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THE TEACHER'S ACTIVITY UNDER A PHENOMENOLOGICAL LENS

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According to phenomenological perspectives, students must be educated to see and focus things in ways coherent with the mathematical notions to learn: for example to see the mathematical properties of a function in a graph representing a certain situation. We refer to Rota's account of mathematical thinking as "disclosure", and give a phenomenological interpretation of the cognitive processes related to graphical modelling activities, and of the role of the context therein. We contrast the classroom discussions of the same task in two different grades (9 and 11), and show how the teacher uses suitable didactic techniques to promote different "layers" of students' disclosures of Calculus concepts.

INTRODUCTION

It may be a truism in mathematical education that students must learn to "see the general in the particular and the particular in the general" (Mason, 1996). Recently such an issue has been analyzed from a phenomenological perspective, deepening in particular the relationships between perception and theoretical issues, and focusing on the role of the teacher in promoting connections between them to foster the students' learning processes. For example, Radford (2010) has pointed out how teachers can

create the possibility for students to perceive things in certain ways and encounter a cultural mode of generalizing. This new way of perceiving (...) in certain efficient cultural ways entails a transformation of the eye into a sophisticated theoretician organ. (*ibid.*, p. 2)

From another perspective, the so-called embodied cognition (Gallese & Lakoff, 2005) claims that the whole of cognition can be understood in terms of perceptuo-motor activity. Nemirovsky (in print) develops a perspective on mathematical embodied cognition consistent with a phenomenological understanding of perception and body motion.

In this paper we follow this last approach and use a phenomenological stance, based on the elaboration given by the outstanding mathematician and philosopher G. C. Rota (1991) to Husserl phenomenology, to analyze how a teacher manages the embodied and theoretical issues while teaching the same lesson in two different classrooms (one at grade 9, the other at grade 11). A major result of the analysis is that many practices of the teacher can be considered examples of what Husserl called

a ‘natural attitude’ towards human phenomena (in our case didactical phenomena). Such actions appear to be very common in everyday didactical practices of mathematics teachers, even if they not necessarily know phenomenology.

“SEEING AS”: A PHENOMENOLOGICAL STANDPOINT

As underlined by Radford (2010, p. 4), students must be taught to “see and recognize things according to ‘efficient’ cultural means” and to convert their “eye (and other human senses) into a sophisticated intellectual organ”. Namely it is necessary to promote a “lengthy process of domestication” (*ibid.*) of the way they are looking at things while learning mathematics. This process is based on the key phenomenological assumption, pointed out by Rota, that there is “no such thing as true seeing”, but “there is only seeing as” (Rota, 1991, p. 239). Hence, learning mathematics requires different modes of *focusing*: “just like seeing is focusing upon some functions which may be present, similarly, remembering, imagining, or visualizing are other modes of focusing” (*ibid.*). Students must be educated to see and focus things in *the right way*, i.e. in ways coherent with the mathematical notions to learn: for example to see the mathematical properties of a function from the graph of a mountain track, as in Fig. 1. This delicate process is far from being natural: on the contrary, to be achieved it requires precise didactical interventions of the teacher.

Consequently, in this paper we refer to two related didactic techniques from the literature: *making present* absent things to students, and *prompting* them in order to direct their *attention*. As pointed out by Ferrara (2006) and following Husserl, we can distinguish two aspects of making present, namely *remembering*, that is “making present the past (the absent being the past)”; and *imagining*, that is “making present the not yet known (the absent being the not yet known)” (*ibid.*). As said above, Rota considers both remembering and imagining as modes of *focusing*.

Focusing *attention* is at the core of the work by Mason (2008), who describes as follows the *prompting* technique of the teacher:

One of the classic interventions used by relative experts to enculturate novices into particular practices, is often referred to as *scaffolding and fading* (Seeley Brown et al. 1989). A teacher repeatedly uses a particular prompt or question with learners, and then begins to use less and less direct prompts or meta-questions such as “what question am I going to ask you?” or “what did you do last time in this sort of a situation?”, until the teacher need only rarely if at all remind learners of the prompt: the prompt has been internalised and become a spontaneous action. (Mason, 2008, pp. 41-42, emphasis in the original)

Prompting the students’ attention to the suitable context, possibly enriched with recalled or imagined elements, supports the students towards a progressive *disclosure* of the mathematical objects at stake. Disclosure is a Husserlian concept further elaborated by Rota (1991). It indicates the process by which people make sense of the world and of the situations in context to which they are exposed:

The world is primarily a world of sense... Our primary concern is with sense itself, how it originates in the world, how it functions in the world. In short how it relevates... The basic relationship to the world is...our senses. (*ibid.*, p. 61).

