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Graphic calculators and connectivity software to be a community of mathematics practitioners

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ABSTRACT

In a teaching experiment carried out at secondary school level, we observe the students' processes in modelling activities, where the use of graphic calculators and connectivity software gives a common working space in the class. The paper shows results, some in continuity with others, which emerged in the last ICMEs and some new, and offers an analysis on how the novelty of the software can introduce new ways to support learning communities in the construction of mathematical meanings. The study is conducted in a semiotic-cultural framework that considers the introduction and the evolution of signs such as words, gestures and interaction with technologies, in order to understand how students construct mathematical meanings, working as a community of practice. The novelty of the results consists in the presence of two technologies for students: the "private" graphic calculators and the "public" screen of the connectivity software. Signs for the construction of knowledge are mediated by both of them, but the second does it in a social way, strongly supporting the learning community work.

KEY WORDS: connectivity, community of practice, multimodality, mathematical laboratory, humans-with-media, modelling, function, calculator, sign, meaning, semiotic-cultural approach.

1 Introduction

In recent years we have a new generation of technology at our disposal, for teaching and learning, with respect to others present (computers and calculators). We are referring to a technology that enables us to work together, to share the products of our solving problem strategies, to discuss around a theme, to give or receive feedback on our work in real-time: in a word, to be a community of practice (Wenger 1998), or, to be more precise, a learning community (Bielaczyc and Collins, 1999). In a learning community the goal is to advance the collective knowledge and nad in that way to support the grouth of individual knowledge.

The defining quality of a learning community is that there is a culture of learning in which everyone is involved in a collective effort of understanding, while the goal of a community of practice can also be different from educational.

A community of learners is something more than a community of practice, because the members do not interact spontaneously; rather, they are induced by an educational aim. This community has the aim to socially construct knowledge and to share the process of construction. Namely, people involved in it are active together (in a synchronous or an a-synchronous way) and have common objectives. We can consider a class of students or teachers as a learning community (Jaworski et al. 2007), for example. This kind of community can work via distance learning, as reported in literature (Borba and Zulatto 2006; Jonassen 2007), in the class with face-to-face interaction (Dougherty and Hobbs 2007; Robutti et al. in press; Hivon et al. 2008), or in a blended approach (face-to-face and distance). What can change from the one to the other approach is the kind of involvement of the participants, more or less participation in the interaction, and the amount of synchronous versus a-synchronous work done. And the common feature is the use of a *new* technology, which is not neutral in the construction of mathematical knowledge. In fact this technology cannot only change the class work, but also the relationships between pupils and mathematics, between teachers and pupils and among pupils themselves (Hivon et al. 2008). As Borba writes: “if process is considered, I believe that we may be on the way to discovering a qualitatively different medium that, like the “click and drag” tool of the dynamic geometry, offers a *new* way of doing mathematics that has the potential to change the mathematics produced” (Borba 2005, p.175, emphasis added). Borba refers in his article to the use of a platform to carry out an online course for teachers in his country. These teachers interact through the platform utilities such as chat rooms, forums and the use of geometry software. The integration of what he calls “old” information and communication technology and “new” one (considering the platform, e-mail and Internet for distance courses) is the difference between the usual technology and a new tool that forces the community to work together, with the coordination of a University professor. And the intriguing challenge for mathematics education is how to introduce and manage these utilities, “which are no longer ‘simple’ tools but *new* working systems” (Hivon et al. 2008, emphasis added). So my interest lies in exploring the

ways in which connectivity software as TI-Navigator is *new*, with respect to the usual technology a class has used until now, like computers or calculators. In fact, even if calculators connected among themselves with a public display in the classroom (thanks to TI-Navigator) are different than a platform, e-mail and Internet, their use is similar to some web based courses. In both cases what is possible is connectivity among students and with the teacher, working together, sharing results and observing others' processes.
The hypothesis presented in this paper is that this connectivity software is *new* in comparison with calculators, in the same way as social networks are *new* in comparison with a static website. This is because it supports the activation of a learning community in a class, where participation, sharing, and the interaction of all students are at the basis of their activities. The new tool consists of a set made of the following: graphic calculators for the students, the teacher's computer with connectivity software installed, and a fire-wire connection between the students' calculators and the teacher's computer, made by specific hubs linked to them. The hubs and the software carry out the connection between the calculators and the teacher's computer. The possibility of sharing the work done on the calculators in the classroom, gives a new resource to construct mathematical knowledge by a community of learners.

A second research hypothesis is that the software is not only a novelty from a technological point of view, but also from a cognitive perspective. In fact, students' processes are different from those present in a usual class working only with calculators or computers, because there is the possibility to share results and discuss them in real-time. What this technology provides is the opportunity for students to do mathematics together and to look at each other's productions, while doing their own activity. This plural interaction between learners and technology makes the difference in comparison with other tools, such as calculators alone, used by individuals or small groups.

2 Teaching experiment and methodology

The research group I belong to is made of two teacher-researchers (Silvia Ghirardi and Marialuisa Manassero), a master student (Maria Teresa Ravera), and myself. We carried out two teaching experiments at secondary school level (two 10th

grade classes), one of which is described in this paper. The students solved modelling activities, working in small groups with graphic calculators TI-84® (Texas Instruments) and the connectivity software TI-Navigator® (Texas Instruments).

In TI-Navigator, the public display consists of a common Cartesian plane (called Activity Center), to which each student and the teacher can give their personal contribution, inserting mathematical objects as points, lines, and so on (Figure 1) from their calculators, thanks to the connectivity. Another environment is the Screen Capture (Figure 2), through which all the students' screens are simultaneously captured and visible on the teacher's screen. Both the environments can be projected on a big screen if the teacher's computer is connected to a video-projector. In this paper I show protocols referring to the first environment (Figure 1), even if we use both of these environments. The use of these two environments has been made in function of the activities.

