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# Human Capital Accumulation in R&D-based Growth Models

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## Abstract

This paper develops a model that reproduces the essential aspects of the recent ICT-based economy using the framework of endogenous growth theory in which a central role is played by human capital accumulation. In particular, it considers a multi-sectoral growth model in discrete time with infinite horizon, endogenous growth, embodied technological progress, horizontal differentiation and "lab-equipment" specification of R&D, and with human capital accumulation (represented by the fact that households devote a fraction of their time to schooling), in order to take into account the crucial role of the latter when new technologies are present. In this model it is possible to obtain some important results, both analytically and through simulations, either in the case of constant productivity of schooling and in the case in which this productivity is a function of technological progress. The first conclusion is that the productivity of schooling affects the long run growth of the economy, contrary to the productivities of the other sectors, hence in this model human capital accumulation is the true engine of growth. It is then possible to study the reaction of the economy to different types of shocks, and to compare the results with the empirical evidence. The conclusion is that the model is able to reproduce such evidence, suggesting that the interaction between ICT and human capital is one of the drivers of the recent economic performance.

**Keywords**: Information Technology, Endogenous Growth, Embodied Technological Progress.

**Journal of Economic Literature**: C63, E22, O40.

# 1 Introduction

This paper presents a multi-sectoral endogenous growth model, that is able to reproduce the most important aspects of an "ICT-based economy", in which a central role is played by human capital accumulation, extending a previous analysis developed in Mattalia (2002). Indeed, one of the characteristics of the so-called "New Eco-nomy" is that human capital can be of great importance, since education is crucial in acquiring the knowledge necessary to use the new technologies, and at the same time an increase in ICT makes it easier to accumulate human capital. For this reason, the adoption of new technologies has often coincided with growing needs in human capital, reflecting the complementarity between these two forms of investment in the production process. There is also a relationship between human capital (i.e. the skills and competencies embodied in workers) and labour productivity, so that improvements in one lead to increases in the other, and therefore human capital is a significant determinant of economic growth. According to recent studies, for example, in OECD countries, one additional year of schooling would, on average, lead to about 6% higher GDP in the long run (OECD, 2000). The productivity-enhancing role of human capital is due to its complementarity with new technology (ICT use requires skills and competencies), and for this reason the demand for knowledge-intensive employment has risen considerably. In effect, in the 1990s, knowledge workers (including computer workers) were the fastest-growing occupational category in the US and the EU area (the average annual change in 1992-99 was 3.3%). In the US the ICT workforce reached more than 6.5 million in 2000, with almost 60% of these workers employed in high-skilled ICT occupations, 25% in medium-skilled jobs and the remaining in low-skilled occupations. From 1995 to 1999, the percentage of computer workers with respect to total employment in the US raised from 1.8% to 2%, furthermore employment growth has been strong throughout 2001 and unemployment rates in ICT professions very low. With reference to the contribution of human capital to economic growth, then, recently it has been observed a shift of labour demand towards high-skilled workers, together with growing ICT skills shortages. In addition, the distribution of high-skill and low-skill ICT-related occupations in the US and the EU shows an interesting pattern, in fact although the share of ICT workers is growing everywhere, in 1999 the US ICT workforce appeared to be relatively more high-skilled (77%) than that of the EU (56%). High-skill ICT workers are the most rapidly growing component of high-skilled workers and in 1999 they represented 2.4% of total employment in the US and 1.6% in the EU. In 2000 the US employed about 5.5 million persons in the ICT sector, and this sector has been a major source of employment growth, in fact over the 1995-2000 period OECD-area employment in the sector grew at an average annual growth rate of over 4% (almost 3 times that of overall business sector employment), and the proportion of ICT specialists has risen between 1995 and 2004 (see OECD, 2005).

Another indicator that can be used as a proxy of the level of human capital of a nation is represented by the share of the population that has attained qualifications at the tertiary level (indeed, this can be considered a key indicator of how well countries are placed to profit from technological and scientific progress). Table 1 reports some statistics concerning this aspect (see OECD, 2008) and it turns out that OECD countries have seen significant increases in the proportion of the adult population attaining tertiary education over the last decades. In 1992, for the 25-64 year-old population, the OECD average of the population having attained the tertiary level was 19%, while in 2005 the average increased to 26%. US performed remarkably high, and the corresponding percentages were 30.2% in 1992 and 39% in 2005. Furthermore, in the youngest age group, 25 to 34 years old, the OECD country mean for tertiary attainment increased from almost 22% to over 32% between 1992 and 2005 (and in three countries - Canada, Japan and Korea - 50% or more of this age group had in 2005 obtained a tertiary qualification).

Human capital includes not only the education workers bring to the job, but also skills learned while working and adapting to new technologies. In particular, training plays a significant role when technological change is rapid and the knowledge necessary to implement the new technologies is very specific. For example, a number of studies referred to Canada have established that the implementation of new technologies in manufacturing firms increased the level of required qualifications and stimulated firms to invest in training (Baldwin and Peters, 2001; Baldwin et al., 1997). Other researches relative to the US have also shown that highly educated workers are more likely to participate in training than those with little education, suggesting a complementary relationship between human capital acquired through the education system and that acquired through in-house training (Bartel and Sicherman, 1998; Lynch, 1992; Leonard et al., 2003). However, further studies have shown that the participation differentials in training between workers with little education and those who are highly educated are mitigated when there is a high rate of technological change (Turcotte and Rennison, 2004). These studies also conclude that investments in education, training and new technologies are closely related, and are associated with higher productivity.