Disclosure happens when one is able to grasp the *functionality* of the objects in the context, for example those in a didactical situation:

Sense-making depends ultimately on our own being-in-the-world, on the situation of our interacting, our dealing with the contextual situation in the world [...]. If you deconstruct the notion of an object, what you find is pure functionality, the pure ‘being good for’ of that object or something. So that the world, instead of being a world of objects, will become a world of functions, of tools. (*ibid.*, pp. 156-159, *passim*).

Such functions are related to each other “by a system of references, a network of references among them. [...] The world is disclosed to us not just as a system of functions, but as a network of related functions” (*ibid.*, p. 159).

Students must be educated by the teacher to make sense of what they perceive/see when exposed to a mathematical situation. Generally a situation may evoke different contexts and so produce a different sense-making, according to the age and the background of the students. For example, the graph in Fig. 1 can evoke a mountain, a graph of a symmetric function, a normal distribution, and so on. Of course, such different contexts are not isolated but are *layered* upon each other; these layers can generate different levels of disclosure in the flow of time:

Side-by-side with our realization that sense is purely contextual goes the realization that contexts are not units. Contexts themselves are layered upon each other in various ways, and to be in a context is not to be in just one context. ... Be-ing in a context does not in any way presume that such be-ing is be-ing in one context at a time. (*ibid.*, p. 126)

Because of the role of contexts, disclosure includes two aspects: grasping a concept requires both an emotional and an intellectual component, which Rota calls *mood* and *grasp*; they can be present in different ways according to the context:

There are phenomena of disclosure where the actual grasp in the context fits the major role and the mood component fits the minimal role – for example, our approach to solving a mathematical problem. This is not saying that we have to like the problem, but the minimum of mood lets us get involved in it. Unless we get really involved in it, we get nowhere. This is the mood-wise component of the mathematical problem disclosing itself. Without this component of mood, no matter how little, the problem will not be disclosed. (*ibid.*, p. 269).

In our analysis, we will show how the teacher uses the techniques of *prompting* and *making present* to evocate suitable contexts in order to support the students towards the *disclosure* of some basic Calculus concepts.

THE TEACHING EXPERIMENT

In 2009 researchers from New Zealand, Israel and Italy started an ongoing common research project, with the aims of studying the possible benefits of approaching the derivative and the primitive concept in a graphic way.

This paper is based on the teaching experiment carried out in Italy in two classrooms (grades 9 and 11) of a scientifically-oriented high school ('Liceo Scientifico') with the same teacher. In particular, it focuses on the lesson that followed the first task of the teaching sequence. The task was composed of two parts: the first asked to interpret a height-distance graph (see Fig. 1) and to draw the graph that represents its slope. The second part proposed a gradient graph (Fig. 2) and asked to draw a graph whose slope was represented by it (inverse problem).

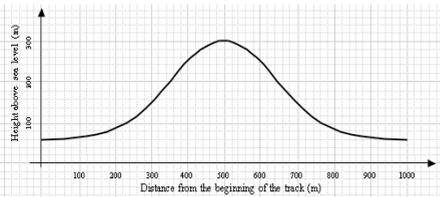


Figure 1: Distance-height graph of track

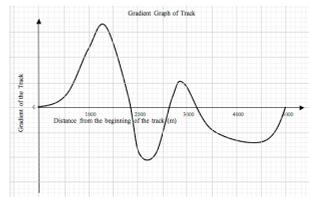


Figure 2: The gradient graph

The students solved the task in groups of 3 or 4, and afterwards were involved in a classroom discussion on the concepts concerned with the task.

The lessons were video-taped by a camera, allowing us to consider the semiotic productions of the teacher and of the students (speech, inscriptions at the blackboard, and gestures). After a first scrutiny, we carried out a semi-structured interview to the teacher to ascertain the reasons of his didactical actions. In the next paragraph we will illustrate and discuss some results of our overall analysis at the light of the phenomenological perspective described above.

