Common Optimal Scaling for Customer Satisfaction Models: a point to Cobb-Douglas' form

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Abstract: The first aim of this paper is to present a singular algorithm of ALSOS's (Alternating Least Squares with Optimal Scaling). It allows to assign the same scaling to all variables measured on the same ordinal scale in a categorical regression. The algorithm is applied to a regression model to measure and evaluate Customer Satisfaction (CS) in a sanitary case. The results seem to support the use of multiplicative models like Cobb-Douglas's, to analyze how the overall CS of goods or services is shaped. According to this evidence, the second aim intend to suggest a theory about the overall CS very similar to theory about utility in Marginal Economics. After a brief introduction to the CS measurement and evaluation methods (Sec. 1), the algorithm is presented on the Sec. 2. Sec. 3 and 4 concern the application and the theory about overall CS. Conclusions are reported in Sec. 5.

1 Features of a Customer Satisfaction model

In the last twenty years several statistical methods have been proposed to measure and to evaluate the satisfaction degree of a customer about goods or services, namely Customer Satisfaction (CS). A brief overview of these methods is not a target of the present paper, nevertheless it is useful to consider some features that can characterize and distinguish a method.
The first feature concern the measurement scale. The natural scale of CS is typically an ordinal scale (for example: very dissatisfied, dissatisfied, neither satisfied nor dissatisfied, satisfied, very satisfied) but, unfortunately, this measurement doesn't always allow very meaningful analysis. The most diffused approaches to overcome this limit are:

- adopting a Likert scale;
- determining a metric scale from a probabilistic model;
- introducing an Optimal Scaling algorithm.

Tabella 1: Features of some popular statistical method for CS

methods	scaling method	observation	free distribution
SERVQUAL	Likert	Indirect	Yes
Categorical Regression	Optimal Scaling	Direct	Yes
Categorical PCA	Optimal Scaling	Indirect	Yes
Rasch Analysis	Probabilistic	Indirect	No
PLS Path Model	Likert	Indirect	Yes
LISREL	Likert	Indirect	No

The Likert scale (see Brasini et al. 2002, pp 164-168) consists on replacing ordinal categories with their ranks. Such transformation is very easy and is adopted by several statistical methods (see moreover Tab. 1), but is obviously arbitrary and can be considered acceptable only if categories are conceptually equidistanced. Probabilistic approaches are the Thurstone's method and the Rasch Analysis model (see Andrich), but either approach imply the choice of distributional assumptions. Optimal Scaling (OS) is instead a class of distribution free methods, that allow to assign numerical values to categorical variables in a way which optimizes an analysis model (see Boch, and Kruskall). Conceptually Rasch Analysis can be considered like a OS method, but historically OS methods are free distribution, while Rasch Analysis is not. An another feature regards if the CS is directly observable or not. In many cases the customer can be asked for his satisfaction degree (direct observation), but this observation can be considered a effective degree of satisfaction only if we can assume the customer's rationality, in other words this means that his answer in not affected by environmental and psychological influences. Otherwise the CS had to be estimated from other observable variables by means of appropriate models (indirect observation). In the following table are reported some popular statistical method used for CS measurement and evaluation. They are compared in regard to the features discussed.

In the following sections a singular Categorical Regression model is proposed for CS evaluation. It is based on an ALSOS alghorithm (Alternating Least Squares with Optimal Scaling, see Sec. 2) and allows to obtain a common scaling for all evaluation model variables measured on the same ordinal scale. This does not normally happens with the standard ALSOS programs.

