Bid pricing in online auctions with "Buy-It-Now" option

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Bid Pricing in Online Auctions

with "Buy-it-Now" Option

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Abstract

The present note suggests a theoretic and practical decision rule for auctioneers to set out prices in online auctions in presence of Buy-It-Now options. The minimum starting bid and the Buy-It-Now price are calculated as an intuitive function of seller's evaluation of the final price stochastic distribution and of seller's attitude toward the price variability. Specifically, the presence of personalized parameters permits to incorporate the seller pessimism in stating the starting minimum bid and the seller optimism in fixing the Buy-It-Now price. The pricing rule is based on the Extended Gini premium principle, a well-known premium principle in non-life insurance literature that has been recently proposed in different areas of finance and portfolio risk management. In addition to give closed-end formulae for the starting minimum bid and Buy-It-Now price, we provide numerical tables for a number of common distributions for the ending price at different levels of seller's risk-aversion and gain-propensity. Discussion on the spread between the starting minimum bid and Buy-It-Now price is also carried out.

Keywords: Online auctions; Starting minimum bids; Buy-it-Now prices; Pricing rules; Extended Gini premium principle; Pessimism and Optimism indices; Bidding spreads

1 Introduction

Due to the enormous success of online auctions, in recent years the study on bidding process is become an important area of academic research and a debated question among business practitioners, we refer to Krishna (2010) for an extensive analysis.

A very popular type of auction includes the possibility of a buyout or Buy-it-Now (BIN) option, that allows a bidder to end the auction immediately by purchasing the auctioned object at a buyout price set by the seller. eBay introduced this option since the 2000s and now it has been adopted by almost all of eBay's U.S. auctions. A number of empirical and behavioral studies carried out in the literature (see Wan *et al*., 2003 and Hardesty and Suter, 2013) have made evidence that the presence of a BIN option facilitates the successful matching between the counter-parties, fasters the auction closing (see Wang *et al*., 2008) and increases the expected revenue (see Jung and Kim, 2004).

The purpose of the current research is suggest a theoretically well-ground decision rule for the seller bid pricing based on the Extended Gini premium principle, a common method in non-life insurance premium theory (see Denneberg, 1990) that in recent years has been proposed in different areas of finance, as in asset pricing (see Shalit and Yitzhaki, 2009, 2010) and in portfolio risk management (see Shalit, 2010 and Cardin *et al*., 2012, 2013).

Our results show that the main components in the determination of auctioneer's minimum starting bid and the BIN price are: (1) the seller ex-ante evaluation of the stochastic distribution of the final price and (2) the seller "pessimism" in pricing the minimum starting bid and "optimism" in pricing the BIN price.
The paper focuses on two research lines.
The first concerns the setup of general closed-end formulae for pricing bids. Then to test the formulae's sensitivity to the parameters, empirical explorations using a number of common distributions at different levels of attitude to variability are carried out. Results are collected in numerical tables.
At second, we focus on the spread between the buyout option and the starting minimum bid. Such price spread is a relevant component in auctions, because may drive the buyers to exercise or not the BIN option. In fact, evidence shows that the larger the spread, the more likely the buyer would choose to bid instead of buying at the buyout price (see Wan *et al*., 2003). Our results confirm what intuition seems to suggest: the price spread increases as the pessimism in setting the minimum starting bid and the optimism in fixing the payout price increase. But that is not the sole driver, in fact the price spread is also influenced by the stochastic distribution of the ending price, and specifically by its skewness. Finally, we prove that no general recursive calculation price formulae exist for high levels of seller's pessimism/optimism.

Findings from this study represent an initial step toward building a generalized theory of making pricing in Internet auction and further empirical investigations are needed, but we postpone them to future research.

The remainder of this paper is organized as follows. Section 2 introduces the risk and the gain premia according to the language of the insurance theory. In Section 3 closed-formulae and tables for a number of common distributions are given. In Section 4 the price spreads among the different bids are calculated. Section 5 concludes the paper. An Appendix collects the proofs.

2   Minimum starting bids and BIN prices

We consider a standard auction including a permanent Buy-It-Now option.   The first choice that the seller has to face is about the most appropriate stochastic distribution for the ending price. Informed evaluations can be drawn from data on transaction history of similar auctioned items at disposal on the leader auction databases (see for example that of eBay).

Denoted by *X* the random positive random variable of the ending price, we assume the following setup:

- the *starting   minimum   bid* be given by   the expected value $E(X)$ of the ending price *minus* a "risk premium", depending on the level of the auctioneer "pessimism" toward the final   auction outcome;

- the *Buy-It-Now* be given by the expected value $E(X)$ of the ending price *plus* a "gain premium", depending on the level of the auctioneer "optimism" toward the final   auction outcome.