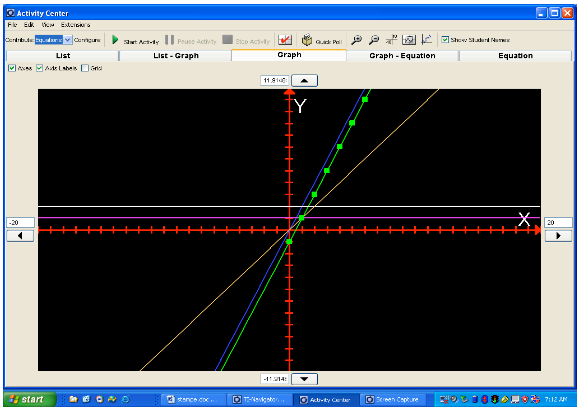

Figure 1: Activity Center

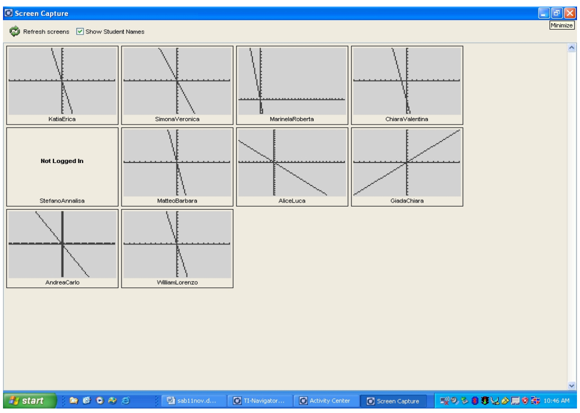

Figure 2: Screen Capture

This software is substantially different from the standard equipment, which is made up of computers or calculators and is used by groups (or individuals), who follow their calculator screens without any information about what is happening in the other groups. With the usual equipment, if the teacher wants information on the processing done by the students he or she has to pass from one group to the other, discussing with each of them without involving other groups. If students want to share their results with the rest of the class, they have to describe them in a class discussion. With Navigator each group may follow his work and simultaneously also other groups' work, looking at the big screen. So, the moment of group work and that of discussion are more integrated together. The teacher her/himself may therefore remain in a central position, following every job on the big screen, discussing with a single group or guiding a class discussion where everyone can take part, because information is shared among all the students.

The methodology of the teaching experiments follows the approach of *mathematics laboratory*, developed in the Italian mathematics education community and presented at ICME10 in various ways: discussion group (Chapman and Robutti 2008), CD-ROM, presentations, and a booklet of recent Italian mathematics education research. A mathematics laboratory is a methodology based on various and structured activities, and aimed at the construction of meanings of mathematical objects. A mathematics laboratory activity involves people, structures, ideas, as well as in a Renaissance workshop, in which the apprentices learn by doing, seeing, imitating, and communicating with each other, namely practicing. In the activities, the construction of meanings is strictly bound, on one hand, to the use of tools, and on the other, to the interactions between people working together.

The students work together in small groups (two, or maximum three members); each group uses one graphic calculator connected to a network hub, which communicates with the teacher's computer, via a wireless connection. The choice to give one calculator to each group and not one per student is made to encourage collaborative work among pupils. Each group has to complete one worksheet on paper, containing questions about the activity that is being solved. Each activity is carried out in a class equipped with the following technology: a computer connected to a projector directed onto a big screen, the calculators and the hubs.

The connectivity software supports the exchange of data between the teacher and the groups.

The activities are followed by collective discussions conducted by the teacher, with the modality of mathematical discussion (Bartolini Bussi 1996). The alternation between group activities and discussions among the whole class has been done in order to share results immediately (in the same room, in the same unit of lesson) among the community of learners.

The role of the teacher is to coordinate social activity in order to lead this construction from subjective meanings towards objective cultural meanings without giving responses, rather by letting the students feel free to discuss, compare, conjecture, imagine, and connect various ideas and concepts. A University masters student is present in the class, in order to record all the activities with a video camera. A researcher (the author) is present during the activity, and helps the teacher in observing the groups or guiding the discussion.

Teacher and researcher planned the activities together, and inserted them in the class curriculum for the year. In the classroom they worked together with synergy, not giving students the right answers, rather posing questions to help them if they have difficulties; making arguments, inviting observation of all the elements of the problem, supporting conjectures, and considering mistakes as occasions to learn. Namely their role is to help and support students, not to judge or to penalise them. In this way, all students can participate in the task, solving problems and constructing meanings of mathematical objects, feeling part of a community of practice.

The topic developed in the teaching experiment is related to the so-called *mathematics of change* (Kaput and Roschelle 1998), with the aim of developing competences related to the number sense (Sowder 1992), the graph sense (Robutti 2006) and the symbol sense (Arcavi 1994) in an integrated approach, as in other studies I have conducted at various school levels (Robutti 2007; Robutti 2009). At the 10th grade one component of the curricula is: functions of first and second degree, with their representations. As cognitive roots (Tall 1989) for the description of a function we choose the qualitative concept of invariance and the quantitative concept of slope (as ratio of increments) and its variation. Related to these roots, we also use other concepts such as: domain, sign, intersection, zero, parallelism, and so on. The activities are centred on families of functions, principally linear, quadratic, and exponential and the construction of meaning starts from modelling problems.

The table shows the plan of activities in the teaching experiment in Silvia Ghirardi's class.