2 Categorical Regression with common optimal scaling

The ALSOS algorithms are OS methods that permit the optimization of a model adopting Alternating Least Squares (ALS) and Optimal Scaling (OS) principles (see Young et all., 1976). More specifically they are based on a iterative two-steps estimation process (fig. 1), which permits to get least squares estimations of scaling values and model parameters. Every algorithm starts with an exogenous scaling and terminates when the iterative solution converges. The models involved are

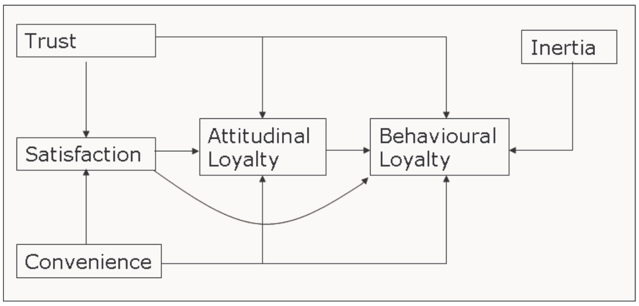

Figura 1: The ALSOS Algorithm

linear models, which can be performed by an optimization (Regression, Principal Component Analysis, ...); the corresponding analysis is also named with the term categorical.

2.1 The pattern of the Model

Let $\tilde{Y}$ the overall satisfaction degree of a customer about a good or service and $\tilde{X}_1, \tilde{X}_2, ..., \tilde{X}_p$ the satisfaction degrees of some aspects of the good or service. All satisfactions are measured on a scale of k ordinal categories $c_1, c_2, ..., c_k$. The target is to convert a qualitative scale into a quantitative one by means of a common transformation $z(\ )$ in order to minimize the error ε of regression:

$$Y = \beta_0 + \beta_1 X_1 + ... + \beta_p X_p + \varepsilon_{ijk} \tag{1}$$

where $Y = z(\tilde{Y})$, $X_1 = z(\tilde{X_1}), ..., X_p = z(\tilde{X_p})$. Practically the transformation $z(\ )$ is defined by k ordered values $z_1 \leq z_2 \leq ... \leq z_k$ corresponding to the k ordered categories. Assuming data are observed on n customers, the score $\mathbf{y}, \mathbf{x}_1, \mathbf{x}_2, ...$ of each scaled variable can be got in the following way:

$$\begin{aligned} \vec{y} &= \vec{U}_{\vec{y}} \vec{z} \\ \vec{x}_{\vec{j}} &= \vec{U}_{\vec{j}} \vec{z} \end{aligned} \tag{2}$$

where $\vec{U}_{\vec{y}}, \vec{U}_{\vec{j}}$ are the typical indicator matrix (the generic element $u_{i,h}$ is 1 if the i-th customer respond c_h about the corresponding variable, else 0); $\vec{z}$ is the vector of the scaling parameters $z_1, z_2, ..., z_k$.
So the model (1) can be described in the classic form:

$$\vec{y} = \vec{X}\vec{\beta} + \vec{\varepsilon} \tag{3}$$

This form is useful for the model step, but not for the OS step, because does not point out the scaling parameters. For it, it needs to rewrite the classic form in the following scaling form:

$$(\vec{U}_{\vec{y}} - \vec{B})\vec{z} = \beta_0 \vec{1} + \vec{\varepsilon} \tag{4}$$

where $\vec{B} = \sum_{j=1}^{p} \beta_j \vec{U}_{\vec{j}}$

2.1.1 The algorithm of the parameters estimation

According the approach of ALSOS (fig. 1), the algorithm is described by the following steps:

Initialisation: an arbitrary $\vec{z}$ is chosen;

Model Step: $\vec{\beta}$ is estimated by classic estimator: $\vec{\beta} = (\vec{X}'\vec{X})^{-1}\vec{X}'\vec{z}$

OS Step: a new $\vec{z}$ is estimated by minimizing SSE in the model (4) with the constrains $z_1 = z_{min}$ and $z_k = z_{max}$

Control Step: if the absolute difference between the two last $\vec{z}$ is less than a suitable convergence, the *Final Results* are obtained; else it need to go back to *Model Step*