The terms "risk premium" and "gain premium" are standard terminology used in Insurance Theory and intuitively grasp the rational seller's decision process in formulating these ex-ante prices.

As mentioned in the Introduction, we propose the Extended Gini (EG) premium principle for the bid pricing. Just to make the paper self-contained we recall some definitions.
In the literature there are different definitions of EG but they all coincide in the case X is a continuous variable. To avoid technical adjustments we assume that X be a continuous variable, if a relaxed assumption is desired some technical adjustments are needed (see Yitzhaki and Schechtman, 2005).

Definition 2.1 *Let X be a continuous random variable. The Extended Gini of X of order k is defined as:*

$$EG_k(X) = E(X) - E\{\min(X_1,...,X_k)\},$$

where k is a positive integer number and $X, X_1,..., X_k$ are identical independent distributed (*i.i.d.*) random variables.

Note that if $k=2$ the $EG_2(X)$ coincides with the Gini index, the most familiar measure of income inequality used in social welfare context (see Gini, 1912).

Analogously, we can define the EG of -*X*.

Definition 2.2 *Let X be the continuous random variable. The Extended Gini of –X of order k is defined as:*

$$EG_k(-X) = E\{\max(X_1,...,X_k)\} - E(X)$$

where k is a positive integer number and $X, X_1,..., X_k$ are *i.i.d.* random variables.

In financial and insurance context the value $EG_k(X)$ is called the *risk-premium* and $EG_k(X)$ the *gain-premium* of *X* of order *k*, respectively   (for applications of risk/gain premium in portfolio selection see Cardin *et al.*, 2013; for those in optimal asset allocation involving hedge funds see Sherman Cheung *et al.*, 2008) .

Definitions of the starting minimum bid and the BIN price follow now.

Definition 2.3 *Let X be the continuous random variable presenting the final price of the auctioned item. The ex-ante bidding prices are:*

- *The starting   minimum   bid:*
  $$START_k(X) = E(X) - EG_k(X) = E\{\min(X_1,...,X_k)\}$$

- *The BIN price:* $BIN_k(X) = E(X) + EG_k(-X) = E\{\max(X_1,...,X_k)\}$

with k a positive integer number and $X, X_1,..., X_k$ *i.i.d.* random variables.

An interpretation of the parameter k follows spontaneously. Let the seller image there exist k independent bidders that share the same views on the distribution of

the ending price X, who independently bid $X, X_1, ..., X_k$. Then, the seller chooses just the expected value of the minimum of $X, X_1, ..., X_k$ as the minimum starting bid; and the expected value of the maximum of $X, X_1, ..., X_k$ as the BIN price. Clearly, the higher the parameter k, the lower the minimum starting bid and the higher BIN price.

Accordingly to the behavioral economic studies, the parameter k can be interpreted as the index of pessimism in eliciting the minimum starting bid and, conversely, as the index of optimism in stating the BIN price (see Chateauneuf *et al*., 2005). Clearly, the auctioneer may display levels of pessimism and optimism which differ each other, so it would be necessary in some contexts to use different indices k_{BIN} and k_{START} instead of k.

It easy to check that $EG_k(X)$ and $EG_k(-X)$ assume non-negative values and, in general,

$$EG_k(X) \neq EG_k(-X).$$

A spontaneous question that may arise is under which conditions the risk and the gain premia coincide. That happens if $k = 2$ and/or the distribution of X is symmetric (see Yitzhaki and Schechtman, 2005).

3. Closed-end formulae for bid prices for common distributions

The bidding prices admit closed-end formulae for many familiar distributions used in risk management modeling. They appear in the literature as special cases of more complicated expressions for moments of order statistics. In Table 1 we collect a number of special cases (for further details and proofs see Cardin *et al*., 2012, 2013). In the following, for simplicity of notation, we skip the under-symbol of k_{BIN} and k_{START} instead of k. To compute the proper bid it is sufficient to substitute the proper values of k_{BIN} and k_{START} in the formulae.

Table 1. The minimum starting bid $START_k(X)$ and the BIN price $BIN_k(X)$.