These researches are in line with recent theoretical and empirical advancements in the endogenous growth literature, according to which not only R&D activity, but also human capital accumulation, is a primary determinant of economic growth. For these reasons, the model presented in this paper studies the role of R&D and of human capital accumulation in the growth process that has characterised the ICT revolution. More precisely, this model extends the one proposed in Mattalia (2002), and introduces the choice of the households to invest in human capital, by devoting a fraction of time to schooling activity. This human capital accumulation, then, turns out to be the true engine of growth of the model, that therefore has important differences with respect to the one presented in the previous paper. In particular, in this case it is possible to obtain explicitely (in the benchmark version) the growth factors of the different variables, contrary to the previous model, where it was possible to obtain only a parameterized solution. The most important contribution, then, is represented by the fact that in its more general formulation the model studies the ICT productivity-enhancing role in education, considering the productivity of human capital (i.e. of schooling) as a function of technological progress, that specifies the complementarity between human capital accumulation and technological progress (since new technologies can allow people to be more educated). In this case the different growth factors and the different long run levels can be found only numerically (and not analytically as in the benchmark version with constant productivity of human capital). This is the true novelty of this model with respect to the existing literature, and represents a very important contribution.

The analytical study of the model allows to find some results concerning the effects on growth of different shocks that can interest the economy, and that can be compared with the available data. These results show in particular that permanent changes in the productivity of the final good sector, in the productivity of the equipment sector and in the productivity of the intermediate good sector do not affect the long run growth rate of the economy (contrary to what happens in the model of the previous paper), while it is affected by the productivity of schooling (that in this case is the true engine of growth).

Resorting to numerical simulations it is then possible to obtain other insights. In particular, it is possible to study how the economy reacts, in the short run, to shocks on the productivities of the different sectors. With reference to this aspect it turns out that an increase in the productivity of the final good sector and a decrease in the cost of a new variety of software, on the one hand, and an increase in the productivity of the equipment sector and an increase in the productivity of the intermediate good sector, on the other hand, have similar consequences. We also have that considering the more general version of the model with productivity of schooling depending on technological progress, the short run behaviour of the variables is different with respect to the benchmark model in which the productivity of schooling is constant.

The paper is organized as follows. Section 2 describes the model with the different sectors that characterize the economy. Section 3 derives the equilibrium conditions, the balanced growth path and the steady state system, with the corresponding analytical results. Section 4 shows the results of numerical simulations on a calibrated version of the model and compares these results with the available data regarding in particular the US. Section 5 concludes, while all the computations are available in a separate Appendix.

## 2 The model

The model presented here builds on Romer (1990) and Boucekkine and de la Croix (2003) for the general structure, but departs from them in some respects. It is a multi-sectoral model written in discrete time with infinite horizon, endogenous growth and horizontal differentiation. The economy is characterized by 4 sectors: the final good sector, the equipment sector, the intermediate good sector and the R&D sector. Technological progress is mainly embodied (the idea is that new softwares need new hardware to work efficiently) and the innovators have a market power represented by copyrights. All these elements are important to reproduce the essential characteristics of the ICT sector. Moreover, the R&D sector assumes the "lab-equipment" specification (first introduced by Rivera-Batiz and Romer, 1991), and a human capital accumulation activity of the kind introduced by Lucas (1988) is considered (assuming, in addition, in the more general version that the productivity of human capital is a function of technological progress). The reason is that, together with the embodied nature of technological progress, the important role of the R&D sector and the link between innovation and market power (all elements that are present in the model introduced in Mattalia, 2002), an ICT economy can be characterized also by an important role of human capital accumulation (for the reasons cited in the Introduction). With reference to this aspect, we have that, despite this importance, it is not assumed that the R&D sector is more intensive in human capital than the final good sector (essentially for algebraic convenience); in any case it turns out that human capital accumulation is the engine of long run growth in the model (so that, even assuming a more human capital intensive technology in the R&D sector, would not qualitatively change the results).

The model has therefore the same general structure of the one presented in Mattalia (2002). The main differences are represented by the fact that human capital is employed as a production factor in the final good sector and in the intermediate good sector, and by the fact that the representative household also invests in human capital, by devoting a fraction of time to non-productive activities (schooling). Moreover, in the more general version the productivity of human capital is a function of technological progress, in order to specify the complementarity between these two elements, and this is the true novelty of the model.

## 2.1 The final good sector

This sector produces the final good (used to consume or to invest in physical capital) using efficient capital (bought from the equipment sector) and human capital, according through a Cobb-Douglas technology:

$$Y_t = z_t K_t^{\alpha} H_{Y,t}^{1-\alpha} \qquad \alpha \in [0,1] \tag{1}$$

where $z_t$ represents total factor productivity (disembodied technological progress), $K_t$ is the physical capital and $H_{Y,t}$ is the human capital used in the final good sector. The stock of physical capital is then defined as:

$$K_t = \sum_{s=-\infty}^{t} E_s (1-\delta)^{t-s} \tag{2}$$

where $E_s$ is the efficient capital bought from the equipment sector at time $s$ and $\delta$ is the physical depreciation rate (constant).

The discounted profits of investing $E_t$ in efficient capital are given by:

$$\pi_t = \sum_{s=t}^{\infty} \left[ Y_s - w_s H_{Y,s} \right] R_t^s - d_t E_t$$

where $R_t^s$ is the discount factor at time $s$, $w_s$ is the wage at time $s$ and $d_t$ is the price of efficient capital at time $t$. The representative firm chooses efficient capital and human capital in order to maximize its discounted profits taking prices as given and subject to its technological constraints:

$$\max_{E_t, \left\{H_{Y,s}\right\}_{s=t}^{\infty}} \pi_t$$
$$s.t. \qquad (1), (2)$$

and from the solution of this problem we get:

$$H_{Y,t} = \left(\frac{(1-\alpha) z_t}{w_t}\right)^{\frac{1}{\alpha}} K_t \tag{3}$$

that is the demand for human capital by the final good sector.

## 2.2 The equipment sector

The equipment sector produces efficient capital (sold to the final good sector) using physical capital (hardware) bought from the final good producers and immaterial capital (software) bought from the intermediate good producers. Efficient capital is produced with a constant return to scale technology:

$$E_t = e_t Q_t^{\lambda} I_t^{1-\lambda} \qquad \lambda \in (0,1) \tag{4}$$

where $e_t$ is a productivity variable (that represents embodied technological progress), $I_t$ is physical capital (hardware) and $Q_t$ is immaterial capital (software), that is built from a series of specialized intermediate goods (horizontal differentiation):

$$Q_t = \left(\int_0^{n_t} x_{i,t}^{\frac{\sigma-1}{\sigma}} di\right)^{\frac{\sigma}{\sigma-1}} \tag{5}$$

where $n_t$ is the number of varieties of intermediate input available in $t$, $x_{i,t}$ is the quantity of intermediate input of variety $i$ used in $t$ and $\sigma > 1$ is the elasticity of substitution between two varieties.

