The Enciclopedia delle Matematiche elementari and the Contributions of Bolognese Mathematicians
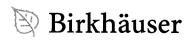
(Article begins on next page)

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The old "Aula Magna" of the University of Bologna and the reading room of the University Library. Photograph courtesy of the Historical Archive of the University of Bologna, Alma Mater Studiorum.

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Mathematicians in Bologna 1861–1960

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A Short Overview on Mathematicians in Bologna in the First Century after the Establishment of Italy

Salvatore Coen

The present volume considers the lives and achievements of mathematicians who studied and worked in various roles at the Bologna University in the century following Italian unification. Most contributions to this volume are historical in character; the few which more closely focus on mathematical research do so strictly in relation to a discussion of the mathematicians concerned.

Without claiming to be exhaustive, the volume deals with many of the most representative mathematicians who worked in Bologna in the period 1860-1960, namely with Luigi Cremona (b. 1830), Eugenio Beltrami (b. 1836), Salvatore Pincherle (b. 1853), Pietro Burgatti (1868), Federigo Enriques (b. 1871), Ugo Amaldi (b. 1875), Beppo Levi (b. 1875), Giuseppe Vitali (b. 1875), Enrico Bompiani (b. 1889), Beniamino Segre (b. 1903), Tullio Viola (b. 1904), Dario Graffi (b. 1905), Gianfranco Cimmino (b. 1908) Bruno Pini (b. 1918) and Lamberto Cattabriga (b. 1930). Sometimes the same mathematician is presented from different points of view by different authors. The work of a number of other mathematicians also comes in for perusal, most notably the contributions made by Giulio Vivanti (b. 1859), Ettore Bortolotti (b. 1866) and Filippo Sibirani (b. 1880).

The University of Bologna is one of the oldest universities in the world (indeed, it is traditionally said to have been founded in 1088) and many well-known mathematicians have studied and taught there. We may recall Luca Pacioli, Domenico Maria Novara, Scipione del Ferro, Girolamo Cardano, Ludovico Ferrari, Rafael Bombelli, Bonaventura Cavalieri, Pietro Mengoli, Giandomenico Cassini, Domenico Guglielmini and Gabriele Manfredi, to mention just a few. It is, however, hard to come up with a corresponding list for the first half of the nineteenth century, a dark and difficult period for mathematics in Bologna, although studies of astronomy and hydraulics continued developing along traditional lines.

In order to overcome the local weakness of mathematical research at Bologna, in 1860, soon after Bologna became part of the Kingdom of Sardinia (by the plebiscite of March 1860), the Minister of Education appointed Luigi Cremona as Professor of Higher Geometry and Matteo Fiorini (b. 1827) as Professor of Geodesy. The presence of Luigi Cremona and Eugenio Beltrami (starting from 1862) brought

The *Enciclopedia delle Matematiche elementari* and the Contributions of Bolognese Mathematicians

Erika Luciano

Abstract It may seem strange, in a volume dedicated to Bolognese mathematicians, to come across a paper about an initiative that, as is well-known, has its roots in Pavia, namely, the *Enciclopedia delle Matematiche Elementari*. Yet the mathematical community of Bologna was in fact involved in this editorial enterprise and several of its authors, including E. Bortolotti, A. Comessatti, S. Pincherle, F. Sibirani, B. Segre and G. Vitali, together with its conceiver R. Bonola and one of its directors G. Vivanti, were either from Bologna or had links with the University there. In the present paper, after summarising the volume's somewhat turbulent publication history, we will focus on the involvement of Bolognese mathematicians and, in particular, on the scientific dialogue which took place between L. Berzolari, G. Vivanti and G. Vitali regarding the coordination of *Enciclopedia* chapters on analysis.

1 The Complicated Circumstances of the Publication

It was at the meeting of the Lombardy Section of the teachers' association Mathesis[1] held in Pavia on 23 May 1909 with Luigi Berzolari as chair, that one of its members, Roberto Bonola,[2] first put forward the idea of publishing an *Enciclopedia delle*

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[1] The Lombardy section had been just founded in Milan the previous month, on 8 April 1909.

[2] Bonola had studied mathematics at the University of Bologna. Assistant to E. Bertini, at Pavia, and teacher at the nearby Scuola Normale, he was a brilliant scholar in non-Euclidean geometry and, a

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Matematiche Elementari under the direction of Mathesis.[3] Bonola noted that Italian mathematical literature to date had not yet produced a comprehensive guide to the mathematics taught in middle and secondary schools.[4] Such an encyclopaedia would provide a point of reference for teachers, students attending the University and the *Scuole di Magistero*, teachers in training, and all those interested in elementary mathematics and wishing to update and improve their teaching methods. Such a tool, he went on, should be developed with a view to:

> spreading knowledge of the exact sciences amongst the many teachers who, due to the posts they hold, live far from intellectual hubs, from libraries, from modern scientific movements. It should, ultimately, forge a kind of intellectual network between all those interested in the advancement of elementary mathematics, and by making valuable information and interesting news about recent developments more widely available, promote research in fields which are neither sterile nor lacking in interest.[5]

From the very beginning, a general overview of the *Enciclopedia* and its aims were well defined. According to Bonola's proposal, the *Enciclopedia* was to contain all the contents of syllabi taught in secondary schools, with those developed in the physics–mathematics section of technical institutes being considered to be particularly exemplary. In addition, it was to incorporate those "complementary" subjects aimed at facilitating a reconsideration of various points of elementary mathematics from an advanced standpoint and which could be used to elaborate exercises for use in the classroom or as homework. Considering the fact that introductory elements of infinitesimal calculus could be usefully added to the curricula of high schools (*Licei*) and technical institutes (*Istituti tecnici*), Bonola suggested that space also be made for theories which had so far been considered as pertaining to the first two years of university, such as infinitesimal analysis and the theories of sets and of analytic functions. Finally, in order to broaden the scope of the references provided, he expressed his desire that the *Enciclopedia* also include chapters on applied mathematics (physics, mechanics) and on history. With reference to the style of exposition, Bonola observed that:

> although they should not eschew the critical and philosophical aspects of certain subjects, entries should be of a primarily descriptive character, avoiding bias towards specific *Schools* or methodologies. For each topic discussed, the most noteworthy relevant propositions should be assembled in a systematic way. Each theory expounded should be accompanied

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few months before his sudden death on 16 May 1911, obtained a chair at the Scuola superiore di magistero femminile in Rome.

[3] See [58, 10]. The report is also published in [11]. See also *Bollettino di Bibliografia e Storia delle Scienze Matematiche* (G. Loria), XI, 1909, p. 91 and XIII, 1911, p. 44–45.

[4] In what follows the expression "middle school", *scuola media*, is used to refer to levels of education between elementary and university.

[5] [10, p. 41]: ... *a diffondere la coltura delle scienze esatte fra i numerosi insegnanti che, per ragioni d'ufficio, risiedono lontani dai centri di studio, dalle Biblioteche, dal movimento scientifico moderno; dovrebbe, infine, formare una specie di legame intellettuale fra quanti s'interessano del progresso delle matematiche elementari e, porgendo a tutti dati preziosi, notizie interessanti, promuovere ricerche in campi non sterili, né privi di qualsiasi interesse.*

by indications as to the various critical modes which can be used to develop a rational and comprehensive treatment of it. As regards the fundamental theorems, especially those which comprise difficulties of concept or development, *Enciclopedia* entries should provide brief proofs. Otherwise, only the simple statement should be provided, together with extensive historical and bibliographical references, which will make it possible for interested readers to find the works that deal with the relevant subjects in detail.[6]

Bonola's proposals, which would be largely adopted during the execution phase of the project, met with unanimous approval. F. Severi, then president of the Mathesis, took it upon himself to involve the association's central office and to "express the Lombardy section's wish that the future committee responsible for bringing the work into being be based in Pavia" [58, p. 36].

The preparations got off to a quick start and, in May of the same year, the executive council of Mathesis reached a unanimous agreement establishing the editorial board for the *Enciclopedia*, which consisted in a group of university and secondary school teachers, all of whom were in Pavia: L. Berzolari, F. Gerbaldi, G. Vivanti, R. Bonola and E. Veneroni.[7] The editorial board was given absolute authority and autonomy in choosing contributors and in deciding on the content and form of the articles.

The board's first task was to find an Italian publishing house willing to underwrite the costs of the *Enciclopedia*, the Mathesis itself lacking the financial resources to fund publication and having therefore simply authorised its publication "under the auspices of Mathesis". In two meetings held in July 1909, the members of the editorial committee refined their intentions in preparation for the Mathesis Congress to be held in Padua the following September. On that occasion, Berzolari and Bonola would make another report, in which they illustrated a preliminary outline of the activities managed up to that time. An addition was made to the original plan, namely, the chapter *Questioni pedagogiche*. The objectives, orientation and contents of the work remained unchanged, although it was now specified that the *Enciclopedia* would be published with a view "not only to saving readers' time and effort in seeking out accurate and reliable information on elementary mathematics, but also and above all to promoting mathematical culture".[8] In the meantime, a three-volume structure had been outlined for the *Enciclopedia*, the first treating

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[6] [10, p. 41]: ... *pur non evitando di toccare in certe quistioni il lato critico e filosofico, dovrebbe essere nettamente espositivo, senza pregiudizi di metodo o scuola. Intorno a ciascun argomento vorrebbe raccolte sistematicamente le proposizioni più notevoli che vi si riferiscono, e per ciascuna teoria indicati i vari modi secondo cui si può darle una trattazione razionale e completa. Delle proposizioni fondamentali, segnatamente di quelle che racchiudono difficoltà di concetto o di sviluppo, vorrebbe che l'«Enciclopedia» porgesse rapide dimostrazioni; delle altre il solo enunciato, insieme a larghe citazioni storiche e bibliografiche, che permettano all'interessato di risalire facilmente alle opere che ne trattano esplicitamente.*

[7] [58, p. 48]. The editorial board changed several times during the years. In 1923 it was composed of L. Berzolari, F. Gerbaldi, G. Vivanti, D. Gigli and R. Serini.

[8] [6, p. 2]: ... *con l'intendimento non solo di risparmiare tempo e fatica a chi desidera notizie precise e sicure su qualche argomento elementare, ma con la mira principale di diffondere la cultura delle matematiche.*

Analisi, the second devoted to *Geometria* and the third to the *Applicazioni, Storia della matematica* and *Questioni didattiche*. In order to speed up publication, with the aim of releasing the entire work within a maximum of three years, the board decided to bring out the *Enciclopedia* in a series of issues, in imitation of the German *Encyclopäedie der Mathematische Wissenshaften*.

