



AperTO - Archivio Istituzionale Open Access dell'Università di Torino

WPHACT 2.0: A Fully massive Monte Carlo generator for four fermion physics at e+ e- colliders

This is the author's manuscript					
Original Citation:					
Availability:					
This version is available http://hdl.handle.net/2318/118519 since					
Published version:					
DOI:10.1016/S0010-4655(02)00685-9					
Terms of use:					
Open Access					
Anyone can freely access the full text of works made available as "Open Access". Works made available under a Creative Commons license can be used according to the terms and conditions of said license. Use of all other works requires consent of the right holder (author or publisher) if not exempted from copyright protection by the applicable law.					

(Article begins on next page)



Computer Physics Communications 150 (2003) 166-196

Computer Physics Communications

www.elsevier.com/locate/cpc

Reference to previous version: Comput. Phys. Comm. 99 (1997)

WPHACT 2.0: A fully massive Monte Carlo generator for four fermion physics at e^+e^- colliders 3,33

Elena Accomando^a, Alessandro Ballestrero^{b,c,*}, Ezio Maina^{c,d}

^a Paul Scherrer Institut, CH-5232 Villigen-PSI, Switzerland
 ^b CERN, EP Division, Geneva, Switzerland
 ^c INFN, Sezione di Torino, Italy
 ^d Dipartimento di Fisica Teorica, Università di Torino, Italy

Received 29 May 2002; received in revised form 1 July 2002

Abstract

WPHACT 2.0 is the new fully massive version of a MC program and unweighted event generator which computes all Standard Model processes with four fermions in the final state at e^+e^- colliders. The program can now generate unweighted events for any subset of all four fermion final states in a single run, by making use of dedicated pre-samples which can cover the entire phase space. Improvements with respect to WPHACT 1.0 include the Imaginary Fermion Loop gauge restoring scheme, new phase space mappings, a new input system, the possibility to compute subsets of Feynman diagrams and options for including ISR via QEDPS, running α_{QED} , CKM mixing, resonances in $q\bar{q}$ channels. © 2002 Elsevier Science B.V. All rights reserved.

PROGRAM SUMMARY

Title of program: WPHACT, version 2.0	Authors of original program: E. Accomando and A. Ballestrero
Catalogue identifier: ADQW	Catalogue identifier of previous version: ADEN
Program Summary URL: http://cpc.cs.qub.ac.uk/summaries/ADQW	Computer: Any computer with FORTRAN77 compiler which ad- mits structures. WPHACT has been tested on Compaq ALPHA and HP stations as well as on Linux Intel PC. For this last type of ma-
Program obtainable from: CPC Program Library, Queen's Univer- sity of Belfast, N. Ireland; also available in http://www.to.infn.it/ ~ballestr/	chines the Portland Group pgf77 compiler has been employed, while the GNU compiler cannot be used as it does not have the STRUC- TURE extension. Compaq and Portland Group FORTRAN90 com- pilers can also be used

270

0010-4655/02/\$ – see front matter © 2002 Elsevier Science B.V. All rights reserved. PII: S0010-4655(02)00685-9

^{*} Work supported by the European Union under contract HPRN-CT-2000-00149, by Ministero dell' Università e della Ricerca Scientifica and by the Swiss Bundesamt für Bildung und Wissenschaft.

^{**} This program can be downloaded from the CPC Program Library under catalogue identifier: http://cpc.cs.qub.ac.uk/summaries/ADQW * Corresponding author.

E-mail addresses: elena.accomando@psi.ch (E. Accomando), ballestrero@to.infn.it (A. Ballestrero), maina@to.infn.it (E. Maina).

Operating systems: UNIX, LINUX, VMS

Programming language used: FORTRAN 77

Memory required to execute with typical data: ≈ 500 KByte

No. of bits in a word: 32

No. of processors used: one

No. of bytes in distributed program: 748754

Distribution format: tar gzip file (≈ 700 Kb including a version of PYTHIA)

Additional keywords: Massive matrix elements, gauge invariance, fermion loop, single W, single Z, running α_{QED} , CKM mixing, resonances

Nature of physical problem

All e^+e^- Standard Model (SM) processes with four fermion final state are necessary for detailed studies of the properties of the W and of the Higgs bosons, for measuring double and single boson production, for high precision tests of the SM and for evaluating backgrounds to searches. One wants therefore a Monte Carlo event generator which provides an accurate description of all four fermion processes in the largest possible region of phase space and which can be used for realistic experimental simulations.

Method of solution

Full tree level matrix elements for all processes are computed by means of subroutines which make use of the helicity formalism of Refs. [24,25]. The speed in computing these amplitudes, that the above mentioned method allows, is essential to take exactly into account fermion masses and to obtain high precision in a reasonable

amount of CPU time also in regions with large and possibly overlapping enhancements.

Reasons for the new version

Extend the code to cover in a reliable way all *t*-channel dominated processes and low $f\bar{f}$ invariant mass regions. Introduce the possibility to simultaneously generate unweighted events for all processes or any user-selected subset. Include photon p_t and CKM mixing.

Summary of revisions

Fully massive matrix elements, new unweighted event generation for any subset of four fermion final states in a single run, implementation of the Imaginary Fermion Loop gauge restoring scheme, new phase space mappings, new input system, possibility to compute subsets of Feynman diagrams and options for including ISR via QEDPS, running α_{QED} , CKM mixing, resonances in $q\bar{q}$ channels.

Restrictions on the complexity of the problem

QED radiative corrections are implemented only in Initial State Radiation and in Coulomb corrections. Final state radiation is not computed when unweighted events are not requested. Only the imaginary part of Fermion Loop corrections is implemented. QCD corrections are introduced in an approximate way.

Typical running time

The running time strongly depends on the process considered and on the precision requested. The example reported in the test run took about 90 minutes on a 500 MHz ALPHA computer, which corresponds to effectively generating more than 1500 weighted $e^-e^+\mu^-\mu^+$ events per second.

Unusual features of the program STRUCTURE declarations are used.

LONG WRITE-UP

1. Introduction

Four fermion final states in e^+e^- collisions, which involve electroweak boson pair production, are of special interest since they allow the mechanism of spontaneous symmetry breaking and the non Abelian structure of the Standard Model (SM) to be directly tested by the experiments. Moreover they provide a very important background to most searches for new physics.

LEP2 has provided in this respect an ideal testing ground for the SM. After the end of the period of data taking, the Collaborations are working towards the final results and combinations. W properties have been measured with great accuracy. New bounds on anomalous trilinear gauge boson couplings and new limits on the Higgs mass and Susy particles have been set. Single W, single Z, ZZ and $Z\gamma^*$ cross sections have been determined for the first time. Hopefully these studies will be continued with much higher statistics and energy at a future e^+e^- Linear Collider.

In parallel and in strict collaboration with the experimental activity, a considerable theoretical effort has been devoted to the analysis of four fermion physics and to the production and improvement of dedicated Monte Carlos. Much of this work is documented in the two yellow reports of the LEP2 CERN Workshops [1,2].

A first version of WPHACT, a Monte Carlo program and unweighted event generator for all processes $e^+e^- \rightarrow 4f$ was published [3] at the beginning of LEP2 activity. It was mainly aimed at describing W pair production and Higgs physics, but it allowed to compute any four fermion final state. WPHACT 1.0 was extensively compared with several other codes [4] during the first LEP2 workshop [5,6], and demonstrated its reliability. All these early codes, with the exception of grc4f [7], were based on massless matrix elements. WPHACT 1.0 took exactly into account only the b mass. The reason for such a choice was essentially the huge amount of CPU time needed for massive calculations and the very good results given by the massless approximation to all WW, ZZ and Higgs physics. As a consequence, however, the codes were inadequate to describe processes dominated by *t*-channel exchanges like single W production $(e^+e^- \rightarrow e^-\bar{\nu}_e f \bar{f}')$ at small electron scattering angle), single Z and $\gamma\gamma$ like configurations $(e^+e^- \rightarrow e^-e^+f \bar{f})$ in which one or both electrons are undetected).

After a few years of data taking it has become evident the necessity of having an accurate description of such regions, and in particular of single W production which is relevant for triple gauge boson coupling measurements and as a background to New Physics searches. Small angle electron scattering is singular in the massless limit and this makes it extremely sensitive to violations of U(1) gauge invariance [8] like those produced by a naive introduction of the decay width in W propagators. It is therefore mandatory to introduce fermion masses and to adopt a consistent gauge restoring scheme. For such a reason a new fully massive version of WPHACT was created and the Fermion Loop method [9] of restoring gauge invariance was extended to the massive case [10,11].

Many fully massive 4f MC programs are now available. WPHACT, grc4f [7], KORALW [12] and COMPHEP [13] are based on Feynman diagrams¹ while NEXTCALIBUR [15] and SWAP [16] are based on the Dyson–Schwinger equations, a set of recursive relations among Green functions. Comparisons between the different codes have shown good technical agreement in several benchmark processes, as recently summarized in [17].

Other advances of great significance documented in [17] regard higher order corrections to four fermion WW processes in the so-called double pole approximation (DPA) [18–20]. This feature, which is not implemented in WPHACT, has led to a major improvement in the theoretical uncertainty for WW cross sections and distributions, which has decreased from the previous estimate of 2%, which was comparable with the experimental errors, to about 0.5%.

In the present paper we present a full description and documentation of the new version of WPHACT in which many new features and improvements have been added.

In order to meet some of the additional requirements put forward by the ever increasing accuracy of experimental measurements at present and future accelerators we have included:

- fully massive matrix elements and phase spaces for all 4 f final states,
- new phase space mappings for low mass and small scattering angle regions,
- Fermion Loop corrections in the IFL scheme [10],
- explicit generation of ISR photons with p_{\perp} effects via an interface to QEDPS [21] which can be used as an alternative to the previous Structure Function (SF) approach,
- possibility to use running α_{QED} for *t*-channel or low invariant masses,
- support of CKM mixing,
- resonances in $q\bar{q}$ channels and new low mass hadronization with an interface to the routines of Ref. [22],
- possibility to compute relevant subsets of Feynman diagrams,
- optional beamstrahlung effects for future Linear Colliders generated with the help of circe routines [23].

¹ grc4f and KORALW adopt the same matrix elements based on GRACE [14].

Since generation of large samples of fully simulated events is best performed if all processes or specific classes of them can be generated simultaneously, we have introduced the possibility of

• unweighted events generation for any subset of massive 4f final states in a single run.

We have moreover implemented new routines which provide

• a simplified and more user friendly procedure for specifying the program input.

These new features and the full set of available input settings is described in the following. One example of test run is given at the end.

2. New and improved features

We have retained all the best features of WPHACT 1.0. One of the main strengths of the code is the use of the helicity amplitude method described in Ref. [24]. The code for WPHACT amplitudes has been completely written with the help of PHACT [25] (Program for Helicity Amplitudes Calculations with Tau matrices). With this formalism it is possible to evaluate tree-level matrix elements in a very fast and efficient way by means of a modular scheme which stores for later use subdiagrams of increasing size and complexity. Moreover the massive case is a rather straightforward extension of the massless one. It is in fact based on the same modular and diagrammatic approach and all helicity combinations are computed simultaneously. As a consequence, the code with massive amplitudes written in this way is only about five times slower than the previous massless one. This is to be compared with the fact that the number of helicity states increases by a factor of eight and new terms in the diagram evaluation appear. Moreover, since 1996 the speed of available computers has increased by a factor of about thirty and massive calculations are presently faster than massless ones in the old days. In any case, since massless amplitudes provide an excellent approximation to the full result in all cases which do not exhibit collinear or mass singularities, we have maintained the option to use the faster massless amplitudes. The fast evaluation of matrix elements, both massive and massless, combined with the use of the adaptive integration routine VEGAS [26], is particularly useful when producing distributions at parton level with very high precision in each bin, or when generating complete and statistically significant samples of unweighted events. As in WPHACT 1.0, any distribution at parton level and any event sample can be produced while evaluating the cross section.

2.1. Processes

WPHACT computes all SM processes with four fermions in the final state at e^+e^- colliders. Final states with *t* quarks are not considered, as the *t*'s are known to decay immediately to 3 other fermions.

