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Combination of Tense and Modality*

R.Thomason

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1

Richmond Thomason's article *Combinations of tense and modality* provides the first clear exposition of $T \times W$ semantics. A $T \times W$ frame is defined in terms of a set T of times, a set W of worlds, a linear order $<$ on T , and a set $\{\approx_t: t \in T\}$ of equivalence relations on W. A $T \times W$ structure is defined by specifying a valuation on the frame that assigns truth-values to formulas relative to time-world pairs. Thus, a formula α turns out true or false at any pair (t, w), where $t \in T$ and $w \in W$. The temporal operators behave as in linear tense logic. For example, $F\alpha$ is true at (t, w) if and only if α is true at (t', w) for some t' such that $t < t'$. In addition, a modal operator $\Box$ is defined in such a way that $\Box\alpha$ holds at (t, w) when α holds at (t, w') for all w' such that $w \approx_t w'$.[1]

Thomason's exposition is accompanied by some comments that are far from enthusiastic. About the systems based on $T \times W$ semantics, Thomason says that they "do not seem particularly interesting from a philosophical point of view". According to him, the only interesting case is that in which the set $\{\approx_t: t \in T\}$ is so defined that for any t, t', if $w \approx_t w'$ and $t' < t$ then $w \approx_{t'} w'$. For in that case the relation $\approx_t$ obtains between w and w' when w and w' "share the same past up to and including t". This way $\Box$ expresses historical necessity, the property that is usually indicated by words such as 'settled' or 'inevitable'[2] .

Moreover, not even in that case Thomason is satisfied with $T \times W$ semantics. He finds more congenial tree-like semantics, where a frame is defined in terms of a set T of times and a non-linear order $<$ on T that branches forward, that is, such that it may happen for distinct $t, t', t'' \in T$ that $t < t'$ and $t < t''$ but neither $t' < t''$ nor $t'' < t'$. Thomason says "I like to think of possible worlds as overlapping, so that the same moment may have alternative futures". And again: "Intuitions may differ, but to me the natural notion is that of a possible future -not that of a possible course of events"[3] .

Tomason's attitude towards $T \times W$ semantics is largely shared. Although some technical work has been done to investigate the properties of systems based on $T \times W$ structures, in the current debate on time $T \times W$ semantics is either ignored or treated with nonchalanche. Tree-

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* In Gabbay Dov and Guenthner Franz (eds) Handbook of Philosophical Logic, vol. 7, Kluwer 1984 (second edition 2002).

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[1] (Thomason , 1984), pp. 207-208.

[2] (Thomason , 1984), pp. 208-209.

[3] (Thomason , 1984), p. 207 fn. 5, and p. 222. Thomason's misgivings concerning sameness of temporal ordering will not be discussed here.

like semantics is by far more popular now. Thus, the historical role of Thomason's article is double: besides being the first that outlines $T \times W$ semantics, it is the first that dismisses it[4].

A question that may be raised, then, is whether this attitude is justified. Here it will be suggested that it is not. In the first place, there is no reason to be unsatisfied with $T \times W$ semantics if □ is taken to express historical necessity. Not only $T \times W$ semantics is at least as good as tree-like semantics in this interpretation, it is even better. Secondly, other interesting interpretations of $T \times W$ semantics are available. One in particular will be considered in which □ expresses an epistemic property called 'definiteness'.

2

Let a *history* be a possible course of events, and let a *moment* be a possible event of null temporal extension that is part of a history. In a tree-like frame histories amount to maximal chains of times, and moments amount to times. Instead, in a $T \times W$ frame histories amount to worlds, and moments amount to time-world pairs. It is quite natural to associate tree-like semantics with a metaphysical view -call it *branching*- according to which two histories can overlap, that is, they can have a temporal part in common. By contrast, it is quite natural to associate $T \times W$ semantics with a metaphysical view - call it *divergence* - according to which there is no overlap, even if two histories can have qualitatively identical temporal parts[5].

Thomason seems to prefer branching to divergence. However, the article provides no argument to justify this preference. Perhaps Thomason regards branching as more "intuitive" than divergence. But intuitions do not help much in these matters. At most, what deserves to be called an intuition is the idea that there are many ways things might go, which by itself does not decide between the two views.

Or perhaps Thomason thinks that indeterminism as it emerges from scientific theories requires branching. But indeterminism is equally compatible with divergence. Determination may be understood in accordance with scientific theories as follows: if t precedes t', the state of the world at t' is determined by the state of the world at t if and only if it is entailed by the state of the world at t and the laws of nature. Assuming that a state is a condition that can be instantiated by histories at times, it is conceivable that two histories are in the same state at any time up to t but differ at t'. This means that the state in which they are at t is compatible with two different states at t'. If indeterminism is phrased in terms of absence of determination so understood, it does not entail branching.

Apart from there being no apparent advantage of branching on divergence, there is an apparent advantage of divergence on branching. Only one among the possible futures will become *actual*. So it is plausible to suppose that only one history is the actual history. But this supposition does not harmonize well with branching. Imagine that two histories h and h'

[4] (Kutschera, 1997) and (Di Maio and Zanardo, 1998) are technical contributions on $T \times W$ logic. (Belnap et. al., 2001) and (MacFarlane, 2003) are recent works that adopt tree-like semantics.