Activity 0	Introduction to the use of graphic calculator in interaction with *Activity Center*.	2 hours
Activity 1	Linear function: straight line	2 hours
Activity 2		4 hours
Activity 3	Quadratic function: parabola	2 hours
Activity 4	Problem solving on linear, quadratic, exponential function	2 hours

There are two key elements of this teaching experiment: the integration of different technologies, managed by students and teacher, and the collective construction of meanings and solutions. The different tools used are: paper and pencil, calculators with various environments (graphs, numerical tables and calculations), software with Screen Capture and Activity Center. During discussion, another tool is the catalyst of attention and the mediator for the collective construction of meaning: the big screen. The big screen contains the work done by all the groups, and it supports comparison among solutions, the invariance of methods and results, or the visibility of errors. It also gives teacher the possibility to guide the discussion, referring to some particulars of the solutions projected, with gestures, words or other signs and so enlarges the semiotic activity in the class. Having the groups' production at their disposal, teacher and students can refer not only to products, but also to processes of solution and construction of meanings, concentrating the attention on what is projected on the screen, and not having to describe it on a blackboard.

Data of the teaching experiment are: written materials (worksheets), the calculator screens or teacher's computer screen, and videos. These data are analysed in terms of students' semiotic resources (language, signs, gestures, actions on artefacts), their introduction and evolution.

3 Calculators in Mathematics Education

According to the main issues of the Rome ICMI Symposium 2008 we can say that every tool introduced in the classroom influences the students while constructing mathematical meanings, and mediates this construction at a cognitive level. This is confirmed by a range of studies (e.g., Bartolini Bussi and Mariotti 2008; Borba and Villarreal 2005; Noss et al. 1997). Traditionally, from the Dienes blocks to the first calculators, from the programming languages of computer science (Logo, Pascal or Fortran) to the mathematics software as CAS, from the spreadsheets to the microworlds, the use of technology in mathematics has mostly been individual or reserved to small groups (two-three students per each) (e.g., Ferrara et al. 2006; Laborde et al. 2006). Of course, it does not mean that the activity cannot be discussed in the class in a collective way, managed by the teacher, but that everyone's work is not visible to everybody. In some cases "the personal or private use of the tool in fact served to breakdown group communications" (Leung 2008, p. 228). The main difference between this use of technology and the one described in this paper is the affordability of the new technology being used, which "acts" in a new way, because of the new interface that allows public display of students' work and connectivity among students.

The use of calculators for doing mathematics is recent: in the last ten to fifteen years lots of experiments have been carried out, in order to investigate the impact of these tools on learning mathematics with different methodologies, mainly at secondary school level. In previous ICMEs there have been several studies, from various perspectives, on computers and calculators and their impact on learning mathematics. They described the kind of tool used, the possible mediation in constructing meanings, the methodology induced by tools, the use of relevant theoretical frameworks to analyse the role of technology in teaching and learning. For example, Lagrange, in ICME8, reports on teaching experiments made in France in order to observe different mediation of CAS in algebra activities, comparing the work done by a class on computers in a laboratory and on calculators in the classroom. His results show that (Lagrange 1997):

- Computer sessions are less frequent, because the laboratory facility is shared amongst several classes. Therefore the computer laboratory sessions have to be alternated with sessions in the classrooms, carried out with traditional tools and methodology. In contrast with using calculators, students can decide at any time to use computer algebra system during each algebra session in the classroom.

- In a computer laboratory it is quite difficult to concentrate the students' attention on a whole-class activity. Whereas in the classroom, the students can participate in a class discussion and pay attention to the work being done on the blackboard, even if they are using portable devices. "Very often, the discussion started about the output on the screen of a pupil's device, when he compared with the screen of the teacher as displayed on the wall, and the other students participated actively" (Lagrange 1997, p. 116-117).

- "The sessions in a computer room were quite long periods of autonomous problem solving. The teacher could very hardly control the advancement of the research in every team. Therefore the work of the students often had not the significance that the teacher expected. Also, the students could rarely make clear this significance. Therefore the collective discussion about this work, that took place in the next session, was very necessary to give the findings of the students a mathematical meaning. With the TI-92, the research steps were much shorter, and students' attention was constantly directed toward the collective advancement of the task. Therefore, the students sometimes could not give this research enough application and reflection, because they were pressed to advance it" (Lagrange 1997, p. 117). In this situation, the teacher can easily follow, redirect, and guide the work done by the students, more than in a computer session.

Other issues which have been raised at ICMEs have been the use of computer or calculators for looking at real phenomena and modelling them, through graphs, or making motion experiments with sensors and calculators and investigate on graphs, making prevision and checking conjectures, describing shapes and relating graphs and number tables of physical quantities, or using graphic calculators to investigate on families of functions, describing the changing of graphs related to the changing of parameters in the equation. Common features of these teaching experiments are that students work in small groups, observing experiments on video, or making them by themselves, collecting data and inferring on models of motion, or solving problems posed by the teacher or by themselves in certain cases. For example, Hudson (1997) refers on "the quite exceptional power of the medium to support and sustain collaborative learning. The fact that groups of 14-year-olds consistently interacted with each other and the system for thirty minutes at a time to sketch, reflect on and discuss graphs of motion, in relatively unsupervised conditions, almost came to be taken for granted during the classroom trials" (Hudson 1997, p. 109). And his evidence is consistent with other results, reported by Teasley and Roschelle (1993), namely the fact that in ordinary circumstances, one cannot imagine two 15-year-olds sitting down for 45 minutes to construct a rich shared understanding of velocity and acceleration. The author shows with examples of protocols the importance of feedback (positive or negative) and the high level of interaction existent in students' groups, not made only of words, actions on the computers, but also of gestures for pointing parts of the screen or for showing shapes and ideas (Hudson 1997).

