TEACHER'S SEMIOTIC GAMES IN MATHEMATICS LABORATORY

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Summary
The paper uses a semiotic lens to describe the teacher's interventions in classroom discussions, with all the students or only o group of them. The frame is semiotic-cultural and considers teacher's production within students' productions, during the development of a mathematical activity. This frame uses the model of the semiotic bundle to describe the various semiotic contributions (by the teacher and the students) and allows focussing some important strategies, called semiotic games, used by the teacher to support students' mathematics learning.

Key words
Semiotic game, role of the teacher, mathematics laboratory, semiotic bundle, multimodality, gesture, embodied cognition.

The role of the teacher

The role of the teacher in promoting learning processes is crucial and has been analysed according to different frameworks in mathematics education. For example the Theory of Didactic Situations, originated by G. Brousseau (1997), defines the teacher as a didactical engineer, who designs the situations and organises the milieu according to the piece of mathematics to be taught and to the features of the students. In this frame, particular attention is due to two phases of didactical engineering: the devolution (when the teacher transfers the activity to the students and they carry it out with the awareness of being in a problematic situation with the responsibility to solve it) and the institutionalisation (when the teacher summarises results, organising them in a theoretical frame and adding what is necessary to complete this frame). The comprehensive theory about teachers' decision making developed by the Teacher Model Group (Schoenfeld, 1998, 1999), named KGB theory - Knowledge, Goals, Beliefs – exemplifies the attempts to merge these different aspects. As underlined by Sriraman and English (2005), the systemic work engaged by Schoenfeld ended up in a teacher's decision-making model of "teaching in context", which provides a fine-grained analysis of the processes of decision making, grounded on teacher's knowledge, goals and beliefs. This model is useful both in the theory of mathematics education and in the teaching practice, for enhancing the professional development.
Other researchers, who work according to Vygotsky's conceptualisation of "zone of proximal development" (Vygotsky, 1978, p. 84), underline that teaching consists in a process of enabling students' potential achievements. The notion of zone of proximal development models the learning process through social interaction and it is defined as: "the distance between the actual developmental level as determined by independent problem solving under the adult guidance or in collaboration with more capable peers" (Vygotsky, 1978, p.86). As this definition states, development is possible, thanks to collaboration between one individual, who has a potential attitude to change, and another individual (or more than one), who intentionally cooperate to accomplish a task. Within such an approach, some researchers (e.g. Bartolini & Mariotti, 2008) picture the teacher as a semiotic mediator, who promotes the evolution of signs in the classroom, from the personal senses that the students give, towards the scientific shared sense.
"The teacher's role in the construction of mathematical understanding is particularly emphasised by researchers who adopt a Vygotskian perspective, and see teachers as a guide in the "zone of proximal development": this role is then crucial in making decisions not only about the tasks but also in choosing the communicative strategies to be adopted in classroom interaction (Bartolini Bussi, 1998; Bauersfeld, Krummheuer & Voigt, 1988)." (Malara & Zan, 2008, p. 537).

Taking into account a post-Vygotskian stream, of semiotic-cultural kind, I analyse the role of the teacher as a semiotic mediator, who promotes students' construction of meaning through signs. In this approach, some changes are proposed with respect to the classical Vygotskian approach.

First, I extend the notion of sign to all semiotic resources used in the teaching activities: words (in oral or in written form); extra-linguistic modes of expression (gestures, glances, …); different types of inscriptions (drawings, sketches, graphs, ...); results of actions on instruments (from the pencil to the most sophisticated ICT devices). Then, I consider the embodied and multimodal ways in which those signs are produced, developed and used, during a mathematical activity, by students or teachers. Within such an approach, I utilise a semiotic lens (the semiotic bundle), to focus the interactions between teacher and students (Arzarello & Robutti, 2008). This semiotic lens allows framing and describing important semiotic phenomena, called semiotic games (Arzarello & Paola, 2007) and referred to the teacher. The semiotic games practice is rooted in the craft knowledge of the teacher, and often is pursued unconsciously by him. If the teacher is aware of semiotic games, he can use them to properly design his intervention strategies in the classroom for supporting students' construction of knowledge.

In the following I discuss three sections: (i) the multimodal paradigm and the semiotic tools suitable for describing mathematics learning processes; (ii) three examples, through which the notion of semiotic games is introduced; (iii) some issues for discussion.

From the multimodality of learning processes to the semiotic bundle

The term multimodality is used in literature with many issues and frames. I will describe particularly multimodality from the point of view of neurology and communication. According to neuroscience, cognitive processes are inherently multimodal, in the sense that they use many different modalities linked together: sight, hearing, touch, motor actions, and so on. Verbal language itself (e.g. metaphorical productions) is part of these cognitive multimodal activities: "...language is inherently multimodal in this sense, that is, it uses many modalities linked together-sight, hearing, touch, motor actions, and so on. Language exploits the pre-existing multimodal character of the sensory-motor system. If this is true, it follows that there is no single "module" for language-and that human language makes use of mechanisms also present in nonhuman primates." (Gallese & Lakoff, 2005. p. 456). These authors draw their statement on the experimental evidence that "...an action like grasping...(1) is neurally enacted using neural substrates used for both action and perception, and (2) the modalities of action and perception are integrated at the level of the sensory-motor system itself and not via higher association areas." (Gallese & Lakoff, 2005, p. 459). "Mirror neurons and other classes of pre-motor and parietal neurons are inherently 'multimodal', in that they respond to more than one modality. Thus, the firing of a single neuron may correlate with both seeing and performing grasping. Such multimodality, we will argue, meets the condition that an action-concept must fit both the performance and perception of the action." (ibid., p. 457-8).