ANALYSIS

Sabena (2010) has analysed the students' processes while solving the task, and observed interesting cases of semiotic resources (typically, words and gestures) that could refer both to the given graph, and to the corresponding imagined track, like:

If the slope of the tangent line is zero the track is parallel to the x-axis.

The graph goes downhill.

In each sentence there is some element of the track treated as it were part of a Cartesian plane (first example), or vice-versa (second example). Since these signs – intended in the sense of Peirce (1931-1958) – blend the references to two different domains, they have been called “*blending signs*” (Sabena, 2010). Blending can happen since the two objects, though being different, share deep relationships of iconic character (e.g. the highest point of the graph corresponds to the highest point

of the track). This feature is specific of the task proposed to the students, and was meant to facilitate their solving activity. On the other hand, it is possible that the students who are using blending signs are not (fully) conscious of their double referential nature.

From a phenomenological perspective, the blending signs can be interpreted as markers for possible *disclosures* towards the meaning of the graph as a mathematical modelling tool. Disclosure can develop because the students become aware of the double polarity between the objects and their functions, for example the track highness as modelled by a function graph. A blending sign reveals this double polarity between what Rota (1991) calls “the facticity of the context” and its “functionality” (that he calls also “function”) that must be disclosed.

The task addresses the students’ attention towards the *facticity* of the context (the track in the mountain), but at the same time it is necessary that this facticity “fades before the function” (*ibid.* p. 127), so that the students can achieve the disclosure of the graph as a model of the track. Let us analyse how the teacher helps the students to accomplish this goal.

During the two classroom discussions that followed the task, the teacher refers to the different contexts of the task, to foster two different *layers* of disclosure: a *factual* layer (that of the track, prevailing at the 9th grade) and a *theoretical* layer (that of the functions, prevailing at the 11th grade). For example, in grade 9, the teacher starts the discussion by recalling in an explicit way the context of tracks. In fact, the context can provide meaning to the graph slope, which the students face for the first time. To do that, he uses some blending signs, like saying “the track did something like this”, while drawing the graph at the blackboard:

Teacher: You had a function, about... the track, do you remember, the track did something like this, isn't it? (*drawing the graph of Fig. 1 at the blackboard*) Ok? Roughly. So we had the graph of a function.

In grade 11 we observe something different. These students have already some competences about functions, and in particular they have studied the slope features of a graph and know how to compute approximated values of the function slopes. Consequently, during the whole discussion the teacher endeavours to underline that the graph and the track are two objects that belong to different domains:

Teacher: Well, this first task talked about trampers that were following some tracks in the mountains. And we have imagined that in profile the graph...do you remember? This (*drawing the graph of Fig. 1 at the blackboard*), ok? This graph represented, varying the positions along the track, x ... x represented the distance from the point?

Students: The starting point [...]

Teacher: Then, how is the graph when the slope of the tangent line is negative? You have said well, you have said that it is...

Students: Downhill.

Teacher: Decreasing. Downhill the track; the graph is decreasing. Remember: the properties of the graph are expressed in mathematical language, those of the track on the contrary can be expressed like uphill, downhill.

The teacher stresses the fact that the graph and its parts are signs that *represent* something. Namely the teacher prompts an *un-blending* process in order to reach a further layer of disclosure.

The overall analysis of the two discussions reveals that, starting from the same task, the teacher is working on two different layers of disclosure. For instance, in grade 9 to refer to the point of highest slope the teacher makes present the experience and the *mood* of biking:

Teacher: Look! Graphically (*pointing at the graph in Fig. 1*), can you see here that the slopes are increasing (*starting to surf with his hand along the graph, Fig. 3*)? Can you see that (*moving his hand along the graph*)? And that at a certain point... (*his hand is near the inflection point*)?

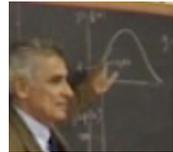


Figure 3

Students: They decrease.

Teacher: They decrease (*taking his hand away from the graph*).

This is the sensation that we feel if we bike along this uphill (*the hand again on the graph*), at the beginning it is very hard, isn't it? At the beginning it is very hard because the slope increases (*with a slanted body and the hand as holding a handlebar, he mimes the act of biking uphill, Fig. 4*), then (*the hand along the graph, after the inflection point*) it becomes less and less hard. In fact at the beginning the function increases more and more and then (*the hand has reached the maximum of the graph*)?



