Statistical Properties of Directional Ocean Waves: The Role of the Modulational Instability in the Formation of Extreme Events

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We discuss two independent, large scale experiments performed in two wave basins of different dimensions in which the statistics of the surface wave elevation are addressed. Both facilities are equipped with a wave maker capable of generating waves with prescribed frequency and directional properties. The experimental results show that the probability of the formation of large amplitude waves strongly depends on the directional properties of the waves. Sea states characterized by long-crested and steep waves are more likely to be populated by freak waves with respect to those characterized by a large directional spreading.

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An important task in the study of surface gravity waves is the determination of the probability density function of the surface wave elevation. The knowledge of the probability of the occurrence of large amplitude waves is essential for different engineering purposes such as the prediction of wave forces and structural responses or the design of offshore structures. A deep comprehension of the physical mechanisms of the generation of such waves is also a first step towards the development of an operational methodology for the probabilistic forecast of freak waves. It is well known that surface gravity waves obey nonlinear equations and, to date, a universal tool suitable for deriving the probability distribution function of a nonlinear system has not yet been developed. Fortunately, water waves are on average weakly nonlinear [1,2] and solutions can be generally written as power series, where the small parameter, in the case of deep water waves, is the wave steepness ε. Strong departure from Gaussian statistics of the surface elevation can be observed if third order nonlinearities are considered. At such order it has been shown numerically [3] and theoretically [4] that, for long-crested waves, a generalization of the Benjamin-Feir instability [5] (or modulational instability [2]) for random spectra can take place [6]. This instability, that corresponds to a quasiresonant four-wave interaction in Fourier space, results in the formation of large amplitude waves (or rogue waves) [7] which affect the statistical properties of the surface elevation (see, for example, [8]). This is particularly true if the ratio between the wave steepness and the spectral bandwidth, known as the Benjamin-Feir Index (BFI), is large [4]. We mention that rogue waves have also been recently observed in optical systems [9] and in acoustic turbulence in He II [10] where giant waves are observed during an inverse cascade process.

We emphasize that in many different fields of physics (plasmas [11,12], nonlinear optics [13,14], charged-particle beam dynamics [15,16]) the modulational instability plays an important role; under suitable physical conditions a nonlinear Schrödinger equation can be derived and the modulational instability can be analyzed directly with this equation [2]. A major question which has to be addressed (and is the subject of the present Letter) concerns the role of the modulational instability in two dimensional propagation. Does the modulational instability play an important role as it does for long-crested, steep waves? Does the probability of formation of extreme waves depend on the directional properties of the waves?

Already a few years ago, using numerical simulations of a higher order nonlinear Schrödinger equation in two horizontal dimensions [17], it was noted that the occurrence of extreme wave events was reduced when the energy directional spreading of the initial condition in the numerical simulations was increased (see also [18]). It should be mentioned that the numerical results have been obtained by simulating envelope equations which are a weakly nonlinear, narrow band approximation (both in frequency and in angle) of the Euler equations. Therefore, *a priori*, it is impossible to be sure that the results are reasonable when large directional spreading is considered.

In order to answer to the aforementioned questions, two experiments have been conducted almost simultaneously but independently: one in Japan at the Institute of Industrial Science, the University of Tokyo (Kinoshita Laboratory/Rheem Laboratory), and the other in Norway, at the Marintek ocean basin. The experiment in Japan was conducted in a facility 50 m long, 10 m wide, and 5 m deep with a segmented plunger type directional wave maker equipped with 32 plungers. The Marintek basin is one of the largest in the world; it is 50 m long and 70 m wide with an adjustable depth of 10 m maximum. The basin is equipped with a new multiflap generator composed by 144 flaps. The present experiment was conducted in 3 m water depth in order to facilitate the positioning of the wave gauges (probes were placed on tripods resting on the bottom).