Other authors refer to the use of graphic calculators for exploring and solving problems on the theme of functions. For example, Borba (1997) at ICME8 discussed the possibilities for graphic calculators to enhance the mathematical discussion in the classroom, reorganising the way knowledge is produced. In the teaching experiment, students use calculators to solve investigation problems (for example, the role of parameters in the equation of a parabola), and then discuss together, coordinated by the teacher. First students use their calculator to explore an open problem in small groups, then intensively discuss on their findings, using gestures and language, and projecting the screen of their calculator through a data-projector, in order to show their work and results. What the author shows in his paper is that calculators support an intensification of the discussion in classroom. "In a sense the calculators represented a new 'authority' in the classroom, in addition to the teacher, as the students found strong support for their positions in the graphical results of their experimentation" (Borba 1997, p.59). They pursue different paths of inquiry, facilitate more independent investigations and generations of conjectures contributing to a certain sense of ownership that may also partially explain the intense discussions. As also suggested in other studies (Sutherland 1993), students feel more comfortable when they can develop their investigation without emotional pressure from teachers.

Other authors refer of the use of graphic calculators for modelling situations of motion or of other phenomena involving physical quantities (e.g. pressure, acceleration, temperature, and so on). These experiments take place in a laboratory in the classroom or outside, for example in an amusement park, measuring atmospheric pressure on a big wheel (Arzarello et al. 2007). Here mathematics laboratory is intended to be a "room without walls", in the sense that methodology is what makes the difference and it is not important where the experiment takes place. However, what is important are perceptuo-motor activities, learning by doing and interacting, observing and collecting data, and then interpreting them in tables or graphs. The experiments are generally carried out with students divided in small groups, which experience a phenomenon and model the relations between the quantities involved, using various kinds of devices. The analysis of students' cognitive processes is made with post-Vygotskian perspectives, along with new theoretical elements such as embodiment, analysis of gestures, and role of imagination. An important issue is that body, language, and instruments mediate and support the transition of students from the perceptual facts to the symbolic representation, also supported by the production of metaphors during the activity (Arzarello et al. 2007).

Frameworks used by researchers in past decades have provided a basis for investigation right up to the present day. Some of them go towards the analysis of gestures and other signs, in a cultural-semiotic perspective (e.g., Arzarello et al. 2009; Edwards 2009; Radford 2009), others analyse the double process of instrumentation and instrumentalisation, following the instrumental approach introduced by Rabardel (e.g., Trouche and Hivon in press).

The studies described above show that:

- the use of technological devices such as calculators has been made in an individual way, or in small groups of students;
- the use of discussion among students, coordinated by the teacher, has been considered essential to share results and solution processes;
- sometimes portable devices are more useful than computers, since they are constantly accessible, not just during particular hours of the week;
- the various environments given by the calculators are catalysts of the students' attention, for many reasons;
- the role of feedback is essential (it can reinforce an idea, or helping in understanding a mistake), since it gives reason of the work done, without requiring a teacher's intervention.

In the next section some theoretical approaches are presented in an integrated way, in order to use their features for analysing the perspective of this study: first of all, the frame of *humans-with-media* and then the *multimodality of production*. They are described and used in the perspective of social interaction in a community of practice, where technology is considered part of it, and all the communication ways are taken into consideration for a semiotic analysis of students' production.

4 Integrating theoretical perspectives

Having a community of practice in a classroom, with students working together is possible thanks to this kind of activity, teacher's support, and methodology. Mathematics education research gives examples in this sense, particularly oriented towards the so-called learning community, namely groups of students (or teachers), oriented on a common task, in which they are engaged and have the possibility of learning (Jaworski et al. 2007). In fact, communities of practice are formed by people who engage in a process of collective learning in a shared domain of human endeavour: they share a concern or a passion for something they do and learn how to do it better as they interact regularly. So, a community of practice is not merely a club of friends or a network of connections between people, rather it has an identity defined by a shared domain of interest. In pursuing their interest in their domain, members engage in joint activities and discussions, help each other, and share information. They build relationships that enable them to learn from each other. A website in itself is not a community of practice, because it does not imply interactions. In fact, members of a community of practice are practitioners, who interact developing a shared repertoire of resources: experiences, stories, tools, and ways of addressing recurring problems (Wenger 1998).

Calculators combined with software Navigator are a support to create a community of practice in the classroom. Some research groups have experienced the use of this software, producing findings that emphasise the role of the teacher in orchestrating more instruments in the class (Hivon et al. 2008), recognising a strong involvement of the teacher in governing complexity and a sure advantage for students in collaborative work, supported by the software and by the methodology of discussion (Robutti et al. in press). In particular, it is recognised that the role of the public screen as catalyst of attention by students and teacher, not only as a traditional blackboard (that is seen as a "inert intermediary between the speaker and his/her advisory", according to Legrand (1993)), but actually as a dynamic, and not inert, space of mutually exchanging information flow in the class.

I will introduce two approaches in order to describe the interaction in classroom: *humans-with-media* and *multimodality of production*. Humans-with-media is a theoretical approach that takes both the subjects and the tools involved in a mathematical activity into account (Borba and Villarreal 2005). It is based on two ideas: first, the construction of knowledge is made in a social way by subjects working together; second, the media involved are part of this construction, because they collaborate to reorganise thinking, with a different role than the one assumed by written or oral language. This point of view focuses on the community of learners (small groups, as well as the whole class or bigger groups), along with the tools, and overcomes the traditional dichotomy between humans and technology. It suggests that learning is a process of interaction among humans as a group, including tools, which are seen as 'actors' in a collective thinking, in the sense that they are carriers of a historical-cultural heritage and mediate the construction of knowledge. Therefore media interact with humans, in the double sense that technologies transform and modify humans' reasoning, as well as the fact that humans are continuously transforming technologies according to their purposes.