At the Padua congress, Berzolari and Bonola were already able to present an index and abstract for each individual volume, close to the definitive one and sufficiently detailed. The volumes were to consist in 12, 17 and 9 chapters, respectively. They also explained to the members of the Mathesis attending the Congress the considerable problems involved in selecting the contributors and the publisher. With regard to the contributors, they announced that they had contacted a set of some forty scholars, both university and secondary school teachers, so as to entrust the various chapters "to academics who have already carried out specialised research on the subjects at hand and will therefore be able to carry out the work required of them rapidly and effectively" [6, p. 4]. The financial side of the project came up for considerable discussion at the congress. Indeed, the entire session of 23 September was dedicated to this matter, and some noteworthy differences of opinion amongst G. Loria, G. Lazzeri, R. Bonola, F. Severi, A. Padoa, G. Castelnuovo, A. Conti, R. Bettazzi and E. Lenzi emerged with regard to the layout, the format, the payments due to contributors, etc. For example, Berzolari and Bonola suggested opening a subscription amongst the member of the Mathesis, so that each member committed himself to purchasing one copy of the entire encyclopedia at a pre-established price payable in instalments over a 3-year period. Volterra and Castelnuovo, in contrast, gave assurances of their willingness to present a request for a publication subsidy to the Ministries of Education and of Agriculture.[9]

Work on the *Enciclopedia* resumed in Pavia in November 1909 and during the first months of 1910 the Committee began distributing assignments for specific entries.[10] In May 1910, for example, Mario Pieri, Berzolari's former assistant at Turin, was assigned to write and edit the chapter on *Logica Matematica*, "including the most recent departures from the Peano system (Russell, etc.)."[11] At the same time, Bonola and Vivanti were sounding out F. Amodeo, A. Perna and G. Vacca as possible authors for the chapters on *Geometria del piano e dello spazio, Teoria della misura*[12] and *Storia della matematica*,[13] respectively. Berzolari would discuss the selection and substitutions of contributors with his colleagues on many occasions over the years to come. Thus, for example, in April 1931 he wrote to Peano:

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[9]In the end, Loria's proposal to offer the same discount to Mathesis members as that offered to booksellers was unanimously approved.

[10]See *Giornale di matematiche ad uso degli studenti delle Università italiane* (Battaglini), 49 (3, 2), 1911, p. 171.

[11]L. Berzolari to M. Pieri, 29 May 1910, in [3, p. 7–8].

[12]G. Vivanti to F. Amodeo, 4 August 1910, in [51, p. 477].

[13]G. Vivanti to G. Vacca, 11 May 1910, in [45, p. 185].

I am very grateful to you for your words and for the interest you have shown in our *Enciclopedia*, which is a source of infinite perplexities for me. I agree with you that it would be an excellent choice to assign the article on *Calcolo approssimato* to Maccaferri. There has been some discussion with Cassina. But if nothing is achieved in that direction, I will surely turn to Maccaferri.[14]

Actually, the Commission

sought the collaboration of the greatest scientific minds of our country, and had the satisfaction of receiving positive responses from almost all the universitary and secondary teachers contacted. On the rare occasions on which, for various reasons, our invitations have not been accepted, we have been able to find alternative candidates quickly.[15]

The publication ran smoothly through the years immediately before the first World War, the "happiest years" [9, p. 396] of the academic careers of Berzolari and Vivanti. However, difficulties soon began to emerge. Bonola, Calò and Veneroni all died young, and even though Berzolari was president of Mathesis from 1915 to 1918, World War I and the post-war crisis paralysed the activities of the association. The editorial board of the *Enciclopedia*

for a long period was reduced to a state of inertia, and was unable to respond to the many enquiries from Italian and foreign mathematicians as to how its endeavours were proceeding. Only when the most acute crisis had been overcome and when it seemed possible that things might gradually return to normal was there a revival of hope among the board members that their still unrealised initiative might be brought to fruition. Negotiations with the publisher were resumed and the contract signed, manuscripts were sent back to their authors for correction and updating, and replacements were found for those contributors who had passed away during the long interval.[16]

F. Enriques' presidency, starting in 1919, saw a certain shift in the matters discussed within Mathesis, although attention continued to be given to the *Enciclopedia di Berzolari*, as it had come to be called, at the various congresses of the association. After having shelved the idea of a serial publication, the first part of the first volume

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[14]L. Berzolari to G. Peano, Pavia, 23 April 1931, Library of Cuneo, Peano Archives, c.p. N. 100299: *Ti sono molto grato di quanto mi scrivi e dell'interesse che mostri per la nostra Enciclopedia, la quale mi procura un'infinità di grattacapi. Sono d'accordo con te che l'articolo del Calcolo approssimato sarebbe al prof. Macc[aferri] ottimamente affidato. In passato ci fu qualche parola col prof. Cassina. Ma, se non si farà nulla con quest'ultimo, mi rivolgerò senz'altro al Maccaferri.*

[15][7, I¹, p. VIII]: *... cercò di mettere a contributo le migliori energie scientifiche del nostro paese, ed ebbe la soddisfazione di vedere accolte le sue richieste di collaborazione da quasi tutti i professori universitari e medi ai quali si era rivolta, e di poter prontamente sostituire quei pochissimi che, per vari motivi, non avevano accettato l'invito.*

[16][7, I¹, p. VIII–IX]: *... ridotta per lunghi anni all'inerzia, e all'impossibilità di dare qualsiasi risposta ai molti che dall'Italia e dall'estero chiedevano notizie dello stato de' suoi lavori. Fu soltanto quando le crisi più acute furono superate e apparve possibile un lento ritorno alla normalità, che rinacque nella Commissione la speranza di poter riprendere l'opera interrotta sul nascere. Si riallacciarono e si condussero a termine le pratiche con l'editore; si rinviarono i manoscritti agli Autori per la revisione e l'aggiornamento; e si sostituirono i collaboratori scomparsi nel lungo intervallo.*

was ready to go to print in December 1923 [5, p. 47], but did not actually come out until June 1929. It was published by Ulrico Hoepli in Milan, was enthusiastically presented the following November by Berzolari at the Unione Matematica Italiana and at the Istituto Lombardo,[17] and was sent off to numerous colleagues.[18] The second part of the first volume appeared between the end of 1931 and the beginning of 1932.[19] However, a series of sudden deaths which struck the editorial board led

to a further suspension of activities until September 1936, the date of publication of the second volume. D. Gigli, one of the editors in chief, died unexpectedly, leaving his article on the *Teoria della misura* incomplete, to be finished by L. Brusotti, on the basis of his deceased colleague's notes. G. Biggiogero found herself in a similar position, when she was called to replace V. Retali in writing the chapter on the *Geometria del triangolo*. P. Benedetti also passed away, leaving his essay the *Fondamenti di Geometria* in proofs.

The Second World War once again brought the publication of the *Enciclopedia* to a standstill. The first section of the third volume, the last to be published under the direction of Berzolari, appeared in 1949. The second was approved and sent to press in the same year by Berzolari's collaborator, Brusotti. It opened with Berzolari's obituary and a warm tribute by Carlo Hoepli to the deceased *Maestro* "on the brink of the end of his 20-year effort dedicated to the direction and publication' of the *Enciclopedia*, which 'is and will remain the most apt and lasting monument to the memory of this great mathematician and teacher" [39, p. n.n].

In 1950, the publication process finally reached its conclusion, with the printing of the third part of the third volume of the *Enciclopedia delle Matematiche Elementari*, dedicated in its entirety to statistics, developed by C. Gini and G. Pompilj. By the end, in its complete form the *Enciclopedia* stood at seven volumes and 63 chapters, for a total of over 4,300 pages.

It occupies a unique place in the panorama of Italian publishing,[20] differing in contents, methodology and style of exposition from the other encyclopaedic works of the same period (F. Enriques' *Collectanea* and *Questioni riguardanti le matematiche elementari*, E. Pascal's *Repertorio di Matematiche superiori*, G. Peano's *Formulario di matematica* and M. Cipolla's *La matematica elementare nei suoi fondamenti*). It met with almost universal approval[21] both in Italy and abroad. The

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[17]See *Bollettino dell'UMI*, s. 1, 8, 1929, p. 171 and *Rendiconti dell'Istituto Lombardo*, s. 2, 62, 1929, p. 802, meeting of 21 November 1929.

[18]See L. Berzolari to G. Peano, 16 November 1929, Library of Cuneo, Peano Archives, c.p. N. 103011: *Carissimo Peano avrai ricevuto da Hoepli, o riceverai presto, la 1ª parte del 1° volume dell'Encicl. delle matem.ᵉ elementari. Un'altra copia ho fatto spedire, in omaggio a codesta R. Accademia delle Scienze. Ti sarei grato se volessi presentarla tu stesso, con qualche "buona parola". Spero che l'opera, malgrado i difetti che potrà avere, farà del bene alla scuola. Questo sarà l'unico compenso che potrò avere per le incredibili fatiche durate nel raccogliere i vari articoli* (Dearest Peano, you will by now have received, or will receive soon by Hoepli, the 1ˢᵗ part of the 1ˢᵗ volume of the *Encicl. delle matem.ᵉ elementari*. I have had another complimentary copy sent off to the R[oyal] Academy of Sciences. I would be grateful if you would present it to them yourself, putting in a "good word". I hope that this work will do some good for the School, despite its defects. That would be the only compensation that I receive for the incredible strain I've been put under while assembling the various articles). As requested, Peano made a formal presentation of the work during the meeting of the Turin Academy of Sciences of 1 December 1929 (see Turin Academy of Sciences Archives, Cat. 3ª, *Adunanze di Classe e Verbali, Classe I, Mazzo 38, Verbali originali della Classe di Scienze Fisiche e Matematiche*, 1920–1933 p. n.n.): *Il socio Peano, per incarico del prof. Berzolari, socio corrispondente dell'Accademia, presenta il primo volume dell'opera Enciclopedia di Matematiche Elementari, a cura di L. Berzolari, G. Vivanti, e D. Gigli, con le seguenti parole: "La matematica elementare è la base della matematica superiore, ed è spesso fine a se stessa. Da alcuni anni, in Italia ed all'estero, si pubblicarono numerosi libri che esaminarono sotto l'aspetto storico e critico i fondamenti della matematica, rilevando definizioni viziose, dimostrazioni illusorie. Le prime 80 pagine del libro costituiscono la Logica del prof. Padoa. Essa tratta le questioni di Logica generale, specialmente quelle che si riferiscono alla matematica, per mezzo d'un calcolo simile al calcolo algebrico. È una chiara ed esauriente esposizione storica e critica di quanto fu fatto finora e contiene vari perfezionamenti dell'autore. Seguono le trattazioni dei Professori Gigli, Bortolotti, Cipolla, Finzi, Tacchella, dell'Aritmetica, Teoria dei numeri, Logaritmi e Calcolo meccanico. Questo libro è di massima importanza per tutti gli insegnanti di matematica, di ogni grado"* (Academy member Peano, at the request of prof. Berzolari, honorary member of the Academy, presented the first volume of the *Enciclopedia di Matematiche Elementari*, edited by L. Berzolari, G. Vivanti, and D. Gigli, with the following words: "Elementary mathematics are the basis for higher mathematics, and are frequently as well an end in themselves. Over the last few years, in Italy and abroad, a number of books have been published that examine the foundations of mathematics from historical and critical points of view. The first 80 pages of this volume consist in an exposition of *Logic* by prof. Padoa. It considers logical issues in general, and particularly those issues that refer to mathematics, through a kind of calculus analogous to algebraic calculus. This is a clear and comprehensive historical and critical overview of research in Logic lead till now and it also provides various particular refinements on the part of the author. This is followed by the articles of Professors Gigli, Bortolotti, Cipolla, Finzi, Tacchella: *Aritmetica, Teoria dei numeri, Logaritmi* and *Calcolo meccanico*. This book is of the greatest value to all teachers of mathematics, at all levels.").