Gallese and Lakoff point out that the sensory-motor system is responsible of containing and characterising action concepts, coming from the perceptuo-motor activity, along with the more abstract concepts. And not only the logic inference paths, grounding the abstract concepts, but also the meanings of the grammatical constructions are located in the sensory-motor regions of the brain. This means that not only the structure, but also the semantic of concepts would be mapped in our brain sensory-motor system. The experimental evidence that activity of doing a thing, or imagining doing that thing, use the same neural substrate shared, supports the previous hypothesis. The authors come to the conclusion that "*understanding is imagination*, and that *what you understand of a sentence in a context is the meaning of that sentence in that context*." (ibid., p. 457, italic in the text).

These authors have the goal of providing a testable embodied theory of concepts, based on reconciling both concrete and abstract concepts within a unified framework. The multimodality as reported by Gallese and Lakoff has important links with the multimodality of communication.

Kress (2004), for example, speaks of multimodality in communication, and says that every mode of communication forces the subject into making certain kinds of commitments about meaning, intended or not. The choice of mode has profound effects on meaning, and textbook designers, for instance, need to be aware of such meaning effects of different modes. For example, certain texts encourage readers to engage in the semiotic work of imagination, following the given order of words on the line, but filling the relatively 'empty' words with the reader's meaning. Contemporary texts (information books, web-pages, the screens of CD-ROMs, etc.), ask the reader to perform different semiotic work, namely to design the order of the text for themselves. The designer of such 'pages'/sites is no longer the author of an authoritative text, but is a provider of material arranged in relation to the assumed characteristics of the imagined audience. The dominant media are now those of the screen (e.g. the gameboy, the mobile phone, the pc, the TV and video). Kress calls these different media "the multimodal landscape of communication", and says that the number of ways of expressing and shaping a message implies choices and questions, which may have different consequences on the users. In this landscape, there is also a place for the teacher as communicator, with his body, involving speech, movement and gesture (as different media). All these media offer specific possibilities, both to the teacher and to students as users of them. For example, the up and down of the voice, which produces the melody of speech; different kinds of writing (words, sketches, and so on); the use of representations; gestures, actions, body movements, ... As human senses all work simultaneously and interdependently, even those modes that might be considered "monomodal" are composed of a variety of modes in concert (gestural language used by hearing and speech impaired are one kind of language that makes this multimodality explicit for the tipically monomodally-conceived term "language").

So, multimodality of communication is pervasive in the tools we use today, but also in human way to communicate, and teachers have to consider this fact and use different ways of communication. They have to make choices, thinking of which modes of expressing and communicating a message are the best for their learners, and these choices should be, as much as possible, conscious.

Scholars of human non-verbal communication used to claim that up to 90% of communicable information is non-verbally signalled, and teacher educators have sometimes tried to help teachers, especially in initial training, to take some control of their non-verbal behaviours when communicating in classrooms. The problem, of course, is that non-verbal behaviour largely operates below the conscious level and becomes different when made the focus of attention. Williams calls "threshold moment" when one somehow crosses the threshold and sees something important differently (Williams, 2008). In a similar way, other scholars identify this particularly important moment and call it "cognitive pivot" (Arzarello & Robutti, 2003), or "initial sign" (Robutti, 2006). This threshold moment is a key "moment" in a teaching experiment, when multimodal communication provides the first objectification of a key idea that eventually may grow if carefully nurtured and tended, like a mathematical germ or seed, into new mathematics. If introduced by students, this moment can be supported by the teacher, in order to help them in constructing mathematical meanings.

The frame of multimodality suggests that "...the understanding of a mathematical concept rather than having a definitional essence, spans diverse perceptuomotor activities, which become more or less active depending of the context." (Nemirovsky, 2003; p. 108). Its main consequence for educational theories consists in stressing the role of perceptuo-motor and multimodal ways of learning. Some researches have been pursued along these lines for general education and for the specificity of mathematical education.

The discovery of printing in the XV century produced a revolution in the ways of conceiving and doing teaching: after Gutenberg the transmission through books assumed a capital relevance, till now. Consequently, from this period on, a symbolic-reconstructive way of learning prevailed (Antinucci, 2001). This way is different from the perceptuo-motor way of learning, prevailing when learners work in an artisan workshop, using perception, interaction, feedback. The two ways differ not in terms of the nature of what is learnt, but in how learning occurs. The first way of learning, which is present from the beginning of the cognitive development of the child, works on symbols (linguistic, mathematical, logical) and reconstructs 'objects', their meanings and mental representations, in the mind. It is a sophisticated way of knowing and requires awareness of the procedures and the appropriation of the symbols used and their meanings.

In this regard, we note that 'traditional' teaching in mathematics, which is usually characterized as 'transmissive', supports this way of learning. However, those students who are not able to learn in this way, necessarily are not involved in the symbolic process, and will try to remember it by rote memorization. The risk of using symbols in a mechanical way is great, and can cause misunderstandings and mistakes (Arzarello, Robutti & Bazzini, 2005).

The perceptuo-motor way of learning involves action and perception and produces learning based on doing, touching, moving and seeing. It does not only characterize the first phase of cognitive development, but it is also involved in the most advanced learning processes. This way of learning challenges the traditional way of teaching mainly based on the transmission of content through formal language.

Perceptuo–motor learning does not exclude the symbolic–reconstructive one: they are two complementary ways of learning that can be integrated, not necessarily in a hierarchy, but with several mutual interactions and enrichment. The way to better integrate these two modalities of learning has been called mathematics laboratory (Anichini, Arzarello, Ciarrapico, & Robutti, 2004), intended as a methodology based on various and structured activities aimed at the construction of meanings, wherever and whenever it is possible, inside or outside school. We can imagine the laboratory environment as a Renaissance workshop, in which the apprentices learned by doing, seeing, imitating, communicating with each other - that is, practicing. In the laboratory activities, the construction of meanings is strictly bound, on the one hand, to the use of artefacts, and on the other, to the interactions between people working together (without distinguishing between teacher and students). "While modulated by shifts of attention, awareness, and emotional states, understanding and thinking are perceptuo-motor activities; furthermore, these activities are bodily distributed across different areas of perception and motor action based on how we have learned and used the subject itself." (Nemirovsky, 2003; p. 108).