		$START_k(X)$	$BIN_k(X)$
Uniform (θ)		$\frac{\theta}{k+1}$	$\frac{k\theta}{k+1}$
Normal (μ, σ^2)	$k=2$	$\mu - \frac{\sigma}{\sqrt{\pi}}$	$\mu + \frac{\sigma}{\sqrt{\pi}}$
Skew-Normal SN $(0, \omega^2, \alpha)$	$k=2$	$\xi + \omega \sqrt{\frac{2}{\pi(1+\alpha^2)}}\left(\alpha - \sqrt{\frac{1}{2}}\right)$	$\xi + \omega \sqrt{\frac{2}{\pi(1+\alpha^2)}}\left(\alpha + \sqrt{\frac{1}{2}}\right)$
Pareto $(\alpha, c=1)$		$\frac{\alpha k}{\alpha k - 1}$	$\frac{\alpha^k k!}{(\alpha k - 1)\cdots(\alpha - 1)}$
Weibull $(m=2, \lambda = 2/\sqrt{\pi})$		$\frac{1}{\sqrt{k}}$	$\sum_{j=1}^{k}(-1)^{j+1}\binom{k}{j}\frac{1}{\sqrt{j}}$
Exponential (λ)		$\frac{1}{\lambda k}$	$\frac{1}{\lambda}\sum_{j=1}^{k}\frac{1}{j}$

4 Spread between the minimum starting bid and the Buy-It-Now price

Due to the non-negativeness of EG_k the following inequality holds

$$START_k(X) \leq E(X) \leq BIN_k(X)$$

Identity holds if and only if $k_{BIN} = k_{START} = 1$, *i.e.* the auctioneer is risk and gain neutral so the minimum starting bid and the buyout price are both equal to the expected value of the ending price. Let now set the formal definition of the price spread.

Definition 4.1 Let a seller with level of pessimism k_{START} in fixing the minimum starting bid and that of optimism k_{BIN} in fixing the BIN price . The width of the price spread of X is given by

$$d = BIN_{k_{BIN}}(X) - START_{k_{START}}(X) \ .$$

A general result can be stated. Since EGs increase as the parameters k increase, the more the seller is pessimistic in stating the starting price and optimistic in stating the buyout, the larger the price spread.

In order to highlight the effects of the levels k on the price-spread, we assume $k_{BIN} = k_{START} = k$ and call

$$d_k(X) = BIN_k(X) - START_k(X)$$

the price spread of order k. A list of properties follows:

1) Simple calculations lead to $d_k = EG_k(X) + EG_k(-X)$. If X is symmetrical and/or $k = 2$, it becomes $d_k = 2EG_k(X)$.

2) d_k is an increasing function of k. Since $BIN_k(X)$ and $START_k(X)$ are, respectively, a decreasing and an increasing functions of k, the higher k the higher the price spread.

3) Although a recursive formula exists for $k = 3$, they do not for d_k and $k \geq 4$. That is shown in the following Theorems 4.2 and 4.3.

Theorem 4.2 *For any final price X, we have*

$$d_3 = \frac{3}{2} d_2.$$

Corollary *For any non-negative random variables X with symmetric distribution around* ⸮

$$EG_3(X) = \tfrac{3}{2} EG_2(X) \quad and \quad d_3 = 3EG_2(X).$$

But, the above recursive relation cannot be extended to higher as shown in the following.

Theorem 4.3 *There is no recursive relation between d_4 and d_k for $k = 1, 2, 3$ that holds for all positive random variables.*

Table 2. Price spreads for some distributions of the final price.

The values d_k can be easily computed for a number of common distributions

Price spread d_k	Mean	$k = 2$	$k = 3$	$k = 4$
Uniform $(\theta = 2)$	1	0.67	1	1.20
Normal $(1, \sigma^2 = 1)$	1	1.12	1.70	2
Skew-Normal $(0, \omega^2 = \pi, 1)$	1	1.40		
Pareto $(\alpha = 2, c = 0.5)$	1	0.67	1	1.26
Weibull $(m = 2, \lambda = 2/\sqrt{\pi})$	1	0.58	0.88	1.07
Exponential $(\lambda = 1)$	1	1	1.50	1.83

using the formulae in Section 3., see Table 2. As expected, the price spread increases as the pessimism/optimism parameter k increases.

Since no recursive formulae exist for price spreads when $k_{START} = k_{BIN} = k$, it seems very unlikely they may exist for $k_{START} \neq k_{BIN}$. Nevertheless, in such case the magnitude of the width of the price spread can be calculated by Definition 2.3 substituting to $BIN_k(X)$ and $START_k(X)$ the values calculated with the proper values of k_{START} and k_{BIN}.

