The Annals of Applied Probability
2014, Vol. 24, No. 1, 447–448
DOI: 10.1214/13-AAP928

A NOTE ON "BAYESIAN NONPARAMETRIC ESTIMATORS DERIVED FROM CONDITIONAL GIBBS STRUCTURES"

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The present note aims at clarifying some possibly confusing notation used in [1], even if its correct interpretation should be clear from context and from the proofs contained therein. In particular, the expression

$$\mathbb{P}[K_n = k, \mathbf{N}_n = (n_1, \ldots, n_{K_n})] \tag{$*$}$$

displayed in (3) of [1] has been used to indicate the probability of observing a specific realization of a random partition of the integers $[n] = \{1, \ldots, n\}$ into k blocks of sizes $(n_1, \ldots, n_k) \in \Delta_{n,k}$. However, the notation in ($*$) may be actually misleading since its natural interpretation is as the probability of all partitions of $[n] = \{1, \ldots, n\}$ into k blocks of sizes $(n_1, \ldots, n_k) \in \Delta_{n,k}$. For this reason, (3) in [1] should be rewritten as

$$\Pi_k^{(n)}(n_1, \ldots, n_k) = \frac{\theta^k}{(\theta)_n} \prod_{j=1}^{k} (n_j - 1)!, \tag{3}$$

where $\Pi_k^{(n)}$ is the *exchangeable partition probability function* notation introduced in Section 2. Hence, if Π_n is a random element taking values in the set of all partitions of $[n]$, one has $\Pi_k^{(n)}(n_1, \ldots, n_k) = \mathbb{P}[\Pi_n = \pi]$ for any partition π of $[n]$ into k blocks $\{A_1, \ldots, A_k\}$ with $|A_i| = n_i$, for $i = 1, \ldots, k$. See also the first displayed equation on page 1522 after (4) in [1]. The probability of all partitions of $[n]$ into k blocks with respective sizes $n_1, \ldots, n_k$ (in exchangeable order) is then equal to $n!\Pi_k^{(n)}(n_1, \ldots, n_k)/(k!n_1! \cdots n_k!)$. The same caveat also applies to (28) and (44)–(46) in [1].

An analogous clarification concerns

$$\mathbb{P}[K_m^{(n)} = k, L_m^{(n)} = s, \mathbf{S}_{L_m^{(n)}} = (s_1, \ldots, s_{K_m^{(n)}}) | K_n = j] \tag{$**$}$$

displayed in (9) of [1]. Indeed, ($**$) has been used to denote the probability of a specific partition of $[s]$ into k blocks with sizes $(s_1, \ldots, s_k)$ in $\Delta_{s,k}$, given any partition of $[n]$ into j parts, thus differing from its natural meaning as the probability

Received February 2013.

of all such partitions. Our intepretation of $(**)$ in [1] is clearly consistent with the right-hand side of (9), that is,

$$\frac{V_{n+m,j+k}}{V_{n,j}} \binom{m}{s} (n - j\sigma)_{m-s} \prod_{i=1}^{k} (1-\sigma)_{s_i - 1}$$

and with the result displayed in Corollary 1, where equation (10) arises after multiplying the right-hand side of (9) by $s!/(k!s_1! \cdots s_k!)$ and, then, marginalizing with respect to all $(s_1, \ldots, s_k)$ in $\Delta_{s,k}$. Similar remarks apply to the notation appearing in the following displayed formulas: (17), (19), equation at the end of page 1528, (21), equation right after Corollary 2 on page 1529, (22), equation at the end of page 1531, (34).

REFERENCES

[1] LIJOI, A., PRÜNSTER, I. and WALKER, S. G. (2008). Bayesian nonparametric estimators derived from conditional Gibbs structures. *Ann. Appl. Probab.* **18** 1519–1547. MR2434179

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