Both basins are equipped with absorbing beaches at one end (opposite to the wave maker) in order to reduce wave reflections. Details of both experiments can be found in the following papers [19,20]. It should be noted that the tank at the University of Tokyo has an aspect ratio of 5.0; therefore, it is unclear *a priori* how this could effect the statistical properties of the surface elevation. In [21] it has been pointed out that the sidewall reflections limit the region where a desired directional spectrum can be generated. The wider the tank, the broader the region where the target wave spectrum can be achieved. On the other hand, in a narrow tank, a previous study [22], positively utilizing the side wall reflections, has found that (i) it is possible to generate a symmetric random directional wave field suitable for sea-keeping tests; (ii) the anticipated cross-tank standing wave modes do not appear for short-crested random waves; (iii) the wave statistics of the short-crested random wave, such as the significant wave height and average wave period, agree well with that of the long-crested wave. Concerning the effects of the paddles in the generator, it was pointed out in [23] that the segmented wave maker generates unidirectional waves, in addition to the intended directional wave. The mean direction of the generated random wave deviates from the target direction. This problem is alleviated as the tank width increases and the number of plungers per width increases. Both studies [22,23] considered short duration experiments. As far as we know, there is not a lot of literature available discussing the role of nonlinearity and the effect of sidewall reflections for long-term experiments. We therefore consider the present Letter comparing wave statistics from different tanks of distinct geometry as quite unique and inspiring.

Both experiments have been performed using wave generators capable of reproducing the surface elevation $\eta(x=0, y, t)$ just in front of it ($x=0$) with the desired amplitude and directional properties using the Fourier series:

$$\eta(x=0, y, t) = \sum_{j=1}^{L}\sum_{i=1}^{M} a_{ij}\cos(k_j y - \omega_{i,j} t + \phi_{i,j}), \quad (1)$$

where $\eta(x, y, t)$ is the surface elevation and $a_{i,j}$ are random amplitudes with average values related to the spectral energy density $E(\omega, \theta)$ as follows: $a_{i,j} = \sqrt{2E(\omega_{i,j}, \theta_{i,j})\Delta\theta\Delta\omega}$. Here $\omega = \sqrt{g|k|}$, with g gravitational acceleration; $\phi_{i,j}$ are random phases distributed uniformly in the $[0, 2\pi)$ interval. Note that the results discussed in this Letter are independent of how the spectrum was discretized because we assured sufficiently large number of wave modes to represent the continuous wave spectrum.

For the input spectral energy density $E(\omega, \theta)$, the frequency and the angular dependence were factorized (as usual): $E(\omega, \theta) = F(\omega)G(\theta)$. The Joint North Sea Wave Project (JONSWAP) formulation [24] has been used to model the wave energy in the frequency domain:

$$F(\omega) = \frac{\alpha g^2}{\omega^5}\exp\left[-\frac{5}{4}\left(\frac{\omega}{\omega_p}\right)^{-4}\right]\gamma^{\exp[-((\omega-\omega_p)^2)/(2\sigma_j^2\omega_p^2)]}. \quad (2)$$

Here γ is called the peak enhancement parameter and usually ranges from 1 to 6 for ocean waves, ω_p is the angular frequency corresponding to the peak of the spectrum, and $\sigma_j = 0.08$. The parameter α is related to the significant wave height $H_s = 4\sigma$ (with σ the standard deviation of the surface elevation) as follows: $\alpha = 5H_s^2\omega_p^4(1 - 0.287\log(\gamma))$.

Fixing ω_p, γ, and H_s, the frequency energy spectral density is univocally defined. In the experiment in Marintek (wider basin) we have chosen the following values $\omega_p = 2\pi$ rad/s, $\gamma = 6$ and $H_s = 0.08$ m. In the experiment in the University of Tokyo (narrower basin) the following values were chosen: $\omega_p = 2\pi/1.23$ rad/s, $\gamma = 3$, and $H_s = 0.05$ m. These initial conditions are characterized by a large BFI; therefore, according to previous dimensional studies, the waves should show large departures from Gaussian statistics. Concerning the angular part of the spectrum, a $G(\theta) = A_N\cos^N(\theta)$ function (with $|\theta| \leq \pi/2$ and A_N a normalization factor) is then applied to model the energy in the directional domain [25]. In order to consider different degrees of directional spreading, several values of the parameter N have been used: $N = 24, 50, 90, 200, 840, \infty$ in the Marintek experiment (here ∞ corresponds to unidirectional waves), $N = 10, 25, 50, 75, 100, 125, 250$ in the University of Tokyo experiment, ranging from short-crested (small N) to fairly long-crested waves (large N). To estimate the relation between N and the actual directional spreading in angle, the reader should refer to Fig. 1.