Final Results: the last $\vec{z}$ and $\vec{\beta}$ are the final results

It is easy to note that the OS model above does not include constrains for the monotonicity of transformation: If initial scaling is monotone and customer responses are rational, they are not needed, but there are no problems to include them. Indeed the minimum and the maximum of scaling parameters are fixed. It is due to avoid the algorithm produces the dummy solution $z_1 = z_2 = ... = z_k$. Generally it needs to fix two constrains to define a metric intervals scale (average and standard deviation, minimum and maximum, etc.) and the constrains adopted are very suitable in a linear optimization problem. The convergence is guaranteed because the sum of squares errors (SSE) decreases at every step and round. There is one hooker: the ALSOS procedure does not guarantee convergence on the global least squares solution. Nevertheless every final scaling is better (in terms of SSE) than an initial, supposed good scaling.

3 Multiplicative Models for CS

The proposed model was applied to a survey on CS in a Piedmont ASL (Local Sanitary Firm): 525 patients were asked about their satisfaction degree on:

- whole service (overall satisfaction);
- some aspects of the service (waiting time, suitable environment, courtesy, professionalism, etc.).

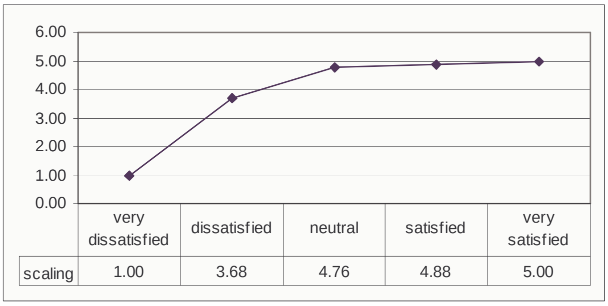

Figura 2: The optimal scaling in a sanitary case

Responses scale was: very dissatisfied, dissatisfied, neutral, satisfied, very satisfied. Here only the final scaling is reported in the figure 2 (for more details see Chirico (2005)).

This result contrasts with idea of conceptual equidistance among categories. Nevertheless it is possible to partially recover equidistance with a power transformation like:

$$\vec{\dot{z}}' = [a^{z_1}, a^{z_2}, ..., a^{z_k}] \tag{5}$$

with $a > 1$. It means that the scaling $\vec{z}$ could be viewed (see fig. 2) as the logarithmic transformation of a more realistic scaling $\vec{\dot{z}}$. Then the model (1) should be the logarithmic transformation of the model:

$$a^Y = a^{\beta_0 + \beta_1 X_1 + ... + \beta_p X_p + \varepsilon} \tag{6}$$

that can be rewritten as:

$$\dot{Y} = \beta_0 \cdot \dot{X}_1^{\beta_1} \cdot ... \cdot \dot{X}_k^{\beta_p} \cdot \varepsilon \tag{7}$$

where variables with the point above correspond to $a^{variable}$. Now the new variables' values, in model (7), better represent the the categories $c_1, c_2, ..., c_k$. This fact suggests that the better relation between overall satisfaction and partial satisfactions might be multiplicative, like a Cobb-Douglas function, rather than linear. The linear model, thanks to the proposed algorithm, is useful to estimate the parameters $\beta_0, \beta_1, ..., \beta_p$ (they do not change in the multiplicative model) and the pre-final scaling $z_1, ..., z_k$.

3.1 Some observations

Final Scaling. The final scaling $\dot{z}_1, ..., \dot{z}_k$ could be get from $z_1, ..., z_k$ by means of a power transformation with basis $a > 1$:

$$\dot{z}_j = a^{z_j} \tag{8}$$

Unlikely it is not clear which value of a is better to get the final scaling, because not every value of a determines the same effects in terms of ratio and intervals among $\dot{z}_1,...,\dot{z}_k$. If a conceptual equidistance among the categories $c_1,...,c_k$ is assumed, a could be chosen in order to minimise the variability of the differences: $\dot{z}_h - \dot{z}_{h-1}$ ($h = 2,...,k$). Other criteria can be adopted; each one determines different final scaling and consequently different values of position indicators like the mean, for example. Indeed the parameters $\beta_0, \beta_1, ..., \beta_p$ (which indicate the importance of every factor $X_0, X_1, ..., X_p$) not change and not their significance (see next section).