The list of available processes is given in Tables 1–3. All possible channels are divided in four classes. The first (CC) and the third (NC) one contain all so-called charged and neutral current processes, respectively. The second (MIX) is the class in which both neutral and charged current contributions are present. These three classes exhaust the four fermion processes. The corresponding matrix elements can be computed with all fermion masses taken into account or in the massless approximation. Contributions from Higgs diagrams are not available in these first three classes. The last class in Table 3 (NC+Higgs) contains again all neutral current processes with *b*'s, *c*'s and τ 's. These are specific for Higgs studies: requesting these last processes (flag iproc ranging from 33 to 53) one can compute the Higgs signal or the background or the complete Higgs+background+interference. In this case the first seven ones (33 to 39 included) are intended for Higgs decaying to $b\bar{b}$, the second group of seven for $H \rightarrow \tau^{+}\tau^{-}$. The corresponding matrix elements are evaluated with massive *b*'s, *c*'s and τ 's, respectively, with all other particles taken as massless both in the matrix elements and phase space.

Table 1

CC and MIX processes. They can be computed with massive particles (imass=1) or in the massless approximation (imass=0). The processes indicated by ** correspond only to the CC contributions induced by CKM mixing. Their normal NC contributions have to be evaluated with proc=24, ich=1,2,4, respectively

CC						
Process type	iproc	ich	Final state	ich	Final state	
CC9	1	1	$\mu^- \bar{\nu}_\mu \nu_\tau \tau^+$	2	$\mu^+ \nu_\mu \ \bar{\nu}_\tau \ \tau^-$	
CC18	2	1	$e^- \bar{\nu}_e \nu_\mu \mu^+$	3	$e^+ v_e \bar{v}_\mu \mu^-$	
		2	$e^- \bar{\nu}_e \nu_\tau \tau^+$	4	$e^+ v_e \bar{v}_\tau \tau^-$	
CC10	3	1	$\mu^- \bar{v}_\mu \ u \ \bar{d}$	11	$\mu^- \bar{v}_\mu c \bar{b}$	
		2	$\mu^- \bar{v}_{\mu} c \bar{s}$	12	$\mu^+ v_{\mu} \bar{u} s$	
		3	$\mu^+ v_\mu \bar{u} d$	13	$\mu^+ v_\mu \bar{c} d$	
		4	$\mu^+ \nu_\mu \bar{c} s$	14	$\mu^+ v_\mu \bar{c} b$	
		5	$\tau^- \bar{\nu}_{\tau} u \bar{d}$	15	$\tau^- \bar{\nu}_{\tau} u \bar{s}$	
		6	$\tau^- \bar{\nu}_{\tau} c \bar{s}$	16	$\tau^- \bar{\nu}_{\tau} c d$	
		7	$\tau^+ v_{\tau} \bar{u} d$	17	$\tau^- \bar{\nu}_{\tau} c b$	
		8	$\tau^+ v_{\tau} \bar{c} s$	18	$\tau^+ v_{\tau} \bar{u} s$	
		9	$\mu^- \bar{\nu}_\mu u \bar{s}$	19	$\tau^+ v_{\tau} \bar{c} d$	
		10	$\mu^- \bar{v}_\mu \ c \ d$	20	$\tau^+ v_{\tau} \bar{c} b$	
CC20	4	1	$e^- \bar{v}_e u \bar{d}$	6	$e^- \bar{v}_e c \bar{d}$	
		2	$e^- \bar{v}_e c \bar{s}$	7	$e^- \bar{v}_e c \bar{b}$	
		3	$e^+ v_e \bar{u} d$	8	$e^+ v_e \bar{u} s$	
		4	$e^+ v_e \bar{c} s$	9	$e^+ v_e \bar{c} d$	
		5	$e^- \bar{v}_e u \bar{s}$	10	$e^+ v_e \bar{c} b$	
CC11	5	1	s ē u d	13	$d \ \overline{c} \ c \ \overline{s}$	
		2	$\bar{s} c \bar{u} d$	14	$b \bar{c} c \bar{s}$	
		3	s c u s	15	$d \bar{c} u \bar{s}$	
		4	$d \bar{c} u d$	16	$b \bar{c} u \bar{s}$	
		5	bāud	17	$d c \bar{u} s$	
		6	$\bar{s} c \bar{u} s$	18	$b c \overline{u} s$	
		7	$d c \bar{u} d$	19	$d \bar{c} c b$	
		8	$\bar{b} c \bar{u} d$	20	$b \ \bar{c} \ c \ d$	
		9	$d \bar{u} u \bar{s}$	21	s ū u š_**	
		10	s ū u d	22	$d \bar{c} c \bar{d} **$	
		11	$s \bar{c} c \bar{d}$	23	bācb **	
		12	s ē c b			
MIX						
Process type	iproc	ich	Final state	ich	Final state	
MIX19	6	1	$\mu^- \mu^+ \nu_\mu \bar{\nu}_\mu$	2	$\tau^- \tau^+ \nu_\tau \ \bar{\nu}_\tau$	
MIX56	7	1	$e^-e^+ v_e \bar{v}_e$			
MIX43	8	1	$d \bar{d} u \bar{u}$	2	$s \bar{s} c \bar{c}$	

With respect to the previous version of WPHACT, new channels have been added. The NC table has been extended by including processes with *b*'s in the final state, while the NC+Higgs table now contains also channels in which the Higgs boson decays into *c*'s and τ 's.

We have introduced the possibility to take into account CKM mixing, which provides a more accurate description of the flavor content of hadronic final states. This might be relevant in evaluating backgrounds to Higgs and new particle searches. In the CC class we have included all additional channels which are induced by $d \leftrightarrow s$ and $s \leftrightarrow b$ mixing. The processes which are induced by $d \leftrightarrow b$, which are much more strongly suppressed, have been neglected, as the multiperipheral type contributions to NC processes $e^+e^- \rightarrow v_e \bar{v}_e q\bar{q}$, in which two W's

			NC		
Process type	iproc	ich	Final state	ich	Final state
NC6	9	1	$v_{\mu} \bar{v}_{\mu} v_{\tau} \bar{v}_{\tau}$		
NC12	10	1	$v_{\mu} \bar{v}_{\mu} v_{e} \bar{v}_{e}$	2	$v_{\tau} \ \bar{v}_{\tau} \ v_{e} \ \bar{v}_{e}$
NC12	11	1	$v_{\mu} \bar{v}_{\mu} v_{\mu} \bar{v}_{\mu}$	2	$\nu_{\tau} \ \bar{\nu}_{\tau} \ \nu_{\tau} \ \bar{\nu}_{\tau}$
NC36	12	1	ve ve ve ve		
NC10	13	1	$u \bar{u} v_{\mu} \bar{v}_{\mu}$	3	$c \bar{c} v_{\mu} \bar{v}_{\mu}$
		2	$u \bar{u} v_{\tau} \bar{v}_{\tau}$	4	$c \bar{c} v_{\tau} \bar{v}_{\tau}$
NC19	14	1	$u \bar{u} v_e \bar{v}_e$	2	$c \ \bar{c} \ v_e \ \bar{v}_e$
NC64	15	1	น นิ น นิ	2	$c \ \bar{c} \ c \ \bar{c}$
NC32	16	1	иūcē		
NC10	17	1	$\mu^- \mu^+ v_\tau \bar{v}_\tau$	2	$\tau^- \tau^+ \nu_\mu \bar{\nu}_\mu$
NC20	18	1	$e^- e^+ v_\mu \bar{v}_\mu$	2	$e^- e^+ v_\tau \bar{v}_\tau$
NC19	19	1	$\mu^- \mu^+ v_e \bar{v}_e$	2	$\tau^- \tau^+ v_e \bar{v}_e$
NC24	20	1	$\mu^- \mu^+ u \bar{u}$	3	$\tau^- \tau^+ u \bar{u}$
		2	$\mu^- \mu^+ c \bar{c}$	4	$\tau^- \tau^+ c \bar{c}$
NC48	21	1	$e^- e^+ u \bar{u}$	2	$e^- e^+ c \bar{c}$
NC19	22	1	$d \bar{d} v_e \bar{v}_e$	3	$b \ \bar{b} \ v_e \ \bar{v}_e$
		2	s s ve ve		
NC10	23	1	$d \bar{d} v_{\mu} \bar{v}_{\mu}$	4	$d \bar{d} v_{\tau} \bar{v}_{\tau}$
		2	$s \bar{s} v_{\mu} \bar{v}_{\mu}$	5	$s \bar{s} v_{\tau} \bar{v}_{\tau}$
		3	$b \bar{b} v_{\mu} \bar{v}_{\mu}$	6	$b \bar{b} v_{\tau} \bar{v}_{\tau}$
NC32	24	1	s ร น น	3	b b̄ u ū
		2	$d \bar{d} c \bar{c}$	4	b
NC24	25	1	$\mu^- \mu^+ \tau^- \tau^+$		
NC48	26	1	$e^- e^+ \mu^- \mu^+$	2	$e^- e^+ \tau^- \tau^+$
NC48	27	1	$\mu^{-} \mu^{+} \mu^{-} \mu^{+}$	2	$\tau^- \tau^+ \tau^- \tau^+$
NC144	28	1	$e^{-} e^{+} e^{-} e^{+}$		
NC64	29	1	$d \ \overline{d} \ d \ \overline{d}$	3	b
		2	s s s s		
NC48	30	1	$e^- e^+ d \bar{d}$	3	$e^- e^+ b \bar{b}$
		2	$e^- e^+ s \bar{s}$		
NC24	31	1	$\mu^- \mu^+ d \bar{d}$	4	$\tau^- \tau^+ d \bar{d}$
		2	$\mu^- \mu^+ s \bar{s}$	5	$\tau^- \tau^+ s \bar{s}$
		3	$\mu^- \mu^+ b \bar{b}$	6	$\tau^- \tau^+ b \bar{b}$
NC32	32	1	$d \bar{d} s \bar{s}$	3	$s \bar{s} b \bar{b}$
		2	$d \ \bar{d} \ b \ \bar{b}$		

NC processes. They can be computed with massive particles (imass=1) or in the massless approximation (imass=0)

attach to a single fermion line and one sums on the flavor of the virtual quark. In the NC class, the contributions due to CKM mixing are doubly suppressed. Nonetheless, we have included the CC exchange contributions to the three processes

 $e^+e^- \rightarrow s \bar{u} u \bar{s}, \ d \bar{c} c \bar{d}, \ b \bar{c} c \bar{b}$

Table 2

which can be produced when CKM mixing is turned on.

In WPHACT the momenta of the final state partons are called p3, p4, p5, p6. They are assigned in the order in which the particles appear in Tables 1–3. Therefore, for instance, the first process in Table 1 reads:

$$e^+(p_1)e^-(p_2) \to \mu^-(p_3)\bar{\nu}_{\mu}(p_4)\nu_{\tau}(p_5)\tau^+(p_6).$$

The order of the particles is important when one wants to impose cuts or to compute distributions at parton level.

NC+HIGGS					
Process type	iproc	ich	Final state	ich	Final state
NC21	33	1	$b \ \bar{b} \ v_e \ \bar{v}_e$		
NC11	34	1	$b \ \bar{b} \ v_\mu \ \bar{v}_\mu$	2	$b \bar{b} v_{\tau} \bar{v}_{\tau}$
NC33	35	1	b b̄ u ū	2	b
NC25	36	1	$b \bar{b} \mu^- \mu^+$	2	$b \bar{b} \tau^- \tau^+$
NC50	37	1	$b \bar{b} e^- e^+$		
NC33	38	1	$b \ \bar{b} \ d \ \bar{d}$	2	$b \bar{b} s \bar{s}$
NC84	39	1	b		
NC21	40	1	$c \bar{c} v_e \bar{v}_e$		
NC11	41	1	$c \bar{c} v_{\mu} \bar{v}_{\mu}$	2	$c \bar{c} v_{\tau} \bar{v}_{\tau}$
NC33	42	1	c		
NC84	43	1	c		
NC25	44	1	$c \bar{c} \mu^- \mu^+$	2	$c \bar{c} \tau^- \tau^+$
NC50	45	1	$c \bar{c} e^- e^+$		
NC33	46	1	$c \ \bar{c} \ d \ \bar{d}$	3	$c \bar{c} b \bar{b}$
NC44		2	$c \bar{c} s \bar{s}$		
NC21	47	1	$\tau^- \tau^+ \nu_e \bar{\nu}_e$		
NC11/NC20	48	1	$\tau^- \tau^+ \nu_\mu \bar{\nu}_\mu$	2	$\tau^- \tau^+ \nu_{\tau} \bar{\nu}_{\tau}$
NC25	49	1	$\tau^- \tau^+ u \bar{u}$	2	$\tau^- \tau^+ c \bar{c}$
NC25	50	1	$\tau^- \tau^+ \mu^- \mu^+$		
NC68	51	1	$\tau^- \tau^+ \tau^- \tau^+$		
NC50	52	1	$\tau^- \tau^+ e^- e^+$		
NC25	53	1	$\tau^- \tau^+ d \bar{d}$	3	$\tau^- \tau^+ b \bar{b}$
		2	$\tau^- \tau^+ s \bar{s}$		

Table 3

NC processes which take into account Higgs contributions. They require imass=0 and have by default massive *b*'s in processes 33–39, massive *c*'s in processes 40–46 and massive τ 's in processes 47–53. All other external particles are massless

It has to be noticed however that for CC and Mixed processes the order of the particles in WPHACT is different from the one used in PYTHIA [27]. We conform to the convention of PYTHIA when in WPHACT the momenta are passed to the high energy standard COMMON/HEPEVT [28]. Moreover, when PYTHIA is called, all 3-momenta are reversed in order to conform to the convention for which e^- is in the positive z direction (WPHACT default is that the incoming e^+ is in the +z direction).

