Tripartite Networks: a first exploratory step towards the understanding of multipartite networks

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Tripartite Networks: a first exploratory step towards the understanding of multipartite networks

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Introduction

In tripartite networks nodes are divided in three different sets and edges are allowed only between couples of nodes belonging to different families. In other words the graph is defined by

$$\mathcal{G} = \{\mathcal{V}, \mathcal{E}\}$$

where

$$\mathcal{V} = \mathcal{V}_1 \cup \mathcal{V}_2 \cup \mathcal{V}_3 \qquad \text{and } \mathcal{V}_i \cap \mathcal{V}_j = \emptyset \qquad \text{for } i \neq j$$

and

$$\mathcal{E} = \left\{ (u, v) : u \in \mathcal{V}_i, v \in \mathcal{V}_j \text{ and } i \neq j \right\}.$$

Tripartite graphs are a natural way to describe a number of real world structures. Recently, we found them useful to describe and analyze the two following systems:

▸ MiTo - Settembre Musica gathers data of more than 30 years of organization of the prestigious international music festival hold in Torino, Italy. The tripartite graph in this case emerges considering as node classes the artists, the concerts or music pieces, and the directors. Edges therefore represent single concerts played in different editions of the festival.
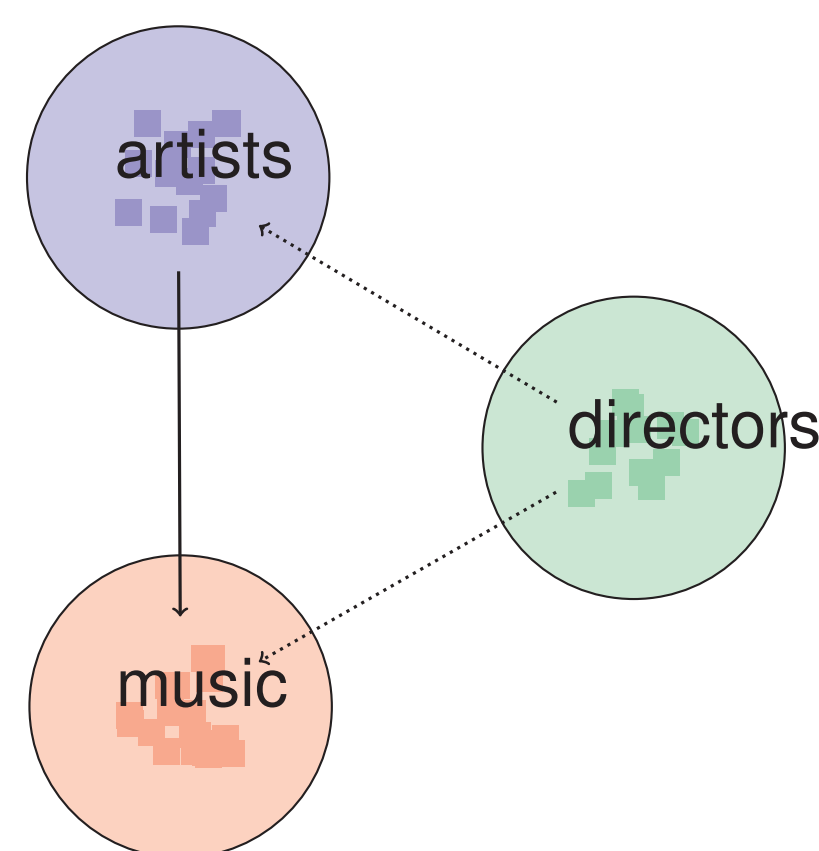
On a second level of complexification, a network representation of this event's history could consideer three additional nodes' classes, representing the authors and gender of musical pieces, and the location or hall of the concerts.

▸ We-Sport.com collects data of the vertical social network http://www.we-sport.com. The main aim of this project is to connect people with sports avoiding one of the main problem of practicing sports: the lack of partner with whom to practice one's favorite sports. Considering as nodes athletes, chosen sports, and places where those are played, the tripartite structures is readily apparent.