Weighting. As Least Squares methods are applied on the linear model (1), the fit of the multiplicative model (7) is worse in correspondence of greater value of $\dot{Y}$. To reduce this effect, it is possible to change the two estimation steps introducing a weighted least squares estimation method.

4 A theory about overall CS

According with the results underlined in the last section, the following theory about the CS is proposed:

- every customer determines his/her own satisfaction about a good or service (Overall Customer Satisfaction: *OCS*) composing the relative evaluations of some fundamental aspects of the good or service (Partial Customer Satisfaction: *PCS*);
- the composition criterion is approximated by a multiplicative model of the Cobb-Douglas type:

$$OCS = \alpha \cdot PCS_1^{\beta_1} \cdot ... \cdot PCS_k^{\beta_k} \tag{9}$$

The first assumption is typical of the most of CS model (SERVQUAL, ACSI, ECSI). The second one shapes the Customer Satisfaction similar to the customer utility in the marginal consumer theory. In fact it is easy to prove that:

$$\beta_j = \frac{d(OCS)/OCS}{d(PCS_j)/PCS_j} \tag{10}$$

that means β_j is the elasticity of *OCS* respect to PCS_j. If customer's responses are rational, all β_j will be positive or null (negative estimates of these parameters could be obtained, but they ought to be not different from zero at the test). Generally $\alpha = 1$ and $\sum \beta_j = 1$ are expected (scale effects do not have sense!). The second assumption involves:

$$0 < \beta_j < 1 \tag{11}$$

Therefore β_j indicates the importance of the j-th aspect for the CS. Another similitude to marginal consumer theory is that the marginal overall satisfaction determined by each partial satisfaction is decreasing. In fact:

$$\frac{d(OCS)}{d(PCS_j} = \beta_j \frac{OCS}{PCS_j} \tag{12}$$

If PCS_j increases, the OCS increases less proportionally (see (9) and (11)) and consequently $d(OCS)/d(PCS_j)$ decreases. This means the improvement of one level from *satisfied* to *very satisfied* in an aspect produces a smaller increase of the overall satisfaction than the improvement of one level from *neutral* to *satisfied* in the same aspect. In other words, improvements from low quality levels are more important for customers than improvements from high quality levels. This deduction from the model (9) is consistent with the psychology of the majority of the customers. If the OCS of a good or service ought to be improved, the best strategy is not always to improve the most important aspect (that one with the biggest β_j). It could be more effective to improve another aspect with a low quality level. Each possible improvement ought to be considered and valued in regard to his marginal satisfaction and, of course, his cost (costs of needed actions to get the improvement).

5 Conclusions

The algorithm presented in this paper has the typical features of ALSOS programs: free distribution method and convergence of estimates obtained by analytic functions. It also ensures a common scaling for all data measured on same ordinal scale, whereas ALSOS programs included in the most popular statistic software do not. In fact these programs, as general approach, assign different scaling to every qualitative variable, whether it is measured on a common scale or not. However the same values should be assigned to same categories, if the scaling gives a metric significance to the measurement of qualitative data (see Chirico 2005).
The application of algorithm in a CS evaluation study has pointed out that the relation between the overall satisfaction and its factors seems to be formalized better by multiplicative models, like Cobb-Douglas ones. In other words: the overall satisfaction and its factors are conceptually comparable to overall utility and its factors in the marginal consumer theory (the Cobb-Douglas' function was originally proposed like production function, but subsequently it was also used to confirm the marginal consumer theory). This model form permits to formalized the concept of *decreasing marginal satisfaction* that involves the strategic importance of improving the low quality aspects. At present, further studies on how to get the final scaling in a multiplicative model are being carried on.

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