The Cutoff Policy of Taxation When Taxpayers are Risk Averse

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The Cutoff Policy of Taxation When Taxpayers are Risk Averse

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Abstract

In this paper we model taxpayers reactions to the possibility of either reporting income as usual and running the risk of an audit or reporting a "cutoff" income and paying a threshold tax that gives the certainty of not being audited. Models of this kind already discussed in the literature assume that taxpayers are risk neutral. We depart from this stream of research by assuming instead that taxpayers have a Constant Relative Risk Aversion (CRRA) utility function and differ in relative risk aversion coefficient and in income. The government can rely upon a signal in order to assess the income class to which the taxpayer belongs and fix the threshold tax. Our main result is that, within each class, the threshold tax is paid by taxpayers whose relative risk aversion lies in a given interval. Taxpayers with high risk aversion and relatively low income file their report as usual. Unlike analogous models under risk neutrality, relatively rich taxpayers with low risk aversion might not pay the threshold tax as well. An equity problem arises.

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1 Introduction

Under a cutoff policy the Tax Administration audits, with a given probability, each taxpayer who reports an income that falls short of a given threshold; if the income report instead meets or is above the threshold, it is not audited. If taxpayers are risk-neutral, and the expected sanction for tax evasion is large enough, the effects of the cutoff rule are that taxpayers whose income (and thus whose tax[1]) is lower than the threshold pay their due tax and risk audits, while those who owe a tax equal to or higher than the threshold pay the threshold tax and avoid audits. Overall payments made by taxpayers are non decreasing in income. This approach has been examined in many respects in the literature (see *e.g.* [11], [13],[5], [12], and, for a generalization, [3]), mainly as an efficient strategy in agency models in which the Tax Administration as a principal can commit to a given audit policy. A cutoff property has, however, been shown to hold also for equilibrium in the taxation game under given conditions [7].

According to the principal-agent approach, the cutoff policy may entail efficiency gains, as it secures savings in terms of audit costs that might exceed the revenue losses in taxation. The mechanism has characteristics similar to those of plea-bargaining in criminal proceedings[2], as taxpayers who pay the threshold tax can be assimilated to those who, by pleading guilty, receive a milder treatment than the due one according to general rules. As in plea-bargaining under risk-neutrality, those who accept the threshold offer are those who would receive a harsher treatment under general rules if the government possessed full information. There might thus be an equity/efficiency trade-off. In taxation the cutoff rule has been criticized from an equity point of view, as it introduces a regressive bias, because taxpayers payments strictly increase in income only until the threshold level. However, an approach based on audit classes, as suggested by Scotchmer [13], might countervail this effect.

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[1]For the sake of simplicity the income tax is described as a function of reported income, disregarding possible differences between reported (gross) income and net taxable income due to exemptions, deductions etc.

[2]See Grossman and Katz [6].

Cremer, Marchand and Pestieau [5], in their conclusions about the cut-off policy, put at the top of the agenda for future research the relaxation of the risk neutrality assumption. Since then, however, as far as we know, their suggestion has been largely neglected. In this paper we try to address the problem, even by adopting a partial equilibrium perspective in order to overcome the technical difficulties involved.

When taxpayers are risk averse, the cutoff system can perform a role which is by definition excluded under risk neutrality: that of collecting risk premia for the insurance against audits provided by the cutoff[3]. When the tax rate and the enforcement parameters are such that the expected yield of tax evasion is positive, this characteristic may render the cutoff policy profitable even disregarding the benefits in terms of reducing the number of audits to run. In view of the exploitation of this opportunity, we assume that the Tax Administration, relying upon a signal, forms taxpayers classes, and credibly sets a threshold tax for each one, by adding a risk premium to the expected payment of the class under general rules. We assume that taxpayers are informed about these thresholds and we focus upon the taxpayers' reaction to such offers.

The signal received by the Tax Administration refers to the optimal report which the taxpayer would choose to submit under general rules (leaving the cutoff offer aside), instead than to the taxpayer's true income, as e.g. in Scotchmer [13]. Since the Tax Administration has a much larger experience in receiving income reports than in assessing true incomes through audits, it seems likely that it should more easily exploit information pertaining to the former than to the latter in order to shape the cutoff policy.

We aim at capturing in the model some features of the practice of tax enforcement. In many countries only a minor share of controls is random, while most are based upon procedures (for the United States, see [2]) that directly or indirectly refer to predictors of a reference income (and hence tax) which is compared to the reported one, for deciding whether to start the control. Thus the taxpayer who meets the tax payment expected from him on the basis of personal and economic characteristics (the threshold tax in the

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[3]For this approach, see also [4].