[5] (Lewis, 1986) spells out the difference, pp. 206-209, and argues in favour of divergence. (Belnap et al., 2001), pp. 206-209, defend branching.

overlap up to a certain moment but branch after that moment, and suppose that *h* is the actual history. Then it is legitimate to ask whether the non-actual branch of *h′* is really a continuation of one and the same past. A negative answer can be given on the assumption that the actual history is the history in which we are. For if we are in *h* and not in *h′*, it is hard to see how *h′* can be connected to our past. On the contrary, divergence is clearly compatible with the supposition that there is a unique actual history. If *h* and *h′* are wholly distinct histories and *h* is the actual history, each moment along *h* has a unique continuation[6].

T × *W* semantics has a related advantage on tree-like semantics. If treelike semantics is phrased without reference to a unique actual history, as usual, there are essentially two ways in which truth can be defined. One is to define truth at a moment-history pair, that is, truth at a time relative to a maximal chain to which the time belongs. The other, which rests on the first, is to define truth at a moment as truth at all moment-history pairs. However, neither of these two definitions seems to account for truth *simpliciter*, the property we have in mind when we wonder whether a sentence about the future is true. Consider

(1) There will be a sea battle

To say that (1) is true today relative to a history in which there is a sea battle is to say that if things go as in that history, (1) is true today. Obviously, there is nothing wrong with ascribing truth so understood to (1), since the ascription requires no more than acceptance of the conditional 'If there will be a sea battle then (1) is true'. The fact, however, is that this is not what one is after when one says that (1) is true. What one seems to claim when one says that (1) is true is that (1) is true without relativization to this or that history. The claim is not about how the sentence is to be evaluated given the hypothesis that things will go a certain way. Rather, it is a claim about what hypothesis is to be advanced on how things will go. In other words, the claim is about which of many possible events will actually occur. It is in virtue of such event that truth is ascribed to the sentence.

The second option fares no better. Historical necessity is not what we are after when we ask whether (1) is true. Saying that (1) is true, one is not committed to the claim that the truth of (1) holds no matter how things will go. That would amount to claiming that the truth of (1) is independent of the way things will go, which is patently incorrect. If (1) is true, it is true in virtue of the way things will actually go. *T* × *W* semantics, by contrast, is able to explain truth simpliciter in terms of actuality, provided that one of the members of *W* is specified as the actual world. A sentence is true *simpliciter* at *t* just in case it is true at (*t, w*) and *w* is the actual world. Or equivalently, a sentence uttered at *t* is true *simpliciter* just in case it is true at (*t, w*) and *w* is the actual world.

[6] (MacFarlane, 2003), p. 325 emphasizes the conflict between branching and the supposition that there is a unique actual history.

3

Independently of the question whether there is reason to be unsatisfied with $T \times W$ semantics if □ is taken to express historical necessity, it should not be presumed that this is the only interesting interpretation of $T \times W$ semantics. Another interpretation that is no less interesting is that in which worlds are understood as courses of events that are possible "for all one knows", that is, histories that are apparent rather than real. In this case □ may be read as 'it is definitely the case that'. Assuming that one is a position to know that *p* when *p* holds in all courses of events that are possible for all one knows, to say that it is definitely the case that *p* is to say that one is in a position to know that *p*. To distinguish an interpretation of this kind from one in which □ expresses historical necessity, call *epistemic* the former and *metaphysical* the latter.

The epistemic interpretation differs from the metaphysical interpretation in at least two crucial respects. In the first place, it does not require that the set $\{\approx_t : t \in T\}$ is so defined that for any t, t', if $w \approx_t w$ and $t' < t$ then $w \approx_{t'} w'$. The quantification involved in the definition of □ is unrestricted, as the relevant differences between worlds are not confined to the future. As far as we know, different histories may have lead to the present state of Affairs. For example, today we are not able to tell whether the number of cats that slept inside the Colosseum on September 4th 1971 is even or odd. This means that at least two histories leading to the present state of affairs are equal for all we know: one in which that number is even, the other in which that number is odd. Moreover, as far as we know there are different ways things may go at present. For example, we don't know the position of a certain whale that is now swimming in the ocean, hence we are not able to discriminate between moments that differ as to the position of that whale.

In the second place, the epistemic interpretation does not allow reference to actuality. A structure represents an epistemic state, and the knowledge of what is actual cannot be included as part of that state. This means that the structure does not tell us which of the members of *W* is the actual world. Accordingly, for any sentence that is true at a moment (t, w) and false at another moment (t, w'), such as (1), the structure does not tell us whether the sentence is true *simpliciter* at *t*. More specifically, the structure does not tell us whether (1) as uttered now is true *simpliciter*, for it doesn't tell us which moment is the present moment. On the indexical account of actuality considered, this is to say that we don't know exactly where we are.