[19]Berzolari presented it as forthcoming at the meeting of the Istituto Lombardo on 5 November 1931, *Rendiconti dell'Istituto Lombardo*, s. 2, 64, 1931, p. 1091. The second part of the first volume was also presented by Peano to the Turin Academy of Sciences, at the meeting of 15

November 1931 (see Turin Academy of Sciences Archives, Cat. 3ª, *Adunanze di Classe e Verbali*, Classe I, Mazzo 38, *Verbali originali della Classe di Scienze Fisiche e Matematiche*, 1920–1933 p. n.n.): *Il socio Peano presenta il Vol. I, Parte II, dell'Enciclopedia delle matematiche elementari, donato all'Accademia dagli autori Berzolari, Vivanti, Gigli, coadiuvati da altri illustri matematici. Già due anni or sono presentò la parte prima di detto volume. Il socio Peano illustra lo scopo e l'utilità di tale Enciclopedia, veramente notevole per chiarezza e rigore* (Academy member Peano presented Vol. I, Part II, of the *Enciclopedia delle matematiche elementari*, donated to the Academy by the authors Berzolari, Vivanti and Gigli, in collaboration with other eminent mathematicians. Two years or so ago he presented the first part of the same volume. Peano explains the aim and the usefulness of this *Enciclopedia*, truly remarkable for its clarity and rigour).

[20]The same might be said upon comparison with similar encyclopaedias from abroad, such as the German and French versions of the *Encyclopäedie der Mathematische Wissenschaften*, F. Klein's volumes of *Elementarmathematik vom höheren Standpunkte aus* or H. Weber and J. Wellstein's *Encyklopädie der elementar Mathematik*.

[21]Enriques is an exception here. He criticised the planning out of the first two volumes, arguing that, in their effort to meet the demands of eclecticism, the aspiration to seek out unity in diversity has been neglected, resulting in an overly fragmented overview which levelled and blurred distinctions between theories which actually varied in outlook and value. His criticisms mainly referred to the treatment of the foundations of mathematics and of didactic considerations, on which latter point he wrote [28, p. 113]: *Mi auguro che queste osservazioni sieno tenute in conto dai redattori dei volumi*

repeated requests received by Berzolari to provide a German edition, published at the same time as the original Italian one, the excellent reviews in the top scholarly journals in Italy, France, Germany and the United States [2,4,8,15,19–22,24,26,28, 30–36,40,41,43,44,46–50,54,56,59,66], and the many anastatic reprints over the years all bear witness to the appreciation of a collection of essays which preserves its indisputable mathematical, historical and cultural value even today.

2 Contents and Contributors

The first historiographical question which the *Enciclopedia delle Matematiche Elementari* presents to us is the attribution of a precise meaning to the term "elementary" in its title. The term is particularly rich in nuances and made frequent appearances in analogous series between the end of the nineteenth and the beginning of the twentieth centuries. Indeed, as E. Bompiani writes:

> When used with reference to geometry (or to mathematics in general), the term "elementary" can assume at least three different meanings. Firstly, it expresses simplicity and straightforwardness, delimiting subjects that are the object of a qualitative middle-school teaching. Secondly, it refers to the elements, to the foundations or bases, and provides an opportunity for critical analysis to enter more deeply into the concepts which underlie common intuition and consequently to broaden its field of research. The third refers to the elementary nature of the tools employed to attain to properties which might be more easily arrived at with other methods. Of these three aspects, the first is necessary, the second is of the greatest interest, because it goes beyond the first, giving it a firm foundation and indicating new lines of development; the third often has to do with technical skills.[22]

Without a doubt, the *Enciclopedia* reflects all of these different connotations, while there are at the same time noticeable differences from one article to the

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in preparazione, e in ispecie di quello che concernerà le questioni didattiche: l'Enciclopedia deve fornire al lettore un'informazione quanto è possibile obiettiva dei varii indirizzi e criterii scientifici, logici e didattici: i redattori non hanno il compito di risolvere, secondo un proprio criterio questioni disputate, che sono passibili soltanto d'un giudizio storico di là del presente (I hope that the above observations will be borne in mind by the editors of the forthcoming volumes, and in particular by those dealing with didactic considerations. The *Enciclopedia* should provide readers with information as objectively detached as possible from the various scientific, logical and didactic schools and criteria. It is not the task of the editors to resolve, with their own criteria, controversies which will only be truly surmounted with the benefit of hindsight).

[22][8, p. 156]: La qualifica di «elementare» data alla geometria (o alla matematica in genere) può assumere almeno tre significati diversi: il primo è quello di semplice, piano, e vale a delimitare qualitativamente la materia che è oggetto dell'insegnamento medio; il secondo è quello di relativo agli elementi, ai fondamenti ed offre all'analisi critica l'opportunità di approfondire le nozioni che soggiacciono all'intuizione comune e di allargare conseguentemente il campo della ricerca; il terzo si riferisce alla elementarietà dei mezzi impiegati per raggiungere proprietà che più rapidamente si troverebbero con altri mezzi. Di questi tre aspetti il primo è necessario; il secondo è il più interessante perché supera il primo dandogli sicuro fondamento e indicando nuove vie di sviluppo; il terzo sa spesso d'artificio.

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other. Thus, while in some sections the treatment seem overly elementary and rather too close in style to a common textbook, in others, to the contrary, it seems too abstract and "high-minded" with regard to the work's aims and the readers targeted. The various authors adopt different solutions to provide a bridge between "elementary" and "higher" mathematics. B. Colombo, for example, seeks to confine "higher matters" to the very end of his article *Sistemi lineari di cerchi e di sfere*, presenting them in the guise of an appendix. In his chapter on *Trasformazioni geometriche elementari*, which are elegantly outlined in accordance with the *Erlangen program*, U. Cassina adopts a different strategy. Here, "higher matters" are dealt with in specifically reserved paragraphs, which are indicated with asterisks and frequently include the results of original research by Cassina himself.

The *Enciclopedia* articles can be divided into four general groups. First, there are those which illustrate theories or methods fundamental for the practice of teaching, such as the discussion of problems and the *Questioni didattiche*, edited by R. Marcolongo and L. Brusotti, respectively. Next, there are those which address particular topics in elementary mathematics from a higher standpoint. Then, we find those chapters which aim at providing a historical framework for given scientific theories. The last typology consists in those articles providing specific aids for teaching, such as Cipolla's essay *Matematica ricreativa*, useful for finding exercises and problems which can make lessons more lively and interesting. In most cases, the choice of themes for entries echoes those included in the syllabi issued by the Italian Ministry for Education for the curricula to be taught at the secondary level of various kinds of schools. They thus reflect school reforms between 1909 and 1950 and, at the same time, economic development during that period. For example, in T. Boggio and F. Giaccardi's treatment of financial and actuarial mathematics it is possible to trace the effects of the improvement of social and industrial conditions after the second World War as well as the changes to the ministerial syllabi for technical and trade colleges. Similarly, C. Gini's chapter on *Metodologia statistica* was added after school reforms in the late 1940s and in order to meet the new demands of industry regarding the production planes. However, the chapters in the *Enciclopedia* are much more than mere summaries of traditional courses[23] or syntheses of treatises.[24] Indeed, the *Enciclopedia* includes a number of valuable and comprehensive articles on little-known and very circumscribed topics, such as G. Biggiogero's *Geometria del tetraedro* and Biggiogero and V. Retali's *Geometria del triangolo*. Moreover, even when they are treating aspects of mainstream didactic tradition, the authors seldom neglect to outline generalisations, new developments,

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[23]Among the more traditional chapters, we can mention Brusotti's article on *Poligoni e Poliedri*, Artom's on *Proprietà elementari delle figure del piano e dello spazio* and Agostini's on *I Problemi geometrici elementari e i Problemi classici*, which, despite their conventional approach, would nonetheless prove useful to teachers in need of rapid and ready guidelines.

[24]Here, we might refer to the articles by C. Burali-Forti, G. Loria, G. Fano and U. Cassina. This last, in particular, closely follows the author's 1928 volume *Calcolo numerico con numerosi esempi e note storiche originali*, Bologna, Zanichelli.

applications and updatings of those theories, also taking into account research in progress at the time of writing.[25] For example, after providing a rather traditional exposition of polyhedrons, linked to the teaching of the theory of polygons at the middle school level, Brusotti goes on to append some interesting considerations on topology. Although the treatments primarily emphasize theoretical aspects, there is no lack of references to the applications of the various themes expounded. Thus, A. Comessatti, after illustrating the general aims and principles of descriptive geometry, dwells on its applications in photography and aerial photography.

The majority of the chapters in the *Enciclopedia* seek to give an account of the various scientific and methodological approaches, without affiliating themselves with any single author or approach and without privileging one kind of School over another.[26] They, thus, fulfil the requirements of the editorial board, whose task it was to supervise the balanced development and the *generale affiatamento*, or overall harmony, of the various sections. There are a few exceptions, the most blatant of which is without a doubt the article *Logica*, entrusted to A. Padoa.[27] The choice to include an entry on logic in an *Enciclopedia delle matematiche elementari* in itself bears witness to the breadth of Berzolari's outlook of mathematics. Despite the general acknowledgment of the contributions of Peano's School in this field, that a section on logic should serve as an introduction to the field of elementary mathematics was by no means a given. Even though the circumstance is important for its cultural meaning, Padoa limited himself to presenting a new system of propositions of ideographical logic based on three primitive concepts. He displayed a rather blinkered vision in neglecting to contextualize the relationship of symbolic to classical logic and to other contemporary systems. Indeed, his chapter was met with a great deal of perplexity and criticism, from those both inside[28] and outside Peano's circle. Enriques' words were particularly harsh:

> But I cannot refrain from making some specific reflections on the article on Logic. First of all, there is the title of the article: simply "Logic", rather than "mathematical logic," "symbolic", "pasigraphic", or "ideographic"! We completely agree with the choice of this title, which implies the concept that there are not two different kinds of logic, one of which is worked by mathematicians, or by those mathematicians who use symbolic languages, while the other is cultivated by other scholars (philosophers, scientists in general or non-symbolist mathematicians). There is only one "Logic" and we recognise that, *for Padoa*, this Logic is given its highest expression in the symbolic treatment and in the system to which Padoa adheres. The full explanation of this view, and a comparison with other approaches, would be very interesting, also for those who do not share Padoa's point of view. Thus, what we really deplore is the fact that the author cloistered himself within a particular School and practically within his own current system.[29]

In other cases, instead, there are shortcomings in the coordination of related articles. Thus, for example, Gigli's chapter on *Aritmetica generale* makes no cross-references to the article on *Logica*, even where the two overlap (for example, in expounding on the notion of finite classes), nor does it take into complete account recent developments in the study of the foundations of arithmetic, providing rather instead vague definition of the concept of number.[30]

Contributors were repeatedly asked to take special care with the historical and bibliographical apparatus, whose breadth and precision was considered essential both for those wishing to carry out further research by going back to the sources and for those in need of a concise overview on the various theories.[31]

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[25]E. Togliatti, for example, includes recent results by O. Chisini in his chapter on *Massimi e minimi*.