Many different final states give the same cross section at parton level up to mass effects. This is the case for instance of $\mu^- \mu^+ d \bar{d}$ and $\tau^- \tau^+ s \bar{s}$ if mass differences are neglected. For this reason each final state is described by a combination of two flags, iproc and ich, which unambiguously identify it. The first flag refers to a group of processes which are described by the same set of diagrams, the second to the specific final state in the group. Processes which differ by a final state CKM rotation are also distinguished by ich. It has to be noticed that for CC processes charge conjugate final states belong to the same iproc. The amplitudes for a CC final state and its charge conjugate, for a given set of four momenta, are not identical. They are related one another by a final state parity transformation. This implies that if cuts are not invariant under *P*, the two cross sections will be different. In WPHACT they are considered separately and are identified by different values of ich. When identical particles appear in the final state the cuts must obviously be symmetrical with respect to their exchange. This is not checked for on input and unphysical non-symmetric cuts can produce unreliable results.

Since our approach is based on Feynman diagrams, it is possible to compute subsets of the diagrams for a given amplitude. While this procedure is not in general gauge-invariant and should as a consequence be used with extreme care, it is nonetheless extremely useful in practice in order to compare and combine the results of several experiments which adopt different selection strategies. In effect, when appropriate cuts evidentiate quasi on shell contributions of intermediate vector bosons, the subset of diagrams which would correspond to

the production times decay approximation gives by far the dominant contribution to the cross section. The most famous example is the so-called CC03 contribution to WW production, but one can of course consider also ZZ or $Z\gamma *$. *t*-channel dominated processes like single Z or single W also belong to this category, as one may consider that the photon in *t*-channel is almost on shell (Weiszacker–Williams approximation). Signal definition in terms of a subset of diagrams is also useful when one can take advantage of a specialized program which describes phenomenologically a set of processes, for instance gamma–gamma physics, which are known to have important non-perturbative contributions. In other cases it has been successfully employed in order to take into account higher order corrections which were available only for the dominant part of an amplitude. In Sections 3.1.2 and 3.1.11 we describe the subsets of diagrams which can be selected in WPHACT 2.0.

All neutral current processes are normally computed at order α^4 . When there are four quarks in the final state, there are however contributions $O(\alpha^2 \alpha_s^2)$ from diagrams with gluon exchange. These contributions are enhanced by the ratio of the strong to the electroweak coupling and can be relevant for some processes, energies and cuts. However, these contributions are already, at least partially, accounted for by two quark final states when parton showering and hadronization are performed. In order to avoid a problematic double counting, in WPHACT 2.0 gluon exchange diagrams have been switched off by default. The corresponding flag is iqu which is set to zero in a DATA statement. Setting iqu=1 enables the computation of these QCD diagrams.

2.2. Phase space and integration

Most processes which have been studied in the first part of LEP2 activity are dominated by the production of boson pairs (i.e. WW, ZZ, Zh or hA). In this case, the four final fermions can be conveniently divided into two sets corresponding to possible decays of a W in CC processes, or to possible decays of a Z or Higgs boson in NC contributions. The cross section is strongly enhanced when the two fermion-pair invariant masses M_1 and M_2 are close to the intermediate boson masses. Appropriate phase space mappings were already available in WPHACT 1.0.

In the new version of WPHACT new mappings dedicated to cover both low invariant mass and small scattering angle regions have been implemented.

In NC processes an additional peak for low invariant masses, due to the photon *s*-channel propagator, may be present. In order to treat all possible kinematical configurations of this kind, a convenient double mapping which accounts for both Z and γ^* peaks on the same variable M_i (*i* = 1 or 2) is performed.

When both the M_1 and M_2 invariant mass peaks are mapped into a uniform distribution we refer to the corresponding phase space as double resonant. If only one or no variable transformation is performed, we have respectively a single resonant or a non resonant mapping.

In WPHACT there is the possibility to choose among these mappings as explained in Section 3.1.3. For Mixed (or NC+Higgs) processes, the peaking structures of CC (or Higgs) and NC contributions are different. WPHACT by default integrates separately the two contributions, adding the interference to one of them. The program uses two different phase spaces for the two different parts of the calculation. When generating events, the interference is treated in such a way to avoid negative weights.

New mappings have also been introduced to deal with forward scattering. The processes in which one or two electrons (or positrons) are emitted at very small angle with respect to the beam present collinear enhancements which are logarithmic in the ratio of the electron mass to the total collider energy. They are therefore extremely sensitive to numerical cancellations which, if not properly treated, can lead to strong numerical instabilities. Our approach to such integration is based on the method of Ref. [29] and can be switched on by the flag ismallangle as it will be described in Section 3.1.3. In this case the user can choose among single or non resonant mappings for the invariant mass of the fermion pair which contains no forward electrons (or positrons). Even with the new tools, if cuts are such that one gets important contributions from extreme regions, it is safer to run WPHACT in quadruple precision (with obvious modifications to the code), or to test that real*8 result agrees with real*16. An example of these possible dangerous contributions is given, for processes with outgoing electrons at LEP2,

by the region in which $|q_{\gamma}^2|$, the absolute value of the squared momentum of the exchanged *t*-channel photon, becomes much lower than 10^{-14} . In such cases, also the calculation in double precision of $|q_{\gamma}^2|$ from the external four-momenta becomes unreliable.

All integrals are computed with VEGAS. With this routine it is preferable to use more than one iteration since between iterations VEGAS adapts its phase space grid to the integrand at hand in an effort to decrease the overall variance. In each iteration the integral is evaluated and in the end the various results are combined together. This allows to optimize the number of points evaluated in each region of the integration variables. It may also be useful to perform some thermalizing iterations with a smaller number of points. The thermalizing iterations are used only to adjust the grid and not for the final result.

The adaptivity of VEGAS is such that, even if most cuts are implemented in the program with the use of i f statements which act as θ functions, usually this does not correspond to a substantial loss in time and precision. The cuts on the invariant masses which are function of integration variables are implemented directly as integration limits.

2.3. Radiative effects

In view of the experimental accuracy reached at LEP2, which will be further increased at future Linear Colliders, the inclusion of QED higher order radiative corrections is an unavoidable step for accurate theoretical predictions of cross sections and distributions.

The exact calculation of $O(\alpha)$ electroweak corrections for four fermion processes is very difficult and at present available only for the WW signal in Double Pole Approximation (DPA) [18] and implemented exclusively in RACOONWW [19] and YFSWW3 [12,20].

In the rest of this section we will describe how important radiative effects as Initial State Radiation (ISR), Final State Radiation (FSR), Coulomb Corrections and Fermion Loops are accounted for in WPHACT 2.0. We will also describe the implementation of running α_{QED} which can give a rather good estimate of higher order effects in *t*-channel or low $f \bar{f}$ mass regions.

2.3.1. Initial state radiation

Initial state radiation is known to give large corrections. The Structure Function (SF) formalism, the Parton Shower (PS), and the Yennie, Frautschi and Suura algorithm (YFS), are the three most widely used methods to describe ISR in e^+e^- annihilation processes. The last two approaches take into account the transverse momentum (p_{\perp}) of the emitted photons. The SF formalism usually generates collinear radiation, but can include p_{\perp} effects via p_{\perp} -dependent SF.

In the previous version of the program, the relevant Leading Log (LL) QED radiative corrections due to ISR were implemented via the Structure Function method according to the soft and collinear approximation of Ref. [30] (p_{\perp} -independent).

In the new version, we have extended the treatment of the QED radiation by incorporating the Monte Carlo program QEDPS [21], based on the parton shower algorithm in QED, which solves the DGLAP equations in the LL approximation and generates photons p_{\perp} . The transverse momentum of emitted photons, neglected in the collinear limit, can have sizable effects on the cross section and might modify the shape of the distributions. This is an important effect to take into account. However one must be somewhat careful because, while in the collinear and massless limit the electrons, after radiating, remain on mass shell, this is no more true when the emitted photons have p_{\perp} and when electron masses are accounted for. In such cases the commonly used approximation of forcing electrons to be on mass shell after ISR emission may not be always reliable.

The user can select between the SF and PS options via the input flag isr (see Section 3.1.4).

An important source of theoretical uncertainty related to radiative effects is the choice of the correct energy scale in forward scatterings, like single W, single Z and $\gamma \gamma$ processes. Commonly applied to annihilation processes, ISR turns out to require a more careful treatment for *t*-channel dominated scatterings, especially when including the photon p_{\perp} . The choice of the scale is determined by physics considerations and, when possible, by comparing with higher order results. The *natural* scale used for *s*-channel processes, that is the total energy squared *s*, might not give a good description of the radiative emission, when applied to multi-scale processes dominated by the low transferred momentum $|q_{\nu}^2|$ of the exchanged *t*-channel photon.

For small angle Bhabha scattering and multi-peripheral $\gamma \gamma$ processes, where exact QED radiative corrections are already known, the full prediction and the result obtained by using SF at the scale $|q_{\gamma}^2|$ are in good agreement [31]. In all other cases, where no exact calculation is available, it has been recently shown that a comparison with the soft limit of the O(α) corrections can determine the scales to be used in SF and PS [32,33]. Detailed analysis, concerning the sensitivity of *t*-channel scatterings to the choice of the energy scale in the presence of ISR, have been performed [15,32,33], leading to a better understanding of the behaviour of the radiation for single W and $\gamma \gamma$ processes. The general finding is that the choice of the scale describing the SF evolution can produce differences up to 8%, and there is a widespread consensus on the fact that the *s*-scale is not the most appropriate.

When non strictly collinear ISR is introduced by any of the methods, PS, YFS or p_{\perp} -dependent SF, the situation is even more delicate. In these approaches in fact, after radiative emission, the off-shell electrons are projected on mass-shell for computing the amplitude, as already mentioned. This approximation is not valid for high p_{\perp} ISR photons and low momentum transfer, $|q_{\gamma}^2| \simeq 0$, as can be easily realized by comparing with the exact kinematic of the virtual *t*-channel photon. Therefore, slightly different p_{\perp} spectra can lead to rather different and sometimes inconsistent results in the extreme forward regions. However, the use of $|q_{\gamma}^2|$ or similar scale implies a high suppression of the radiation in such regions. As a consequence, when taking into account the p_{\perp} of ISR photons, one can avoid the above mentioned problem because the low momentum transfer events are almost never accompanied by high radiation and therefore by high p_{\perp} . In this sense, choosing an ISR radiation scale different from *s* is unavoidable, and not just a possible option as for the SF collinear case.

A more consistent theoretical study of the problem of radiation in multi-scale processes has been recently performed in Ref. [34], but it has not yet been implemented in a MC code.

In WPHACT 2.0, ISR is treated as follows. When ISR is included via SF (isr=1), the energy scale relevant to the radiative emission is fixed to be $q^2 = s$, independently of the process. When ISR is implemented via PS (isr=2), WPHACT follows instead a strategy in part similar to that of Ref. [33]. If the final state does not contain any electrons or positrons, the energy scale is fixed to be $q^2 = s$. For processes involving at least one electron (or positron), the program generates, as a first step, the complete kinematics in the absence of photon emission. The transferred momentum $|q_{\gamma}^2|$ of the photon exchanged in the *t*-channel is then computed. If $|q_{\gamma}^2| \leq 10m_e^2$ (m_e is the electron mass), the program evaluates the differential cross section without ISR. In all other cases, WPHACT 2.0 goes back to the starting point, switches on photon emission via QEDPS, recomputes the kinematics and evaluates the differential cross section. If $10m_e^2 \leq |q_{\gamma}^2| \leq s/100$, the scale $q^2 = |q_{\gamma}^2|$ is used, otherwise $q^2 = s$.