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model), does not run the risk of being audited.

The assumption that taxpayers are perfectly informed about the cutoff system is fully realistic when the threshold tax offer takes an explicit form, like in the FATOTA (Fixed Amount of Taxes or Tax Audit) system analyzed by Chu [4], in which the taxpayer must opt for the FAT if she wants to insure against tax audits. The assumption is quite reasonable also for countries in which the Tax Administration publishes reference guides for assessing income on the basis of economic indicators, or largely resorts to the cooperation of employees, business and professional organizations, which perform a filter role in assessing incomes and collecting reports (examples can be found in France, Italy and Spain). Even in countries, like the US, in which the IRS guards the secret about the details of the mechanisms that trigger controls, taxpayers gain some information by *ex-post* observation of the IRS behavior[4]. The topic thus seems relevant, as the cutoff approach represents a policy option that some countries, more or less thoroughly, adopt and others might find it profitable to take into consideration.

In this paper we analyze one component of the problem of a cutoff policy under risk aversion: the taxpayers reaction to the threshold tax offer. This is a limited but not trivial task, as we assume that taxpayers are heterogenous in terms of income and relative risk aversion. Hence, for given parameters that characterize the tax system (tax function, penalty and probability of control), a given tax report that does not meet the threshold may hide many possible tax evasion levels. Our model aims at characterizing those taxpayers who choose to pay the threshold tax, in comparison with those who refuse it, in terms of income and relative risk aversion.

The main result of the paper is that, when taxpayers have a CRRA utility function, the threshold tax is paid by an intermediate interval of taxpayers. Taxpayers with low income relative to their class and high risk aversion file their report as usual and evade only a small amount of their due tax. Unlike

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[4]We have found on the Internet free-of-charge sites that provide statistics about factors that increase the audit probability and, on the basis of the declaration one submits, assess the audit probability.

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in models under risk neutrality, rich taxpayers with low risk aversion might also refuse to pay the threshold tax. This fact gives rise to equity problems within each class, while among classes vertical equity, in a mild sense that will be clarified in Section 5, is being preserved. The taxpayers reaction takes at any rate a clearly predictable pattern, which can form the basis for assessing the equity-efficiency trade-off of the cutoff policy.

The paper is organized as follows. In Section 2 we describe the taxpayer's problem with reference to both the optimal income report and the conditions needed for the acceptance of a cutoff proposal. In Section 3 we characterize, in terms of both true income and relative risk aversion, those who pay the threshold tax, within each class. A numerical example referred to a single class is presented in Section 4. In Section 5 the general properties of a cutoff policy based on classes are characterized. Some conclusions are drawn in Section 6, while the Appendix contains most of the proofs.

2 Modelling the Taxpayer's Problem

Let us study the taxpayer's optimal report, with reference only to general tax rules, and setting the cutoff policy aside for the moment. It is assumed that the taxpayer's true income is a random variable $\widehat{w}$, whose realization w is known only by himself. The utility that the taxpayer enjoys out of her income w is assumed to be of the standard CRRA form, with constant relative risk-aversion coefficient α:

$$u\left(w\right) = \frac{w^{1-\alpha} - 1}{1 - \alpha}. \tag{1}$$

In this class we also include the case $\alpha = 1$ by taking the limit of the right hand side in (1) for $\alpha \to 1$, obtaining $u(w) = \ln w$.

A proportional tax system[5] is considered: the income tax is given by $t(y) = ty$, where y denotes the *reported income* and $0 < t < 1$. We assume

[5]A version considering a progressive tax and a sanction linear in concealed income, available from the authors upon request, leads to the same main results of the present paper and is much more cumbersome.

also that the sanction to be paid in case of audit[6] is proportional to the amount of the evaded tax:

$$S\left(w, y\right) = (1 + s)t\left(w - y\right), \qquad (2)$$

where $s > 0$ is a penalty rate.

As we rule out rewards to honest taxpayers by assumption[7], a taxpayer will report $y \leq w$, where $w > 0$ denotes the *true income*. A rational taxpayer who earned a true income w will choose to report the income y^* that maximizes her expected utility

$$\mathbb{E}u(y) = \frac{(1 - p)\left(w - ty\right)^{1-\alpha} + p\left[w - ty - (1 + s)t\left(w - y\right)\right]^{1-\alpha} - 1}{1 - \alpha} \qquad (3)$$

with respect to y, where $0 < p < 1$ is the probability of detection. Note that, by considering $u(w) = \ln w$ when $\alpha = 1$, $\mathbb{E}u(y)$ is well defined for all $\alpha > 0$ and for all feasible y.