[26]Notwithstanding this, in the paragraphs on *Progressioni and Logaritmi*, A. Finzi paid no heed whatsoever to the contributions of the Peano School, such as those of T. Boggio, E. Viglezio, R. Frisone, A. Borio etc. which, instead, are expounded in detail in Cassina's chapter on numerical calculus.

[27]A similar evaluation might be attached at Benedetti's article entitled *Fondamenti di Geometria*, which Enriques strongly criticizes, arguing that [28, p. 113]: ...*l'esposizione dell'argomento per l'Enciclopedia delle matematiche elementari non aveva da conformarsi a vedute subiettive, anzi doveva seguire un ordine storico* (The treatment of this subject for the *Enciclopedia delle matematiche elementari* must not comply with subjective points of view. It should rather follow the historical order of developments). Enriques' criticism was not without foundation. Indeed, as a kind of supplement to Benedetti's article, an appendix on *Fondamenti della Geometria*, edited by G. Giorgi is inserted in the *Enciclopedia*. Here, the various aspects of foundational problems are discussed from an abstract, experimental and psychological point of view, also taking into account contributions to the field given by the logic and the advanced mathematics.

[28]See G. Vacca to G. Peano, 24 November 1929, Turin, Peano Archives, *Vacca correspondence*, c.p.: *Non ho ancora visto l'Enciclopedia del prof. Berzolari, se non di sfuggita. Ho avuto l'impressione che la Logica del Padoa sia troppo lunga, e forse anche, per quanto me ne ha detto il Padoa stesso, non corrispondente in tutto al mio modo di vedere* (I have not as yet had more than a fleeting glance at prof. Berzolari's *Enciclopedia*. My first impression was that Padoa's *Logica* was too long and perhaps also, from what Padoa himself has told me, rather dissonant with my own ideas on the subject). See also [19, p. LV–LVI].

[29][26, p. 40–41]: *Ma non posso astenermi da alcune considerazioni particolari sull'articolo relativo alla logica. Anzitutto sul titolo dell'articolo: "Logica", anziché "Logica matematica" o "simbolica" o "pasigrafia" o "ideografia"! Approviamo toto corde la scelta di codesto titolo che implica il concetto non esservi due diverse logiche, ad una delle quali lavorano i matematici o quei matematici che usano linguaggi simbolici, mentre l'altra sarebbe coltivata dagli altri (filosofi, o cultori delle scienze in genere o anche matematici non simbolisti). C'è una sola logica, e si capisce anche che, per il Padoa, questa tocchi la più alta espressione nella trattazione simbolica e nel suo proprio sistema. La spiegazione d'una tale veduta, col confronto delle idee altrui, sarebbe riuscita interessantissima anche e specialmente per chi ne dissenta. Perciò deploriamo tanto più che l'autore si sia chiuso in una scuola particolare e quasi nel suo sistema attuale.*

[30]Similarly, Artom's article on *Proprietà elementari* ... makes only passing references to that of Benedetti on the foundations of geometry. By contrast, Gigli's *Teoria della misura* is outstanding in its constant cross-referencing to other articles.

[31]Of particular value are the "many thousands" of bibliographical citations, fruit of extensive and careful research by Berzolari himself who, in order to simplify this task, over the years had gradually created and enriched a miscellanea of 3,176 booklets and off-prints. The *Miscellanea*

The comprehensiveness of bibliographical data, which also include references to manuscripts, has, in general,[32] been recognised as one of the *Encicopedia*'s greatest merits.

With its 45 authors and 8 board members, the *Enciclopedia delle Matematiche Elementari* was one of the most "choral" and interactive forms of collaboration of Italy's mathematical community. Its true team-leader was Berzolari. A pupil of Pincherle, Berzolari benefited from his early experience in secondary school teaching in Pavia (1885–1887) and Vigevano (1888). This was followed by a period of university teaching in Turin (1893–1899), where he made contact with many scholars involved in the Schools of Segre, Peano and Volterra, before winning, in 1899, a chair in Algebraic analysis and Analytical geometry in Pavia, where he would remain until his retirement. Berzolari's research in the field of higher geometry, and, in particular, differential-projective geometry, is well-known, but it represents "a small part of Berzolari's services to mathematics" [9, p. 400]. What made him such an excellent candidate for the role of editorial director of the *Enciclopedia* was his involvement in "a particular form of academic endeavour" which began around 1906 and continued from then on "intensifying, increasingly absorbing almost all his time free from academic tasks".[33] This "particular form

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Berzolari was donated to the Collegio Ghislieri and the University of Pavia, where it is still treasured today, together with a general miscellany of 1,000 booklets and with the Casorati miscellany, donated by G.A. Maggi, which consists in 2001 off-prints.

[32]See by contrast [20, p. LXVIII]: ... *estesissime, fin troppo a mio avviso (in un'opera di cultura generale, dedicata specialmente ai neo-laureati e professori di scuole medie), perché la sovrabbondanza straordinaria ... finisce con lo smarrire più che coll'aiutare il lettore novizio. E la consueta ricchezza di indicazioni bibliografiche fa poi maggiormente risaltare le lacune, talvolta non trascurabili* (They are immensely thorough (almost too thorough for a work of general education, aimed at new graduates and middle school teachers), because the extraordinary abundance ends up baffling rather than helping the novice reader. And the wealth of bibliographical data usually provided tends to throw light on shortcomings which are often far from negligible).

[33][14, p. XXVII]: On this point, Bompiani writes [9, p. 402–403]: ... *ché anzi essa è di gran lunga superata, sia per l'ampiezza dei campi considerati sia per mole di lavoro, dalla organizzazione in vaste opere enciclopediche del sapere raggiunto. Questa attività richiede anzitutto il sacrificio del piacere e dell'orgoglio della ricerca scientifica personale per porre la propria dottrina a servizio e a profitto di altri ricercatori; e non dà neppure la soddisfazione che prova l'A. di un libro nel legare il proprio nome ad un'esposizione originale della materia al cui sviluppo abbia contribuito. Ma oltre a questa mortificazione della propria personalità come ricercatore e come autore, essa richiede una completa conoscenza della materia da raccogliere, chiari criteri ordinativi secondo cui raggruppare i prodotti delle varie fantasie individuali, senso di equilibrio onde contemperare le necessità di un'esatta e minuta documentazione bibliografica con l'esigenza di porre in rilievo le idee direttrici, vigilanza continua delle proprie preferenze, obiettività e serenità di giudizio* (That activity is indeed exceeded, in terms of both the immense range of fields considered and the sheer amount of work, by the activity of organisation of the knowledge obtained into vast encyclopaedic works. This activity requires sacrificing the pleasure and pride of individual scientific research in order to place one's own learning at the service and profit of other scholars. Nor does it give the satisfaction which the author of a book feels in seeing his name connected with an original treatment of a subject to which he himself has made some contributions. But aside from this mortification of one's own identity as researcher and author, this activity requires a complete

of academic endeavour" was his involvement in the main encyclopaedic projects of the period: the *Encyclopädie der Mathematischen Wissenschaften*, for which he prepared three articles on algebraic curves in the plane and in space (with K. Rohn), E. Pascal's *Repertorium der höheren Mathematik*, for which he was responsible for ten essays, and the *Enciclopedia italiana Treccani*, to which he contributed his masterful 49-column entry on *Algebra*. All of these experiences, undertaken in a patriotic spirit with the intention of performing a "great national service" [14, p. XXVII], predated his involvement with the *Enciclopedia delle Matematiche Elementari* and represented a kind of apprenticeship, perfecting his "mastery in fully satisfying both the aims of an entry of an encyclopedia: the need for a broad, all-encompassing perspective, and that for detailed bibliographical information. He was able to conciliate these requirements, which appear at first glance antithetical, into an harmonic and effective framework".[34] It was Berzolari who, with his sensitivity to the intellectual needs of teachers, his erudition and his organisational capacities and tenacity, outlined the *modus operandi* for the *Enciclopedia*'s preparation, established the editorial plan, selected most of the contributors, wrote various articles of his own and continued to correct manuscripts and proofs until the very final days of his life, suggesting changes, integrations, links among the articles, etc. Again, it was Berzolari who strictly dictated the form, extension, content and style of the work, thus maintaining its unity of vision, together with the coordination and balance of the various parts. It is no coincidence that many of the contributors had had some form of specific contact with Berzolari: Brusotti was his student at secondary school, then his assistant and colleague at Pavia; Biggiogero had also worked as his assistant; Boggio and Severi had been his students in Turin; R. Serini, E. Daniele and F. Sibirani served as professors when the Mathematical Institute in Pavia was under his direction; A. Palatini and L. Gabba were his colleagues at Pavia; Loria his fellow student, and so forth with others. This notwithstanding, the selection of contributors was remarkable in its breadth of perspective. The *Enciclopedia* included the involvement of scholars representative of the various *Schools* of mathematics present in Italy, even those whose outlooks and attitudes were diametrically opposed, and involved the universities of Turin, Pavia, Milan, Bologna, Genoa, Naples, Padua, Rome, Pisa, Modena and Trieste.

Duilio Gigli (1878–1933) and Giulio Vivanti (1859–1949) would share the direction of the *Enciclopedia delle Matematiche Elementari* with Berzolari. Gigli, Berzolari's son in law, after having been his student at Pavia's *Liceo* and then his assistant at the University there for the chair of Infinitesimal calculus, taught and

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knowledge of the subjects that must be illustrated, clear regulating criteria according to which the products of many individual personalities are to be grouped, and an ability to find the right balance between the need to provide exact bibliographical information and the need to highlight the guiding principles. And finally this activity requires a constant supervision of one's own preferences, objectivity and serenity of judgement).