However, many other example of tripartite, or even multipartite, networks could be found in real world. In eco-epidemiological frameworks a tripartite network could be useful to describe the transmission cycle of *Taenia Solium* between raw pork meat, humans and, swine. Another interesting example can be the ubiquitylation, the main processes used by cells to mark proteins in order to degrade them. In fact the process requires three kind of enzymes and the marking of a particular protein is mediated by particular enzymes in a hierarchical manner.
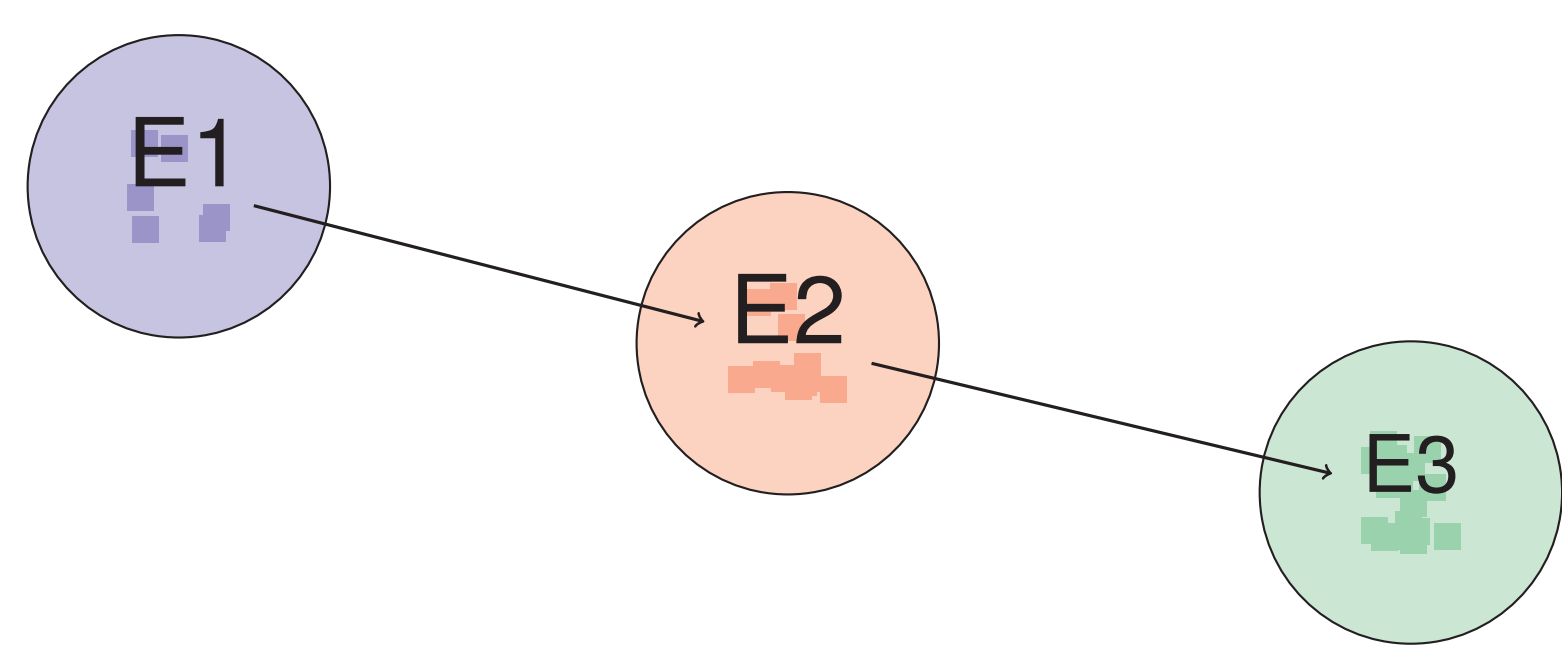
Therefore, the understanding of the network behind it, of its modular structure, and of its principal features could be of great importance for public health.