Studies in neuroscience tell us that the sensory-motor system of the brain is multimodal rather than modular (Gallese and Lakoff 2005): "an action like grasping ... (1) is neurally enacted using neural substrates used for both action and perception, and (2) the modalities of action and perception are integrated at the level of the sensory-motor system itself and not via higher association areas." (Gallese and Lakoff, p. 459). "Accordingly, language is inherently multimodal in this sense, that is, it uses many modalities linked together—sight, hearing, touch, motor actions, and so on. Language exploits the pre-existing multimodal character of the sensory-motor system." (Gallese and Lakoff, p. 456). If the sensory-motor system of human brain is multimodal, also human activity is multimodal, and we can analyse all the modalities in order to understand cognitive processes. (Arzarello and Edwards 2005). During the mathematical activities with media, students produce a variety of signs as words, gestures, and actions on the tools, interactions, written or oral signs of whatever nature.

These two approaches are the basis for my analysis of the process of knowledge construction through a semiotic-cultural frame, as developed in my research group (Arzarello et al. 2009). The experience of learning together (learning to be with others in mathematics, as written by Radford (2006)) with the use of a technological tool, can be described by a frame which takes the multimodal production of the students into account, as well as the teacher and the technology itself. In this approach, learning mathematics is a matter of being-in-mathematics (Radford 2006), living in a classroom as a community (Jaworski et al. 2007), working together and sharing activities and results.

In this paper I analyse the cognitive activity of the students, describing the signs involved; gestures, words, gazes, actions on the paper, in the air, on the artefacts, interactions with the teacher, and whatever sign they use in their activity (Arzarello et al. 2009). In doing this, I make use of the *semiotic bundle* as a model which takes the multimodality of production of signs by the students or the teacher during an activity into account (Arzarello 2006). The semiotic bundle of a group of subjects must not be considered as a juxtaposition of signs, but a systemic structure to describe the activity of the group, in terms of the signs used and their relationships. This structure is dynamic and shows the evolution of subjects' activity over time, showing the variety of signs involved, their relationships and their transformation. The relationships concern signs produced in different times: for example, a sign made by a subject can influence the sign made by another subject, or a sign is transformed into another sign (think of a gesture converted into a written sign on the paper) by the same subject, or two signs which are made simultaneously by the same subject or by two different subjects. This description passes through dynamic elements of evolution in time of the signs used (description as a movie), along with the complex interaction at certain instants (description as a picture), giving reason of the multimodal aspects of the learning processes (Arzarello 2006). Some of these signs are particularly significant, because they introduce new elements (previously not present) for the construction of meaning: in this sense, they are considered semiotic means of objectification (Radford 2006), because they introduce a new element of knowledge, not present before.

With signs we also include those coming from the technology, considering not only the community of students and teacher, but also the media (Borba and Villarreal 2005). Using the semiotic bundle we can describe the multimodality at an instant of the activity (in a static way, as a picture), or the evolution over time of signs and their mutual relationships (in a dynamic way, as a movie). Within this framework, it is interesting to describe when and how the students, during a group activity, make something visible which was not visible before. Namely, how they introduce a new piece of meaning in the construction of knowledge.

We know that students' processes evolve both individually and collectively, but we are particularly interested in those that evolve collectively. With this in mind, some research questions of this study are as follows:

- What are the social ways to produce knowledge in this teaching experiment?
- What are the features of the technology which influence and support the collective production of knowledge?

- What are the teacher's uses of the technology to support the collective production?
- Is there an "added value" to the teaching/learning processes, thanks to the new technological devices (connectivity software and calculators), with respect to traditional technological equipment (only calculators)?
- How do the rhythms change with respect to a more traditional activity?

Considering that the novelty of the software is not a mere technological novelty, rather it is also a cognitive novelty, in the sense that it has a deep impact on the students' cognitive productions, I want to analyse the elements of this novelty. In fact, these elements can influence research into mathematics education, but also teaching methods and learning ways. Therefore it may be possible to discover new and different features in the use of calculators combined with connectivity software: they deal with communication, sharing, working together, sign production, practices in a community, rhythms of working.

5 Activities and discussion

The activity presented here is one of the first of the teaching experiment, after an introduction on the software and some exercises with the environments of the calculator. The students have to find the various terms of this sequence as coordinates of points, and to send them to the public screen, where they are represented altogether (Figure 3).

Consider the point P_0 (0, -1). Find the coordinates of P_1 , by adding 1 to the abscissa of P_0, 2 to its ordinate. Represent the point on the Cartesian plane. Find P_2, adding 1 to the abscissa of P_1, 2 to its ordinate and represent P_2. Now find P_3, P_4 and so on. Write the sequence of the points P_0, …, P_6. How do you pass from one point to the subsequent? What are the coordinates of P_{10}? Explain how to determine P_{100} and what is the rule.

The aim of the activity is the model (linear) of the situation, expressed not only in a recursive form ($x_n=x_{n-1}+1$, $y_n=y_{n-1}+2$; with $x_0=0$, $y_0=-1$), where each element of the sequence is written in function of the previous element, but also with a formula ($x_n=n$, $y_n=2n-1$), where each element of the sequence is determined in function of its position in the sequence. Both the symbolic expressions, as well as the meaning of the relation among the abscissas and ordinates of the points in the sequence, are important.

The students, divided in small groups (of two or three), carry out the activity with one graphic calculator connected to the public screen and one papersheet to be filled in. At the end of the group work a discussion takes place. First the discussion has the aim of writing the formula (this is the last question on the paper-sheet), then of describing the model from a graphical point of view. In the group session nothing particularly new in the construction of learning has been introduced with respect to other experiences described in #3. During the class discussion students and teacher refer to the Activity Center (Figure 3, projected on a big screen), to which every group has previously sent the results (coordinates of the points of the sequence).