Figure 4

Students: It increases less and less.

To grasp the different increasing modalities of the function, and specifically to focus the attention to the point of inflection, the teacher makes present with his words and his body posture the physical sensation of the fatigue ones feels when climbing a steep hill with the bike: it is the *mood* component in Rota's account. Such a sensation may be well known to the students, since they live in a hilly territory: therefore the making present may be accomplished through remembering their lived experiences.

In grade 11, we do not find such perceptual references for the same mathematical concepts. While solving the task, in fact, the students have identified that the point with the steepest uphill corresponds to the point of inflection. In the discussion, the teacher reads their answers and prompts to the fact that they have well done, without making present any experience related to the track. This interpretative hypothesis is confirmed by the interview to the teacher:

[Asked about the bike episode in grade 9] I often search for the situation that I think it is nearest to their experience: for instance I speak of skiing for those who go skiing, biking, climbing, surfing... so to think to an experience that is very concrete, very perceptible.

[Referring to grade 11] It is true that the situation was that of tracks, but now the students should understand that they have modelled it with a graph, so we speak of the properties of a graph, with adequate language. [...] I think that for the students at grade 11 the concrete situation has not helped them so much. They had the tools to speak in terms of graphs. For the students of grade 9 it is different. I imagine that they have indeed started from the concrete situation and have imagined the person who was climbing with all the problems, then [tried] to eliminate the inessential things and so to think simply to the outline of the track that becomes exactly the outline of the graph.

It is important to notice that the progressive disclosure of the graphs as mathematical objects does not imply a definitive discharge of blending signs. On the contrary, the teacher comes back to blending signs when teaching about new (possibly difficult) properties of the mathematical objects to be disclosed. For instance, in the second part of the task the students have to draw the graph of a function, starting from the graph of its slopes (i.e. to draw a primitive function). Being an inverse problem, this question can raise some difficulty also for the students at grade 11. After drawing a primitive graph, the teacher puts on the table the issue of the y-s of the primitive:

Student 1: [We cannot know the y-s] because the slope graph does not give us that information

Student 2: The differences could be both from 0 to 1 and from 100 to 101

Teacher: By the way, this graph here (*pointing at the primitive graph drawn at the blackboard*) could be an underwater mountain, below the sea-level (*the hand mimes the action of moving the graph below the x-axis*)

It is the teacher himself to introduce a blending sign: in fact, he blends the references to the graph (by means of gestures) and to a concrete imagined context (the underwater mountain). In this way, he has provided a new context by which the students may give sense to a property that regards a relationship between two mathematical objects, i.e. a graph and its primitives.

DISCUSSION

We have sketched an interpretation of the teacher's actions in the classroom, based on the phenomenological notion of *disclosure*, as defined by Rota (1991). Disclosure happens because people are able to grasp the *functionalities* of the context. In this process, both emotional and cognitive aspects are involved. Our analysis was meant to show how the teacher uses didactic techniques like *prompting* and *making present* (i.e. *remembering* or *imagining*) to promote the different layers of students' disclosures. Precisely, we have seen that he fostered the notion of slope of a function in a point through the steepness of a road, on which students are asked to remember/imagine to bike (grade 9); or the fact that all the primitives of a function differ by a constant, imagining a mountain that sinks under the sea (grade 11). In

making present these contexts and promoting the disclosure of the related mathematical concepts, we have identified the production of *blending signs* (Sabena, 2010): for the teacher they are tools for fostering students' disclosure process.

Our analysis suggests that in the learning processes, contexts are layered upon each other, and that they are never completely discharged. This causes a complex dynamics in teaching actions. From the one side, when the teacher judges that students have reached a sufficient disclosure of a concept, he pushes towards a more abstract layer, where further disclosure processes can start. From the other side, when some more difficult concept must be faced, the teacher can go back to a previous layer to provoke suitable disclosure processes (e.g. imagining the mountain under the water level to support the disclosure of the existence of infinite "parallel" primitives).

Space does not allow to present things from the side of students, namely to illustrate the extent to which the teacher's actions aimed at provoking learners' disclosures are successful. This problem will be the object of our future research.

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