Close Tripartite Networks

We call "Close Tripartite" those networks for which there is no restriction on the definition of the tripartite structure. Below we show a graphical example of these networks and the structure of their adjacency matrix.
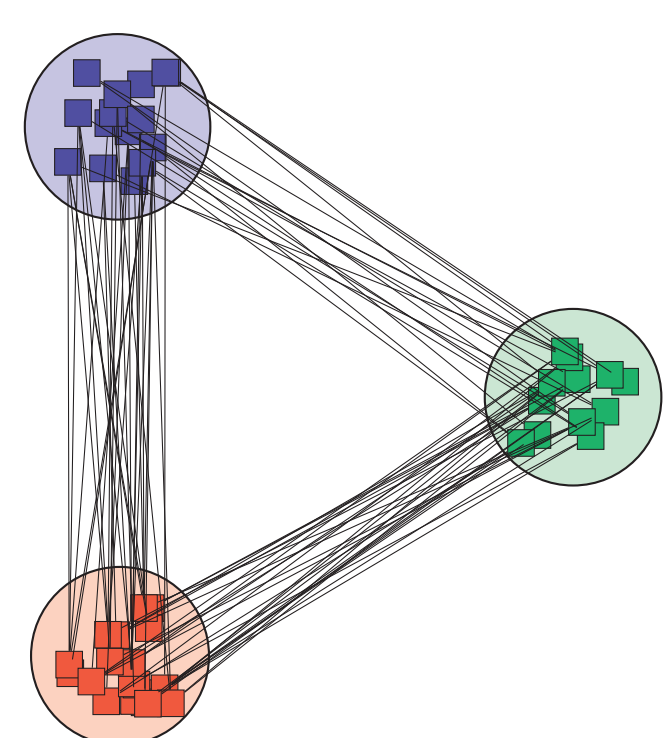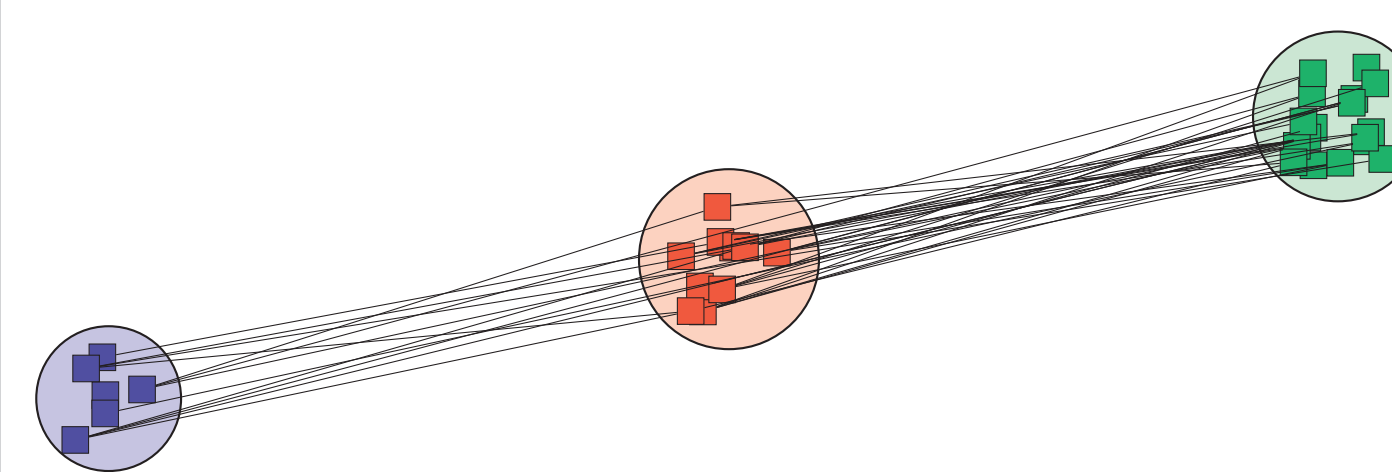

$$\begin{pmatrix} 0 & A & C \\ A^t & 0 & B \\ C^t & B^t & 0 \end{pmatrix}$$

A real-world example of that network could be that of MiTo - Settembre Musica. The available projections of such structure are different according to the needs and questions of the researchers. For example a two-mode projection is possible, and we will explore it in next section. Furthermore, a one-mode projection is possible. The adjacency matrix of this projection would be obtained as the matrix multiplication ABC^t. Thus, the i,j element of such matrix indicates the number of path of length three between nodes i and j belonging to the same partition. For examples, on the *Taenia Solium* spreading network, projecting on swine nodes will give the number of possible path of the parassite between different swines. Therefore, on that, simplified, graph we could perform a number of epidemiological speculations.