The profits of the equipment sector at time $t$ are:

$$\pi_t' = d_t E_t - I_t - \int_0^{n_t} p_{i,t} x_{i,t} di$$

where $p_{i,t}$ is the price of software of variety $i$ at time $t$. The representative firm chooses the investment in physical capital and in immaterial capital in order to maximize profits taking prices as given and subject to its technological constraints:

$$\max_{I_t, x_{i,t}} \pi_t'$$
$$s.t. \qquad (4), (5)$$

and from the solution of this problem we get:

$$x_{i,t} = \left(\frac{\phi}{q_t}\right)^{\sigma} Q_t p_{i,t}^{-\sigma} \tag{6}$$

(where $\phi = \frac{\lambda}{1-\lambda}$ and $q_t = \frac{Q_t}{I_t}$) that is the demand for intermediate input $i$ by the equipment sector at time $t$.

## 2.3 The intermediate good sector

This sector produces immaterial capital (software, sold to the equipment sector) and it researches for new varieties, in order to expand the range of software (horizontal differentiation).

### 2.3.1 The production activity

The variety $i$ of software is produced according to a linear technology that uses human capital as the only input:

$$x_{i,t} = \tau_t H_{i,t} \tag{7}$$

where $H_{i,t}$ is the human capital used in the intermediate good sector and $\tau_t$ represents productivity of human capital. The producer behaves monopolistically (since market power is given by the presence of copyrights which have an infinite lifetime - i.e. the inventor of a new variety of software obtains these copyrights forever -) and its profit is:

$$\pi_{i,t}^{''} = p_{i,t}x_{i,t} - w_t H_{i,t} = \left(p_{i,t} - \frac{w_t}{\tau_t}\right) x_{i,t}$$

The price of output is chosen so as to maximize this profit subject to the demand formulated by the equipment sector, hence the problem solved by the firm is:

$$\max_{p_{i,t}} \ \pi_{i,t}^{''}$$
$$s.t. \qquad (6)$$

from which we get:

$$p_{i,t} = \left(\frac{\sigma}{\sigma-1}\right)\frac{w_t}{\tau_t} \qquad \forall i \in [0, n_t] \tag{8}$$

i.e. the output price is a mark-up over unit human capital cost (where the mark-up rate depends on the price elasticity of demand, that is given by $-\sigma$).

### 2.3.2 The research activity

The intermediate good sector also researches for new varieties of immaterial capital, in order to expand their range. As in Mattalia (2002) we assume the “lab-equipment” specification of R&D (first introduced by Rivera-Batiz and Romer, 1991) With reference to this aspect it is possible to observe that, even if human capital is very important to model an ICT economy, here we do not assume that the R&D sector is more intensive in human capital than the final good sector, and this is done essentially for algebraic convenience. In any case, the fact that $H$ is crucial in the ICT economy (and especially in the innovation side of it) is confirmed by the result of this model (and in particular also in the lab-equipment version presented here), according to which human capital accumulation turns out to be the engine of long run growth. Therefore, also modifying the technology of the R&D sector in order to have a more $H$-intensive technology would not qualitatively change the results. Furthermore, a channel trough which human capital interacts with ICT is represented by the fact that, as shown in the next Subsection, in the more general version of the model ICT enhance productivity in the education sector.

The "lab-equipment" specification of R&D considered here implies that the cost to create a new variety of software is equal to $\eta$ units of $Y$, and there will be entry of new firms in the economy until this cost is equal to the discounted flow of profits linked to one invention:

$$\eta = \sum_{z=t}^{\infty} R_t^z \pi_{i,z}^{''} = \sum_{z=t}^{\infty} R_t^z \frac{1}{\sigma - 1} \frac{w_z}{\tau_z} x_{i,z}$$

that is the free-entry condition.

## 2.4 Household behaviour

After the 4 sectors that characterize the economy it is also possible to consider the household present in this economy. The representative household consumes, saves for future consumption, supplies human capital for productive activities and accumulates human capital by devoting a fraction of its time to non-productive activities (schooling). The utility of the representative household at time 0 is:

$$u_0 = \sum_{t=0}^{\infty} \rho^t \ln C_t$$

i.e. it is the discounted sum of instantaneous utilities from 0 to $\infty$, where $\rho$ is the psychological discount factor and the instantaneous utility function is logarithmic. The budget constraint faced by the household is:

$$A_t = (1 + r_t) A_{t-1} + w_t u_t H_{t-1} - C_t \tag{9}$$

where $A_t$ represents the assets held by the household at time $t$, $H_t$ is the total human capital and $u_t$ is the fraction of time devoted to productive activities. The accumulation of human capital is then described by the following equation:

$$H_t = \left[1 + \varepsilon_t \left(1 - u_t\right)\right] H_{t-1} \tag{10}$$

where $\varepsilon_t$ is the productivity of schooling and $1 - u_t$ is the fraction of time devoted to non-productive activities (schooling).