[34][14, p. XXVII]: ... *maestria nel soddisfare pienamente ad entrambi gli scopi di un articolo di Enciclopedia: una larga veduta d'assieme ed una minuta informazione bibliografica, conciliando tali esigenze a prima vista antitetiche in un quadro armonico ed efficace.*

served as deputy head teacher of the secondary school in Pavia. He contributed three articles to the *Enciclopedia* and dedicated "all the brief moments of respite from his arduous school commitments to the scrupulous correction of bibliographical references, ensuring the standardisation of notations and symbols and compiling the indices of authors and periodicals" [65, p. 256].

From a scientific point of view, Giulio Vivanti's role in the publishing project was more significant. After graduating in Civil engineering from the University in Turin (1881), Vivanti went on to study Mathematics at the University of Bologna, where he was a student of C. Arzelà and S. Pincherle, graduating on 30 June 1883. He obtained his habilitation (*libera docenza*) in Infinitesimal calculus on 13 May 1892. During this period, Vivanti taught a course in Bologna on algebraic numbers, of which a precise documentary record remains. On 23 October 1892 he sent Peano a note from Mantua entitled *Sull'uso della rappresentazione geometrica nella teoria aritmetica dei numeri complessi* for publication in the *Rivista di Matematica*, of which Peano was the editor-in-chief. Vivanti introduced the note as a digest of his lessons held on this topic:

> While teaching a course on the general theory of algebraic numbers, I came to observe that the theory of complex whole numbers assumes a fairly clear and intuitive form thanks to the constant recourse to geometrical representation. Here I outline the part of my lessons that refers to the above mentioned theme, in the hope that it might be of some interest to readers of the *Rivista* from a didactic point of view. For brevity's sake, I will omit all that reflects the extension of elementary operations to complex numbers, an extension to be carried out in conformity with Hankel's *principle of the permanence of formal rules*.[35]

The treatment begins with the definitions of the system T of whole complex numbers and the definitions of sum, difference, product and division, of which the primary properties are enounced. Vivanti then proceeds to outline an original geometric interpretation of the product of numbers of the system T. Geometric interpretations, visualized in the form of diagrams, are used for the introduction of the concept of lowest remainder of a number, concluding with the study of the ideals of the T system and of various theorems on congruence classes, including that of Wilson.

Vivanti moved to Pavia in 1893, where he held a chair at the *Scuola Normale* annexed to the Faculty of Sciences, remaining there until 1895, when he took over the chair in Infinitesimal calculus at the University of Messina, winning a full professorship in 1901. He returned to Pavia in 1907, as full professor of Infinitesimal calculus. He would hold this chair uninterruptedly, together with that in Higher analysis, until 1924, when he transferred to the University of Milan. There he taught

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[35][60, p. 167]: *Nell'occasione d'un corso sulla teoria generale dei numeri algebrici tenuto nell'Università di Bologna, ebbi ad osservare che la teoria dei numeri interi complessi prende una forma assai chiara ed intuitiva mediante l'uso costante della rappresentazione geometrica. Espongo qui quella parte delle mie lezioni che si riferisce all'accennato argomento, nella lusinga che essa possa riuscire di qualche interesse ai lettori della Rivista dal punto di vista didattico. Ometterò per brevità tutto quanto riflette l'estensione ai numeri complessi delle operazioni elementari, estensione la quale deve farsi conformemente al* principio della permanenza delle leggi formali *di Hankel.*

Analysis (algebraic, infinitesimal and higher) and was dean of the faculty until his retirement in 1934. The author of some 200 publications,[36] dedicated prevalently to the theory of analytic functions, Vivanti, like Berzolari, also had to his credit some prior experiences of collaboration on specialised encyclopedias, having been one of the editors of Peano's *Formulario* and of the *Enciclopedia Treccani*, for which he prepared the entries *F. Brioschi*, *Equazioni* and *Calcolo infinitesimale*.

Thanks to his mastery of modern languages, he was solidly inserted in the international mathematical community, above all in that of Germany, where his indefatigable activity as reviewer for the *Jahrbuch über die Fortschriften der Mathematik* was well-known (he published a total of 1,740 reviews between 1884 and 1938).

Vivanti was, moreover, sensitive to didactic issues. The clearest testimony to his teaching skills lies in his 19 university and six secondary school textbooks, together with the clearly-explained booklets published by Hoepli on analytic functions, on integral equations, and on polyhedric and modular functions, some of which were translated into German and favourably received in Italy and abroad.

Given his training and his academic activities, it was natural that Vivanti's collaboration on the *Enciclopedia delle Matematiche Elementari* was primarily focused on analytic slope. He contributed two articles on this matter and oversaw their coordination with other related entries. Member of the Pavia section of Mathesis, and later, starting from 1925, President of the Milan section, he was nonetheless only fully involved in the first volume of the *Enciclopedia*. Subsequently, his role became increasingly marginal and eventually ceased altogether, partly as a result of his blindness, and partly because of the racial laws, which led to his marginalisation from the academic world.

3 The Contribution of Bolognese Mathematicians

In the first decades of the twentieth century, Bologna was a well-known hub for the debates on pedagogical issues. Here, the Mathesis had its headquarters from 1900 to 1902, during the presidency of G. Frattini, and then again, from 1919 to 1932, under the direction of F. Enriques. The Federazione Nazionale degli Insegnanti di Scuola Media, for a long time directed by Vitali, placed here its first congress in 1902. The presence of publishing houses such as Zanichelli, sensitive to didactic issues and committed to the publication of textbooks, and the fact that the *Periodico di Matematiche* had its publishing seat in Bologna were components that together contributed to create fertile ground for the promotion of interactions between the world of secondary schools and that of the university, whose members were called

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[36]Today Vivanti is principally remembered for the theorem which carries his name, which states that for a function represented by a series of real nonnegative coefficients, the intersection of the circumference of convergence with the real-positive semi-axis is a singular point. On Vivanti's scientific output see [23, p. 184–205].

to face themselves on methods and program contents, on the initial training of teachers and their continuing education after graduation. It is not surprising, then, that in the "geographic distribution" of the members of the team of the *Enciclopedia*, the contribution of Bolognese community is prominent. This involvement assumed the widest possible range of forms and titles and enabled the scholars involved to develop or reinforce their network of professional relations with colleagues throughout Italy and, above all, in Pavia. The contributors with ties to Bologna range from those – such as Vivanti – who had carried out here the entirety or a part of their studies at the University, to those – such as Bortolotti, B. Segre, Sibirani, Vitali and Pincherle – who taught at the University of this town during the period in which they were involved in the *Enciclopedia*, to those who were linked to this project in a more indirect and tangential way, such as Enriques, in his role as Mathesis president, as driving force behind a similar editorial enterprise – the *Questioni riguardanti le Matematiche elementari* – as well as a critic and reviewer of the *Enciclopedia delle matematiche elementari* itself.

The occasions of contact between the editorial board of the *Enciclopedia* and the Bolognese community passed presumably through Pincherle, to whom both Berzolari and Vivanti had ties. Pincherle, the "Nestor of Italian mathematicians", had taught Vivanti at university, and Berzolari – who would succeed him as president of the Unione Matematica Italiana – in secondary school. In addition to this trait d'union, we should however also recall the large number of former students and colleagues of Berzolari and Vivanti from their periods in Turin and Pavia who had become fixtures to a greater or lesser extent in the Bolognese milieu. B. Levi and F. Sibirani all fell into this category, as did U. Amaldi and F. Enriques, who were also in their turn involved in the *Enciclopedia*, albeit in a less formal way.[37]

From a historiographical point of view, the analysis of the participation of a *local* community in a such *national* and polyphonic enterprise as the *Enciclopedia* implies:

- examining whether and to what extent the chapters reflect the authors' training received at the University of Bologna and/or their teaching practices there;
- reconstructing, through recourse to relevant documentary sources, the reasons that justify the choice of these authors and the desire of the editorial board to involve them in the *Enciclopedia*'s team; and finally,
- establishing the existence of any common ground between these mathematicians, beyond the obvious geographical location, evaluating the presence and nature of

[37]It was not by coincidence that, on the occasion of celebrations of Berzolari's 50 years as a teacher, in October 1935, many of those who participated came from a Bolognese background. Together with the mathematicians mentioned above, there were also G. Charrier, E. Lodi, M. Manarini, G.B. Zecca, and messages arrived from Pincherle's family, from the University, from the Unione Matematica Italiana, from the Istituto superiore di scienze economiche e commerciali and from the Accademia delle Scienze dell'Istituto di Bologna, of which Berzolari was a corresponding member (*socio corrispondente*).

any direct relations or interactions between them over the period in which they were working on the *Enciclopedia*.

Due to a plethora of internal and external factors, it is impossible to answer all these questions. The often quotidian and informal nature of the relationships which develop among a group of colleagues and scholars working in the same place, the lack of archival materials and the component of chance that mark the ties to Bologna of some of these mathematicians all impede us from being exhaustive. For example, it is impossible, on the basis of our extant historical knowledge, to establish whether and in what way Agostini's years in Bologna, first as a student in Mathematics and then as assistant and lecturer in free courses of History of mathematics, influenced him as he was preparing his chapter on *Problemi geometrici elementari e problemi classici*. Similarly, we do not fully understand the reasons which led E. Artom, who had spent two years in Bologna as Enriques' assistant, to request a transfer immediately after the First World War to become secretary of the editorial board of the *Enciclopedia delle matematiche elementari*.

The picture is clearer for the six full professors linked to Bolognese milieu of whom whole entries were commissioned: G. Vitali, who prepared the article on *Limiti, serie, frazioni continue e prodotti infiniti* (I, 2, p. 391–439), G. Vivanti, who wrote the *Elementi di analisi infinitesimale* (I, 2, p. 441–547) and the *Teoria degli aggregati* (I, 2, p. 549–563); S. Pincherle who was responsible for *Le funzioni analitiche da un punto di vista elementare* (I, 2, p. 565–597), B. Segre who prepared the entry on *Geometria analitica* (II, 2, p. 141–249), F. Sibirani who outlined the *Calcolo delle probabilità* (III, 2, p. 193–244) and finally E. Bortolotti, of whom was commissioned the *Storia della matematica elementare* (III, 2, p. 539–750).

Without attempting to reconstruct the genesis and stages of development for each of these articles, we will seek to identify their distinguishing features and their relationship to the other sections of the *Enciclopedia*, indicating, at the same time, the plausible reasons, both scientific and not, why Bolognese mathematicians were chosen as their authors.