Open Tripartite Network

"Open Tripartite Networks" are a particular case of the tripartite family. For them, the circular structure is not allowed: connections are only between $\mathcal{V}_1$ and $\mathcal{V}_2$ and between $\mathcal{V}_2$ and $\mathcal{V}_3$.


$$\begin{pmatrix} 0 & A & 0 \\ A^t & 0 & B \\ 0 & B^t & 0 \end{pmatrix}$$

That tripartite graph could projected in two-mode network. For instance, the projection on $\mathcal{V}_1$-$\mathcal{V}_3$ is build connecting nodes n and m a number of time equals to the number of shared nodes belonging to $\mathcal{V}_2$. The adjacency matrix of that weighted projection is obtained as AB.

Triangular Tripartite

Another interesting family of tripartite graph is what we called a "Triangular Tripartite". In that network a couple of node, v_1 and v_2, belonging to the partition $\mathcal{V}_1$ and $\mathcal{V}_2$, are connected if and only if at the same time they share at least a neighbor in $\mathcal{V}_3$.
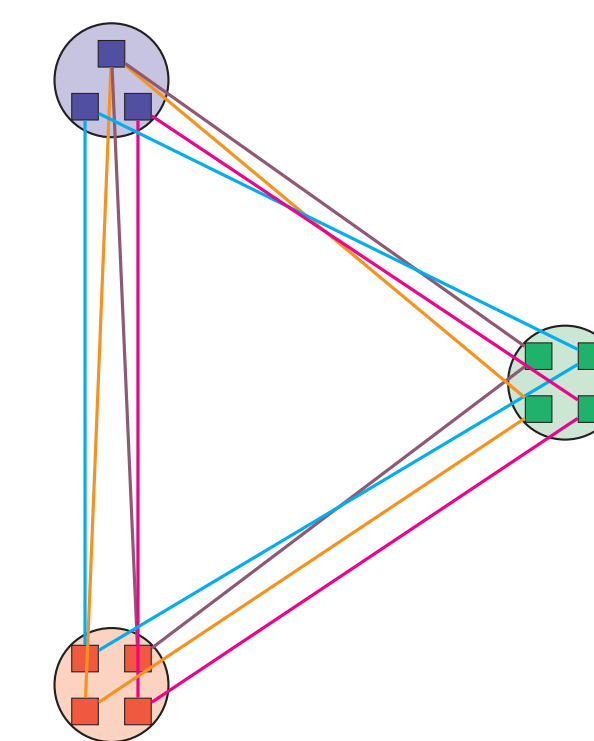
The aforementioned structure is the one we observe when modeling the We-Sport network. Indeed, an athlete is connected with a sport and a location only if the sport is available in that location. This kind of structure provides a deeper level of information that could also be expanded to the network projection.
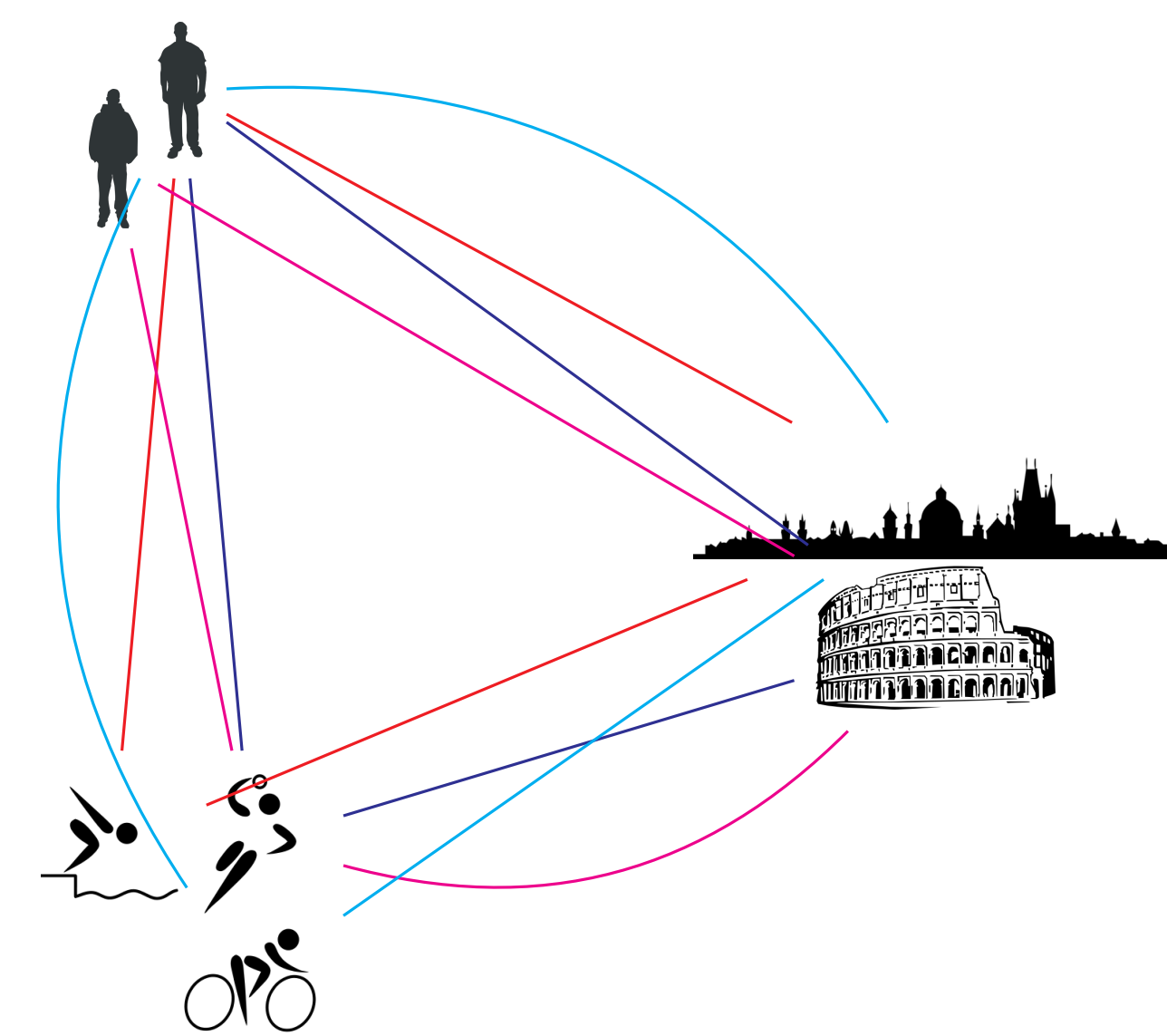
For instance, considering only the bipartite network we could say that two athletes are connected if they share a common sport. But the inferences on connections among athletes could be different and some questions need a deeper level of investigations: the fact that athletes share the same sport in the same location definitely has a different value and for both sociological researches and for policy makers.