The two sides of multimodality (from neuroscience and from communication) can be integrated in a unitary frame, which has a double counterpart: the perceptuo-motor learning of the students and the interaction of the teacher with the students. The teacher involved in mathematics laboratory is not a transmissionist-instructor, but a mediating communicator who uses different methodologies in the classroom, not only the frontal lesson, but also working groups, discussions, problem solving, activities with technologies, and so on: in a word, a teacher who uses the mathematics laboratory. And the interaction between teacher and students occurs in a multimodal way, with the use of different semiotic resources at the same time. For this reason, a semiotic frame seems to be a proper tool to analyse such an interaction. In fact:

(i) Students and teachers use a variety of semiotic resources in the classroom: speech, gestures, glances, inscriptions and extra-linguistic modes of expression (including signs from the instruments used). Namely, the interaction in the class is multimodal.

(ii) Some of these resources do not satisfy the requirements of the classical definitions for semiotic systems as discussed in the literature (e.g. Duval, 2006), which are very structured systems, whose rules of sign production and manipulation are precise algorithms (think of the oral and written language, or the algebraic register).

(iii) In order to study the relationships within and among these resources, active at the same moment, and their dynamic developing in time, it seems that the *semiotic bundle* (Arzarello, 2006) can be the proper model. It includes all signs produced by actions that have an intentional character (e.g. speaking, writing, drawing, gesturing, handling an artefact, etc.) and whose modes of production and transformation (e.g. gesturing or drawing) may have or not specific rules. It is a dynamic structure, where such different resources coexist and develop with their mutual relationships. An example of semiotic bundle is represented by the unity speech-gesture. It has been written that: "gesture and language are one system" (McNeill, 1992, p.2): from our point of view, gesture and language are two components of the same semiotic bundle.

(iv) The semiotic bundle gives a model to describe the phenomena observed, in terms of students' activity of construction of meaning (Arzarello et al., 2008), or role of the teacher (what made in this paper), because it is richer than other tools (as the semiotic systems), in giving reason of different types of signs.

(v) The role of the teacher can be interpreted in the semiotic bundle describing the semiotic activity he makes, in terms of what are called semiotic games (Arzarello & Paola, 2007).

The semiotic bundle is a way to describe the rich and complex sign production and transformation of a group of subjects during social mathematical activities. It must not be considered as a juxtaposition of signs, but a systemic structure to describe the activity of the group, in terms of the signs used, their relationships, their evolution. The relationships concern signs produced in different (near or far) times: for example, a sign made by a subject can influence the sign made by another subject, or a sign can be transformed into another sign (think of a gesture converted into a written sign on the paper) by the same subject, or two signs are made simultaneously by two different subjects. Using the semiotic bundle we can describe the multimodality at an instant of the activity (in a static way, as a picture), or the evolution of the signs and their mutual relationships (in a dynamic way, as a movie). Within this framework, it is interesting to describe when and how the students make visible in the group activity something that was not before, namely, they introduce a new piece of meaning in the construction of knowledge (cognitive pivot, as described above). And in doing so, they evolve from the particular meaning they give to a mathematical object, towards the historical-cultural meaning, shared by the community of the mathematicians. In the semiotic bundle we also include the signs from the teacher, if he participates to the discussion of the small group or if he guides a collective discussion of the class, and the signs coming from the tools used (a representation, or a result of a calculation, a symbol, and so on).

In this paper, the interest is most on the semiotic activity of the teacher, involved in interacting with the students without being directive not authoritative, but a mediator from their individual meanings to the mathematical meanings, what Sinclair and Schiralli (2003) call *ideational mathematics* (i. e. how an individual represents concepts to herself) towards *conceptual mathematics* (i.e. mathematics as a subject matter or discipline, shared by the community of mathematicians).

The teacher can interact with the students in different ways, namely using different semiotic games: repeating a sign (gesture, or word) made by one of the students, to render it more incisive; making a question on a sign introduced by students; introducing a specific mathematical term (to substitute a generic term used by the students); acting on a tool; introducing a metaphor, and so on. These games consist in using a resource (a word, a symbol, a gesture, a sign coming from the activity, and so on), in order to support students in constructing meanings, and are activated by the teacher when he notices that students are in a zone of proximal development. The same games, introduced by the teacher in interaction with students, can give us elements for understanding: how the teacher's signs can influence students' cognitive processes and mathematical production; if the methodology of mathematics laboratory can support students' construction of meaning, through the use of tools and materials.

Analysing different semiotic games by various contexts and various teachers can give us ideas and suggestions not only for research, but also for teaching practice. In fact, if teachers are aware of the semiotic power of their interventions in a discussion with the students, they can pass from an attitude to a transmissive lesson to an interactive lesson, or from judging students with positive/negative feedback, to debating with them about the solution of an activity.

In the next paragraph I present three examples of different classes at different school levels (from kindergarten to secondary school).

The semiotic games: three examples

The first example refers to an activity at kindergarten, with a group of children of 3 years old, in the context of the fairy tale of the three little pigs. The aims of the teaching experiment is to give the children a context where counting, in progressive and regressive way, from 1 to 4 and back to 0. The activities are carried out in a perceptuo-motor way (as usual, in the kindergarten) and the teachers who guide the class are two, with 23 children. The first activity is a simulation of the little pigs' walk with a bundle, containing some things for a snack.
The children are filling the bundle with snacks, in subsequent steps from 1 to 4, and in each of them they put 1 more snack into the bundle, starting from an empty bundle.
In the episode below, the children have already one object in the bundle, and they have to take one more object from a set o snacks, in order to have totally two objects in the bundle. The result is not anticipated by the teacher, whose role is to guide the experience of taking objects and putting them in the bundle and to discuss with the children. This experience can take place at this early age, thanks to the results in embodied cognition, which explain our number sense and arithmetic competence in terms of the metaphor "Arithmetic is object collection" (Lakoff & Nùñez, 2000, p. 55). This metaphor states that we have the concept of addition, thanks to experiences where we collect objects and put together objects from different collections.