The problem solved by the representative household is then:

$$\begin{aligned} &\max_{\{C_t, u_t, A_t, H_t\}_{t=0}^{\infty}} \quad u_0 \\ &s.t. \qquad (9), (10) \end{aligned}$$

from which we get:

$$\frac{C_{t+1}}{C_t} = (1 + r_{t+1})\rho \tag{11}$$

$$\frac{w_{t+1}}{w_t} = \frac{1 + r_{t+1}}{1 + \varepsilon_{t+1}} \cdot \frac{\varepsilon_{t+1}}{\varepsilon_t} \tag{12}$$

In this context it is also possible to consider a more general version of the model, in order to have a link between human capital accumulation and technological progress, that is typical of ICT (since, as outlined above, in general new technologies allow people to be more educated). This more general version considers the productivity of human capital (i.e. of schooling) as a function of technological progress, in order to specify the complementarity between these two elements. In particular, the productivity of schooling $\varepsilon_t$ is assumed to be a function of $g_t = g_n = \frac{n_t}{n_{t-1}}$ (the growth factor of the number of varieties of software), so that we have $\varepsilon_t = \varepsilon(g_t)$. For instance, it is possible to assume the following functional form:

$$\varepsilon_t = \varepsilon(g_t) = a - a' e^{-b g_t} \qquad a > 0,\ a > a',\ b > 0$$

for which we have $\varepsilon'(g_t) > 0$, $\varepsilon''(g_t) < 0$ and that for $a' = 0$ gives the particular case (benchmark case) in which $\varepsilon_t$ is constant ($\varepsilon_t = a$).

## 3 The equilibrium

The equilibrium of the economy in the model considered is determined by the equilibrium on the human capital market and on the final good market.

Equilibrium on the human capital market implies that human capital used in productive activities is employed either in the final good sector or in the intermediate good sector, i.e.:

$$H_{Y,t} + \int_0^{n_t} H_{i,t} di = u_t H_{t-1} \tag{13}$$

while equilibrium on the final good market implies that output is employed for consumption, investment and research, i.e.:

$$Y_t = C_t + I_t + \eta \bigtriangleup n_t \tag{14}$$

where $\eta \bigtriangleup n_t$ is the cost of research for new varieties (for all the algebraic details concerning this Section see the separate Appendix).

## 3.1 The equilibrium conditions

The equilibrium conditions summarize the first order optimality conditions and the market equilibrium relationships shown above. The results obtained are summarized in the following Proposition:

**Proposition 1** *Given the initial conditions* $K_{-1}$ , $H_{-1}$ *and* $n_{-1}$ *an equilibrium is a path:*

$$\{w_t, q_t, C_t, I_t, K_t, H_t, n_t, u_t, r_{t+1}\}_{t\geq 0}$$

*that satisfies the following equations:*

$$\left(\frac{(1-\alpha)\,z_t}{w_t}\right)^{\frac{1}{\alpha}} K_t + n_t \left(\frac{\sigma-1}{\sigma}\right)^{\sigma} \left(\frac{\phi}{w_t}\right)^{\sigma} q_t^{1-\sigma} I_t \tau_t^{\sigma-1} = u_t H_{t-1} \tag{15}$$

$$z_t^{\frac{1}{\alpha}} K_t \left(\frac{1-\alpha}{w_t}\right)^{\frac{1-\alpha}{\alpha}} = C_t + I_t + \eta \bigtriangleup n_t \tag{16}$$

$$\alpha z_t^{\frac{1}{\alpha}} (1-\lambda) e_t q_t^{\lambda} \left(\frac{1-\alpha}{w_t}\right)^{\frac{1-\alpha}{\alpha}} = 1 - \left(\frac{1-\delta}{1+r_{t+1}}\right)\left(\frac{e_t}{e_{t+1}}\right)\left(\frac{q_t}{q_{t+1}}\right)^{\lambda} \tag{17}$$

$$H_t = \left[1+\varepsilon_t(1-u_t)\right] H_{t-1} \tag{18}$$

$$\frac{C_{t+1}}{C_t} = (1+r_{t+1})\rho \tag{19}$$

$$\frac{w_{t+1}}{w_t} = \frac{1+r_{t+1}}{1+\varepsilon_{t+1}} \cdot \frac{\varepsilon_{t+1}}{\varepsilon_t} \tag{20}$$

$$K_t = (1-\delta)K_{t-1} + e_t q_t^{\lambda} I_t \tag{21}$$

$$\frac{w_t q_t}{\tau_t \phi} = n_t^{\frac{1}{\sigma-1}} \left(\frac{\sigma-1}{\sigma}\right) \tag{22}$$

$$\frac{\tau_t^{1-\sigma}\,(\sigma-1)^{1-\sigma}\,\sigma^{\sigma}}{\phi^{\sigma}} \cdot \frac{\eta r_{t+1}}{1+r_{t+1}} = w_t^{1-\sigma} q_t^{1-\sigma} I_t \tag{23}$$

In the more general case in which the productivity of schooling is given by $\varepsilon_t = a - a' e^{-bg_t}$ the equilibrium conditions are the same, except for equations (18) and (20) that are replaced by:

$$H_t = \left[1 + \left(a - a' e^{-bg_t}\right)(1-u_t)\right] H_{t-1} \tag{24}$$

$$\frac{w_{t+1}}{w_t} = \frac{1+r_{t+1}}{1+a-a' e^{-bg_{t+1}}} \cdot \frac{a - a' e^{-bg_{t+1}}}{a - a' e^{-bg_t}} \tag{25}$$

## 3.2 The balanced growth path

After the characterization of the equilibrium, the balanced growth path of the model can be analysed. In this case we assume that the variables:

$$z_t, e_t, \tau_t, \varepsilon_t, r_t, u_t$$

are constant in the long term, while each endogenous variable grows at a constant rate along a balanced growth path. In this case, when the productivity of schooling $\varepsilon_t$ is assumed to be constant (i.e. $\varepsilon_t = a$), it is possible to determine explicitely the different growth factors.