In Fig. 1 some results regarding the so-called single Z processes are reported. In the upper part the cross section for $e^-e^+\mu^-\mu^+$ is given as a function of the energy. In the lower part we present the cross section for the sum of all processes $e^-e^+q\bar{q}$ (q = u, d, c, s, b). The following cuts have been applied: $m(f\bar{f}) > 60$ GeV ($f \neq e$), e^- within 12 degrees from its forward direction, e^+ between 60 and 168 degrees (WPHACT convention is incoming e^+ at 0°). In the left hand side plots we compare different treatments of ISR. From them one can see the numerical relevance of QEDPS as implemented in WPHACT with respect to SF with *s*-scale. The QEDPS curve falls in between the SF one and the curve in which no ISR effect is considered. For these set of cuts the difference between SF (*s*-scale) and QEDPS (*t*-scale) is of the order of 4%. All three curves have been computed using running α_{em} coupling in photon vertices (see Section 2.3.5).

2.3.2. Final state radiation

Radiative corrections and photon emission in the final state are not directly computed in WPHACT, unless unweighted events are generated. In this case the variable irad which is in a DATA statement of the routine wph.f, determines how FSR is computed. If irad=0, no FSR photons are emitted. If irad=1 and PYTHIA is called (ijetset=1), FSR radiation is performed via PYTHIA itself. Having final state radiation performed by an hadronization package can be preferable in case of quarks in the final state, where interplay between strong and



Single Z in ee $\mu\mu$

Fig. 1. Cross sections for $e^-e^+\mu^-\mu^+$ (upper) and for the sum of $e^-e^+q\bar{q}$ (q = u, d, c, s, b). Cuts for single Z region are defined in the text. Left-hand plots compare different ISR implementations, right-hand ones show the effect of running α_{QED} .

electromagnetic corrections takes place. In the case of emission from leptons, one can instead prefer to have FSR from the PHOTOS [35] dedicated package. For purely leptonic processes, this is achieved by setting irad=2 and ijetset=1. For semileptonic channels, the same set of flags produces PHOTOS FSR for the leptonic pair and PYTHIA FSR for the quark pair.

2.3.3. Coulomb correction

The Coulomb singularity is relevant only for CC and Mixed processes near the W-pair threshold. However, in that kinematical region it gives a sizable correction of a few percent. In WPHACT the Coulomb correction can now be computed with four different methods. The input parameter icoul (see Section 3.1.5) allows one to choose among the different implementations here below summarized. We define the following quantities:

E. Accomando et al. / Computer Physics Communications 150 (2003) 166–196

$$p^{2} = \frac{1}{4s} \left(s^{2} - 2s(s_{1} + s_{2}) + (s_{1} - s_{2})^{2} \right); \qquad E = \frac{s - 4M_{W}^{2}}{4M_{W}}, \tag{1}$$

177

$$\beta_M = \frac{2}{\sqrt{s}} \left(\sqrt{\frac{M_W}{2} \left(\sqrt{E^2 + \Gamma_W^2} + E \right)} + i \sqrt{\frac{M_W}{2} \left(\sqrt{E^2 + \Gamma_W^2} - E \right)} \right),\tag{2}$$

where $\sqrt{s_{1,2}}$ are the W boson invariant masses and \sqrt{s} is the center of mass energy.

The general first-order formula for the Coulomb correction then reads:

$$F_{\text{coul}} = \frac{\alpha_{\text{QED}}\sqrt{s}}{4p} \left(\pi - 2\delta^2 \arctan\left(\frac{s|\beta_M + \Delta|^2 - 4p^2}{4p\sqrt{s}\operatorname{Im}(\beta_M)}\right) \right),\tag{3}$$

where for the two parameters Δ and δ we have included the following options:

- icoul=1: $\Delta = |s_1 s_2|/s$ and $\delta = 1$ In this case the above formula reproduces the one of Ref. [36] which was the only one available in the previous version of WPHACT.
- $icoul=2: \Delta = |s_1 s_2|/s$ and $\delta = (1 2p/\sqrt{s})$ This factor δ implements the so-called Khoze Chapovsky ansatz [37] which mimics the effect of non factorizable WW corrections. They are known to be almost negligible in total cross sections but relevant for quantities as W angle and mass distributions.
- icoul=3: Δ = 0 and δ = 1
 With this choice, the results of the Coulomb correction of Ref. [36] agree with those obtained in Ref. [38], as therein explained.
- icoul=4: $\Delta = 0$ and $\delta = (1 2p/\sqrt{s})$ This corresponds to the Khoze Chapovsky ansatz on the Coulomb correction of Ref. [38].

The Coulomb factor F_{coul} , strictly related to the WW subset of diagrams (CC03), is implemented in the program as an additive correction

$$|M_{\rm CC,Mix}|^{2} = |M_{\rm CC,Mix}^{\rm Born}|^{2} + F_{\rm coul}|M_{\rm CC03}^{\rm Born}|^{2}.$$
(4)

2.3.4. Fermion loop

The inclusion of weak boson finite-width effects requires a careful treatment, as these effects are strictly related to the gauge invariance of the theory and even tiny violations of Ward identities can lead to totally wrong predictions in many cases.

The most appealing approach which preserves gauge invariance is the Fermion Loop (FL) scheme [9] which consists in the resummation of the fermionic one-loop corrections to the vector boson propagators and in the inclusion of all remaining fermionic one-loop corrections, in particular those to the Yang–Mills vertices (see Fig. 2). Several realizations of this general scheme have been proposed and used in numerical computations.



Fig. 2. The extra fermionic diagrams needed to cancel the terms which break gauge invariance.



Fig. 3. The t-channel component of the CC20 family of diagrams: fusion, bremsstrahlung and multi-peripheral.

The so-called Imaginary Fermion Loop (IFL), includes only the imaginary part of the loops, which is the minimal set of one-loop contributions needed to preserve gauge invariance. The complete FL includes instead all contributions from fermionic loop corrections.

Initially applied only to the case where weak currents that couple to fermionic loops are conserved, both IFL and FL have been recently extended to non-conserved currents, which are present when one takes into account external fermion masses [10,11].

A class of processes where these corrections become particularly relevant is the one related to single W production, for example $e^+e^- \rightarrow e^-\bar{\nu}_e u\bar{d}$ whose complete set of *t*-channel diagrams is shown in Fig. 3. These processes, dominated by the four *t*-channel photon diagrams when the electron is emitted at very small angle, diverge in the massless limit $m_e \rightarrow 0$. Moreover, their apparent q_{γ}^{-4} behaviour gets reduced to q_{γ}^{-2} by gauge cancellations. If the electron mass is taken into account there are however terms proportional to m_e^2/q_{γ}^4 . This kind of processes therefore requires the inclusion of fermion masses and the use of a gauge preserving scheme when taking into account finite width effects.

The IFL scheme for non-conserved currents has been implemented, as an option, in the new version of WPHACT.

As an alternative option, we have also included in WPHACT a simplified gauge-preserving scheme. An easier way to preserve gauge invariance, commonly adopted for practical purposes, is in fact the use of the so-called fixed width approximation (FW), which gives unphysical widths for space-like momenta but retains U(1) gauge invariance. In the unitary gauge, this *naive* scheme consists in replacing M_W^2 with $M_W^2 - iM_W\Gamma_W$ both in the denominator and in the $p^{\mu}p^{\nu}$ term of the bare W propagator. Note that neglecting to modify the latter could

lead to large errors which increase with energy. This effect, negligible at LEP2, could be greatly enhanced in the high energy range of future colliders.

A comparison between the IFL and FW schemes for single W production at small angle (collinear electron), where one expects gauge invariance issues to be essential, has shown very good agreement for total cross sections but not for distributions.

2.3.5. Running of a QED

The running of the electromagnetic coupling constant α_{QED} can give rise to important corrections, especially for small momentum transfer ($q^2 \simeq 0$) kinematical configurations as in small angle scattering and small invariant mass regions.

The G_{μ} scheme, which fixes the value of α_{QED} to be $\alpha_{G_F} \simeq 1/132$, is appropriate for high energy *s*-channel scattering processes, like WW, ZZ and *h*Z vector boson pair production. However, it is not adequate for multi-scale processes as single W and single Z production which are determined by both the vector boson mass scale and the small transferred momentum $|q_{\nu}^2|$ in the *ee* γ vertex.

As it is known, the choice of the correct scale for the couplings is related to the real part of Fermion Loop radiative corrections. Presently, the complete FL has been computed only for single W processes [11], leading to a drastic reduction of the theoretical uncertainties on the cross section, and is implemented exclusively in WTO [39]. Since a detailed analysis of these corrections has shown that the running of α_{QED} accounts for a large part of FL effects [17], we have included in the new version of WPHACT two additional options for the determination of α_{OED} .

For CC processes with an electron (or positron) in the final state, a well-behaved, even if *ad hoc*, prescription for the running of α_{QED} can be selected via the input parameter ialftsw (see Section 3.1.5). Since *s*- and *t*-channel diagrams constitute two independent gauge invariant subsets, it is possible to use two different values of α_{QED} : $\alpha_{\text{QED}}(q_{\gamma}^2)$ for the *t*-channel (applied to two vertices), and α_{G_F} for the *s*-channel. In this way, $SU(2) \times U(1)$ is preserved. A comparison between the results obtained with FL and IFL plus running α_{QED} has shown that the agreement between the two schemes, even if process dependent, is generally better than 2–3% for single W production at LEP2, i.e. within the present experimental precision tag.

More generally, it has been also introduced the possibility of having α_{QED} -running in all photonic vertices $\gamma f \bar{f}$ (see also Section 3.1.5). For each vertex the scale is taken equal to the square of the photon momentum. This approach does not preserve gauge invariance. However, at LEP2 energies, it can give an adequate description of NC and Mixed processes dominated by low q^2 photons, such as single Z, $Z\gamma^*$ or $\gamma^*\gamma^*$. An example of the relevance of such an approach can be found in Fig. 1. In the first two plots to the right, cross sections with α_{em} running are compared with the analogous ones in G_{μ} scheme for single Z processes, which are dominated by low scale photon propagator contributions. In such a case one can find differences between the two approaches of the order of 5%.

2.4. Resonances and interface to hadronization

In WPHACT 2.0 there is the possibility to account for the resonant structures in $\gamma^* \rightarrow q\bar{q}$ near threshold through a link to R_RES [22]. These routines provide a parametrization of the low mass $q\bar{q}$ resonances and continuum which reproduces the R-factor. This feature is useful for phase space regions where the $\gamma^*\gamma^*$ or $Z\gamma^*$ contributions are dominant. However, in final states with an e^+e^- pair, low $m(q\bar{q})$ dominant contributions may also come from multiperipheral diagrams when at least one electron is produced at small angle. In this case the above parametrization may not be adequate. One may however decide to evaluate the two contributions, applying appropriate cuts, in two different runs, and eventually generate events simultaneously in the two regions with the one-shot procedure described in Section 2.7.

Hadronization in WPHACT is provided via a link to PYTHIA. The interface is provided by the routine AB_LU4FRM which is derived from PYTHIA'S PY4FRM. When the final state consists of two identical pairs of quarks or of four quarks in a Mixed final state, different color singlets can be formed. In the first case, WPHACT 2.0 evaluates for every generated phase space point the separate contributions of the diagrams

corresponding to the different pairing, and assigns the color flow with a probability proportional to their weights. In the Mixed case, the separate generation of charged and neutral currents, which is the standard procedure in WPHACT, is used to get the correct proportion of colour pairings.

The hadronization of quark pairs with $m(q\bar{q}) < 2$ GeV is not handled by PYTHIA, but by the routine Hadgen of the R_RES package. The description of this region by theoretical models is in fact known to be problematic, while Hadgen produces final states according to measured exclusive e^+e^- cross sections compiled for this purpose.

2.5. Susy and anomalous couplings

Besides SM Higgs processes with two b's (c's or τ 's) plus two other fermions in the final state, WPHACT computes also SUSY neutral Higgs production in the same final channels.

As for other previously discussed processes, in the new version of WPHACT the contributions of all diagrams containing the Higgs can be optionally separated from the rest or individually selected, as explained in more detail in Section 3.1.11.

Moreover, in WPHACT 2.0 we have introduced new options for the SUSY parameters to be given as input (see also Section 3.1.11). We have in fact updated the one-loop corrections to the lightest neutral Higgs mass M_h used in the previous version of the program, by including the two-loop RG improved effective potential results as given in Ref. [40]. As a further option, the user can also give directly the three parameters M_h , M_A and tg(β) as independent inputs, where M_A is the CP-odd Higgs mass and tg(β) is the ratio between the two vacuum expectation values.

As far as Anomalous Couplings effects are concerned, we have implemented those relative to the trilinear vertices WW γ and WWZ. Among the various possible parametrizations, we have included the one given in Ref. [41].