The epistemic interpretation and the metaphysical interpretation agree on two basic facts. The first is that $\Box\alpha \rightarrow \alpha$ is true at any moment in any structure, while the converse does not hold. Suppose that $\Box\alpha$ is true at (t, w). Then α is true at (t, w), for α is true at (t, w') for every w' such that $w \approx_t w'$. Now suppose that α is true at (t, w) but false at (t, w'). Then α is true at (t, w) but $\Box\alpha$ is false at (t, w). The second fact is that □ does not distribute over disjunction: for some structure and some moment (t, w), $\Box(\alpha \vee \beta)$ is true at (t, w) while $\Box\alpha$ and $\Box\beta$ are false at (t, w). Suppose that $W = \{w, w'\}$ and consider t, t' such that $t < t'$. Let α be true at (t', w) but false at (t', w') and at any later moment in w'. Then $F\alpha$ is true at (t, w) and $\neg F\alpha$ is true at (t, w'). It follows that $\Box(F\alpha \vee \neg F\alpha)$ is true both at (t, w) and at (t, w'). But $\Box F\alpha$ is false both at (t, w) and at

(t, w'), and the same goes for $\Box\neg F\alpha$.

These two facts are understood differently in the two interpretations. Definiteness is an epistemic property, historical necessity is a metaphysical property. Definiteness entails historical necessity, but is not entailed by it. Imagine that only one course of events is possible, even though many courses of events appear possible. A sentence may be historically necessary without being definitely true, if it is true in the only possible course of events but false in some apparently possible course of events. Since definiteness and historical necessity are distinct properties, the two interpretations account for different data. Consider

(2) Either there will be a sea battle or there will not be a sea battle

There is a clear epistemic difference between (1) and (2). We are not able to tell whether (1) is true, while we are certain that (2) is true. This is easily explained in terms of definiteness: (2), unlike (1), is definitely true. Another datum is that the present state of the world, independently of what we know about it, seems to leave unsettled whether (1) is true, while it seems to settle that (2) is true. This may be explained in terms of historical necessity: (2), unlike (1), is historically necessary. The second explanation presupposes indeterminism, while the first is compatible both with determinism and with indeterminism.

Certainly, tree-like semantics may as well be interpreted epistemically, with $\Box$ read as 'it is definitely the case that'. But again, there is reason to think that $T \times W$ semantics is preferable. A modified tree-like frame could represent a plurality of apparently possible pasts, in addition to a plurality of apparently possible futures, in terms of an order that branches backward and forward. In that case there would be a point at the centre of the tree that stands for the present moment. But we saw that not even the present is unique from an epistemic point of view. It is plausible to assume that there are distinct moments such that, for all one knows, it is indetermined which of them is the present moment. Therefore, even if tree-like semantics is as good as $T \times W$ semantics as an epistemic representation of the past, it has more limited resources as an epistemic representation of the present.

$T \times W$ structures are more general than tree-like structures in the sense that every set of histories - real or apparent - that can be accommodated in a tree-like structure can also be accommodated in a $T \times W$ structure, but not the other way round. This sense is stated in more rigorous way as follows. Let A be a tree-like structure formed by a set of times T_A, a non-linear order $<_A$ and a valuation V_A. Let B be a $T \times W$ structure where M_B is the set of moments, $<_B$ is a relation on M_B such that $(t, w) <_B (t', w')$ if and only if $w = w'$ and $t < t'$, and V_B is a valuation. We say that B *transfers* A if for every subset S of T_A that is a maximal set of $<_A$-related times, there is an isomorphism f from S on a subset S' of M_B ordered by $<_B$, and for any formula α that includes no temporal or modal operator and any $t \in S$, $V_B(\alpha, f(t)) = V_A(\alpha, t)$. It is provable that for every tree-like structure, there is a $T \times W$ structure that transfers it. Let A be a tree-like structure as above, and call H_A the set of maximal sets of $<_A$-related members of T_A. Given a set of moments M_B and a relation $<_B$ defined on M_B as a union of disjoint linear orders, let f be a function that satisfies the following conditions:

1 f is injective;
2 f maps H_A into the power set of M_B;
3 for every $h \in H_A$, $f(h)$ is a maximal set of $<_B$-related moments isomorphic to h.

Since $<_B$ is a union of disjoint linear orders, from 3 we get that the range of f is a set H_B such that for any $f(h), f(h') \in H_B$, $f(h) \cap f(h') = \emptyset$. Now let V_B be a function such that for any $h \in H_A$, any formula α that does not contain temporal or modal operators and any $t \in h$, $V_B(\alpha, m) = V_A(\alpha, t)$, where m is the moment of $f(h)$ that the isomorphism from h on $f(h)$ assigns to t. For every $T \times W$ structure B whose set of moments includes M_B, whose order on times conforms to $<_B$ and whose valuation is V_B, we get that B transfers A. It is easy to see that the converse relation does not obtain. There is no sense in which tree-like structures can be shown to transfer $T \times W$ structures. Any $T \times W$ structure that includes distinct worlds that differ at every point has no equivalent tree-like structure.

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