The results obtained are expressed in the following Proposition:

**Proposition 2** *In the model considered with human capital accumulation and constant productivity of schooling, the growth factors of the different variables are the following:*

$$g_Y = g_C = g_I = g_n = [\rho(1+a)]^{\frac{1}{\omega_4}}$$

$$g_q = [\rho(1+a)]^{\frac{\omega_1}{\omega_2 \omega_4}}$$

$$g_w = [\rho(1+a)]^{\frac{1}{\omega_2 \omega_4}}$$

$$g_K = [\rho(1+a)]^{\frac{\omega_3}{\omega_4}}$$

$$g_H = \rho(1+a)$$

*where:*

$$\omega_1 = \frac{1-\alpha}{\lambda\alpha} \qquad \omega_2 = \frac{(\sigma-1)(1-\alpha+\lambda\alpha)}{\lambda\alpha}$$

$$\omega_3 = \frac{\lambda(1-\alpha)}{(\sigma-1)(1-\alpha+\lambda\alpha)} + 1 \qquad \omega_4 = \frac{\sigma(1-\alpha+\lambda\alpha)-1+\alpha-2\lambda\alpha}{(\sigma-1)(1-\alpha+\lambda\alpha)}$$

*Furthermore, the optimal long run value of $u$ (the fraction of time devoted to productive activities) is given by:*

$$u^* = \frac{(1+a)(1-\rho)}{a}$$

*and since $0 < u^* < 1$ this requires $\frac{1}{1+a} < \rho < 1$.*

From this Proposition it turns out that, in the present model, along a balanced growth path the growth factors of the different quantities depend only on the parameters $\alpha, \lambda, \sigma, \rho, a$ (i.e. technological and preference parameters). In particular, as in the basic Lucas (1988) model, human capital accumulation depends on the parameters describing preferences and the skill acquisition technology.

In the more general case in which the productivity of schooling is a function of technological progress the different growth factors can be obtained only as functions of $g_q$, and the corresponding results are expressed by the following Proposition (it is also possible to observe that if $a' = 0$, so that the productivity of schooling is constant, these results coincide with those of Proposition 2):

**Proposition 3** *In the model considered with human capital accumulation and productivity of schooling depending on technological progress, the growth factors of the different variables are the following:*

$$\begin{aligned} g_Y = g_C = g_I = g_n &= (g_q)^{\frac{\omega_2}{\omega_1}} \\ g_w &= (g_q)^{\frac{1}{\omega_1}} \\ g_K &= (g_q)^{\frac{\omega_2\omega_3}{\omega_1}} \\ g_H &= (g_q)^{\frac{\omega_2\omega_4}{\omega_1}} \end{aligned}$$

*where $\omega_1, \omega_2, \omega_3, \omega_4$ are the same expressions of Proposition 2.*

*Furthermore, $g_q$ satisfies the equation (that can be solved numerically, and whose solution exists and is unique):*

$$(g_q)^{\frac{1}{\omega_1}} = \frac{(g_q)^{\frac{\omega_2}{\omega_1}}}{\rho\left(1 + a - a'e^{-b(g_q)^{\frac{\omega_2}{\omega_1}}}\right)}$$

*and the optimal long run value of $u$ is given by:*

$$u^* = 1 - \frac{(g_q)^{\frac{\omega_2\omega_4}{\omega_1}} - 1}{a - a'e^{-b(g_q)^{\frac{\omega_2}{\omega_1}}}}$$

It is then possible to study the comparative statics, in particular of the growth factor of human capital $g_H$, both in the case of constant productivity of schooling and in the more general case of productivity of schooling function of technological progress. The following results hold:

**Proposition 4** *In the model considered with human capital accumulation and constant productivity of schooling, the comparative statics of the growth factor of human capital $g_H$ are given by:*

$$\frac{\partial g_H}{\partial a} > 0 \qquad \frac{\partial g_H}{\partial \rho} > 0$$

*i.e. $g_H$ increases with the increase in the productivity of schooling and with the increase in the psychological discount factor.*

*In the same model but with productivity of schooling depending on technological progress, the comparative statics of $g_H$ are given by:*

$$\frac{\partial g_H}{\partial a} > 0 \qquad \frac{\partial g_H}{\partial a'} < 0 \qquad \frac{\partial g_H}{\partial b} > 0 \qquad \frac{\partial g_H}{\partial \rho} > 0$$

*i.e.* $g_H$ *increases with the increase in the parameters* $a$ *and* $b$ *that affect the productivity of schooling and with the increase in the psychological discount factor, and it decreases with the increase in the parameter* $a'$.

## 3.3 The stationarized dynamic system and the steady state system

Even if the various growth factors can be determined explicitely (in the case in which the productivity of schooling is assumed to be constant), the equilibrium system in terms of levels formed by equations (15) − (23) is undetermined, in fact rewriting these equations by substituting the generic variable $x_t$ with the expression $\overline{x}g_x^t$ we get 9 equations for 10 unknowns ($\overline{w}$, $\overline{q}$, $\overline{C}$, $\overline{I}$, $\overline{K}$, $\overline{H}$, $\overline{n}$, $\overline{r}$, $\overline{u}$, $g$). In order to eliminate the indeterminacy and to find a solution is therefore necessary to stationarize the system by means of some auxiliary variables (i.e. to rewrite the system in terms of variables that are stationary - that is constant - in the steady state), obtaining the stationarized dynamic system corresponding to the original one.

In the same way it is then possible to obtain the steady state system corresponding to the stationarized one. In this case we have a system of 9 equations with 9 unknowns ($\widehat{q}$, $\widehat{C}$, $\widehat{I}$, $\widehat{K}$, $\widehat{H}$, $\widehat{n}$, $g$, $r$, $u$) that can be solved explicitely. Indeed, first of all it is possible to obtain functions expressing the long run levels ($\widehat{q}$, $\widehat{C}$, $\widehat{I}$, $\widehat{K}$, $\widehat{H}$, $\widehat{n}$, $r$, $u$) exclusively in terms of $g$, then in the benchmark version of the model (when the productivity of schooling is assumed to be constant) it is also possible to get an explicit expression for $g$ (depending only on the parameters of the model).