26. **TEACHER:** *Here Giorgio is taking another thing, then Gabriele, and Daniela, then Alessandro a thing to eat, only one. Now count them, how many things do you have into the bundle, let's count.*
27. **CHILDREN:** *Two, two* (among them Alberto, who shows his hand with only two fingers open, the index and the medium finger, and the others close. Near him there is a child counting the objects, pointing to them with her index finger, then representing "two" with her index and medium finger and the other fingers closed.)
28. **TEACHER:** *Show me "two". Two like this? Let's see with the hand how is "two".*
29. **CHILDREN:** *Like this!*
30. **TEACHER:** *Also like this, or like this, as a pistol, we can do it also like this. Two. Very good. Now let's take another thing to eat.*

The teacher is guiding the activity, which consists here in taking one more object to add to the one already present in the bundle. She asks the children to count the objects in the bundle, to verify that they are exactly two (#26). Some of the children are trying to represent two with their fingers, and this gesture is copied from one to the other (#27), and soon becomes shared in the classroom. The teacher uses this moment to share this gesture with all the children, so she asks the children to show two with the fingers, in order to see how it is made. Many children participate in the task, and show different ways to represent two with their fingers (#29). So, the teacher profits of this situation to show with her hand the different possibilities to represent two, using the children's ways.
The semiotic bundle of this episode is made of gestures and words used by the children and by the teacher. If we want to describe an evolution in the use of signs, then we start from "taking another thing", which is the first step to obtain two objects in the bundle (the children here started from the result of the previous experiment, in which they have taken one object). Note that the word "two" (already present in children' experience, in or out school) is introduced by the children themselves and not by the teacher. The semiotic game of the teacher here consists in using the signs (both gestures with fingers and words) introduced by the children and reproducing them, in order to share them with all the children. In some cases, the children use only a gesture to represent the quantity (#29), in other cases they use gesture and words together (#27). The teacher's game consists in supporting children in their representations, showing all of them, and adding others ("also this"), giving a metaphor to remember it ("as a pistol"). Teacher's aim is to gave the idea of invariance of quantity, which is independent form the representation used, namely it does not depend on the fingers used, what it counts is only the number of fingers. Then the teacher invites children to take another snack and put it in the bundle.

36. **TEACHER:** *one eh, one, one. Then, babies... Sara, one more. Now, let's try to count how many things we have, do we count together?*

37. **CHILDREN:** *one, two, ...*
38. **TEACHER:** *let's lift up the fingers, then we count them.*
39. **ALL TOGETHER:** *one, two, three* (teacher and children count together showing the objects with the index finger).
40. **TEACHER:** *how many?*
41. **SOME CHILDREN:** *three.*
42. **TEACHER:** *how many?*
43. **SOME CHILDREN:** *like this.*
44. **TEACHER:** *yes, like this, good, or like that,* (she shows the children what is a gesture for three), *also like this is good.* While the teacher is addressing to other children, Alberto is trying to link his index, medium and ring finger to three objects (Figure 1b). Doing this, his hand opens with respect to the previous position, where he was keeping closed thumb and little finger (Figure 1a). So, the difficulty for him is to let his fingers (index, medium and ring) correspond to the three objects in the bundle. *You have to add one finger to the pistol, like this, very well! Three, very good* (Figure 1c). (The child behind Alberto shows only the index and medium finger open, and the rest of the hand close). *One, two, three, add a finger, like this.*

[…]

48. **TEACHER:** *three. But how many things to eat had the piglets? Who remembers how many?* (In the meanwhile a child helps another to open thumb and index finger letting close the others, then he tries to show three with the hand, Figure 2).
49. **CHILD:** *four.*
50. **TEACHER:** *yes, they had four! Now let's go on, and take one more thing.*

Figure 1 a-b-c

Figure 2 a-b

To count three objects, the teacher involves the children in gesturing and saying numbers altogether, starting to touch or to point objects (#36-39). Then, her semiotic game consists in asking questions about the number of objects (#40-43). Children answer the question using a word ("three") or using a gesture ("like this"). They answer about the quantity of objects, after the count. So, their answer corresponds to the cardinality of the set of objects, recognised with the last number pronounced ("three").

In the semiotic bundle, where the teacher is using her semiotic game, also the signs introduced by children are important. For example, Alberto correctly represents three (closing thumb and little finger with the other hand), but to be sure that his representation is correct, he searches for a correspondence between fingers and objects, and his difficulty is to move his fingers in order to

show the three objects, letting thumb and little finger closed (#44). Teacher's attention moves from the class to Alberto, and she claims for a representation "far from" the objects, abstract in itself. In fact, she helps Alberto to represent three with other fingers, in a simpler way, "adding one finger to the pistol" used before to represent two (#44 and 48) (Figure 1c). Recalling the previous metaphor, she gives continuity from the previous experience (of two objects) to the present one (of three objects), and enlarges the set of possible representations of three.
The semiotic bundle consists of words and gestures from children and teacher, mixed together in an evolution towards the meaning of two and then of three, as cardinal numbers of a set of objects.

The second example refers to an activity at higher secondary school (Robutti, 2003), in the context of mathematics of change, particularly focussed to construct meanings of the area under a graph of a function, as cognitive root (Tall, 1989) of definite integral. The teaching experiment is made of various activities based on approximate measures of areas under curves in the Cartesian plane, using before paper and pencil, then a technological artefact, namely the symbolic-graphic calculator TI-89. The students, at the 12th grade of a scientific-oriented Italian school (17-18 years old), have 3 classes of mathematics per week, and are used to work in small groups, then to share the results in a class discussion led by the teacher.
In the example described here, the students are working to evaluate the area under the graph of a given function. The task consists in the determination of the work made by a perfect gas during an isothermal transformation, represented by a hyperbola on the Cartesian plane (Figure 3a). From the discussion about different procedures (obtained by the students in the groups) to determine the work (the area under the hyperbola), the need of an algorithmic formula arose. A formula has an advantage with respect to other non-algorithmic methods: it can be implemented in a program on the calculator. The teacher guides the various students' interventions, to converge on the method of rectangles under and over the function to approximate that area. Then the students use two programs based on this calculation, in a group activity, to evaluate the area under the graph with different numbers of rectangles, including or included in the graph (Figure 3b-c).