Before starting the discussion on the probability of occurrence of extreme waves, we will furnish shortly some information of the frequency wave spectra estimated from time series measured in the tank. The wave maker is

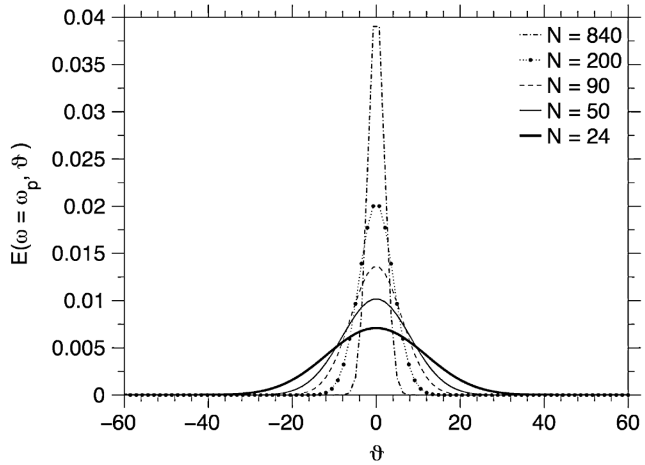

FIG. 1. Energy directional distribution as a function of angle θ for different values of the parameter N, see text.

programmed to generate a JONSWAP-like spectrum as described in Eq. (2). According to the weak turbulence theory (see [26]), the four-wave resonant interaction process (see also [1]) should be responsible for the formation of an ω^{-4} tail. We anticipate that in the present experiment we do not observe the formation of such a tail. Our wave spectra are steeper than the weak turbulence prediction; they range from $\omega^{-5.8}$ to $\omega^{-4.7}$ (see [19]). As far as we know experiments in wave tanks, where waves are generated mechanically, have been unsuccessful at verifying the weak turbulence prediction or the prediction by Phillips [27]. According to the most recent experiment [28], the slope is very much dependent on the forcing, ranging from ω^{-6} for weak forcing to almost ω^{-4} for strong forcing.

To give an idea of what large amplitude waves look like in our experiment, we report in Fig. 2 a typical time series of an extreme wave. The wave height of the largest wave is about 15 cm for a background significant wave height of 6 cm. A statistical measure of the presence of such waves in a time series is the kurtosis (the fourth order moment of the probability density function). In order to achieve statistically significant results, at least 4000 waves have been collected for each experiment. Waves generated just in front of the wave maker have Gaussian statistics (i.e., kurtosis is equal to 3) because they are generated as a linear superposition of sinusoidal waves with random amplitudes and phases. As waves propagate nonlinearly along the basin, the kurtosis also evolves in space. In Fig. 3 we have plotted the maximum value achieved by the kurtosis along the tank for different directional spreading from the two different wave tanks. The figure shows that the kurtosis remains more or less constant up to $N = 100$ and then grows with N; i.e., it increases as the waves become long crested. Extreme waves are more probable for long-crested waves. We then consider the exceedence probability for wave crests measured at the probe where the kurtosis reaches its maximum. In Fig. 4 we show the results for $N = \infty$ and $N = 24$. In the figure the Tayfun distribution, derived from a second-order solution of the water wave problem under the narrow band approximation, is also included. The figure shows two important results: (i) for quasi-long-crested waves, the appearance of extreme waves can be underestimated up to an order of magnitude if linear or second-order theory (Tayfun distribution) are considered; (ii) for large directional distribution, the probability of occurrence of extremes is well described by second-order theory.

To summarize, we have performed two completely independent experimental investigations whose role was to address the statistical properties of the surface elevation for different degrees of directional energy distribution. The used experimental facilities have different sizes and are equipped with different wave makers. Nevertheless, results obtained are very consistent: the modulational instability process, which is one of the main mechanisms of formation of extreme waves in deep water, random, long-crested waves, seems to be quenched when short-crested waves are considered. We believe that, after many years of numerical and theoretical research, this result represents an important step towards the understanding of the physics of extreme waves.