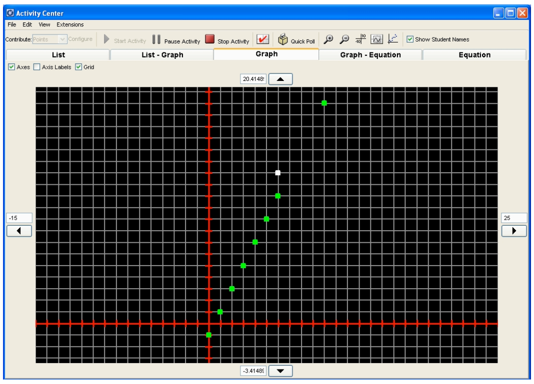

Figure 3: the Activity Center

Figure 4: Gesture in the discussion

The Activity Center is the catalyst of gazes, gestures and words of the students during the discussion, and it supports the teacher in the mediation of meaning construction. The teacher starts the discussion with attention to the objects on the Cartesian plane (Figure 3). In the following, ***Th*** means teacher, ***St*** a group of students answering together, and other names denote particular students.

1. ***Th:*** *What do you observe in the points you found?*
2. ***Ca:*** *They are a straight line* (*Figure 3*).
3. ***Th:*** *Yes, they are a straight line. Except that one, which seems to be out of its place. Why is it out of place?*
4. ***Ma:*** *We calculated incorrectly.*
5. ***Th:*** *You calculated incorrectly. Which coordinates does that point have? The one which seems out of place?*
6. ***Ma:*** *(6,13).*
7. ***Th:*** *Why doesn't it work?*
8. ***Ma:*** *Because I added ... I had to put (6,14), then it resulted to be more in this direction.* [with a gesture he shows the direction, which is wrong (Figure 4)].
9. ***Th:*** *(6,14) do you agree? Also you put (6,14)?*

The teacher goes back to the first point of the sequence, in order to understand the process Ma and his classmate followed to obtain such a wrong value (6,13). Along with the class, the teacher comes to the point: Ma and Ba always added 2 to the abscissa and 1 to the ordinate, but exchanged x and y, obtaining a wrong table of numbers, with a pattern in itself that makes sense even if not correct (Figure 5).

X	***y***
0	**2**
1	**4**
2	**6**
3	**8**
4	**10**
5	**12**
6	**14**

Figure 5: Ma's wrong table

Let me now attempt to analyse the elements of the semiotic bundle which *are* important in correcting the answer given by the student. When examining the group work, there are two more variables during the discussion: the teacher involved in the discussion and the public screen (Activity Center). For this reason, the multimodality involves also the teacher's production, and the humans-with-media is intended as: all the students, the teacher and the technologies (calculators and software) used.

The public screen offers the environment to share the previous experience, where every group worked with the calculator without knowing what the other groups were doing. Therefore the discussion is a process of sharing results and justifying them with a rule, a pattern the students choose to send such points. The teacher has the role of mediator in this discussion and supports this students' evolution in sharing and justifying. In this way, the description of the semiotic bundle takes into account this evolution and looks at the signs that mark its various steps.

At the beginning of the discussion, the teacher's question (#1) calls for attention to the public screen, where the graph shows the points of the sequence sent by the groups. The students answer looking at these signs (the points) and introduce a new sign, the word: "a straight line" that represents a pattern through the points, and marks the first step in the construction of knowledge. Another sign is remarked by the teacher, who says that one of the points seems to be "out of place" (#3). The group responsible of this point says that it is not correct (#4), because they calculated it wrongly. The graphical representation on the public screen actually gives feedback that can be used by the teacher not to remark upon a mistake, rather to ask why it is out of the pattern (#3, 5 and 7).

Another sign marks the second step: Ma's gesture that correctly identifies the direction for the point to be moved, in order to have it in the right place (Figure 4). Thanks to the public screen that represents all the points given by the groups, the student is aware of his mistake and makes this gesture, to correct the mistake itself. In fact, he compares the different position of his point with respects to the points of other groups. Then, he makes a gesture correct in the direction (the correct point should be aligned with the others), but wrong in coordinates. The student is influenced in fact by the process followed during the group activity. This is the reason of the new mistake. Here the signs introduced by the teacher are strategic, to understand why the point is wrong.

The third step begins with a question directed to the class as a whole: “do you agree?” (#9). This question, along with the repetition of the wrong coordinates (6,14), and the word “why” later, are the signs introduced by the teacher into the semiotic bundle that marks the beginning of a new reasoning. In a process of going back, till arriving to the first point of the sequence, the teacher supports the students in comparing the correct with the wrong coordinates, discovering the pattern followed by Ma and Ba. This comparison is not only useful to this pair of students, but also to the others, who can also be aware of the processes.

In the semiotic bundle, the first signs are the straight line and the point “out of place” in the graph on the public screen. This one is transformed into Ma’s gesture (Figure 4), to “put the point into place” aligned with the others, and this gesture is then replaced by the coordinates (6,14) of the new point (#8). These coordinates claim for an explanation rich of signs (words, numbers, Ma’s table (Figure 5, and so on), which lead the class to understand why Ma sent them and why they are wrong. So the semiotic bundle is made of gesture, words, and mathematical signs written, spoken and represented on the screen. These signs are related to each other, because one is substituted by another or is transformed into another. The role of the public screen is strategic, to make the contributions of all the groups visible, with the possibility of discussing them immediately. The signs coming from the public screen are part of the semiotic bundle, as well as those introduced by the teacher. Therefore its presence is not neutral, and it gives the students the possibility of sharing results, having immediate feedback, and introducing new signs (the straight line, or the point not aligned). The differences between the group results offer the teacher the occasion to discuss why there are such differences and to analyse the students’ processes in obtaining them. With the calculators alone, without the connectivity software, this sharing would have been more difficult to obtain, for reasons of both time and space.