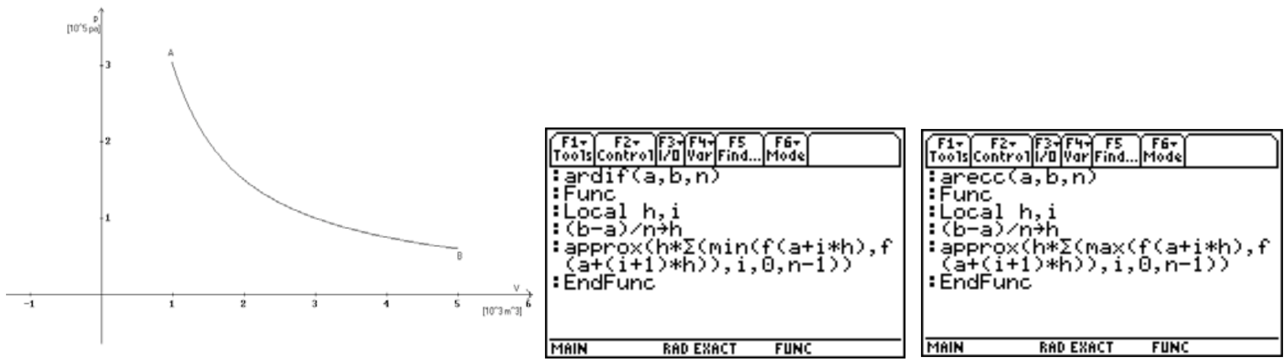


Figure 3a-b-c

The discussion following this activity was aimed to reflect on the degree of approximation with respect to the number of rectangles.

<u>TEACHER:</u> *Which was the best we said?*
ANDREA: *The last!*
<u>TEACHER:</u> *Why?*
ANDREA: *Because it has more intervals and then ...*
STELLA: *Because it gets nearer to the area.*
<u>TEACHER:</u> *But why is it so precise, if there are more intervals?*
ANDREA: *Because ... with more intervals ... it is possible to give a better approximation of the curve with a line going to a more ... microscopic, and then ... nearer.*

The last phrase reveals a passage from the global to the local properties of functions, as if Andrea could notice the local properties of a graph, after having observed the global ones, thanks to the sub-division of the interval on the x-axis. The student has the intuition that the more the intervals, the better is the approximation of a curve with segments, which are closer to the curve. This intuition marks a first step in the conceptualisation of definite integral. The word "microscopic" reminds to the local approximation of curves with lines, that is the theoretical base of Calculus.
The teacher in this episode is asking questions, in order to go deeper in the concept of approximation: if the program of subdivision of the area under the graph in a certain number of rectangles is applied, then the most precise value is the one with the maximum number of rectangles. Both the programs, the one with rectangles included in the graph and the one with rectangles that include the graph, are most precise with the maximum number of subdivision. And the two values become more and more close. Since we are in a class of scientific lyceum, the teacher's aim is to let the students explain why there is such best approximation of the area under the graph. So, her semiotic game consists in rephrasing students' words and posing questions to explain ("Why is it so precise?"). The students participate in this discussion, giving the semiotic bundle signs as words of explanation ("Because it gets nearer to the area", "it is possible to give a better approximation of the curve with a line going to a more … microscopic"). The words "nearer" and "microscopic" are fundamental in the construction of a meaning for the process of calculation of the area of rectangles, as approximation of the area under the graph.
The discussion continues with the next excerpts:

<u>TEACHER:</u> *The last is more precise: what does it mean saying more precise?*
ANDREA: *That it gets nearer to the average value.*
STUDENTS: *That it gets nearer to the real value. That it gets closer to the real value.*

The students come to a second step in the conceptualisation process: the idea that the last result of the program, which approximates the area, is more precise than all the previous results, because "it gets nearer to the real value". This step is characterised by the consciousness that there exists a "real value" for the area, even if they do not have it, at the moment, because they have seen a succession of values approximating the area, but not the area itself. The teacher is insisting in the concept of "more precise", now asking "what does it mean", and the students come to the idea of "real value" of the area, and consequently "more precise" means "closer to the real value". Here the teacher's semiotic game consists in re-using students' words ("more precise"), going on in asking what does it mean. So, the teacher is continuing the game of the previous episode, where she was guiding the reflection of the students in the direction of understanding how to obtain the best approximate value for the area under the graph.

<u>TEACHER:</u> *What do we remember thinking back to this situation?*
STELLA: *The square root of 2.*
<u>TEACHER:</u> *The square root of 2. That is, when did we construct what?*
FRANCESCO: *The contiguous classes.*

In this discussion, the students are guided by the teacher toward connecting the approximate value of the area to a theoretical content, developed in the previous year (the concept of real number as a pair of contiguous classes). The teacher is stimulating this connection, to let the students recognise that the two sequences of rational numbers given by the areas of rectangles and the two sequences of rational numbers that approximate an irrational number as ℘2 have the same meaning as real numbers: separation elements of two contiguous classes.
The students recognise the construction of a real number, namely ℘2, and this is the third step in the construction of meaning of the area as separation element of two sequences of areas of rectangles. But it is not sufficient, because, if they are able to link the approximate measures of the area of

rectangles and a real number, they are unable to bridge the gap between the approximation process and the exact value of the area, namely between finite and infinite.
The students need to extend the possibilities of the real calculator in order to reach infinity, because at a certain moment Francesco says, substituting *n* with the symbol ° in the program of rectangles on the real calculator:

FRANCESCO: *I put infinite instead of a number n, and the calculator answers "undef".*

Instead of giving the program on the calculator a finite number of rectangles (under or over the graph), Francesco put a symbol he knows: "°", because he has the intuition that, as the process of area calculation increases its precision while increasing the number of rectangles, the most precise (the exact) value should be the last one, and the last is infinite. He is expecting a numeric value as a result of the calculation, and when the response of the calculator is "*undef*", he shares his surprise with his mates.
At this point, the teacher decides to use this sign "*undef*", given by the calculator, in order to help students in bridging the gap between finite and infinite. Therefore, she introduces an ideal calculator, which can do the same calculation and program of the real calculator, but without limitations, namely a calculator that does not give the answer "*undef*", but an answer in terms of numeric result. This metaphoric calculator can work with infinite values and do infinite computations.

