\\
&\quad\cdot\left(\frac{\beta\left(1-\delta\right)\left(1-\beta\delta\right)}{\left(1+\beta-2\beta\delta\right)^{2}}\right)^{\frac{\xi(1-\alpha)}{\xi-\alpha}}\left(\alpha\left(\xi+\theta\right)-\xi\right)^{\frac{\xi-\alpha(\xi+\theta)}{\xi-\alpha}}\left(\alpha\eta\right)^{\frac{\alpha\eta}{\alpha-\xi}}\left(\alpha\left(\xi+\eta+\theta\right)-\xi\right)^{\frac{\alpha(\xi+\eta+\theta)-\xi}{\xi-\alpha}}
\end{aligned}$$

and finally:

$$
\begin{aligned}
\frac{\partial a}{\partial d} &= \frac{\alpha}{\xi-\alpha}\cdot\frac{\beta}{1-\beta}\left(\frac{\xi-\alpha}{\xi}L-1+\alpha\right)\left(\frac{\alpha d}{\xi}\right)^{\frac{\alpha}{\xi-\alpha}-1}\frac{\alpha}{\xi}A^{\frac{\xi}{\xi-\alpha}}E^{\frac{\xi(1-\alpha)}{(1-\delta)(\xi-\alpha)}}L^{\frac{\alpha(1-\xi)}{\xi-\alpha}}\cdot \\
&\quad\cdot\left(\frac{\beta(1-\delta)(1-\beta\delta)}{(1+\beta-2\beta\delta)^2}\right)^{\frac{\xi(1-\alpha)}{\xi-\alpha}}(\alpha(\xi+\theta)-\xi)^{\frac{\xi-\alpha(\xi+\theta)}{\xi-\alpha}}(\alpha\eta)^{\frac{\alpha\eta}{\alpha-\xi}}(\alpha(\xi+\eta+\theta)-\xi)^{\frac{\alpha(\xi+\eta+\theta)-\xi}{\xi-\alpha}} = \\
&= \frac{\alpha^2}{\xi(\xi-\alpha)}\cdot\frac{\beta}{1-\beta}\left(\frac{\xi-\alpha}{\xi}L-1+\alpha\right)\left(\frac{\alpha d}{\xi}\right)^{\frac{2\alpha-\xi}{\xi-\alpha}}A^{\frac{\xi}{\xi-\alpha}}E^{\frac{\xi(1-\alpha)}{(1-\delta)(\xi-\alpha)}}L^{\frac{\alpha(1-\xi)}{\xi-\alpha}}\cdot \\
&\quad\cdot\left(\frac{\beta(1-\delta)(1-\beta\delta)}{(1+\beta-2\beta\delta)^2}\right)^{\frac{\xi(1-\alpha)}{\xi-\alpha}}(\alpha(\xi+\theta)-\xi)^{\frac{\xi-\alpha(\xi+\theta)}{\xi-\alpha}}(\alpha\eta)^{\frac{\alpha\eta}{\alpha-\xi}}(\alpha(\xi+\eta+\theta)-\xi)^{\frac{\alpha(\xi+\eta+\theta)-\xi}{\xi-\alpha}}
\end{aligned}
$$

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