The results obtained (in the case of constant productivity of schooling) are summarized in the following Proposition:

**Proposition 5** *In the model considered with human capital accumulation and constant productivity of schooling, there exist explicit functions expressing the long run levels* $\widehat{q}$, $\widehat{C}$, $\widehat{I}$, $\widehat{K}$, $\widehat{H}$, $\widehat{n}$, $r$, $u$ *exclusively in terms of* $g$:

$$\widehat{q} = \Psi_{\widehat{q}}(g) \qquad \widehat{C} = \Psi_{\widehat{C}}(g) \qquad \widehat{I} = \Psi_{\widehat{I}}(g) \qquad \widehat{K} = \Psi_{\widehat{K}}(g)$$

$$\widehat{H} = \Psi_{\widehat{H}}(g) \qquad \widehat{n} = \Psi_{\widehat{n}}(g) \qquad r = \Psi_r(g) \qquad u = \Psi_u(g)$$

*with:*

$$g = [\rho(1+\varepsilon)]^{\frac{1}{\omega_4}}$$

In the more general case in which the productivity of schooling is a function of the technological progress, the same Proposition holds, with the difference represented by the fact that the value of $g$ is given by the solution of the equation:

$$g^{\frac{1}{\omega_2}} = \frac{g}{\rho(1+a-a'e^{-bg})}$$

(it is possible to prove that this solution exists and is unique).

From the analysis of the expressions $\Psi_{()}(g)$ it is possible to deduce the effects of the different exogenous variables on the long run levels in the economy. In particular, it turns out that the function $\Psi_{\widehat{q}}(g)$ depends on $z$, $e$ and $\varepsilon$ and the functions $\Psi_{\widehat{n}}(g)$, $\Psi_{\widehat{I}}(g)$, $\Psi_{\widehat{K}}(g)$, $\Psi_{\widehat{C}}(g)$ and $\Psi_{\widehat{H}}(g)$ depend on $z$, $e$, $\tau$ and $\varepsilon$ (the productivity variables in the different sectors). In addition, the long run growth factor $g$ is not affected by the productivity of the final good sector, of the equipment sector and of the intermediate good sector (contrary to what happens in Mattalia, 2002), while it is affected by the productivity of schooling, as expressed in the following Proposition:

**Proposition 6** *Assuming that a solution for the steady state system exists, the long run values of $z$ (the productivity in the final good sector), $e$ (the productivity in the efficient capital sector) and $\varepsilon$ (the productivity of schooling) affect the stationary values $\widehat{q}$, $\widehat{n}$, $\widehat{I}$, $\widehat{K}$, $\widehat{C}$ and $\widehat{H}$ and the long run value of $\tau$ (the productivity in the intermediate good sector) affects the stationary values $\widehat{n}$, $\widehat{I}$, $\widehat{K}$, $\widehat{C}$ and $\widehat{H}$. Furthermore $z$, $e$ and $\tau$ have no impact on the long term growth factor $g$, while $\varepsilon$ affects it.*

In this model, therefore, permanent changes in $z_t$ (the productivity in the final good sector), in $e_t$ (the productivity in the efficient capital sector) and in $\tau_t$ (the productivity of human capital in the intermediate good sector) will not affect the long run growth rate of the economy, and only a change in $\varepsilon_t$ (the productivity of schooling) will have an impact on this growth rate. In conclusion, therefore, the true engine of growth is represented by the accumulation of human capital, through the schooling activity. This is the central difference of this model (that is based on the accumulation of human capital) with respect to the one introduced in Mattalia (2002), in which human capital was not present. According to these results, the recent ICT-based economic expansion should have a permanent effect on growth only when ICT investment is combined with other organizational assets, in particular human capital. This is consistent with the available empirical evidence, as it will be illustrated in the next Section.

# 4 Simulation of the model

The model described above can then be simulated numerically in order to get information concerning, in particular, the behaviour of the economy (both in the long run and in the short run) as a consequence of shocks that can hit the system. The results of the simulations are also compared with the available data in order to verify the ability of the model to reproduce the real situation. This requires a calibration, and the values of the different parameters are chosen in such a way that the model is able to reproduce the most relevant empirical data that are available.

## 4.1 Calibration

The calibration of the model is based essentially on the recent data of the US economy, so that the different parameters are fixed to values that can be considered reasonable on the basis of the empirical evidence. In addition, these values are chosen in such a way that they match a series of moments of the steady state of the model. In particular, these target moments are those based on the recent study of Atkinson and Andes (2010).

First of all, concerning the parameters related to the technology, in the benchmark model (with constant productivity of schooling) the capital share in the final good sector $\alpha$ is equal to 0.35, close to the value of 0.30 used in their model by Boucekkine and de la Croix (2003) and to the values chosen in other studies by different authors (see for example Alvarez and Lucas, 2007). With reference to the share of software in the production of efficient capital $\lambda$, this parameter is used to calibrate the size of the new economy in terms of the human capital employed in the intermediate good and in the R&D sector. In particular, this parameter is set to the value 0.85 (significantly different from the value 0.5 used in another paper of Boucekkine, de la Croix and Vailakis, 2002). The consequence of this assumption is that the elasticity of substitution between varieties of softwares $\sigma$ is equal to 1.40 (again significantly different from the value of 3 of the model of Boucekkine and de la Croix), so that the mark-up rate is 3.5.

Two other important parameters are then the rate of depreciation of physical capital $\delta$ and the psychological discount factor $\rho$. The first is set equal to 10%, consistent with some other studies (see for instance Nadiri and Prucha, 1997, that estimate a rate of depreciation for physical and R&D capital in US of 6% and 12% respectively), while the second is fixed at 96% (very close to the value of 97% chosen by Boucekkine and de la Croix in their work).

Another set of parameters, then, is represented by the values that express the productivities in the different sectors. In particular, first of all the productivity in the final good sector $z$ is equal to 3 (while in the model of Boucekkine and de la Croix this value is normalized to 1). The productivity in the equipment sector $e$, then, is fixed to 12 (this value is such that the capital/output ratio turns out to be equal to 2, with respect to the value of 3 used in the model of Boucekkine, de la Croix and Vailakis). The human capital productivity in the intermediate good sector $\tau$, then, is fixed to 0.25 (close to the value of 0.2 used by Boucekkine, de la Croix and Vailakis) because in this way (together with the value chosen for the parameter $\lambda$ considered above) we have that about 5% of human capital is employed in the intermediate good and in the R&D sector (again, these results are compatible with the available data, see for instance Atkinson and Andes, 2010). The last productivity considered is the productivity of schooling $\varepsilon$, that is fixed to 0.044 (from one of the relations of the steady state). Finally, the last parameter to be fixed is the cost of a new variety of software expressed in units of output $\eta$ (that derives from the "lab-equipment" specification for R&D); this parameter is used to calibrate the size of the R&D sector and is equal to 20, so that the R&D expenditure is approximately equal to 3.5% of

GDP (again consistent with available studies, see for instance Atkinson and Andes, 2010). The interest rate, finally, is 5%.