TEACHER: *Now I am in an ideal calculator, which doesn't exist of course, and I imagine doing the calculation* (Figure 4a).
FRANCESCO: *At the end we will have a root.*
TEACHER: *A root?*
FRANCESCO: *No, a number … What is the name of those numbers?*
TEACHER: *Real. And do the two sequences coincide?* (Figure 4b).

Figure 4a-b

Through the ideal calculator, conceived as an instrument that does the same calculation as those done by the real calculator, but without limitations, neither in quantities, nor in the number of operations, it is possible to bridge the gap between finite and infinite. This is the fourth cognitive step in this activity: to recognise that the exact measure of the area is a real number, limit of the approximate calculations made by the programs of the area of rectangles.
What the teacher did in her semiotic game is introducing a metaphor, taken from the previous activity of the students with the calculator: the metaphor of ideal calculator, conceived with infinite potentiality and the aim to support the link between the exact area and real number, which are the same concept (Robutti et al., 2004). While introducing the metaphor, she turns her arm (Figure 4a), in order to show the process that goes on and on, without limitation. Then she uses another gesture, with the two hands very close, in order to shape two sequences (of areas) that coincide (Figure 4b).
The teacher's semiotic game is here very powerful, because this concept of real number is decisive

to approach the exact value of the area under a graph, with the use of definite integral, which will be introduced only the following school-year. But the laboratory activities made before the formal introduction of integral are the productive context where constructing meanings, based on the area under a graph as cognitive root of integral. The students are able to follow the teacher in this semiotic game and to bridge the gap between two finite sequences of approximate numbers and the real number (separator of two contiguous classes) as exact value of the area.

The third example comes from an activity at secondary level, with students attending the third year of secondary school (11th grade; 16-17 years old). They attend a scientific course with 5 classes of mathematics per week, including the use of computers with mathematical software. These students are early introduced to the fundamental concepts of Calculus since the beginning of high school (9th grade); they have the habit of using different types of software (Excel, Derive, Cabri-Géomètre, TI-Interactive, Graphic Calculus: see Arzarello et al. 2006) to represent functions, both using their Cartesian graphs and their algebraic representations. Students are familiar with problem solving activities, as well as with interactions in small groups. The methodology of mathematics laboratory is aimed at favouring the social interaction and the construction of a shared knowledge.
Here I present some excerpts from the activity of a group of three students: C, G and S. They are clever pupils, who participate to classroom activities with interest and active involvement. In these episodes there is also the teacher, whose role is crucial: he is not always with these students, but he passes from one group to the other (the class has been divided into 6 small groups of 3-4 students each). The excerpts illustrate what is happening after the group has done some exploring activities on one PC, where Graphic Calculus produces the graphs of Figure 5, with a given function (cubic) and the related function (parabolic), described by the "quasi-tangent" line in a point of the given function.

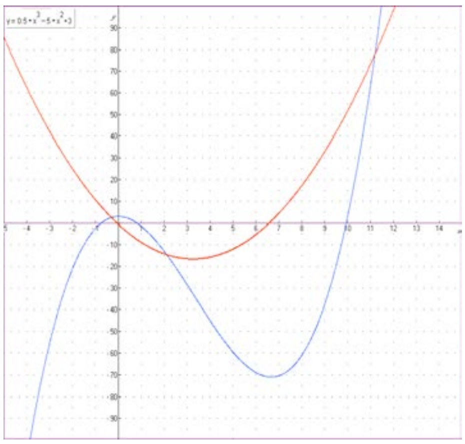
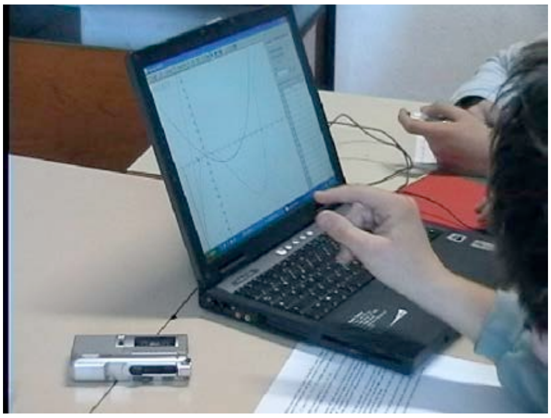


Figure 5

Their task is to explain the reasons why the slope of the "quasi-tangent" is changing in that way (a parabolic shape). The students know the concepts of increasing/decreasing functions, but they do not know yet the formal definition of derivative. Moreover, they are able in using Graphic Calculus and know that the "quasi-tangent" is not the real tangent, because of discrete approximations (it is in fact the secant line in two close points).

