Modeling epidemic spreading in star-like networks

This is the author's manuscript

Original Citation:

Availability:
This version is available http://hdl.handle.net/2318/127882 since

Terms of use:

Open Access
Anyone can freely access the full text of works made available as "Open Access". Works made available under a Creative Commons license can be used according to the terms and conditions of said license. Use of all other works requires consent of the right holder (author or publisher) if not exempted from copyright protection by the applicable law.

(Article begins on next page)
Modeling epidemic spreading in star-like networks

Luca Ferreri, Paolo Bajardi, Mario Giacobini

GECO - Group of Computational Epidemiology
Department of Veterinary Sciences
CSU - Complex Systems Unit
Molecular Biotechnology Center
ARC$^2$S - Applied Research on Computational Complex Systems Group
Department of Computer Science University of Torino

Milano, 5 aprile 2013
Tick-Borne Encephalitis

- endemic in Eurasia from Europe, through Russia To China and Japan
- the virus causes potentially fatal neurological infection
- in last years emergenge of the virus in new area and increase of morbidity
- maintained in nature by complex cycle involving Ixodid ticks (*I. ricinus* and *I. persulcatus*) and wild vertebrate hosts
Systemic Transmission

(time $t$)

(time $t + 1$)
Non-Systemic Transmission
Non-Systemic Transmission
Non-Systemic Transmission
Our research question

how do the non-systemic transmission together with the different aggregation patterns influence the pathogen spreading?
Spreading Model

- at time $t$ a fraction, $\pi(t)$, of passengers (ticks) are infectious
Spreading Model

- at time $t$ a fraction, $\pi(t)$, of passengers (ticks) are infectious
- $\mathbb{P}(k)$ probability that a bus (mouse) transports $k$ passengers (ticks) of them
Spreading Model

- at time $t$ a fraction, $\pi(t)$, of passengers (ticks) are infectious
- $P(k)$ probability that a bus (mouse) transports $k$ passengers (ticks) of them

\[ \pi(t+1) = f(\pi(t)) \]
Spreading Model

- at time $t$ a fraction, $\pi(t)$, of passengers (ticks) are infectious
- $\mathbb{P}(k)$ probability that a bus (mouse) transports $k$ passengers (ticks) of them
- $\beta$ transmission probability for infectious path
- $\mu$ recovery probability
\textbf{Spreading Model}

- at time $t$ a fraction, $\pi(t)$, of passengers (ticks) are infectious
- $\mathbb{P}(k)$ probability that a bus (mouse) transports $k$ passengers (ticks) of them
- $\beta$ transmission probability for infectious path
- $\mu$ recovery probability

$$\Rightarrow \pi(t + 1) = f(\pi(t))$$
the probability that a **susceptible passenger**, having $h$ travel mates, gets the **infection** is

$$1 - (1 - \beta)^h$$
the probability that a **susceptible passenger**, having $h$ travel mates, gets the **infection** is

$$1 - (1 - \beta)^h$$

Let $\pi(t)$ be the **prevalence of infection** among passengers at time $t$, the probability for a susceptible passenger on a bus transporting $k$ individuals including himself to be **infectious** at time $t + 1$ is

$$1 - (1 - \beta)^{(k-1)} \cdot \pi(t)$$
Recalling that $\mathbb{P}(k)$ is the probability for a bus to have $k$ passengers, the probability for a passenger to be on a $k$-bus is

\[
\frac{\text{#passengers on a } k\text{-bus}}{\text{#passengers}} = \frac{k \cdot \text{#}k\text{-bus}}{\text{#passengers}} = k \cdot \frac{\text{#}k\text{-bus}}{\text{#bus}} \cdot \frac{\text{#bus}}{\text{#passengers}} = k \cdot \mathbb{P}(k) \cdot \frac{1}{\langle k \rangle}
\]
thus, the probability for a **susceptible** passenger at time $t$ to be **infectious** at time $t + 1$ is

$$\sum_{k=1}^{\infty} \left[ 1 - (1 - \beta)^{(k-1) \cdot \pi(t)} \right] \cdot \frac{k}{\langle k \rangle} \cdot \mathbb{P}(k)$$