The commission for the chapter on *Calcolo delle probabilità* went to Filippo Sibirani who was, together with Bortolotti, the most "authentically" Bolognese of the team involved in the *Enciclopedia*. Born in S. Agata in 1880, Sibirani had graduated in the Emilian chief town in 1902, remaining there as assistant in Infinitesimal calculus and Rational mechanics until 1907 and then returning in 1929 as dean of the Istituto Superiore di Scienze Economiche e Commerciali. A former colleague of Berzolari in Pavia, where he taught from 1915 and 1922 and again in the 1926–1927 academic year, Sibirani's credentials as a mathematician were already firmly established by the time of his involvement with the *Enciclopedia*, as was his reputation as a scholar with superb organisational capacities, as his activity as *Commissario prefettizio* (Prefectorial Commissioner) of the Unione Matematica Italiana in 1945-46 attests. He could boast a wealth of significant findings on ordering functions and many years of teaching experience both at the secondary school and the university level, which particularly qualified him to be the author of an article for an *Enciclopedia* especially geared towards teachers.

Sibirani's chapter, which was quite similar to that which he would contribute to M. Villa's *Repertorio di Matematiche* (1951), developed a theme which had been inserted in ministerial programmes for secondary schools thanks to the insistence of G. Vailati and, above all, G. Castelnuovo. The treatment begins with the notion of an event's probability, which is considered from a historical perspective, moving from the classical and statistical definitions given by P.S. de Laplace and L. Cournot respectively, to the critiques by H. Poincaré and G. Castelnuovo to the axiomatic definition of F.P. Cantelli and to the abstract theory of R. Von Mises. There follow the fundamental theorems (of total probability, of repeated trials, of Tchebychef, Bernoulli, Bayes, Poisson etc.) and, in conclusion, the applications of Gauss's theory on errors of observation and the method of least squares. Sibirani's primary emphasis is on the contributions of the Italian School to the probability theory[38] from the eighteenth to the twentieth century, and he makes repeated references to the results of Castelnuovo, Cantelli, Tedeschi and De Finetti.[39] He does not neglect to place his own article in relation to those on financial and actuarial mathematics by T. Boggio and F. Giaccardi. The insights into the applicative relapses of the calculus of probability provided by Sibirani would be further enriched by F. Severi and F. Conforto who, in the third volume of the *Enciclopedia*, would illustrate the significance that this field was acquiring in the experimental sciences:

> especially in relation to physics during the great crisis that brought us from a decidedly deterministic perspective to those more recent conceptions, whereby the universe is thought to be governed by laws of an indeterministic and therefore probabilistic character (quantum physics, atomic physics).[40]

The editorial board's choice regarding the commissioning of the chapter on *Storia della matematica elementare* appears to be "natural" from a scientific point of view. In the by no means crowded ranks of historians of mathematics at that time, Ettore Bortolotti, who taught Analytical geometry in Bologna from 1919 to 1936, was one of the most illustrious figures. It is therefore no surprise that, after Giovanni Vacca's refusal, the choice fell on Bortolotti. With a wealth of publications to his name, focusing above all on the history of algebra, Bortolotti also fully shared Berzolari's patriotic fervour and, although sometimes prone to lapse into nationalistic tones, seemed the scholar most qualified to highlight the contribution of Italian mathematicians to the history of elementary mathematics. His article, indeed, stands as a true monograph, accompanying in the *Enciclopedia* that of M. Gliozzi on the *Storia del pensiero fisico*. After providing an overview

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[38]The contributions of Italians are also emphasised in Berzolari's articles, although without nationalistic asides.

[39]He includes some suggestive reflections on the subjective theory of probability by B. De Finetti, who had been student of Vivanti at the University of Milan. These thus make their first appearance in an *Enciclopedia* aimed at teachers.

[40][57, p. 809]: ... *accompagnando in ispecie la fisica nella grande crisi che ha portato dalle vedute decisamente deterministiche alle più recenti concezioni, nelle quali si pensa l'universo retto da leggi di tipo indeterministico e quindi probabilistico (teoria dei quanti, fisica atomica).*

of mathematics in ancient civilizations (Chinese, Egyptian, Sumerian), Bortolotti, rightly, dwells at some length on the description of Greek mathematics, from its "formative" period (sixth to fourth centuries B.C.) to the classic and Alexandrian periods. He then provides a detailed examination of the problems of Renaissance mathematics and arrives at the first origins of infinitesimal calculus. There is no lack of references to the history of mathematics beyond Europe – in China, India and the Islamic world, in particular. His treatment of the subject is systematic, harmonious and engaging, echoing the author's own research and thoughtfully selecting only general methods and ideas, without getting lost in superfluous details or erudite excursus. The precision with which the author refers back to the sources, of which he often also provides significant excerpts, is exceptional. However, various canons of historiographical inquiry, now outdated but typical of that time, crop up, such as the use of rather imprecisely-defined categories such as *School* and *Maestro*, the tendency to search for precursors and to make claims about the contribution of Italian mathematical Schools compared to those in other states.

More closely related to the author's didactic experiences than to his research activity is B. Segre's chapter on *Geometria analitica*. In fact, at the time of his involvement in the *Enciclopedia delle matematiche elementari*, and prior to the exile imposed on him as a consequence of the racial laws, he had already taught courses on this topic both in Turin, as assistant from 1925–1926, and in Bologna, where he had been awarded a chair in Analytic and Descriptive geometry in 1931. He too had been invited to collaborate on the *Enciclopedia Treccani*, for which he prepared the entry on *Coordinate*, the commission for which had first been offered to Berzolari, but refused by him.

Although there is no lack of interactions between these authors, all of whom were colleagues working together at the University of Bologna in the early 1930s, the contributions of this community show themselves to be particularly close-knit with regard to analysis, the sector with regard to which it had the most opportunities for dialogue.

The second part of the first volume of the *Enciclopedia delle matematiche elementari* is devoted to analysis, both in its "classical" part and in those themes called at that time "complimentary" (i.e., the theory of analytic functions and set theory). This section was coordinated by Berzolari, who was, however, flanked by Vivanti somewhat more strongly than elsewhere.

Even though it had always been a fundamental discipline in the curricula and one of the most important courses for the training of not only pure but applied mathematicians as well (engineers, scientists, naturalists, etc.), infinitesimal analysis was inserted into the syllabi for Italian secondary schools only after 1909,[41] thanks

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[41]The history of the insertion of the first elements of analysis into Italian secondary school syllabi is fairly complex. As early as 1867, L. Cremona had recommended introducing the concept of function and the method of limits. In effect, the first elements of infinitesimal calculus made their appearance in algebra textbooks written for the physics-mathematics section of *Istituti Tecnici* in around 1871. In 1887, however, Italian school syllabi were noticeably behind the times, especially in comparison to those of Prussia where, thanks to F. Klein's interventions, differential calculus had

to the efforts of Vailati and Castelnuovo, who had, in turn, been strongly influenced by Klein's opinions in this regard. It is therefore natural that Berzolari and Vivanti should seek to bring out the best of the calculus section of the *Enciclopedia*, assigning it to an individual who was prepared to take into account the different methodological approaches which had emerged over the time. Between 1880 and the first two decades of the 1900s, Italy had witnessed fierce debates between mathematicians of contrasting tendencies concerning calculus, with regard to both the choice of research themes and its teaching. The role of logic, set theory and foundational criticism as theories preparatory to calculus, the relationship with numerical and physical applications, and the connection between the courses on Infinitesimal calculus and those on Algebraic and Higher analysis are just a few of the themes which came up for discussion amongst renowned scholars such as G. Peano, E. Cesàro, C. Arzelà, S. Pincherle, G. Fubini, L. Tonelli and F. G. Tricomi. This, then, is the reason why, for the treatment of such a delicate topic, the editorial board of the *Enciclopedia* selected two scholars known in the academic world for their balanced outlook: Vitali and Vivanti, both of whom had deliberately remained on the sidelines, so to speak, of all those polemics.

By the beginning of the 1930s, both could boast experience in the teaching of calculus which spanned decades. Vitali had taught at high schools in Voghera and Genoa for more than 20 years before being given the chair left vacant by Tonelli at the University of Bologna in 1930. Vivanti had won general approval as a teacher of Differential and Integral calculus in all three universities where he had taught during his career: Messina, Pavia and Milan.

Both Vitali and Vivanti adopted a traditional content structure. Vitali offered a schematic development of the main properties of the infinite algorithms of sequences of real numbers and the properties of limits of numerical sequences and series. The paragraphs on the convergence criteria for series with positive terms and double series are particularly meticulous, as are those on continuous fractions and infinite products, which were completed by Cassina in his article on *Calcolo numerico*. Vivanti instead expounds the elements of infinitesimal calculus, stretching so far as to make a brief reference to the calculus of finite differences. A distinguishing feature of his exposition, as had already been the case in his university textbooks, is the historical notes provided, fruit of his own research activity in the sector.

The genesis and the various rewritings of these entries of the *Enciclopedia* can be well documented, thanks to various letters by Berzolari and Vivanti, collected in the *Fondo Vitali* conserved at the Archives of the Unione Matematica Italiana and as yet only published in part.

Vivanti and Vitali began corresponding in 1904, when Vivanti, who was still teaching at Messina at the time, asked Vitali for an extract of his note *Sopra le serie di funzioni analitiche*, which he intended to review for the *Jahrbuch*.[42] It therefore seems highly probable that the fact that Vitali was called on to contribute to the *Enciclopedia* from the first decade of the twentieth century resulted from his relationship with the Mantovan mathematician. We need, instead, to look ahead to January 1928 to trace Vitali's first contact with Berzolari, as a consequence of the latter's need to inform him of the economic conditions which the editorial board of the *Enciclopedia* had managed to "wrench" from the Hoepli publishing house:

> For the publication of the first volume of the Enciclopedia delle mat.ᵉ elem.ⁱ I had contacted Zanichelli, as well as Hoepli. However, Zanichelli rejected the proposal, as they already had many other commitments. Our only option, then, is Hoepli, who offers us the following conditions: *that each author should receive 10 per cent of the cover price, payable in three instalments: one third upon publication of the volume, another after the first thousand copies sold (the first print run will be of 1,500 copies) and the last when the first print run has sold out. Each author will have 25 free off-prints of his article.* I do not like the fact that payment is in three instalments, but Hoepli is unshakeable on this point. The publisher also wishes to have a written declaration of acceptance of the aforementioned conditions from each author. If you accept, please send the declaration (referring to the conditions) to me and I will send it on to Hoepli together with those of the other authors.[43]

Vitali's involvement in the *Enciclopedia* project strengthened his ties to Berzolari, as shown by another letter, written a year later, in which Berzolari thanked Vitali for the gift of his important monograph, *Geometria nello spazio Hilbertiano*, about which he expressed a fairly positive opinion.[44]

In spring 1930, the dialogue between the two was further confirmed in consideration of Vitali's work on the chapter on *Limiti*, the manuscript version of which had already been submitted to Berzolari in the previous months. On 4 February, the Pavese mathematician contacted Vitali, indicating his desire that the first draft be modified and partially extended, in order to also take into account the complex field:

> Dear Professor, I must trouble you once more in order to ask you a favour. It would be good if in one of the articles in our *Enciclopedia* there appeared at least the most elementary notions pertaining to limits and series in the complex field. It seems to me that the most

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been part of the syllabus since 1882. Under the influence of Klein, Vailati introduced the concept of differentiation (1909–1910) in all three kinds of *Liceo* and, at the *Liceo scientifico*, added that of integration. This was the object of lively debates. In 1911, Castelnuovo released the new syllabi for the *Liceo moderno*, which included the concepts of function, derivative and integral. Soon after, these concepts appeared in textbooks by F. Enriques and U. Amaldi (1914) and by S. Catania (1914). See [37, p. 5, 13, 17–26, 29, 35–37, 43–47, 51–52].