a b


c d

Figure 6a-b-c-d

Typically, their first explanations compose a semiotic bundle, where the speech is not the fundamental part, but the main component is the multimodal use of different resources, especially gestures, to figure out what happens on the screen. Figures 6 show how C captures and embodies the inscriptions in the screen through his gestures. More precisely, the evolution of the gesture from Figure 6a to 6d (from pointing to shape an interval in an iconic way) shows a concept that is expressed by words. It could be phrased in this way: "the quasi-tangent is joining pairs of points whose x-coordinates are equidistant, but it is not the same for the corresponding y-coordinates: the farther they are the steepest is the quasi-tangent". But the words of the student are: "Let us say towards this side. When, here, …when …however it must join two points, which are farther, that is there is less...less distance". C wants to express the fact that the interval Δx is always the same, while Δy changes, but his speech is not clear, while the gesture incorporates the meaning of different intervals. The gesture in Figure 6b is the basic sign (the thumb and the index getting near each other): in fact it starts a semiotic production strictly related to the construction of meaning for the quasi-tangent line. And this gesture will be shared among the other students of the group and the teacher too. In this episode the teacher echoes C's words, introducing a more technical word (delta-x), namely he gives the scientific name to the concept expressed by C and C shows that he understands what the teacher is saying. C's attention is concentrated on the relationships between the Δx and the corresponding Δy variations. Gesture and speech both contribute to express the covariation between Δx and Δy, underlining the case when the variations of Δy become bigger corresponding to fixed values of Δx.

a b

c d

Figure 7a-b-c-d

In the following the excerpts of a new episode, referring to Figure 7:

18 **TEACHER:** *Hence let us say, in this moment if I understood properly, with a fixed delta-x* (Figure 7a)*, which is a constant,…* (Figure 7b)
19 **C:** *Yes!*
20 **S:** *Yes!*
21 **TEACHER:** *It… is joining some points with delta-y* (Figure 7c), *which are near* (Figure 7d).
22 **C:** *In fact, now they* [the points on the graph] *are more and more…*
23 **TEACHER:** *It is decreasing, is it so?* [with reference to Δy]
24 **S:** *Yes!*
25 **C:** *…they* [their ordinates] *are less and less far. In fact, the slope... I do not know how to say it,……the slope is going towards zero degrees.*
26 **TEACHER:** *Uh, uh.*
27 **C:** *Let us say so…*
28 **S:** *Ok, at a certain point here delta-y over delta-x reaches here…*
29 **C:** *…the points are less and less far.*
30 **TEACHER:** *Sure!*

31 **S:** *...a point, which is zero.*

This episode shows an important aspect of the teacher's role: his interventions are crucial to foster the positive development of the situation. This appears both in his gestures and in his speech. In fact he summarises the results the students have already pointed out: the covariance between Δx and Δy and the trend of this relationship nearby the stationary point. To do so, he exploits the expressive power of the semiotic bundle used by C and S, using the basic sign (#21) to refer to the corresponding Δy and to its smallness nearby the local maximum x. In the second part of the episode (from #22 on) we see the immediate consequence of the strategy used by the teacher. C has understood the relationship between the covariance and the phenomena seen on the screen nearby the stationary point. But once more he is (#25) unable to express the concept through speech. On the contrary, S uses the words previously introduced by the teacher ("delta") and converts what C was expressing (in a multimodal way through gestures and speech) into words. His words in fact are an oral form of the symbolic language of mathematics: the semiotic bundle now contains the official language of Calculus (#28 and 31).

The episode illustrates the semiotic games of the teacher. Typically, the teacher uses the multimodality of the semiotic bundle produced by the students to develop his semiotic mediation. Let us consider #18 and #21 and Figure 7. The teacher mimics one of the signs produced in that moment by the students (the basic sign), but simultaneously he uses different words: while the students use an imprecise verbal explanation of the mathematical situation, he introduces precise words to describe it (#18, #21, #23) or to confirm the words of S (#30). Namely, the teacher uses one of the shared resources (gestures), to enter in a resonant communication with his students and another one (speech), to push them towards the scientific meaning of what they are considering. This strategy is developed when the non-verbal resources utilised by the students reveal to the teacher that they are in a zone of proximal development.

Discussion

Teachers able to use in very effective way the zone of proximal development where their students are active at a certain moment are not ordinary teachers. What are the elements to say this: the fact that these teachers have many years of experience (in the first and third examples, more than 25, in the second example, more than 15), and during these years they have used all the possible occasions to improve their professionalism, to learn new things, to put in discussion themselves. Teachers, who never stay still to wait some help from outside, but believe in their job and do it at the best. These teachers get involved in research groups with university researchers and participate to the teaching experiments also in the phase of planning activities, methodologies, use of materials and technologies. In many cases they are also teacher trainers, or are involved in the organisation of seminars, meetings, congresses and professional events. These teachers during their long experience always have the curiosity to experiment new technologies, in various ways: software or handheld technology, licensed or opens source, and have also the courage to experiment different methodologies of teaching. Due to their participation to research groups, they are very aware of the importance of process observation, so have the habit to observe their students during the activities and also themselves (in the video-recorded data).

For this reason, they gave us such interesting examples of semiotic games, which let the students able to construct meanings of mathematical objects.

Semiotic games are typical communication strategies among subjects, who share the same semiotic resources in a specific situation. They make use of different semiotic resources, integrated together in a multimodal kind of communication. Using them, teachers can develop semiotic mediation, which pushes student' individual knowledge towards the scientific one. Roughly speaking, semiotic games seem good for focussing further how "the signs act as an instrument of psychological activity in a manner analogous to the role of a tool in labour" (Vygotsky, 1978, p. 52) and how the teacher can promote their production and internalisation (Bartolini Bussi & Mariotti, 2008). A first point is

that students are exposed in classrooms to cultural and institutional signs that they do not control so much. A second point is that learning consists in students' personal appropriation of the signs meaning, fostered by strong social interactions, under the coaching of the teacher. As a consequence, their signs in the semiotic bundle (along with the relationships with other signs in the bundle) become a powerful mediating tool to construct meanings of mathematical objects. These signs can act as "personal signs", and the semiotic game of the teacher starts from them to support the transition to other signs, with more scientific meaning, till to the signs shared by the scientific community (conceptual mathematics, according to Sinclair and Schiralli, 2003, or culture, in the sense of Radford, 2006). Therefore, semiotic games constitute an important step in the process of appropriation of the culturally shared meaning of signs (think of "delta", for example). They give the students the opportunity of entering in resonance with teacher's language and through it with the institutional knowledge. In order that such opportunities can be concretely accomplished, the teacher must be aware of the role that a multimodal production of signs can play in communicating and in productive thinking. Awareness is necessary for reproducing the conditions that foster positive didactic experiences and for adapting the intervention techniques to the specific didactic activity (Arzarello & Paola, 2007).
In this paper I have considered teacher's interventions in small collaborative groups and in the whole class discussion. The typology of semiotic games to develop is not so different, what changes is the number of people who share the same semiotic bundle, and the role of the teacher is fundamental in this process of sharing.