In WPHACT 2.0, we have extended the implementation of Anomalous Couplings to the new fully massive matrix elements.

2.6. Distributions and unweighted event generation

Even if no particular new features have been introduced as far as distributions and unweighted event generation of a single process are concerned, we recall here their main properties for completeness and as an introduction to the next subsection.

WPHACT has a built-in mechanism for computing any distribution at parton level. To this aim, the user has to define the variables for which differential cross sections need to be produced in the routine f xn f, and to specify in the *input cards* their number, their sub-intervals and binnings. The program produces them automatically during integration.

The results for all distributions, together with an estimate of the error for each bin, are written in the file ABDIS.DAT.

Distributions can also be produced generating unweighted events. This is particularly useful in experimental studies involving hadronization and detector simulations. WPHACT can produce unweighted events both at parton and at hadron level (with a link to PYTHIA). In the new 2.0 version, it can also produce unweighted events for as many simultaneous massive processes as desired. This last feature is described in the next section.

As explained in [3], unweighted events for a single process at a time can be produced with the hit-or-miss method while evaluating the integral with VEGAS. In this case the last but one effective iteration after thermalization is used to find the maximum rmaxfxn of the integrand and the last one to generate unweighted events.² The

 $^{^{2}}$ It has to be noticed in this respect that in version 1.0 the search for the maximum was performed in the first effective iteration and the generation in the second one, so that the user was forced to use only two iterations when producing unweighted events. We have removed this restriction as we find sometimes useful to have some more iterations (other than thermalization) before actually generating events in order to fully exploit VEGAS adaptivity.

drawbacks of this procedure are that the number of generated events cannot be predicted and that one may have some generated events whose weight is larger than rmaxfxn × scalemax (where scalemax is the factor used to tune the efficiency of the hit-or-miss selection). We therefore recommend the use of a two step procedure for generation: after a first run with iflat=1, istorvegas=1 and irepeat=0 one can perform a second run with iflat=1 and irepeat=2 (cf. Section 3) in which one requires, via nflevts, a predefined number of events. These will be produced using the maximum and the integration grid determined in the last iteration of the first run.

Usually in such a two step procedure, one does not require hadronization and does not store the events generated in the first run, which then simply evaluates the integral and determines the "best" adaptive grid. The necessary information for actual generation is stored in the file ABVEGAS.DAT. It is obvious that any such file can be reused for generation as many times as one wants. The multi-process simultaneous generation described in the next section is in practice just a way of combining and using the information of many such files at the same time. It can also be used for only one process and in such a case it constitutes a convenient replacement for the second run above.

2.7. One-shot generation

Besides the event generation described in the previous section and already present in the first version of WPHACT, a much more powerful method has been implemented in WPHACT 2.0 for all completely massive processes. This allows the user to generate unweighted events not on a process by process basis but for any possible set of processes and channels and cuts in a single run, giving eventually a complete event sample in which all included 4-fermion final states are produced with the correct frequency.

In order to use this feature, single run inputs (hereafter we will call them *input files*) must be present. The best way to understand what they are is just to describe how they can be produced. For every process (and cuts) one wants to consider, one has to perform a run exactly as the first of the two step procedure described in the previous section. This is by all practical means a normal integration run with the flags iflat=1, istorvegas=1, irepeat=0 (cf. Section 3). One then appends the normal output file with integration results of every run to the corresponding ABVEGAS.DAT and saves it with an appropriate name (say procl.inp, proc2.inp,...). Each such *input file* will therefore contain all information about the input of the first run (process, energy, options, cuts, etc.), as well as the result of the numerical integration, the best grid and the maximum for the single process.

If one now chooses to run in the "one-shot" mode (ionesh=1), one specifies in the input the number of the *input files* and their names. WPHACT will then read from the files all necessary information and build up for every single process its probability and a new maximum normalized to it. According to this probability, one final state will be extracted at a time. For the selected process the relative cuts and options will be used, and an event will be generated with a frequency in phase space determined by its flags and grid, and this event will be compared with the normalized maximum in order to keep or reject it. The procedure will then be repeated starting from the process selection until the required number (nunwevts) of unweighted events is produced.

The efficiency of this method relies heavily on an accurate preparation of the *input files*. The choice of the grids and of the maxima affects the efficiency but not the correctness of the result, provided that one eventually checks the number of events exceeding the maximum and if necessary modifies scalemax.

As a consequence, one may use the same *input files* for energies which are different from the energy at which they were produced. However, if the energy difference becomes too large the efficiency may become unacceptably small. In the same spirit, one does not need to produce a single *input file* for every final state, and there is indeed the possibility to use only one representative process for every set of processes with the same <code>iproc</code> value, whose matrix elements differ only for charge conjugation or for final particle masses. In this case the representative process must be the one with <code>ich=1</code>. In our experience this faster possibility is quite safe and efficient.

Any subset of the *input files* can of course be used, so that once the files for all processes have been prepared, one may use them for partial generations as well. As an example one may imagine to generate only neutral current final states, or only processes involving electrons, and so on. Any final state may also be divided, with respect to

input files, in different exclusive complementary parts with different cuts. For instance one may think of separating regions with electrons lost in the beampipe from ZZ or WW contributions. Therefore one can have several *input files* for a single process and use them all together or not.

3. Input

WPHACT 2.0 is much more flexible in accepting input parameters than the previous version of the program. The syntax is almost identical to the one required by the CERN library routine FFREAD. New routines internal to WPHACT are however used (iread, rread), so that real variables can (and must) be given in double precision. The difference between single and double precision can be relevant for some cuts or parameters in particular regions.

All lines in the file of input (hereafter called *input card*) must not exceed 80 characters. A * or C character at the beginning of a line identifies it as a comment line. Comment lines can be freely interspersed within the *input card*, with the only obvious exception that they must not interrupt a list of input values for a single array variable. The name of the variable to be read must be specified as the first word of a line (needs not to begin in column 1). Its value (values) must follow it. The list of values can span several lines. Variables which are not needed for the process under study will be ignored. They can be left in the *input card* without harm. All variables actually read from the *input card* will be reproduced in the output. Since several related runs are often needed, we find it convenient to have a master *input card*. For each run only a small number of variables are modified and all other input values, including those which are irrelevant for the run at hand, are left in place for possible further use. The order in which variables appear in the *input card* is immaterial. The user may freely change it. In output, however, variables will be ordered as they are read by the program.

Every parameter whose initial is i or n is of type integer. All others are real*8. When a variable has a yes/no option the value 1 corresponds to YES, 0 to NO. All energies and masses must be expressed in GeV.

We recall that WPHACT 2.0 can be used in two different modes. The program can compute or generate events for a single process or it can generate events for a user-selected list of processes, provided the corresponding *input files* are available. Different sets of inputs must be specified for the two modes. In the following we discuss each input variable, its meaning and purpose separating the single process and the one-shot cases.

We begin obviously with:

ionesh: this yes/no flag determines whether the program is supposed (1) to generate unweighted events for several massive processes simultaneously, using a list of *input files*, or it is required (0) to compute only one (massless or massive) process. *Input files* are prepared using this second option.

3.1. One process case: ionesh=0

If ionesh=0 the following flags are required:

e_cm: the center of mass energy.

imass: if $iproc \leq 32$ imass=0 means massless fermions and imass=1 means fully massive fermions. If $iproc \geq 33$, which corresponds to the Higgs signal and its irreducible background processes, imass is ineffective: the final states of Table 3 have *b* massive in processes 33–39, *c* massive in processes 40–46 and τ massive in processes 47–53 while all other external particles are massless.

3.1.1. Selection of the final state

iproc, ich: the possible values for these two parameters can be found in Tables 1–3. iproc selects a group of processes with the same Feynman diagrams, while ich distinguishes a specific final state within the group.

The choice of ich is relevant also in the massless case, if the program is used as an event generator and hadronization is performed via PYTHIA or cuts which differentiate between different particle types are used. ickm: this flag determines whether the Cabibbo–Kobayashi–Maskawa matrix is taken into account (ickm=1) or

it is taken equal to the unit matrix (ickm=0). As far as the three processes $e^+e^- \rightarrow s\bar{u}u\bar{s}$, $d\bar{c}c\bar{d}$, $b\bar{c}c\bar{b}$ are concerned, the CC exchange contribution is obtained with ickm=1 and iproc=5, ich=21, 22, 23, while the usual NC part is obtained with iproc=24 and ich=1, 2, 4, respectively. The interference between these two contributions is ignored.

3.1.2. Selection of a subset of Feynman diagrams

In some cases the user may want to compute only a subset of the Feynman diagrams which describe the production of a physical final state. This is useful for instance when heavy intermediate particles can go on their mass-shell. The most commonly used subsets can be obtained with the flags described below. One should however be aware that many of the subsets are not gauge invariant. The results should therefore be taken with great care, bearing in mind that our matrix elements are computed in the Unitary Gauge.

iccnc: in mixed processes this flag allows the user to compute the CC (iccnc=1), NC (iccnc=2) or full CC+NC+interference (iccnc=3) contribution. Let us recall that, if iccnc=3, WPHACT by default integrates separately the two contributions, CC and NC, adding the interference to one of them. When generating events (iflat=1), in order to avoid negative weights we adopt the following procedure, which regards as well iccnc=1 and iccnc=2. The interference is initially summed to the CC contribution (which is generally the larger one). If the result is negative, the quantity CC+Interference is set to zero while the full CC+NC+Interference contribution is assigned to the NC part. For one-shot generation it is more efficient to have separate *input files* for the CC (iccnc=1) and NC (iccnc=2) parts. For this reason only this possibility has been implemented and must be used.

For the following group of flags there are four possible values. If F is the full set of Feynman diagrams for the process and S is the relevant subset

- flag=0 means computing $|F|^2$,
- flag=1 means computing $|S|^2$,
- flag=-1 means computing $|(F S)|^2$,
- flag=2 means computing $|F|^2 |S|^2$.

These four possible values and the corresponding subsets are implemented only for massive matrix elements (imass=1). In the massless case (imass=0), one must set flag=0, with the only exception of icc3 where icc3=1 can also be selected.

icc3: the subset is CC03.

- izz: the subset is NC02 (see Fig. 4 with a = b = Z).
- inc08: the subset is NC08 (see Fig. 4 with $a, b = \mathbb{Z}, \gamma$).
- izg: the subset is the set of four NC08 diagrams in which one pair of final state fermions is connected to an intermediate Z-boson and the other to an intermediate photon (see Fig. 4 with a = Z, $b = \gamma$ or $a = \gamma$, b = Z).
- izg34: the subset is the set of two NC08 diagrams in which the pair of final state fermions with momenta p3 and p4 is connected to an intermediate photon and the other to an intermediate Z-boson (see Fig. 4 with $a = \gamma, b = Z$).
- izg56: the subset is the set of two NC08 diagrams in which the pair of final state fermions with momenta p5 and p6 is connected to an intermediate photon and the other to an intermediate Z-boson (see Fig. 4 with $a = Z, b = \gamma$).



Fig. 4. The sets of diagrams which can be singled out in NC processes with the flags izz, inc08, izg, izg34, izg56, igamgams are generated by the two diagrams above for different choices of the vector bosons a and b. If identical particles are present in the final state there are two additional diagrams which can be obtained exchanging p3 with p5. In this case the number of diagrams selected is twice the number mentioned in Section 3.1.2.



Fig. 5. The multi-peripheral type diagrams.

igamgams: the subset is NC02 (see Fig. 4 with $a = b = \gamma$).

- igamgam: the subset is the set of multiperipheral diagrams characterized by two *t*-channel photon propagators (i.e. $\gamma \gamma$ channel, see Fig. 5 with $a = b = \gamma$).
- imulper: the subset is the set of all multiperipheral diagrams characterized by two *t*-channel neutral vector boson propagators (see Fig. 5 with all possible intermediate neutral vector bosons). This flag is active only for NC processes or the NC part of Mixed processes.
- itch: the subset is the set of so-called *t*-channel diagrams, those in which there is at least one *t*-channel vector boson propagator (see, for example, Fig. 3).
- itchnomp: the subset is the set of diagrams in which there is at least one *t*-channel vector boson propagator but which are not of the multiperipheral type (see Figs. 5 and 3). This flag is active only for NC processes or the NC part of Mixed processes.