Having two completely independent experiments suggesting the same conclusion, but with different tank geometries and wave generation methods, encourages us to extend the result to ocean waves generated under the influence of wind. Therefore, it is our interest to investigate

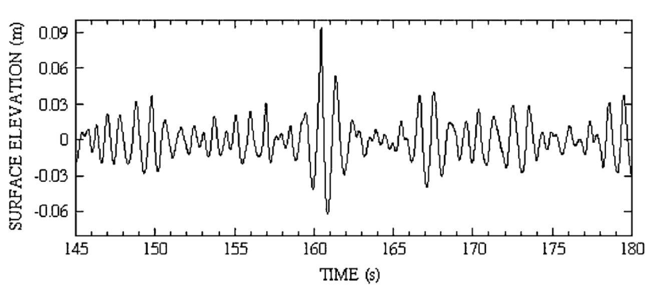

FIG. 2. Time series showing an extreme wave.

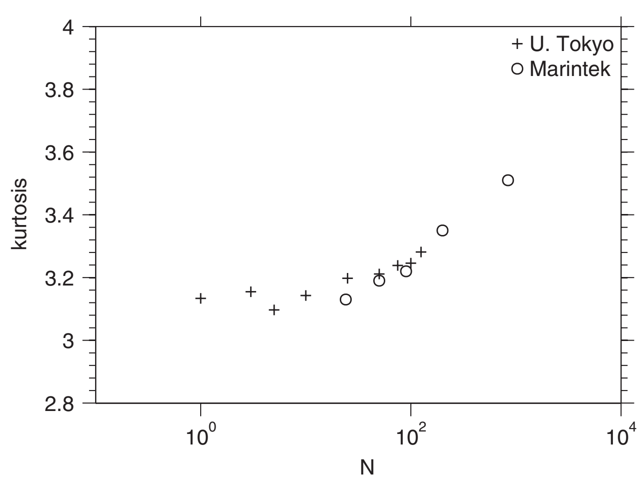

FIG. 3. Maxima of kurtosis as a function of N.

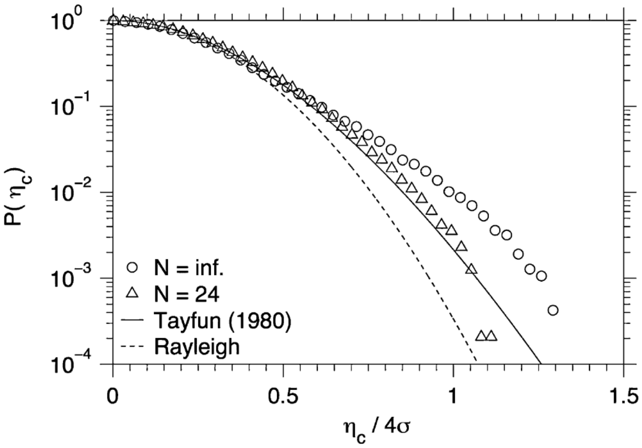

FIG. 4. Exceedence probability for wave crests estimated from time series recorded at the probe where the maximum of kurtosis is observed.

further the directional properties of the ocean waves and the associated probability of large amplitude waves. However, this is an area that is poorly addressed in the research of ocean waves. At the moment the *in situ* directional measurements of ocean waves are limited. Nowadays, numerical simulations of the Euler equations in two horizontal dimensions are feasible (see, for example, [30] or [31]); therefore, we are planning in the near future to perform a detailed comparison between experimental and numerical results.

It should also be mentioned that the effect of wind has not been considered in the present experimental investigation.

Nowadays, wave forecasts (wave heights, dominant wave periods, and dominant wave directions) are based on the numerical integration of the energy balance equation [24], that provides an evolution equation for the wave spectrum including nonlinear interactions, wind input, and dissipation. At each time and for each location in the ocean, the spectral shape is available from wave models. Assuming that the models are capable of reproducing accurately the directional distribution of the wave spectrum, a probability density function for wave heights/crests could be associated, making the forecast of extreme waves possible (at least in a statistical sense). To conclude, we mention that such a scheme has recently been introduced at the European Centre for Medium-Range Weather Forecasts (ECMWF) and it is based on a parameterization of kurtosis obtained from the observations here described and from simulations with the $2D + 1$ nonlinear Schrödinger equation.

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