and therefore the prevalence among passenger at time $t + 1$ is

$$\pi(t + 1) = f(\pi(t)) = (1 - \mu) \cdot \pi(t) + [1 - \pi(t)] \cdot \left\{ 1 - \sum_{k=1}^{\infty} (1 - \beta)^{(k-1) \cdot \pi(t)} \cdot \frac{k}{\langle k \rangle} \cdot \mathbb{P}(k) \right\}$$
Equilibria

imposing the stationary condition $\pi(t + 1) = \pi(t) = x$ we can derive the equilibria as solutions of the following equation

$$x = f(x) = (1 - \mu) \cdot x + [1 - x] \cdot \left\{ 1 - \sum_{k=1}^{\infty} (1 - \beta)^{(k-1) \cdot x} \cdot \frac{k}{\langle k \rangle} \cdot P(k) \right\}.$$
Equilibria

imposing the stationary condition $\pi(t + 1) = \pi(t) = x$ we can derive the equilibria as solutions of the following equation

$$x = f(x) = (1 - \mu) \cdot x + [1 - x] \cdot \left\{ 1 - \sum_{k=1}^{\infty} (1 - \beta)^{(k-1) \cdot x} \cdot \frac{k}{\langle k \rangle} \cdot \mathbb{P}(k) \right\}.$$ 

Now:

- $x = 0$ is a solution,
- $f(1) = 1 - \mu \leq 1$,
- $f''(x) < 0$. 
Equilibria

imposing the stationary condition $\pi(t + 1) = \pi(t) = x$ we can derive the equilibria as solutions of the following equation

$$x = f(x) = (1 - \mu) \cdot x + [1 - x] \cdot \left\{ 1 - \sum_{k=1}^{\infty} (1 - \beta)^{k-1} \cdot x \cdot \frac{k}{\langle k \rangle} \cdot \mathbb{P}(k) \right\}.$$ 

Now:

- $x = 0$ is a solution,
- $f(1) = 1 - \mu \leq 1$,
- $f''(x) < 0$.

assuming $\langle k \rangle$ and $\langle k^2 \rangle$ finite

\[ f'(0) > 1 \]

\[ f'(0) < 1 \]
Equilibria

imposing the stationary condition $\pi(t + 1) = \pi(t) = x$ we can derive the equilibria as solutions of the following equation

$$x = f(x) = (1 - \mu) \cdot x + [1 - x] \cdot \left\{ 1 - \sum_{k=1}^{\infty} (1 - \beta)^{(k-1)} \cdot x \cdot \frac{k}{\langle k \rangle} \cdot P(k) \right\}.$$

Now:

- $x = 0$ is a solution,
- $f(1) = 1 - \mu \leq 1$,
- $f''(x) < 0$.

assuming $\langle k \rangle$ and $\langle k^2 \rangle$ finite

therefore conditions to have one, and only one, solution $\hat{x} \in (0, 1)$ is that $f'(0) > 1$ or

$$-\frac{\ln (1 - \beta)}{\mu} > \frac{\langle k \rangle}{\langle k^2 \rangle - \langle k \rangle}.$$
Stability

recalling that $f''(x) < 0$ and that

$\bullet$ $f'(1) = -\mu + \sum_k (1 - \beta)^{k-1} \frac{k}{\langle k \rangle} P(k) > -\mu > -1$

$\bullet$ $f'\left(\hat{x}\right) < 1$

hence $\hat{x}$ is asymptotically stable when it exists. Therefore:

$\bullet$ disease-free equilibrium is asymptotically stable when $\hat{x}$ does not exist.

$\bullet$ disease-free is unstable when $\hat{x}$ exists. Furthermore, when $\hat{x}$ exists it is also asymptotically stable.
Conclusion and Discussions

- co-feeding transmission
- spreading on star-like networks
- spreading dynamic on dynamic bipartite networks
- analytical result confirmed by simulations
Acknowledgements

Source of inspirations for this work come from a scientific project hold with the Fondazione Edmund Mach, San Michele all’Adige (TN). In particular, we are grateful to Anna Paola Rizzoli, Roberto Rosà e Luca Bolzoni.