3.1.3. Selection of a phase space mapping

ips_cc, ips_nc: these flags select among the various phase space mappings for the integration over invariant masses. ips_cc refers to the phase space of CC or Higgs signal contributions. ips_nc to the NC contributions. Only one of the two flags is relevant if the process is not of the Mixed (or NC+Higgs) type. For Mixed processes the two flags can be chosen independently. Both parameters can assume 4 values: 1 for double resonant mapping, 2 for resonant mapping on the 34 (36 for CC part of Mixed processes) invariant mass and non-resonant distribution of the 56 (54 for CC part of Mixed) invariant mass, 3 for resonant mapping on the 56 (54 for CC part of Mixed) invariant mass, 3 for CC part of Mixed) invariant mass, 4 for non-resonant distribution of both invariant masses. We recall that a resonant mapping is intended to flatten out the relevant mass peaks of the pair of final particles at hand. For neutral

currents this includes the Z resonance, together with a possible photon enhancement at low invariant masses or an additional propagator enhancement corresponding to a Higgs resonance.

- ismallangle: it must be set 0 if no electrons or positrons are present in the final state. ismallangle \neq 0 selects the mapping of the angular distribution of final state electrons and/or positrons which can be strongly affected by *t*-channel enhancements. For such a reason we recommend to use these latter values only together with imass=1.
 - ismallangle=0 is appropriate when the final state e^{\pm} are at large angles with respect to the beam.
 - ismallangle=1 is appropriate when the process is dominated by final states with one e^- (or e^+) close to its initial direction. If ismallangle=1 and there is at least one e^-e^+ pair in the final state, the next flag ismallangle_ee must be set.
 - ismallangle=2 is appropriate when the process contains at least one e^-e^+ pair in the final state and it is dominated by final states with both e^- and e^+ close to their initial direction.

If ismallangle is non zero ips_nc (ips_cc) must be equal to 3 or 4 for NC or NC part of a Mixed process (CC or CC part of a Mixed process).

ismallangle_ee: only relevant if ismallangle=1 and iproc=7,18,21,26,28,30. ismallangle_ee=3(4) selects a phase space which is appropriate for a final state $e^-(e^+)$ close to its initial direction. According to our convention, before ISR the e^+ momentum is in the +z-direction. It has to be noticed that for the CC part of the Mixed process $e^-e^+v_e\bar{v}_e$, the choice ips_cc=3 corresponds to the W-resonant mapping on the e^+v_e pair for ismallangle_ee=3 (electron at small angle), and on $e^-\bar{v}_e$ for ismallangle_ee=4 (positron at small angle).

3.1.4. Selection of electromagnetic radiation

- isr: isr=0 means no Initial State Radiation (ISR). isr=1 means ISR using Structure Functions [30]. isr=2
 means ISR using QEDPS [21].
- ibeam: yes/no beamstrahlung correction [23]. This effect is relevant only for Linear Collider energies. ibeam=0 must be chosen in all other cases.

3.1.5. Higher order corrections and scheme selection

- icoul: if icoul=0, no Coulomb corrections to the CC03 part (Eq. (4)) are included. If one wants to compute them, one can choose among 4 possible values (1, 2, 3, 4). For their meaning one has to refer to Section 2.3.3.
- istrcor: yes/no 'naive' QCD corrections to the cross sections. It has to be noticed that if istrcor=0, quarks are present in the final state and igwcomp, igzcomp or ighcomp are equal to 1 the widths are computed for consistency without QCD corrections.
- ifloop: yes/no IFL corrections to CC processes or to the CC part of Mixed processes with at least an electron (or positron) in the final state. ifloop=1 requires the W width to be s-dependent (see below ipr=2). ifloop=1 is relevant for low angle electron final states and it is therefore only implemented for the massive case (imass=1).
- ialftsw: yes/no running $\alpha_{\text{QED}}(q_{\gamma}^2)$ applied to two vertices in *t*-channel diagrams of CC processes or CC part of Mixed processes, with at least an electron (or positron) in the final state. Implemented only for the massive case (imass=1).
- ialfar: yes/no running $\alpha_{\text{QED}}(q_{\gamma}^2)$ applied to all photon vertices $\gamma f \bar{f}$. This option, even if not gauge invariant, can be useful for processes dominated by electromagnetic contributions. Implemented only for the massive case (imass=1).

ipr: this flag selects among running or constant Z, W, Higgs widths in s-channel propagators:

- ipr=0 Z, W, H have constant width.
- ipr=1 Z, W, H have *s*-dependent width.
- ipr=2 Z, H have constant width; W has *s*-dependent width.

If ifloop=1, ipr is set automatically to 2. A warning will appear in the output.

- iswgcomp: If this flag is set to 1, $\sin^2 \theta_W$ and g are computed in terms of the Z mass, the W mass and G_f . This corresponds to use the " G_{μ} scheme". If it is set to 0, the values for $\sin^2 \theta_W$ and α_{QED} are taken from the DATA and $g^2 = 4\pi\alpha_{\text{QED}}/\sin^2 \theta_W$.
- igwcomp: if igwcomp=1 the W width is computed using standard formulae. If igwcomp=0 the W width is taken from the DATA.
- igzcomp: if igzcomp=1 the Z width is computed using standard formulae. If igzcomp=0 the Z width is taken from the DATA.
- ighcomp: if ighcomp=1 the H width is computed using standard formulae. If ighcomp=0 the H width is taken from the DATA.
- iresonance: yes/no interface to the routines of R_RES [22], an experiment-based parametrization of lowmass quark pairs coupling to a virtual photon. R_RES provides a reweighting factor which reproduces the experimentally observed resonant structures (R-factor) in $\gamma^* \rightarrow q\bar{q}$ near threshold. When iresonance=1 is set, the factor is applied to each $q\bar{q}$ pair with $m(q\bar{q}) < 12$ GeV. One should be careful about the use of iresonance=1 in phase space regions where important low $m(q\bar{q})$ contributions are multiperipheral.

3.1.6. Anomalous couplings

ianc: this yes/no flag determines whether anomalous couplings are used. If ianc=1 all of the following parameters must be set, otherwise a fatal error will occur. They are defined in Ref. [41] delz, xf, xz, yf, yz, zz.

3.1.7. Cut selection

- icut: this yes/no flag determines whether a predefined set of cuts is implemented. If icut=1 all of the following parameters must be set, otherwise a fatal error will occur.
- e_min(e_max): minimum (maximum) energy for the final state fermions. Four values are required in the order in which the particles appear in the tables.
- rm_min(rm_max): minimum (maximum) invariant mass for pairs of final state fermions. Six values are required. They must be given in the following order: m(34), m(35), m(36), m(45), m(46), m(56).
- pt_min(pt_max): minimum (maximum) transverse momentum for the final state fermions. Four values are required.
- icos: if icos=0 the limits on angles are given in degrees; if icos=1 they refer to the cosine of the relevant angle. Notice that the lower limit always refers to the smaller angle in degrees. For example, if $0 \le \theta \le \pi$ and icos=1 then the following flags thbeam_min and thbeam_max must be set to 1 and -1, respectively.
- thbeam_min(thbeam_max): minimum (maximum) angle of the final state fermions with respect to the e^+ beam. Four values are required. The accepted region is controlled by the following flag iext.
- iext: if iext=0 the angle θ passes the cut if theta_min $\leq \theta \leq$ theta_max while if iext=1 the angle θ passes the cut if $\theta \leq$ theta_min or theta_max $\leq \theta$ that is with iext=0 the interval between the limits is accepted while with iext=1 the acceptable region is the one outside the limits. Four values are required, one for each final fermion. This flag is useful for instance for processes with final particles lost in the beam pipe.
- thsep_min(thsep_max): minimum (maximum) angle between pairs of final state fermions. Six values are required. They must be given in the following order: (34), (35), (36), (45), (46), (56).
- ielost: additional angular cuts for the process $e^+e^- \rightarrow e^-e^+e^-e^+$. An electron is considered visible if it makes an angle larger than thetabeam (see below) with both beams. ielost=0 should be set when all four final particles are required to be visible. ielost=1 requires instead only one invisible e^- (the other three particles being outside the blind cone), ielost=2 only one invisible e^+ . ielost=3 is for only a visible pair e^-e^+ , ielost=4 requires the like-sign e^-e^- pair to be visible, and ielost=5 the like-sign e^+e^+ pair visible. ielost=6 is for only a visible e^+ , the other three particles being lost in the beam pipe, and finally ielost=7 is for only a visible e^- . If ielost>0, the following input flag must be set.

thetabeam: separation angle between visible and invisible e^{\pm} , to be specified if ielost>0. The value of thetabeam must be always expressed in degrees, independently on the flag icos.

Additional cuts may be implemented by the user in WPHACT 2.0 (routine fxn.f) just after the line

* here define additional cuts

where a commented example is reported. The cuts must be specified using the momenta of the final particles in the collider frame $p_3(0:3)$, $p_4(0:3)$, $p_5(0:3)$, $p_6(0:3)$ and the incoming positron $p_1(0:3)$ and incoming electron $p_2(0:3)$. They are assigned in the order in which the final state particles appear in Tables 1–3.

3.1.8. Distributions

If distributions have to be automatically computed, they must first be defined by the user in routine f xn.f, after the line:

* here define weighted distributions

where some commented examples of distributions are given. The resulting cross sections corresponding to every single bin will be stored in the file ABDIS.DAT. Each line will contain 3 numbers: the value of the central point of the bin, the distribution for the bin (i.e. the cross section divided by the width of the bin) and the statistical error. The next entries are unchanged from WPHACT 1.0.

- idistr: this yes/no flag determines whether distributions are desired. If idistr=1 all of the following parameters must be set, otherwise a fatal error will occur.
- ndistr: the number of distributions to be generated (limited by PARAMETER ndismax in routine wph.f, whose present value is 50).

For each required distribution the following set of parameters is mandatory (the string *i* must be replaced by the actual label of the distribution $1 \le i \le ndistr$):

- nsubint(i): number of sub-intervals with different binning (limited by PARAMETER nintmax in routine
 wph.f, whose present value is 10). In normal plots one just uses one subinterval with a prescribed number
 nbin_number(1) (see below) of bins. If one chooses different binning for different subintervals one can
 evidentiate in a plot particular regions of interest.
- distr_estrinf(*i*): lower limits of each sub-interval (which coincide with the upper limit of the previous one) plus the upper limit of the last subint.nsubint(*i*)+1 values are required.
- nbin_number(*i*): the number of bins for each sub-interval. nsubint(*i*) values are required (the total number of bins is limited by PARAMETER nbinmax in routine wph.f, whose present value is 500).

3.1.9. Unweighted event generation

This set of flags is relevant when generating unweighted events for a single process or when preparing *input files* for one-shot generation.

- iflat: this yes/no flag determines whether unweighted event generation is desired. If iflat=1 all of the following parameters must be set, otherwise a fatal error will occur. iflat must be equal to 1 when preparing input files.
- scalemax: factor by which the largest generated value of the differential cross section is multiplied for the hitor-miss selection. This coefficient can be used to tune the efficiency of the hit-or-miss selection.

- istorvegas: this yes/no flag determines whether VEGAS data are stored after the last but one iteration in ABVEGAS.DAT (or in ABVEGAS_CC.DAT and ABVEGAS_NC.DAT for NC+Higgs and for Mixed processes when iccnc=3). Stored VEGAS data are necessary if one wants to rerun the program to generate again unweighted events. When the program is rerun using stored VEGAS data, the maximum of the last iteration will be automatically used as the new maximum. istorvegas must be equal to 1 when preparing *input files*.
- irepeat: irepeat has to be set to 0 for the first run. It has to be set to 1 if one wants to rerun the program exactly with the same input and grid starting from the last iteration. In this case the same weighted points will be reproduced unless scalemax is varied. In the first two cases the number of attempts is fixed but not that of generated unweighted events. irepeat=2 has to be chosen if one wants to rerun with the same input and grid as before, but letting the program run until a requested number of events nflevts is reached. For both cases irepeat=1 and 2 one might of course vary scalemax, ijetset and istormom (see below as to the last two entries) with respect to the first run with irepeat=0. irepeat must be equal to 0 when preparing input files.
- nflevts: the number of unweighted events to be generated if irepeat=2.
- istormom: this yes/no flag determines whether the momenta of the generated unweighted events have to be stored or not. If istormom=1, the momenta of the unweighted events are generally written in single precision in the ABMOM.DAT. In the particular case of Mixed (when iccnc=3) or NC+Higgs processes, the ABMOM_SIGN.DAT and ABMOM_BACK.DAT files will be produced.ABMOM_SIGN.DAT is used for CC or Higgs events, ABMOM_BACK.DAT for NC events.