The moments of the steady state that have to be reproduced and the values of the different parameters that allow to reach these targets are reported in Table 2.

With these values, therefore, the model leads to a growth rate of output equal to 1.04% per year, that is in line with the available data (see for instance Greenwood et al., 1997). At this point, the benckmark case can be used to study the effects of different types of shocks that can interest the economy. More precisely, after that the steady state for the initial calibration of the model has been computed, a shock on a particular variable is considered (here all the shocks have an intensity equal to 1%), then the new steady state is obtained and the dynamics of transition from the old steady state to the new one is determined. In this way it is also possible to determine the magnitude of the effects of the shocks on the relevant variables in the short and in the long run, and to compare the results with those of the model presented in Mattalia (2002) - where human capital accumulation was not present - and with the available data, in order to verify the ability of the model to replicate the real situation.

## 4.2 Productivity shocks

The simulations performed in the model allow to study the effects of different shocks in the productivity variables, initially in the benchmark version of the model, with constant productivity of schooling; all these shocks are permanent (from $t = 0$) and have an intensity of 1%.

The first simulation considers an increase in $z$ (the productivity of the final good sector). From the analytical results derived above (Proposition 6) we know that this increase has no effect on long term growth, and in fact in the long run the growth rates of output, physical capital and human capital don't change with respect to the initial situation. This is a central difference with respect to the model (presented in Mattalia, 2002) without human capital accumulation, where an increase in $z$ increases long term growth. The reason is that, like in that model, the lab-equipment specification implies that the production function in the R&D sector is the same as in the final good sector, so that an increase in the productivity of the latter is equivalent to an increase in the productivity of R&D; nevertheless, this is no longer the engine of growth of the model (that is now represented by the accumulation of human capital), and hence an increase in its productivity doesn't have long run effects on growth.

With reference to the short run, there is initially a strong increase in the human capital fraction employed in education (about 10% with respect to the initial steady state value) and a consequent reduction in the fraction employed in production (where the increase in the productivity of the final good sector reduces the cost of production of the final good and determines a reallocation of human capital favourable to the final good sector and at the expenses of the intermediate good sector). As a consequence, initially there is a strong increase in the growth rate of human capital (because of the increase in the human capital fraction employed in education) and a strong decrease (about 15% with respect to the initial steady state value) in the growth rate of output (because human capital accumulation influences growth with a lag, since education requires time before producing its effects on the economy). After some time, then, the situation reverses, the human capital fraction in education and the consequent growth of human capital reduce, while the growth rate of output increases (because the previous accumulation of human capital produces its effects) and returns to its initial level.

The same type of dynamics arise when we consider, instead of an increase in $z$, a decrease in $\eta$ (the cost of a new variety of software). Indeed, a reduction in this cost (that is expressed in terms of output, as a consequence of the lab-equipment assumption) is analogous to an increase in the productivity of the final good sector, and this explains why the results obtained are essentially of the same kind.

The second simulation considers an increase in $e$ (the productivity of the equipment sector). Also this kind of shock has no long run effect, because an increase in the productivity of the equipment sector increases the production of efficient capital and therefore of the final good, that corresponds (as a consequence of the lab-equipment assumption) to an expansion in the R&D sector, but since this is no longer the engine of growth of the model there is no effect on long term growth.

With reference to the short run behaviour of the different quantities, in this case initially there is a strong increase in the human capital fraction employed in education (more than 20% with respect to the initial steady state value) and a consequent decrease in the fraction employed in production (where the increase in the profitability of producing efficient capital and of the production of sofwares determines a rise in the human capital fraction employed in the intermediate good sector, at the expenses of the fraction employed in the final good sector). In this situation the growth of output initially slightly decreases and then increases (of about 5% with respect to the initial value), then the situation partially reverses, the human capital fraction in education decreases and the growth rate of output returns to its initial level.

The same kind of dynamics can be obtained considering an increase in $\tau$ (the productivity of the intermediate good sector). In fact, this rise reduces the cost of softwares and increases their production, originating the same type of behaviour observed in the case of an increase in $e$.

The third simulation considers an increase in $\varepsilon$ (the productivity of schooling). In this case, contrary to the previous shocks, there are long run effects on growth, since the accumulation of human capital through education is the true engine of growth of this model. In particular, the rise in the productivity of schooling determines a gradual increase in the human capital fraction employed in education and a consequent decrease in the fraction employed in production (where initially the fraction of human capital in the final good sector increases, at the expenses of the fraction employed in the intermediate good sector). This originates an increase in the growth rate of output (initially very strong, and that then partially reduces), so that the final result is a permanent positive effect on the growth of the economy (more precisely, as a consequence of a 1% increase in the productivity of schooling, the growth rate of output increases from 1.04% to 1.24%).

It is then possible to analyse these shocks in the more general version of the model, in which the productivity of schooling is a function of technological progress (the values chosen for the new parameters are $a = 1.037$, $a' = 1$ and $b = 0.007$, in any case other values of $a$, $a'$ and $b$ can be considered, but the results of the simulations do not change in a substantial way). In this case the long run effects are the same as in the benchmark version, while the short run dynamics are different. In particular, considering an increase in the productivity of the final good sector $z$, in the long run the various growth rates (of output, of physical capital, of human capital) do not change with respect to the initial situation. The short run behaviour, on the other hand, is characterised first of all by a decrease in the human capital fraction employed in education (contrary to an increase of the same fraction in the version of the model with constant productivity of schooling) and a consequent increase in the fraction employed in production (where in particular there is an initial reallocation of human capital favourable to the intermediate good sector, at the expenses of the fraction employed in the final good sector). As a consequence, initially there is a reduction in the growth rate of human capital (because of the decrease in the human capital fraction employed in education), together with an increase in the growth rate of output. After some time, then, the situation reverses, the human capital fraction in education and the consequent growth of human capital increase, while the growth rate of output strongly decreases and then partially increases, until it returns to its initial level.