In the first example, the main difficulty for the children is realising that the last number pronounced is the quantity there are counting, namely the cardinal number. In fact, at that age it happens that, at the question "How many", children answer with the counting sequence (because they know it by memory), but are not able to say how many with a number (the last of the sequence of counting). Another difficulty is to put in correspondence one sign (number) and one object. Often, counting, they go on with the sequence of number, without be conscious of that correspondence. For that reason, in the teaching experiment we chose only small numbers, till 4, in order to avoid complexity. The role of the teacher, in participating in the activity and entering the semiotic bundle of their children, is to help them in counting the objects, starting from zero and always adding one, till reaching 4 objects, then counting again leaving one object at a time, till reaching zero again (simulating to eat them). The game of the teacher in the protocol above consists in putting various signs into the semiotic bundle, to favour a perceptuo-motor way of learning: pointing with fingers to the objects, opening fingers while counting, and simultaneously saying numbers, showing various ways to represent the same numbers. She sometimes introduces a new sign that the students can reproduce and use (two fingers open, the word "two"), some others she echoes the word or the gesture made by a child ("like this"), or link two representations in a multimodal way (e.g., gesture plus word: "Two like this"). Children construct their meaning of the quantity "two" or "three", participating to the semiotic game of the teacher, they actually "play the game", in the sense that share with the teacher rules and conditions of the game: to represent the number of objects at their disposal in various ways. All their signs are in the semiotic bundle. But the teacher does also another important thing: she tries to move children from their individual meaning of "two", or "three" (Alberto is trying a physical correspondence between objects and fingers) and their scientific meaning, where numbers are abstract and represent quantities of things (even if things are different for kind and position). The passage from the physical correspondence objects-fingers to the word "three" is mediated by other experiences: the position of three fingers open, independently by which fingers one is using. And the teacher supports this passage, with gestures and interactions with her pupils (Figure 1c).
In the second example, students are at secondary school level and the teacher cannot guide their fingers, but she can guide their signs in the same way. In the semiotic bundle, during the activity of finding the area under a graph, we shall consider not only the sign introduced by students and

teacher, but also those introduced by the calculator. And the calculator is responsible of the results of calculation of the areas of rectangles that approximate the area requested. The students interact with the program in the calculator putting a sign (number of subdivisions in rectangles: 2, 5, 10, 100, 200, 1000, and so on) and receiving a sign (area of rectangles). In so doing, they notice that these areas approximate better and better the area under the graph, and they are able to formulate the conjecture that the most precise is the last, with the maximum number of subdivision in rectangles. One of them go further: he put the sign "°" as input in the program, because he has the intuition that, going on, the "last" is an infinite quantity of rectangles. But in the semiotic bundle shared by the class (they are in a collective discussion) there is an unexpected event: the sign of output by the calculator is "undef". So the last term of the process seems to be a non-sense, from the calculation point of view.
The teacher takes in her hands this opportunity, giving an echo of the two signs: the one from the student ("°"), and the other from the calculator ("undef"). She uses the metaphor of the ideal calculator for doing this. Having noticed that her students are in a zone of proximal development for bridging the gap from finite to infinite values, she forces the situation introducing the metaphor of ideal calculator in order to imagine the value to be obtained when using "°" as input. And she uses gestures to support her challenge: a metaphoric gesture with the hand, to simulate the ongoing process to increase the number of rectangles, and finally a gesture of juxtaposition of the hands to give the idea of converging sequences of areas of rectangles at the same value. The elements of the semiotic bundle (other than the inscriptions on the blackboard written before the episode here reported, visible in Figure 4) are then words and gestures of the teachers and students, but also signs from the calculator (an input and an output).
In the third example the students explain a new mathematical situation producing simultaneously gestures and speech (or other signs) within a semiotic bundle: their explanation through gestures seems promising, but their words are very imprecise or wrong and the teacher mimics the former but pushes the latter towards the right form. Teacher's intuition that his students are in a zone of proximal development let him choose a strategy to enter their semiotic bundle with both gestures (repeating students' gestures) and words (introducing new terms: e.g. "delta"). This choice, to repeat a sign of the students and to introduce a new sign, more precise, seems to be winner in the process of constructing meanings, because then students use the new sign introduced by the teacher. Therefore, the teacher uses one of the resources already present in the semiotic bundle (gestures), to share an element with the students, and introduces another one (speech), to direct them towards a mathematical term. These two resources are used together in a multimodal communication.
If this study can give us information about the use of the zone of proximal development in a semiotic game chosen by an expert teacher, there are still many open problems that it could be interesting deepening. One of these problems is: are ordinary teachers able (and also interested in) to use semiotic games in the zone of proximal development of their students, in order to support their construction of meanings? Or their are more interested in frontal lessons, where they are the only active people in the classroom, but can be sure to finish the curricular program of their class, without any interests in the students' processes of construction of meanings? And: is it possible to train normal teachers in this form of discussion that uses semiotic games, with the introduction of multimodal forms of communication, starting from the semiotic resources present in the students' activity?
One possible future research in this field seems to be the observation of teachers during their professional pre-service training, during which the videos and materials of this study are used in order to let them be aware of the semiotic games. In this way, a productive link between research and teaching practice can be constructed.

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