Conclusions and Future Works

The use of tripartite, or tetrapartite or even multipartite graphs to analyze network models could give many details on their implicit characteristics. For instance, referring to the We-Sport network, the concurrent use of partition nodes such as sites, sport facilities, schedules and sport skills could explain the behavior of network nodes. In fact, with a sufficiently high number of nodes it is possible to investigate the different ways in which, for example, a network evolves in time, how a specific characteristic of a node impacts the relationships with other nodes, and how the evolutionary behavior of the nodes are predictable or not.

A thorough knowledge of the behavior of this social network can also provide information to the policy makers on the use of sport facilities, schedules of utilization, types of sports preferred, the number of practitioners in a specific geographical areas as well as on the perception that citizenship has on public administration in the specific context. Sport is one of the primary prevention of degenerative diseases[1]. Thus, monitoring frequency of practice and understanding the mechanisms of cohesion and social influence could be fundamental elements to direct targeted policies to support the practice, as well as an analysis tool of effect of existing policies, and improve and increasing effectiveness of government policies along time.

Furthermore, the analysis of the evolutionary history of MiTo - Settembre Musica network could result in new insights on musical preferences of artists, directors, and public.

A key feature observed through the complex analysis of this social networks is a innovative characteristic of networks themselves: it is possible to validate an intrinsic feedback mechanism between the web and the real world, where political and social dynamics of the real world affect the web, which in turn, through the exchange of information between nodes and diffuse mechanisms of participation, can affect the real world in a pattern like: real world-web-real world, or web-real world-web.

[1] Matheson et al. - Responsibility of sport and exercise medicine in preventing and managingchronic disease: applying our knowledge and skill is overdue - Br. J. Sports. Med. 2011