[42] See G. Vivanti to G. Vitali, Messina, 16 October 1904, in [12, p. 447–448].

[43] L. Berzolari to G. Vitali, Pavia, 20 January 1928, Bologna, UMI Archives, *Vitali correspondence*, c.p.: *Per la pubblicazione del 1° volume dell'*Enciclopedia delle mat.ᵉ elem.ⁱ *mi sono rivolto, oltre che all'Hoepli, allo Zanichelli, ma questi ha rifiutato, avendo già molti altri impegni. Non resta dunque che Hoepli. Il quale pone le condizioni seguenti. Dare ad ogni autore il 10 per cento sul prezzo di copertina, pagando in tre volte: un terzo appena uscito il volume, un altro terzo dopo vendute mille copie (l'edizione sarà di 1500 copie) e l'ultimo terzo ad edizione esaurita. Ogni autore avrà 25 estratti gratuiti del suo articolo. La ripartizione in tre rate non mi piace ma l'Hoepli è irremovibile. Egli desidera pure avere da ogni autore la dichiarazione scritta dell'accettazione delle dette condizioni. Se Ella le accetta, La prego di mandare a me la dichiarazione (contenente le condizioni) ed io le trasmetterei all'Hoepli con le altre degli altri autori.*

[44] L. Berzolari to G. Vitali, Pavia, 27 December 1929, in [12, p. 505–506].

fitting place for this would be in your article. Would you mind adding a few pages dealing with these subjects?[45]

This request met with a warm response from Vitali, who immediately declared himself willing to modify his original manuscript. On 10 March, Berzolari thanked him "profoundly for your cordiality in showing yourself so to be so willing to help me in the best interests of the *Enciclopedia*"; at the same time, he also put forward another request:

> Your proposal to modify the current text of your article here and there so as to embrace the complex field to the greatest possible extent seems an excellent one to me. However, I must appeal to your good will again in requesting one further small addition. The last article in the first volume is that by Pincherle (*Le funzioni analitiche da un punto di vista elementare*). It is therefore necessary that there should be some treatment of function series, and in particular of power series, circle of convergence, etc., in one of the articles that precedes it. Vivanti's entry (*Elementi di analisi infinitesimale*) should already touch on these matters, but only with limited reference to the field of real numbers. Nor would it be possible, without perverting the nature of that article and without rendering it over-long, to incorporate there the treatment of the complex field. As a consequence, I would ask you to leave your own article in its current form, with the exception of the additions which you yourself have suggested, and to add a brief chapter, in which the properties of limits, of function series and, in particular, of power series are extended to the field of complex numbers. It would be good if you could also draw forth the definitions of power and logarithm, which are deduced in Gigli's article on the basis of the permanence principle. To minimise overlaps with Vivanti's article, you could get in direct touch with him, to whom I will write myself, in order to inform him about this question.[46]

Berzolari, then, invited Vitali to get in touch with Vivanti and, at the same time, urged Vivanti to agree with his colleague upon the subdivision of topics which the two entries might both cover so as to minimise overlaps and gaps. Vitali's chapter was thus revised once more, this time by Vivanti who, in addition to noting, like Berzolari, some lacunae with regard to complex sequences and series, also pointed out the lack of a section on the theory of function series, which, in his article on infinitesimal calculus, he had assumed had been treated elsewhere. Thus he expressed his willingness to add these missing sections – to his mind essential in such a didactic enterprise – himself, while leaving the decision as to where it should best be placed within the *Enciclopedia* up to Vitali.[47]

After this exchange of letters, the two directors of the *Enciclopedia* agreed to contact Vitali, requesting that he fill the gaps indicated. This new request, too, was complied with, and Vitali provided the additions, receiving thanks once more from Berzolari for his promptness.[48]

By 1 April 1930, Vivanti had already examined the second version of the article on the *Limiti*, which he now found to be excellent, and was going to make the necessary adjustments to his own chapter.[49] By September, the division of the contents had become definitive, thanks to the immediate understanding between Vitali and Vivanti. The former would concentrate on the numerical aspect, the latter on the functional ones:

> My esteemed colleague, I am in full agreement with what you have written to me. It is arranged that you will be concerned with the numerical part and I with the functional. Since you tell me that you yourself will write to prof. Berzolari, I see no use in me writing as well to inform him of the agreement reached. I have started reading your fine essay, but I have been sidetracked by other occupations. I hope to get back to it as soon as possible.[50]

In October 1931, a few months before Vitali's sudden death, his correspondence with the editorial board of the *Enciclopedia* came to a close, with the board expressing their recognition of the excellent quality of his work and of his proactive collaboration.[51] In addition to the *Elementi di Analisi infinitesimale*, Vivanti also

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[45]L. Berzolari to G. Vitali, Pavia, 4 February 1930, Bologna, UMI Archives, *Vitali correspondence*, c.p.: *Caro Professore, debbo nuovamente disturbarla per pregarla d'un favore. Sarebbe bene che in qualche articolo della nostra* Enciclopedia *comparissero almeno le cose più elementari relative ai limiti e alle serie nel campo complesso. E mi pare che il posto più adatto sia il Suo articolo. Le spiacerebbe aggiungere ad esso poche pagine, dove quegli argomenti fossero esposti?*

[46]Translation of the letter by L. Berzolari to G. Vitali, Pavia, 10 March 1930, in [12, p. 507–508].

[47]G. Vivanti to G. Vitali, Milan, 17 March 1930, in [12, p. 508–509]: *Bisognerà poi dire qualche cosa delle serie di funzioni, e in particolare delle serie di potenze a variabili reali o complesse (la cui teoria occorre per l'articolo del Pincherle); questo capitolo dovrà trovar posto, mi sembra, nello stesso articolo in cui si parlerà dei limiti delle funzioni. Attendo di sapere quello che Lei propone di fare a questo riguardo* (You should then say something about function series and, in particular, about power series with real or complex variables (whose theory is required for

Pincherle's article). This chapter should, to my mind, be placed in the same article in which the limits of functions are treated. I look forward to hearing what you propose to do on this point).

[48]L. Berzolari to G. Vitali, Pavia, 25 March 1930, Bologna, UMI Archives, *Vitali correspondence*, c.p.: *Carissimo Professore, oggi Le ho spedito – in franchigia, a mezzo dell'Università – le bozze del Suo articolo. Non so come ringraziarla della Sua cortesia e della buona volontà che mi dimostra, e soltanto sono dolente che tutto ciò aumenti il Suo lavoro e contribuisca ad accrescere la Sua stanchezza. Non lavori troppo, e si prenda un po' di riposo!* (My dear Professor, today I have sent off the drafts of your article, with postage paid via the University. I cannot thank you enough for your courtesy and for the goodwill that you have shown me. I am only sorry to have added to your workload and increased your fatigue. Do not work too hard and get some rest!).

[49]G. Vivanti to G. Vitali, Milan, 1 April 1930, Bologna, UMI Archives, *Vitali correspondence*, c.p.: *Carissimo Collega, La ringrazio di quanto mi scrive. La nota sul* Limite *va benissimo. Da parte mia sto preparando le modificazioni e le aggiunte al mio articolo, in modo che alla fine nulla abbia a mancare* (My dear Colleague, thank you for your letter. The note on the *Limite* is absolutely fine. At my end, I am preparing the modifications and additions to my article, so that nothing will be lacking in the end).

[50]G. Vivanti to G. Vitali, Milan, 23 September 1930, Bologna, UMI Archives, *Vitali correspondence*, Bologna, c.p.: *Egregio Collega, sta bene quanto Ella mi scrive; resta fissato che Ella si occuperà della parte numerica, mentre io provvederò alla parte funzionale. Poiché Ella mi dice che scriverà al prof. Berzolari, credo inutile informarlo io pure degli accordi presi. Ho incominciato a leggere il Suo bel trattato, ma poi altre occupazioni sono venute ad interrompermi. Spero riprendere al più presto.* The correspondence between the two includes another missive from Milan, dated 31 December 1928.

[51]L. Berzolari to G. Vitali, Pavia, 23 October 1931, Bologna, UMI Archives, *Vitali correspondence*, Bologna, c. 1r: *Caro Professore, con la presente Le mando un vaglia della Banca d'Italia (N. 67188) di lire 340,95 come compenso per il Suo articolo inserito nell'*Encicl. di matem.[e] elem.[i]. *Poiché le copie in vendita sono 1500 e ognuna costa lire 82 ad ogni rata di pagamento*

prepared the *Enciclopedia* entry on the *Rapporti fra la teoria degli aggregati e la matematica elementare*, thus confirming his reputation as a *Maestro* in this field, a reputation which he had begun to earn in the last decade of the nineteenth century, starting from the publication of a *Notice historique sur la théorie des ensembles* in G. Eneström's *Bibliotheca Mathematica* [Vivanti 1892, VI, p. 9–25]. That had been followed by his involvement in Peano's *Formulario di Matematica*, an encyclopedic treatise written in ideographic language, for the first edition of which Vivanti had written the chapter on the *Teoria degli aggregati*.[52] Referring back to the works of C. Burali-Forti, here Vivanti focused primarily on the theory of transfinite numbers and order types, on well-ordered sets, on the genetic definition of real numbers and on the *classi ordinate secondo n dimensioni*, to which Peano had alluded at in his *Lezioni di Analisi infinitesimale* (Torino, Candeletti, 1893, vol. 2, p. 1–30).[53] He completed the exposition with a *Lista bibliografica* on the theory of sets, which appeared in the *Formulario*,[54] as an article in the *Rivista di Matematica* directed by Peano [61, p. 189–192], and finally, after having been updated to include the

period 1893–1899, in Eneström's *Bibliotheca Mathematica* [64, p. 160–165]. Here, he not only quoted the papers of G. Ascoli, G. Loria, R. Bettazzi, C. Arzelà, R. De Paolis, L. Milesi and F. Giudice, together with his own works, in which Cantor's theory of infinite sets had been assimilated and employed in analytical and geometric contexts, but he also mentioned those university textbooks in which these themes were already partially treated, such as U. Dini's *Fondamenti per la teorica delle funzioni di variabili reali*, G. Veronese's *Fondamenti di geometria*, and Peano's *Arithmetices Principia* and *Lezioni di Analisi*. Although carefully examined and annotated by Peano,[55] Vivanti's chapter did not appear in later editions of the *Formulario*, as did not that by G. Fano on algebraic numbers. However, Vivanti translated another famous article by Cantor, which again appeared in the *Rivista di Matematica* [16], thus contributing, together with Peano and F. Gerbaldi [17, 18] to the spread of Cantorian set theory in Italy. Vivanti had also entered into direct epistolary correspondence with Cantor[56] and on several occasions between 1891 and 1894 he had taken part in the thorny discussions on the use of actual infinitesimals in mathematics, adopting original critical positions.