Considering other possible shocks in the productivities of the different sectors, similarly, we have that in the short run there are some changes with respect to the benchmark version of the model with constant productivity of schooling, but the long run effects are the same (in particular, changes in the productivity of the equipment sector, in the productivity of the intermediate good sector and in the cost of a new variety of software, like in the productivity of the final good sector, do not affect the long run growth rates).

## 4.3 Empirical data and simulations

With reference to the empirical evidence available, it is possible to analyse the most important data concerning the behaviour of the US economy during the ICT revolution, in oder to verify the capability of the model to replicate at least some of the stylized facts that have characterized this period (see OECD 2000, 2001, 2003, 2004).

In particular, the data show that GDP growth in the US has been very strong in the last decades: in fact, it has been equal, on average, to 3.2% per year in 1980-90, to 2.5% per year in 1990-95 and to 4.2% per year in 1995-2000, with a contribution deriving from ICT capital significant and rising.

Another important element that emerges from the data is that in the US the level of GDP per capita in 1990 was already the highest in the world, and nevertheless the growth accelerated strongly in the second half of the decade, due to the fact that US improved labour productivity and labour utilisation at the same time. Labour productivity in particular can be increased in several ways: by improving the quality of labour (human capital) used in the production process (and it is precisely this element that is central in the model proposed in this paper); by increasing the use of capital and improving its quality (investments in physical capital are crucial, especially investments in ICT and also investments in software); by attaining greater overall efficiency in how the factors of production are used together (multi-factor productivity, that reflects many types of efficiency improvements, such as improved managerial practices, organisational changes and innovative ways of producing goods and services).

Taking into account all these elements, it turns out that the third simulation considered (that studies the effects of an increase in the productivity of schooling) is the most realistic, since it shows that the role of human capital is crucial in order to have permanent effects on the growth of the economy. In effect, as outlined above, GDP growth in the US has been remarkable in the last decades and this performance has been due to the improvements both in labour productivity and in labour utilisation. In particular, a fundamental role in labour productivity growth has been played by the improvement in the quality of labour (i.e. human capital) used in the production process, as shown by the rise in educational attainment among workers over the 1990s. For instance, as reported in the Introduction, in the US in 1992 for the 25-64 year-old population 30.2% had attained qualifications at the tertiary level, and in 2005 this percentage increased to 39%, showing a clear and strong improvement in the level of human capital. Another result that confirms the importance of human capital for growth is the one according to which in OECD countries one additional year of schooling would lead, on average, to about 6% higher GDP in the long run (OECD, 2000).

All these data show that human capital is really important for economic growth, that is exactly what emerges from the model presented in this paper, and from the third simulation considered above. The model and the simulations are therefore consistent with the empirical evidence, according to which ICT use does have impacts on firm performance when it is accompanied by other changes and investments, in particular on skills. In effect, many empirical studies (see OECD, 2003) suggest that ICT primarily affects firms where skills have been improved, and therefore one of the drivers of economic performance is represented by the interaction between ICT and human capital, as the third simulation presented above confirms.

# 5 Conclusion

This paper has proposed a model that reproduces the essential characteristics of the recent ICT-based economy using the framework of endogenous growth theory. More precisely, it is a multi-sectoral growth model with embodied technological progress, horizontal differentiation and "lab-equipment" specification of R&D, and with human capital accumulation, in order to take into account the crucial role of the latter when new technologies are present.

In this context, some important results can be obtained, both analytically and through simulations, either in the benchmark case in which the productivity of schooling is constant and in the case in which this productivity is a function of technological progress (in order to specify the complementarity between human capital accumulation and technological progress).

First of all it is possible to show that the productivity of schooling affects the long run growth of the economy, while the productivities of the other sectors don't have these effects (contrary to what happens in the model without human capital), hence in this case human capital accumulation is the true engine of growth. It is then possible to analyse the reaction of the economy, also in the short run, to different types of shocks (in particular shocks on the productivities of the different sectors, and also on the cost of research), and to compare the results with the available empirical evidence. These results show that the model is able to reproduce quite well such evidence, suggesting that one of the drivers of economic performance is represented by the interaction between ICT and human capital.

# 6 Biography

Claudio Mattalia is Assistant Professor (Ricercatore) of Mathematical Methods Applied to Economics at the University of Torino (Italy). He graduated in Economics at the University of Torino (1994), then he earned a Doctorate in Mathematics Applied to Economics at the University of Trieste (2000) and a Master of Arts and a PhD in Economics at the Université Catholique de Louvain (2008). His research interests concern mathematical methods applied to economic theory, with particular reference to optimization and endogenous growth. He teaches several undergraduate and graduate courses of mathematics applied to economics and finance.

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Table 1: Tertiary attainment for age group as a percentage of the population of that age group

| | Age group 25-64 | | Age group 25-34 | |
|---|---|---|---|---|
| | 1992 | 2005 | 1992 | 2005 |
| United States | 30.2% | 39% | 30.2% | 39.2% |
| OECD average | 19% | 26% | 21.9% | 32.2% |

Source: OECD Factbook (2008)

Table 2: Relevant moments of the steady state and values of the parameters, benchmark case

| Moments of the steady state | | Parameter values | |
|---|---|---|---|
| Interest rate | 5% | $\alpha = 0.35$ | $e = 12$ |
| Share of human capit. in interm. and R&D | 5% | $\lambda = 0.85$ | $\tau = 0.25$ |
| Mark-up rate | 3.5 | $\sigma = 1.40$ | $\varepsilon = 0.044$ |
| Capital/output ratio | 2 | $\delta = 0.10$ | $\eta = 20$ |
| R&D expenditure in terms of GDP | 3.5% | $\rho = 0.96$ | |
| Growth rate of output | 1.04% | $z = 3$ | |