For $k > 2$ and asymmetrical distributions, the risk-premium $EG_k(X)$ and the gain-premium $EG_k(-X)$ may impact differently on the price spread d. An intuitive idea if $EG_k(X) > EG_k(-X)$ or vice versa, can be drawn as we think on the original meaning of EGs as measures of income inequality. The larger the number of incomes below the average income, the higher the index of income inequality and $EG_k(X) > EG_k(-X)$. Let note that the left tail of X coincides with the right tail of $-X$. We can conclude that the more the probability mass is on the left-tail of X (such as the right-skewed exponential and Pareto distributions), the higher $EG_k(X)$ than $EG_k(-X)$. For a negatively-skewed asset the result is reversed. That is confirmed by the results in Table 3.

Table 3. The risk premium and the gain premium.

		$EG_k(X)$	$EG_k(-X)$
Uniform (θ)		$\frac{\theta(k-1)}{2(k+1)}$	$\frac{\theta(k-1)}{2(k+1)}$
Normal (μ,σ^2)	$k=2$	$\frac{\sigma}{\sqrt{\pi}}$	$\frac{\sigma}{\sqrt{\pi}}$
Skew-Normal SN $(0,\omega^2,\alpha)$	$k=2$	$\frac{\omega}{\sqrt{\pi(1+\alpha^2)}}$	$\frac{\omega}{\sqrt{\pi(1+\alpha^2)}}$
Pareto $(\alpha,c=1)$		$\frac{\alpha(k-1)}{(\alpha k-1)}$	$\frac{\alpha^k k!}{(\alpha k-1)\cdots(\alpha-1)}-\frac{\alpha}{(\alpha-1)}$
Weibull $(m=2,\lambda=2/\sqrt{\pi})$		$1-\frac{1}{\sqrt{k}}$	$\sum_{j=1}^{k}(-1)^{j+1}\binom{k}{j}\frac{1}{\sqrt{j}}-1$
Exponential (λ)		$\frac{k-1}{\lambda k}$	$\frac{1}{\lambda}\sum_{j=2}^{k}\frac{1}{j}$

5 Conclusion

Our paper contributes to give a theoretical framework to sellers' rule for setting proper pricing of minimum starting bids and Buy-It-Now prices. Through the Extended Gini premium principle it is possible to elicitate the price bids taking into account the seller evaluation on the ending price distribution and the level of the seller pessimism and optimism in calculating the minimum starting bid and Buy-It-Now price, respectively.

Numerical explorations have been carried out and results are collected in numerical Tables.

We left to future research the experimental applications of the pricing methodology proposed.

Appendix

Proof of Theorem 4.2

$$d_3 = BIN_3(X) - START_3(X) = E(\max(X_1,X_2,X_3)) - E(\min(X_1,X_2,X_3))$$
$$= \int_0^\infty 1 - F^3(x) - [1-F(x)]^3 dx = 3\int_0^\infty F(x) - F^2(x)dx.$$

On the other hand we have

$$\mu - E(\min(X_1,X_2)) = \int_0^\infty 1 - F(x) - [1-F(x)]^2 dx = \int_0^\infty F(x) - F^2(X)dx.$$

So,

$$d_2 = BIN_2(X) - START_2(X) = E(\max(X_1, X_2) - E(\min(X_1, X_2)))$$

$$= \int_0^\infty 1 - F^2(x) - [1 - F(x)]^2 \, dx = 2 \int_0^\infty F(x) - F^2(x) dx .$$

Proof of Corollary

Due to symmetry, $EG_k(X) = \frac{1}{2} \left[BIN_k(X) - START_k(X) \right]$ for all k. Then

$$EG_3(X) = \frac{1}{2} \left[BIN_3(X) - START_3(X) \right] = \frac{1}{2} \frac{3}{2} \left[BIN_2(X) - START_2(X) \right] =$$

$$= \frac{1}{2} \frac{3}{2} \left[2EG_2(X) \right] = \frac{3}{2} EG_2(X)$$

Due to the symmetry, $d_3 = 2EG_3(X) = 3EG_2(X)$.

Proof of Theorem 4.3

Since $d_1 = 0$ and $d_3 = (3/2)d_2$, if such a recursive relation exists, then we can write $d_4 = f(d_3)$ for all random variables *X*. If *X* is uniform on [0,1], then $d_3 = 3/4 - 1/4 = 1/2$. If *X* is exponential with $\mu = 1/3$, then $d_3 = (1/3)(1 + 1/2 + 1/3 + 1/4 - 1/3) = 1/2$ as well. So if the recursive relation holds in general, then $d_4 = f(1/2)$ in both cases. But, if *X* is uniform on [0,1]. then $d_4 = 4/5 - 1/5 = 3/5 = 0.60$ whereas if *X* is exponential with $\mu = 1/3$, then $d_4 = (1/3)(1 + 1/2 + 1/3 + 1/4 - 1/4) = 11/18 = 0.61 \neq 0.60$.

Acknowledgments

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