The sum of these activities, at once scientific and didactic, placed him in an ideal position to fashion a suggestive chapter on set theory for the *Enciclopedia delle Matematiche Elementari*, which met with the special praise of Enriques for its remarkable sobriety:

> And we would ascribe to the illustrious authors the merit of having contained these developments – always clearly and precisely – within the appropriate limits for a book aimed at teachers in our middle schools.[57]

Adopting a completely different style from that used in the *Formulario*, Vivanti not only completely dropped here the logical and ideographic notations, but also shed a somewhat different light on the various issues dealt with. Foregoing any forays into the historical or foundational aspects of set theory, he instead dwelt, in a deliberately elementary manner, only on those aspects which were most useful from a teaching perspective. He, therefore, placed his primary emphasis on the differences between the properties of finite and infinite sets, the concepts of cardinal and ordinal number, the definitions of bijective and continuous correspondences, and the notion of dimension. In doing so, he relinquished opportunities to make even passing reference to the expositions of set theory which he himself had prepared for university teaching and assembled in his textbooks on infinitesimal calculus.

Indeed, to find in the *Enciclopedia delle Matematiche Elementari* a broader reference to the links between the set theory, the critique of principles, the theory of real functions and the functional analysis, one must consult F. Severi and

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ciascuna pagina viene compensata con lire 7.45. Il Suo articolo, compreso l'indice è di 46 pagine; quindi le sono dovute lire 342,70. Detraendo lire 1,75 per la spedizione, rimane la detta somma di lire 340,95. Per mia tranquillità, La prego di farmi avere un cenno di ricevuta. Con i saluti più cordiali e con vivi ringraziamenti per il valido contributo dato alla nostra Enciclopedia *mi creda Suo aff.mo L. Berzolari* (Dear Professor, please find enclosed a bank order from the Banca d'Italia (N. 67188) for 340.95 lire, in payment for your article published in the *Encicl. di matem.ᵉ elem.ⁱ*. Since 1,500 copies are on sale and the cost of each is 82 lire, 7.45 lire will be paid per page per instalment. Your article, including the index, is 46 pages long, so you are due 342.70 lire. Subtracting 1.75 lire for postage, the sum due stands at 340.95 lire. For my peace of mind, I would ask you to send me notice of receipt. With my kindest regards and heartfelt thanks for your valuable contribution to our *Enciclopedia*, I am truly, affectionately yours, L. Berzolari). See also L. Berzolari to Vitali, Pavia, January 1939, Bologna, UMI Archives, *Vitali correspondence*, Bologna, c. 1r: *Chiar.mo Professore, della 2ᵃ parte del I volume dell'*Enciclopedia delle Matematiche Elementari *si sono sinora vendute 1360 (delle 1500) copie, e l'editore Hoepli mi ha mandato il compenso da distribuire ai vari autori (limitatamente alle copie vendute). Poiché l'articolo redatto dal compianto suo fratello è di 46 pagine, gli spettano lire 589,70. Le mando perciò un vaglia della Banca d'Italia N. 0.010.090. per la somma di lire 587,70, avendo trattenuto due lire per le spese postali. Con distinti saluti e auguri mi abbia suo dev.mo L. Berzolari. P.S. Le sarò grato di un cenno di ricevuta. Veramente la spesa fu di 1.75 quindi le accludo un francobollo da 0.26. Risponde 25 gennaio 1939.* (Most esteemed Professor, so far 1,360 (of the 1,500) copies of the 2nd part of the first volume of the *Enciclopedia delle Matematiche Elementari* have been sold, and the publisher Hoepli has sent me the payment to distribute to the various authors (limited to the number of copies sold). Since the article by your late lamented brother is 46 pages long, he would be due 589.70 lire. I therefore enclose a bank order from the Banca d'Italia N. 0.010.090 for the sum of 587.70 lire, having detracted 2 lire to cover postal expenses. With sincere best wishes, I remain yours truly, L. Berzolari. P.S. I woud be grateful if you could send me notice of receipt. Postal expenses in fact stood at 1.75, so I have attached a 0.26 lire stamp. Answer 25 January 1939).

[52] G. Vivanti, *Teoria degli aggregati*, *Chap. VI* in [52, p. 65–70].

[53] Instead no appearance is made in the *Formulario* of those "new denominations... whose introduction would not have been advantageous in the exposition with the logical symbols" [62, p. 135]. Amongst the absent concepts, we can cite, for example, the notions of closed, perfect, isolated, separate and concentrated sets.

[54] Vivanti in [63, p. 71–74].

[55] See [53, p. 65–69].

[56] See G. Cantor to G. Vivanti, 3 December 1885, 6 November 1886, 30 January 1888, 2 April 1888, in [42, p. 250–251, 269–270, 300–301, 302–306].

[57] [27, p. 125]. ... *Ed ascriviamo a merito degli illustri autori di avere contenuto questi sviluppi - sempre chiari e precisi - nei limiti che si convengono ad un libro rivolto agli insegnanti delle nostre scuole medie.*

F. Conforto's article on the *Caratteri e indirizzi della matematica moderna*, written in a time of fully Bourbakist *Weltanschauung*. There, completing the propaedeutic exposition laid out by Vivanti, set theory is presented as a possible basis for mathematics and a demonstration given of how, from this starting point and by adding specific postulates, one can attain to a *matematica generale* or *assiomatica* or *astratta*

> in which various theories and concepts which were previously developed autonomously and independently from one another fit together in a unitary frame, thus obtaining an economy of thought which, given the ever-growing field of knowledge and mathematical developments, is increasingly necessary, and managing at the same time to grasp the common logical weave of a group of theories, which historically often grew out of intuitive data.[58]

In the same article, Severi and Conforto also illustrated how set theory becomes modern algebra, at the moment in which operations having all or some of the properties of the four fundamental arithmetic operations are defined as abstractly as possible for the sets on which one is working.[59] The calculus section of the *Enciclopedia* closes with an overview on analytic functions. Here again, the choice of author was a "foregone conclusion", with the commission going to Pincherle, who had been involved in the field since his youth, when he had been invited to give a series of lessons on the general theory of analytic and elliptic functions in Pavia, expounded on the basis of the notes which he himself had taken attending Weierstrass' lessons in Berlin [1, p. 4–5]. Over the course of the years spent as full professor of the chair of Calculus in Bologna, he had had many occasions to expound upon this theory, and lithographs of his *Lezioni sulle funzioni analitiche* had circulated widely in Italy, meeting with large praise, as did his university textbook, *Gli elementi della teoria delle funzioni analitiche*, published by Zanichelli in 1922. To the Bolognese mathematician, we owe a masterful exposition of the principles of this theory, which Enriques judged to be "ingenious" [27, p. 125] because it bridged the gap between the mathematical notions taught in the first two years of university and the higher branches of analysis without omitting the

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[58][57, p. 806]: ... *nella quale s'inquadrano in modo unitario svariate teorie e concetti che prima erano stati svolti in modo autonomo ed indipendente gli uni dagli altri. Si ottiene in tal modo un'economia di pensiero ... sempre più necessaria di fronte al continuo allargarsi delle conoscenze e degli sviluppi matematici; e si riesce a cogliere la comune trama logica di svariate teorie, spesso sorte storicamente da dati intuitivi.*

[59]Thus, they conclude [57, p. 809]: *Già nella seconda metà del XIX secolo era stata pubblicata la grande Enciclopedia Matematica Tedesca. Ma si può ben dire che l'odierno tentativo di Bourbaki si presenta del tutto nuovo e significativo per il chiaro e rigido concetto informatore dell'intera opera; sicché sembra di poter sicuramente affermare che questa rappresenterà ognora un documento molto interessante delle matematiche dei nostri giorni, come ai suoi tempi il* Formulaire de mathématique *di Peano* (In the second half of the nineteenth century, the great *Enciclopedia Matematica* was published in Germany. Yet we may well argue that Bourbaki's enterprise today is wholly new and significant in the clear and rigid concept which informs the whole work. Hence, it seems that we can safely affirm that this will represent a lasting source of great interest on the mathematics of our time, as in its own day Peano's *Formulaire de mathématique* did).

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elementary implications, which were of more direct utility to teachers. Of particular value are the development of the concepts of generalized dependence following Dirichlet and of analytic dependence, his treatment of functional equations for the exponential, of circular functions and of the classical procedure of recurrent series, for the solution of the differential equation of the logarithmic function.

The intertwining of scientific collaboration and personal relationships which underlies the publication of the *Enciclopedia delle Matematiche Elementari* enables us not only to analyse the dynamics of comparison between a single local mathematical community, such as that of Bologna, and the broader mathematical world at that time, but also to reconstruct how, in the terms of Enriques' telling metaphor:

> over the course of history, mathematicians, who were from time to time students and *Maestri*, have offered the spectacle of a universal reason which formulates eternal truth, one that rises above the differences and weaknesses of men. The *School* tends to extend itself beyond its own original environment, and its influence on the scholar is mingled with other, various forces which render it fertile. This is the fulfilment of the law of approach which Klein brought to light. That is to say, that the development of mathematical Schools, subject to alternating periods of progress and decadence within national limits, is revivified through the passage from one nation to another, almost as though the spirit of the world participated more broadly towards a common goal.[60]

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[60][29, p. 181]: *Nella continuità della storia i matematici, a volta a volta successivamente scolari e Maestri, offrono lo spettacolo di una ragione universale che elabora la verità eterna, sopra alle differenze e alle debolezze degli uomini. La scuola tende ad allargarsi fuori del proprio ambiente d'origine, ed allora l'influenza sullo scolaro viene a comporsi con altri motivi diversi che la fecondano. Perciò si avvera la legge di avvicinamento che Klein ha messo in luce, cioè che lo sviluppo delle scuole matematiche, soggetto ad alternanze di progresso e di decadenza nei limiti di una nazione, si ravviva passando da una nazione ad un'altra, quasi a far partecipare più largamente all'opera comune lo spirito del mondo.*

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