As to the entries relative to this subsection, no substantial change has been introduced with respect to the previous version of WPHACT, with the following exception. In the new version the number of iterations required for the unweighted event generation is not constrained anymore to be itmx=2, but can be freely chosen as in all other cases. Every unweighted event is passed to the standard COMMON/HEPEVT.

ijetset: this yes/no flag determines whether the program should be interfaced to PYTHIA. If ijetset=1 the subroutine AB_LU4FRM is called for interfacing PYTHIA (or Hadgen for $m(q\bar{q}) < 2$ GeV), otherwise HEPEVT is still filled but the interface to PYTHIA is not called.

3.1.10. Integration

Unchanged from WPHACT 1.0.

acc: the integration accuracy. When accuracy acc is reached the program stops.

- iterm: this yes/no flag determines whether VEGAS has to adapt its integration grid by performing thermalization iterations whose outcome will not be used in the final result.
- ncall_term: the maximum number of points for each grid refinement during thermalization.
- itmx_term: the number of times the grid is adapted before starting the actual integration (limited by PARAMETER nitmax in routine wph.f, whose present value is 10).

ncall: the maximum number of points for each iteration of the actual integration.

itmx: the maximum number of iterations used to evaluate the integral (limited by PARAMETER nitmax in routine wph.f, whose present value is 10). A value among 3 and 5 is normally the best choice. If higher precision is requested it is usually more convenient to increase ncall rather than itmx.

VEGAS will in general use a number of ncall_term and ncall lower than the input ones. The actual value is written in output, where also the number of points which survive all the cuts (effective ncall) is reported.

As a final remark about the choice of these parameters, one must be aware of the fact that final results with a χ^2 much greater than the number of iterations are not to be trusted. When this happens, one has to increase ncall.

3.1.11. Higgs

All the final states in the NC+Higgs group are also included in the NC class. The difference is the following. The matrix elements present in the NC group do not include the Higgs boson as an intermediate state and all particles can be either massive (imass=1) or massless (imass=0). The class of processes given by iproc≥33 includes instead the Higgs boson as an intermediate particle and are optimized for Higgs searches. In these processes, all initial and final particles are massless, except those which might come from the Higgs decay, i.e. massive *b*'s, *c*'s or τ 's. In Table 3 the complete list of these channels is shown. For each channel with no identical particles in the final state, the order in which the final particles appear is such that the first $f \bar{f}$ pair is the massive one (i.e. the pair that could come from the Higgs decay). For processes with identical particles, like $e^+e^- \rightarrow b\bar{b}b\bar{b}$, the final state is completely massive.

rmb (rmc, rmtau): sets the b (c, τ) mass in both phase space and matrix element for processes where the Higgs could decay into $b\bar{b}$ ($c\bar{c}$, $\tau^{-}\tau^{+}$).

rmbrun (rmcrun): sets the b(c) mass in the Higgs coupling, which might be different from rmb (rmc).

The processes listed in Table 3 can receive contributions either from the SM Higgs or from MSSM neutral Higgses (the CP-odd A and the lightest CP-even h).

isusy: isusy=0 corresponds to SM Higgs whereas isusy=1 to the MSSM Higgs sector.

Also for these channels the user can select subsets of Feynman diagrams.

icch: if icch=1 the subset is the Higgs signal. If icch=2 the subset is the Higgs background (i.e. the complete set of Feynman diagrams without the Higgs). If icch=3 the full process is computed (Higgs+Background+interference).

Moreover, when computing the pure Higgs signal (icch=1) for the three processes $b\bar{b}b\bar{b}$, $c\bar{c}c\bar{c}$ and $\tau^{-}\tau^{+}\tau^{-}\tau^{+}$, in the new version of WPHACT the user can also choose:

iha: for SM Higgs, iha=1 gives the complete amplitude, while iha=2 computes only the Feynman diagrams corresponding to hZ production and decay. For MSSM Higgs, iha=1 gives as before the complete amplitude. iha=2 selects only the diagrams which describe single h production (i.e. diagrams which are singly-resonant on the h mass), iha=3 the diagrams for single A production (i.e. diagrams which are singly-resonant on the A mass). iha=4 computes the subset for hZ production, iha=5 gives the hA associate pair production and finally iha=6 generates the sum of the two contributions hZ + hA.

rmh: the mass of the SM Higgs or the mass of the lightest MSSM CP-even Higgs.

irmhcomp: this flag allows the user either to choose arbitrary values for the lightest MSSM CP-even Higgs rmh
 (irmhcomp=0) or to compute rmh using standard formulae (irmhcomp=1). In the first case, one should
 specify the chosen rmh value, followed by

rma: the MSSM CP-odd Higgs mass

tgb: the value of $tg(\beta)$, the ratio between the two vacuum expectation values.

In the latter case (irmhcomp=1), given rma and tgb as input, the value of rmh is automatically computed. The calculation is controlled by:

iloop: if iloop=1 the lightest CP-even Higgs mass is determined at 1-loop (see WPHACT 1.0). If iloop=2, rmh is derived according to the 2-loop RG improved effective potential results given in Ref. [40].

In the iloop=2 case, the user must specify one of the following possible scenarios:

imixing: imixing=1 corresponds to the no mixing case $(A_t = A_b = 0 \text{ and } |\mu| \ll M_S$ where M_S is the SUSY scale), imixing=2 corresponds to maximal mixing $(A_t = A_b = \sqrt{6}M_S \text{ and } |\mu| \ll M_S)$, and imixing=3 corresponds to typical mixing $(A_t = A_b = -\mu)$. Finally, imixing=4 allows the user to select arbitrary values for

At: the trilinear soft susy breaking term A_t ;

Ab: the trilinear soft susy breaking term A_b ;

rmyou: the SUSY Higgs mass parameter μ .

In addition to the parameters in input, other physical quantities are fixed in the routine wph.f by a DATA statement, where

rmw, rmz, rmt, rmb, rmc, rms, rmu, rmd, rme, rmmu, rmtau are respectively the W, Z, t, b, c, s, u, d, e, μ and τ masses;

rmb_run is the quark *b* mass used for the Higgs coupling;

gamw, gamz, gamh are the total W, Z and Higgs width;

gf is the Fermi coupling constant; alfainv is $1/\alpha_{\text{QED}}$ at the appropriate scale; alfas_cc and alfas_nc are $\alpha_s(M_W)$ and $\alpha_s(M_Z)$;

s2w is the Weinberg $\sin^2(\theta_W)$ and rmsus the SUSY scale;

vud, vus, vcd, vcs, vcb are the CKM matrix elements.

3.2. Multiple process case: ionesh=1

ionesh=1 can be used only for massive processes (imass=1)
If ionesh=1 the following flags are required:

- e_cm: the centre of mass energy. This does not need to be equal to the energy at which the *input files* have been produced.
- scalemax: factor by which the largest generated value of the differential cross section is multiplied for the hitor-miss selection. This coefficient can be used to tune the efficiency of the hit-or-miss selection. It prevents generating too many events with weight larger than the maximum stored in the *input file*. In our experience scalemax=1.1 is usually a good choice.

nunwevts: number of unweighted events to be generated.

- iallme: this yes/no flag determines whether, for every unweighted event, the matrix element squared for all applicable subsets of Feynman diagrams are computed. If iallme=1 the partial results are stored in the variable SQMEL(19) passed to the COMMON/WPHSME.
- istormom: this yes/no flag determines whether the momenta of the generated unweighted events have to be stored or not. If istormom=1, the momenta of the unweighted events are written in ABMOM.DAT.
- ijetset: this yes/no flag determines whether the program should be interfaced to PYTHIA. If ijetset=1 the subroutine AB_LU4FRM is called for interfacing PYTHIA (or Hadgen for $m(q\bar{q}) < 2$ GeV), otherwise HEPEVT is still filled but the interface to PYTHIA is not called.
- iallch: if iallch=0 only those processes for which an *input file* is present in the input list are generated. If iallch=1, for each process for which an *input file* is present in the input list, the full set of processes which have the same iproc will be included in the generation. In this case, the processes for which *input files* are present must have ich=1 and ickm=0. The subsequent flag ickm, which has to be specified in the one-shot input, determines then whether the full set must take into account CKM matrix effects.

190

- ickm: this flag determines whether the Cabibbo-Kobayashi-Maskawa matrix is taken into account (ickm=1) or it is taken equal to the unit matrix (ickm=0). Notice that for ickm=1 the number of channels in each CC group increases. This flag is effective only if iallch=1. If iallch=0, only the processes of the input files are generated, and their ickm flags determine eventual CKM effects.
- nfiles: the number of *input files* to be used for one-shot generation (limited by PARAMETER nmaxproc, whose present value is 150).

The nfiles line must be immediately followed by as many filenames (with complete relative path), each on a separate line, as indicated by the nfiles value. All other input variables are read directly from the *input files*.

4. Conclusions

We have described version 2.0 of WPHACT, a fully massive MC program and unweighted event generator which computes all Standard Model processes with four fermions in the final state at e^+e^- colliders. Thanks to the new features like fermion masses, the IFL gauge restoring scheme and new phase space mappings WPHACT has been extended to all regions of phace space, including kinematical configurations dominated by small momentum transfer and small invariant masses like single W, single Z, $Z\gamma^*$ and $\gamma^*\gamma^*$ processes. Special attention has been devoted to QED effects, which have a large numerical impact, with new options for the description of Initial State Radiation and of the scale dependence of the electromagnetic coupling. The program is now better suited to generate large samples of fully simulated events since it can produce unweighted events for any user selected subset of 4 f final states in a single run.

The program is available from http://www.to.infn.it/~ballestr/wphact/2.0/ together with fully commented examples of input files. All future improvements and bug fixes will be distributed through the site.

Acknowledgements

We gratefully acknowledge useful discussions and comparisons with Giampiero Passarino and Roberto Pittau. A.B. wishes to thank P. Bambade, M. Boonekamp, R. Chierici, F. Cossutti, E. Migliore and many other members of the Delphi Collaboration for advice, suggestions and discussions. We are grateful to F. Cossutti also for providing the code used to interface PHOTOS.

References

- [1] G. Altarelli, T. Sjöstrand, F. Zwirner (Eds.), Physics at LEP2, CERN 96-01 (1996).
- [2] S. Jadach, G. Passarino, R. Pittau (Eds.), Reports of the Working Groups on Precision Calculations for LEP2 Physics, CERN 2000-009 (2000).
- [3] E. Accomando, A. Ballestrero, Comput. Phys. Comm. 99 (1997) 270, hep-ph/9607317.
- [4] T. Sjöstrand, Comput. Phys. Comm. 82 (1994) 74;
 - G.J. van Oldenborgh, P.J. Franzini, A. Borrelli, Comput. Phys. Comm. 83 (1994) 14, hep-ph/9402298;
 - F.A. Berends, R. Pittau, R. Kleiss, Comput. Phys. Comm. 85 (1995) 437, hep-ph/9409326;
 - F. Caravaglios, M. Moretti, Phys. Lett. B 358 (1995) 332, hep-ph/9507237;
 - F. Caravaglios, M. Moretti, Z. Phys. C 74 (1997) 291, hep-ph/9604316;
 - M. Skrzypek, S. Jadach, W. Placzek, Z. Was, Comput. Phys. Comm. 94 (1996) 216;
 - G. Montagna, O. Nicrosini, F. Piccinini, Comput. Phys. Comm. 90 (1995) 141, hep-ph/9506208;
 - G. Charlton, G. Montagna, O. Nicrosini, F. Piccinini, Comput. Phys. Comm. 99 (1997) 355, hep-ph/9609321;
 - H. Anlauf, P. Manakos, T. Ohl, H.D. Dahmen, hep-ph/9605457;
 - J. Fujimoto, et al., Comput. Phys. Comm. 100 (1997) 128, hep-ph/9605312;