The discussion continues with two aims: first, to have a symbolic way to write the pattern of the points; and second, to find a general way to represent a straight line, linking its graphical and symbolic features. Students are able to: say this expression: “you always add 1 to x and 2 to y”, calculate some specific points, write the rules in symbols. After those results, the teacher guides the discussion towards a formula that gives the coordinates of whatever point, knowing its place in the sequence. She introduces the sign *tot* (a generic point P_{tot}), in order to give generality to the reasoning, then she asks for the coordinates of a particular point P_{15}. The strategies followed by the students are twofold: to add to the abscissa of the point, the abscissa minus one, or to double the abscissa and then subtract 1. The final strategy used by most of students is: “The double minus 1”. The passage from natural language to symbolic expression (Figure 6) is shared, and the formula applied to a various set of points.

$$\boxed{P_m\,(m;\ 2m-1)}$$

Figure 6: the formula written by a group

The discussion continues, focusing the students’ attention onto the public screen, where the representation of the points is projected. The aim is now to transfer the construction of meaning on the graph itself, in relation to the formula discovered previously (Figure 7). The discussion begins with some considerations about the number of conditions to give, in order to identify a unique straight line. The students themselves say they need two points or one point and a rule, as we can see below.

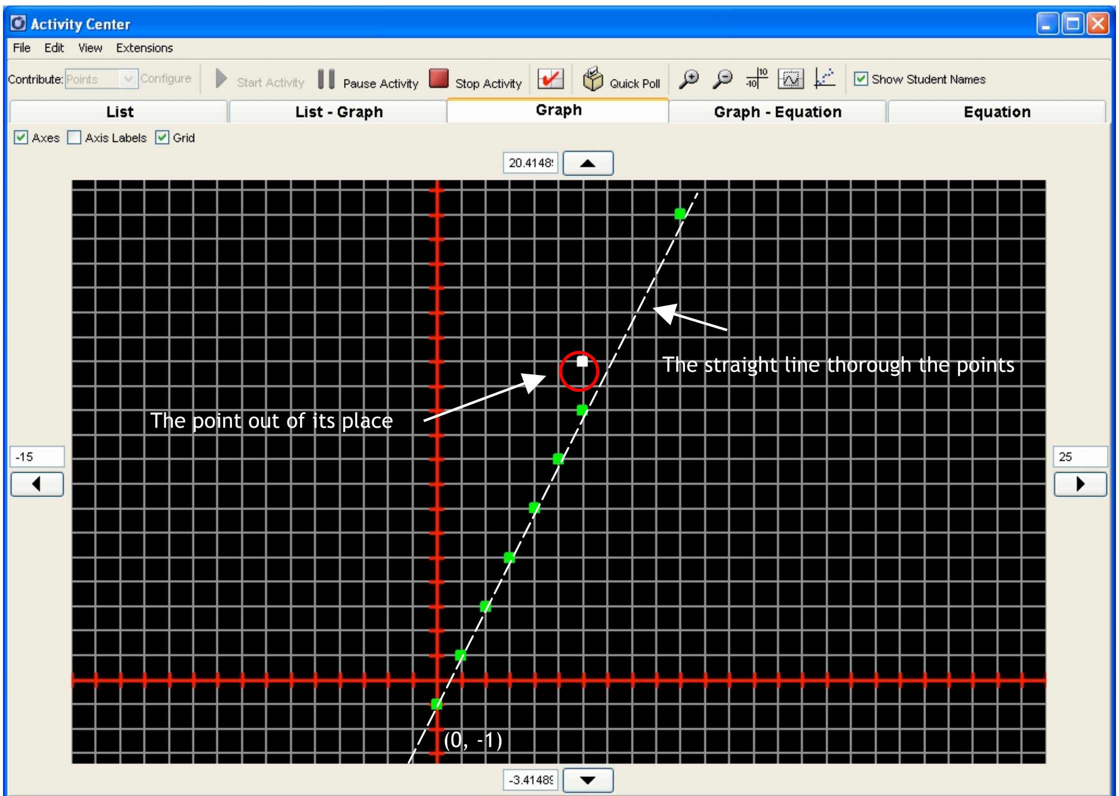

Figure 7: The Activity Center with the sequence of points

118. ***Th:*** *To identify this straight line, we gave you some information ...*

119. ***Lo:*** *How x and y increase ...*

120. ***Th:*** *How they increase. So, you see, to identify a unique line we can give you two points, and through two points there exists only one straight line, or we can say: "I give you one point and how x and y increase". So, how do x and y increase?*

121. ***Ma:*** *How the line is traced* [with his hand he traces the line in the air (Figure 8)].

122. ***Th:*** *Right. This gesture you made is important, why?*

123. ***Lu:*** *How the straight line continues.*

124. ***Th:*** *Another similar gesture by him. How the straight line continues.*

125. ***Th:*** *You made this gesture* [increasing line], *not that one* [decreasing line]. *Why?*

126. ***Ma:*** *Because, being positive, it is oriented in this way* [he repeats the previous gesture to show the direction of the line].

127. ***Th:*** *Right. And how can I understand that it is positive?*

128. ***Ma:*** *From the numbers.*

129. ***Th:*** *From the positive numbers or from the kind of calculation I do?*

130. ***St:*** *Because I always add positive numbers...*

131. ***Th:*** *Add, it is the right word. While I add to x, I add to y. Otherwise, if I add to x and subtract to y ...*

132. ***Ma:*** *It is like this* [he shows the new direction, decreasing, with a gesture].