- G. Passarino, Comput. Phys. Comm. 97 (1996) 261, hep-ph/9602302;
- D. Bardin, et al., Nucl. Phys. Proc. (Suppl.) 37B (1994) 148, hep-ph/9406340;
- D.Yu. Bardin, et al., Comput. Phys. Comm. 104 (1997) 161, hep-ph/9612409.
- [5] D. Bardin, et al., Event generators for WW physics, hep-ph/9709270 in Ref. [1], Vol. 2, p. 3.
- [6] M. Mangano, et al., Event generators for discovery physics, hep-ph/9602203 in Ref. [1], Vol. 2, p. 299.
- [7] J. Fujimoto, et al., Comput. Phys. Comm. 100 (1997) 128, hep-ph/9605312.
- [8] F.A. Berends, G.B. West, Phys. Rev. D 1 (1970) 122;
- Y. Kurihara, D. Perret-Gallix, Y. Shimizu, Phys. Lett. B 349 (1995) 367, hep-ph/9412215.
- [9] E.N. Argyres, et al., Phys. Lett. B 358 (1995) 339, hep-ph/9507216;
 W. Beenakker, et al., Nucl. Phys. B 500 (1997) 255, hep-ph/9612260;
 - U. Baur, D. Zeppenfeld, Phys. Rev. Lett. 75 (1995) 1002, hep-ph/9503344.
- [10] E. Accomando, A. Ballestrero, E. Maina, Phys. Lett. B 479 (2000) 209, hep-ph/9911489.
- [11] G. Passarino, Nucl. Phys. B 574 (2000) 451, hep-ph/9911482; Nucl. Phys. B 578 (2000) 3, hep-ph/0001212.
- [12] S. Jadach, et al., Comput. Phys. Comm. 140 (2001) 475, hep-ph/0104049; Comput. Phys. Comm. 119 (1999) 272, hep-ph/9906277.
- [13] A.S. Belyaev, et al., hep-ph/0101232;
 D.N. Kovalenko, A.E. Pukhov, Nucl. Instrum. Methods A 389 (1997) 299;
 - A.E. Pukhov, et al., hep-ph/9908288;
 - E.E. Boos, et al., hep-ph/9503280.
- [14] T. Ishikawa, et al., KEK Report 92-19, 1993, The GRACE Manual Ver.1.0;
 H. Tanaka, Comput. Phys. Comm. 58 (1990) 153;
 H. Tanaka, T. Kaneko, Y. Shimizu, Comput. Phys. Comm. 64 (1991) 149.
- [15] F.A. Berends, C.G. Papadopoulos, R. Pittau, Comput. Phys. Comm. 136 (2001) 148, hep-ph/0011031.
- [16] G. Montagna, et al., Eur. Phys. J. C 20 (2001) 217, hep-ph/0005121.
- [17] M.W. Grünewald, et al., Four-fermion production in electron-positron collisions, hep-ph/0005309 in Ref. [2], p. 1.
- [18] R.G. Stuart, Phys. Lett. B 262 (1991) 113;
 A. Aeppli, G.J. van Oldenborgh, D. Wyler, Nucl. Phys. B 428 (1994) 126, hep-ph/9312212;
 W. Beenakker, F.A. Berends, P. Chapovsky, Nucl. Phys. B 548 (1999) 3, hep-ph/9811481;
- A. Denner, S. Dittmaier, M. Roth, Nucl. Phys. B 519 (1998) 39, hep-ph/9710521.
 [19] A. Denner, S. Dittmaier, M. Roth, D. Wackeroth, Phys. Lett. B 475 (2000) 127, hep-ph/9912261; J. Phys. G 26 (2000) 593, hep-ph/9912290; Eur. Phys. J. direct C 4 (2000) 1, hep-ph/9912447; Nucl. Phys. B 587 (2000) 67, hep-ph/0006307.
- [20] S. Jadach, et al., Comput. Phys. Comm. 140 (2001) 432, hep-ph/0103163; Phys. Lett. B 417 (1998) 326, hep-ph/9705429; Phys. Rev. D 61 (2000) 113010, hep-ph/9907436.
- [21] J. Fujimoto, Y. Shimizu, T. Munehisa, Prog. Theor. Phys. 91 (1994) 333, hep-ph/9311368;
 T. Munehisa, J. Fujimoto, Y. Kurihara, Y. Shimizu, Prog. Theor. Phys. 95 (1996) 375, hep-ph/9603322;
 Y. Kurihara, et al., Prog. Theor. Phys. 103 (2000) 1199, hep-ph/9912520; Eur. Phys. J. C 20 (2001) 253, hep-ph/0011276.
- [22] M. Boonekamp, hep-ph/0111213.
- [23] T. Ohl, Comput. Phys. Comm. 101 (1997) 269, hep-ph/9607454.
- [24] A. Ballestrero, E. Maina, Phys. Lett. B 350 (1995) 225, hep-ph/9403244.
- [25] A. Ballestrero, PHACT 1.0—Program for helicity amplitudes calculations with Tau matrices, hep-ph/9911318, in: B.B. Levchenko, V.I. Savrin (Eds.), Proceedings of the 14th International Workshop on High Energy Physics and Quantum Field Theory (QFTHEP 99), SINP MSU, Moscow, p. 303.
- [26] G.P. Lepage, J. Comput. Phys. 27 (1978) 192.
- [27] T. Sjöstrand, et al., Comput. Phys. Comm. 135 (2001) 238, hep-ph/0010017;
 T. Sjöstrand, L. Lönnblab, S. Mrenna, hep-ph/0108264.
- [28] Report on QCD Event Generators in Ref. [1].
- [29] Bhattacharya, J. Smith, G. Grammer Jr., Phys. Rev. D 15 (1977) 3267.
- [30] E.A. Kuraev, V.S. Fadin, Sov. J. Nucl. Phys. 41 (1985) 466;
 G. Altarelli, G. Martinelli, in: J. Ellis, R. Peccei (Eds.), Physics at LEP, CERN Report 86-02, CERN, Geneva, 1986, p. 47.
- [31] W. Beenakker, F.A. Berends, S.C. van der Marck, Nucl. Phys. B 349 (1991) 323;
 Y. Kurihara, et al., Prog. Theor. Phys. 103 (2000) 1199, hep-ph/9912520.
- [32] G. Montagna, et al., Eur. Phys. J. C 20 (2001) 217, hep-ph/0005121.
- [33] Y. Kurihara, et al., Eur. Phys. J. C 20 (2001) 253, hep-ph/0011276.
- [34] G. Passarino, Nucl. Phys. B 619 (2001) 313, hep-ph/0108255; hep-ph/0101139.
- [35] E. Barberio, Z. Was, Comput. Phys. Comm. 79 (1994) 291;
- E. Barberio, B. van Eijk, Z. Was, Comput. Phys. Comm. 66 (1991) 115.
- [36] D. Bardin, W. Beenakker, A. Denner, Phys. Lett. 317 (1993) 213.
- [37] A.P. Chapovsky, V.A. Khoze, Eur. Phys. J. C 9 (1999) 449, hep-ph/9902343.

- [38] V.S. Fadin, V.A. Khoze, A.D. Martin, A. Chapovsky, Phys. Rev. D 52 (1995) 1377, hep-ph/9501214;
 V.S. Fadin, V.A. Khoze, A.D. Martin, Phys. Lett. B 311 (1993) 311.
- [39] G. Passarino, Comput. Phys. Comm. 97 (1996) 261, hep-ph/9602302.
- [40] M. Carena, J.R. Espinosa, M. Quiros, C.E.M. Wagner, Phys. Lett. B 355 (1995) 209, hep-ph/9504316.
- [41] M. Bilenky, J.L. Kneur, F.M. Renard, D. Schildknecht, Nucl. Phys. B 409 (1993) 22;
 F.A. Berends, A.L. van Sighem, Nucl. Phys. B 454 (1995) 467, hep-ph/9506391.

TEST RUN

We report here an example of test run for one of the single Z cross sections presented in Fig. 1.

Input

WPHACT requires from standard input the name of the file from which all flags are read. The content of this file for this case is reported below. In the distribution of WPHACT a complete and fully commented example of input file is included. The comments, which remind the meaning and the available choices of the various flags, are not reproduced here, but they can be left in place in actual computations as they are ignored by the program.

ionesh	0				
e_cm	199.5d0				
rmw	80.40d0				
iproc	26				
ickm	0				
ich	1				
imass	1				
icc3	0				
izz	0				
izg	0				
izg34	0				
izg56	0				
igamgams	0				
inc08	0				
imulper	0				
igamgam	0				
itch	0				
itchnomp	0				
ips_cc	3				
ips_nc	3				
ismallangle	1				
ismallangle_e	ee 3				
ialftsw	0				
ialfar	1				
iresonance	0				
isr	2				
ibeam	0				
icoul	0				
istrcor	1				
ianc	0				
ipr	1				
iswgcomp	1				
igwcomp	1				
igzcomp	1				
ighcomp	1				
icut	1				
e_min	0.d0 3.d0 0	.d0 0.d0			
e_max	5000.d0 5000.	d0 5000.d0	5000.d0		
rm_min	0.d0 0.d0 0	.d0 0.d0	0.d0 60.d0	C	
rm_max	5000.d0 5000.	d0 5000.d0	5000.d0	5000.d0	5000.d0
pt_min	0.d0 0.d0 0	.d0 0.d0			

pt_max	5000.d0	5000.d	l0 5000.	.d0 5000	.d0	
icos	0					
iext	0	0 0	0			
thbeam_min	168.d0	60.d0	0.d0 ().d0		
thbeam_max	180.d0	168.d0	180.d0	180.d0		
thsep_min	0.d0 0	.d0 0.	d0 0.d0	0.d0	0.d0	
thsep_max	180.d0	180.d0	180.d0	180.d0	180.d0	180.d0
idistr	0					
iflat	0					
acc	0.d	C				
iterm	1					
ncall_term	2000	000				
itmx_term	4					
ncall	4000	0000				
itmx	4					

Output

In the output file produced by WPHACT one will recover all relevant input flags (not reported here) followed (for ionesh=0) by the process name, a list of physical parameters, the result of the thermalization iterations and finally the result of the iterations for the actual integration.

NC48) e-(p3) e+(p4) mu-(p5) mu+(p6)

INPUT			
cm energy	=	0.1995000D+03 GeV	
DATA			
Z mass	=	0.9118700D+02	GeV
W mass	=	0.804000D+02	GeV
electron mass	=	0.5109991D-03	GeV
mu mass	=	0.1056583D+00	GeV
tau mass	=	0.1777000D+01	GeV
up mass	=	0.500000D-02	GeV
down mass	=	0.100000D-01	GeV
charm mass	=	0.130000D+01	GeV
strange mass	=	0.200000D+00	GeV
top mass	=	0.1750000D+03	GeV
bottom mass	=	0.480000D+01	GeV
Gf	=	0.1166390D-04	GeV-2
alfas_nc	=	0.1230000D+00	
DERIVED QUANTITIES			
W width	=	0.2101131D+01	GeV
Z width	=	0.2506693D+01	GeV
s2w	=	0.2225969D+00	
1/alfa_em	=	0.1323609D+03	
$1/alfa_em(q**2=0)$	=	0.1370360D+03	

OPTIONS

```
Z s-dependent width
QEDPS
Naive QCD corrections included
Single resonant (56) "NC" phase space
_____
Thermalization
input parameters for vegas: ndim= 7 ncall=
                                           156250.
                         it= 1 itmx= 4
iteration no. 1:
                          effective ncall=
                                            60114
iteration no. 1: integral = 0.2211926E-01+/- 0.37E-03
all iterations: integral = 0.2211926E-01+/- 0.367E-03 chi**2/it'n = 0.00E+00
iteration no. 2:
                         effective ncall=
                                            93901
iteration no. 2: integral = 0.2232726E-01+/- 0.15E-03
all iterations: integral = 0.2229726E-01+/- 0.139E-03 chi**2/it'n = 0.27
iteration no. 3:
                         effective ncall=
                                           95374
iteration no. 3: integral = 0.2234192E-01+/- 0.14E-03
all iterations: integral = 0.2231946E-01+/- 0.987E-04 chi**2/it'n = 0.16
iteration no. 4:
                         effective ncall=
                                            93411
iteration no. 4: integral = 0.2228780E-01+/- 0.11E-03
all iterations: integral = 0.2230523E-01+/- 0.733E-04 chi**2/it'n = 0.12
NC process
input parameters for vegas: ndim= 7 ncall= 3294172.
                         it= 1 itmx=
                                          4
iteration no. 1:
                         effective ncall= 2027248
iteration no. 1: integral = 0.2237196E-01+/- 0.21E-04
all iterations: integral = 0.2237196E-01+/- 0.208E-04 chi**2/it'n = 0.00E+00
iteration no. 2:
                          effective ncall= 2012237
iteration no. 2: integral = 0.2238736E-01+/- 0.20E-04
all iterations: integral = 0.2237985E-01+/- 0.145E-04 chi**2/it'n = 0.28
                          effective ncall= 1985637
iteration no. 3:
iteration no. 3: integral = 0.2238947E-01+/- 0.24E-04
all iterations: integral = 0.2238239E-01+/- 0.124E-04 chi**2/it'n = 0.20
iteration no. 4:
                         effective ncall=
                                         1964560
iteration no. 4: integral = 0.2238508E-01+/- 0.21E-04
all iterations: integral = 0.2238308E-01+/- 0.107E-04 chi**2/it'n = 0.14
```

Sigma = 0.2238308D-01 +/- 0.107D-04 (pb)

196