133. ***Th:*** *Why?*

134. ***Ma:*** *Because the x is going here* [he shows with the hand the right direction] *and the y is going there* [he shows the bottom direction with his hand] *and so it becomes this one* [he traces the line in the air]. *The x increasing goes to the right, while the y decreasing goes to the bottom.*

Figure 8: Ma's gestures for the straight line

The straight line (not present on the public screen) is the pattern students recognise in the points represented on the public screen (Figure 7). Teacher's question determines Lo's words: "How x and y increase" (#119) are the first signs of the semiotic bundle, starting a chain of other signs. This sign is re-used by the teacher, with a question: "How they increase?". Ma introduces the second sign: an iconic gesture for a line, also repeated later, to show "how" (Figure 8). This gesture is then substituted with a word: "positive", which should explain "how". But it needs more explanation, so the teacher asks for a meaning of "positive". Some students introduce other words about "positive": "Because I always add positive numbers". These words are captured by the teacher, who outlines them and remarks them in the semiotic bundle, in order to distinguish between an increasing and a decreasing straight line. Again Ma introduces a gesture in the semiotic bundle, corresponding to a decreasing straight line (#132). Linking the increment of x and y, Ma gives the starting point for a new discussion, centred on the meaning of slope of a straight line, in order to recognise the "rule" followed by all the points on the line. His gestures correspond perfectly with the meaning of increment, respectively positive or negative, and they make it possible to distinguish between a positive or negative slope for a straight line.

6 Conclusions and open problems

I believe this paper can show how new technology not yet analyzed at ICMEs can make different contributions than the other ones previously presented.

The protocols analysed above give evidence for a new tool that has a profound effect on the interactions in the mathematics class. We can describe some new elements in comparison with a laboratory which includes calculators alone.

a) The students work in this mathematics laboratory with two resources: the "private" screen of the calculator in the groups, and the big public screen, while in a laboratory with calculators or computers only, they have at disposal only the private screen.

b) Both of these resources, private and public, give signs for the semiotic bundle, but the second does it in a social way. So, one of the social ways to produce knowledge is through the public screen (to answer the first research question). This is a new development, compared with those which only have calculators in the class, where signs are produced in the small groups or individuals in front of their private screens, and then eventually shared in a class discussion.

c) The class discussion can be intrinsically intertwined with the group work, because at any moment the public screen gives information on what students do on their private screens.

d) The teacher introduces signs not only referring to what groups are doing, but she can also refer to the public screen giving comments and posing questions, supporting the discussion and the construction of meanings, with particular attention to the students' processes.

e) The use of the public screen is a new tool the classroom, because it shows the work done by all the groups in real-time and it gives feedback by itself, with the possibility of making comparisons and connections among works of different students or groups.

f) The way to interact in the class in "blended" not in the sense of merging activities face-to-face with activities at distance, but in the sense of a blended collaboration among students, namely at the level of small groups (mediated by calculators) and at level of the whole class (mediated by calculators and connectivity software). And this interaction adds new opportunities for students to learn by comparing, sharing, discussing and arguing, if appropriately guided by the teacher. So, including the connectivity software and the graphic calculators in the community of subjects interacting in the classroom (Borba and Villarreal 2005), and adding the signs coming from the technology to the multimodal production of the students (Arzarello et al. 2009), the semiotic bundle is very rich. We observe this fact in the last protocols when gazes, gestures and words often are influenced by the public screen. And this is mainly due to the fact that the public screen contains the work done by all the groups, so the students can compare results and share the production of the groups. The straight line, introduced as model of the points, is the main sign that determines various other signs: the point not aligned, the gesture to align it (Figure 4), the idea of direction, of slope, and gestures and words related to them, the recursive law and its symbolic representation. And this is possible, thanks to the immediate sharing of results on the public screen.

g) This multimodal production of signs is not only more efficient than the use of calculators alone in sharing results in the learning community, but also the rhythms of work are faster. This velocity in exchanging signs is due to the presence of the public screen that shortcut the time usually necessary to describe the solution of a group to the audience. These elements are present and evident to everybody on the public screen; therefore the public screen is a space where everybody can contribute simultaneously to the activity. This space gives cognitive support in the construction of meanings in new ways. For these reasons, this kind of technology introduces new supports for a learning community.

As these results can show, an improvement from the simple use of calculators in learning and teaching, and also for research is possible. Every student receives a feedback which reinforces an idea or evidences a mistake; every group shares its production with the others. The class community has a public screen which is the catalyst of attention; the teacher refers to the work done by every group in real-time on the public screen.

There are several further possibilities for interesting new research into the kinds of software which introduce new ways of learning mathematics in the classroom and support new didactical methodology. For example, the role of the teacher is not subordinated to the use of technology. In fact, if she has more time and energy for the students (because the public screen let the students share results and speeds up feedback), she also has to dedicate time for managing the technology and simultaneously guiding a discussion. Therefore, new questions should be investigated in the future, such as: what changes in the teaching processes with this technology? How can we support teachers in introducing these media in the classroom, along with more traditional tools? How should we change the tasks with the use of these tools? And what implications are on the curricula? Are we beginning new trends in mathematics education, for studying the mediation of these new media?

Although it is not possible to read this study from an instrumental perspective, it would be interesting to develop this study in the frame of orchestration of instruments (Trouche 2004). This frame describes the set of the classroom with the positions of devices, students, teachers, screen of projection. It is then possible to analyse how different ways of orchestration (Drijvers et al. in press) can influence instrumental genesis.

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