The feasible set contains values for y that satisfy

$$(1 + s)t\left(w - y\right) < w - ty. \qquad (4)$$

In other words, we need to assume that the taxpayer can always bear the loss in case of detected evasion. Inequality (4) defines a lower bound for the feasible reported income

$$m_w = \frac{\left[(1 + s)t - 1\right]w}{ts}$$

such that $m_w < y$. Since we are interested in a strictly positive income report, we shall assume that

$$(1 + s)t > 1.$$

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[6] We stick to the standard assumption that detection of tax evasion occurs with probability 1 whenever the tax report is false and an audit is run.

[7] This is also a standard assumption, even if the theory of optimal auditing provides reasons in favor of rewards to audited honest risk-averse taxpayers (see Mookherjee and Png [10]).

This implies that sanctions are large enough to exclude full evasion[8]. There-fore, the feasible set of values for the reported income y is the interval $(m_w, w] \subset \mathbb{R}_{++}$.

In accordance with empirical evidence, we assume that the tax system parameters have values such that cheating in reporting income has a positive expected return. In other words, the expected sanction is assumed to be less than the expected gain for each dollar invested in tax evasion: $sp < 1 - p$. Hence, also the case of full compliance, $y = w$, is ruled out, as it is readily seen by noting that the limit of the marginal expected utility $[\mathbb{E}u(y)]'$ as $y \to w^-$ is negative whenever $sp < 1 - p$.

Since, on the other hand, $\lim_{y \to m_w^+} [\mathbb{E}u(y)]' = +\infty$ and $\mathbb{E}u(y)$ is strictly concave over (m_w, w) for all $\alpha > 0$, there exists a unique value $m_w < y^* < w$ that maximizes the expected utility, which is completely characterized in terms of F.O.C. applied to (3). That is,

$$\frac{w - ty^* - (1+s)t\,(w - y^*)}{w - ty^*} = r^{\frac{1}{\alpha}} \tag{5}$$

where

$$0 < r = \frac{ps}{1 - p} < 1.$$

By solving (5) for y^* it turns out that the optimal reported income is a fixed share of the taxpayer's true income w, that depends on the risk aversion coefficient α:

$$y^* = \frac{(1+s)t + r^{\frac{1}{\alpha}} - 1}{t\left(s + r^{\frac{1}{\alpha}}\right)} w. \tag{6}$$

Note that a larger risk aversion implies a larger share.

2.1 Introducing a Cutoff

For simplicity of presentation we assume in this Section that the Tax Admin-istration is able to exactly identify all the taxpayers whose optimal reported

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[8]The literature that considers optimal income report under risk aversion has routinely focussed upon strictly positive reports, see, *e.g.*, Allingham and Sandmo [1].

income is y^*, independently on the actual submission of the report. This assumption shall be relaxed in Section 5.

For future use, we note that the government can make some inference about the true income w of taxpayers whose optimal reported income is y^*. By solving equation (5) for w, when y^* is reported, we obtain a relation expressing the true income of (optimizing) taxpayers as a function of their relative risk aversion α:

$$w\left(\alpha\right) = \frac{\left(r^{\frac{1}{\alpha}} + s\right) t y^*}{r^{\frac{1}{\alpha}} + (1+s)\, t - 1} \qquad (7)$$

As can be easily checked, the function $w\left(\alpha\right)$ in (7) is strictly decreasing.

We assume also that, in order to implement the cutoff policy, the Tax Administration, which is risk neutral, sets a threshold tax that amounts to $C(y^*) = t y^* + x$, where the premium x is strictly positive and is aimed at cashing the expected value of the sanction plus the risk premium.

The expected reactions of the taxpayer are as follows. She will accept the offer if she is at least indifferent as whether to pay the requested amount $C(y^*)$ or to pay only $t y^*$ and risk an audit. Thus, for a given premium $x > 0$, in order to choose to pay the threshold tax the following condition must be met:

$$\frac{[w - (ty^* + x)]^{1-\alpha}}{1-\alpha} \geq \frac{(1-p)\,(w - ty^*)^{1-\alpha}}{1-\alpha} \qquad (8)$$
$$+ \frac{p\,[w - ty^* - (1+s)t\,(w - y^*)]^{1-\alpha}}{1-\alpha},$$

where the additive constants $-\left(1-\alpha\right)^{-1}$ have already been dropped from both sides.

By considering jointly the optimal condition (7) and the threshold condi-

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tion (8), we are led to the following system:

$$
\begin{cases}
w = \dfrac{\left(r^{\frac{1}{\alpha}} + s\right) ty^*}{r^{\frac{1}{\alpha}} + (1+s)\,t - 1} \\[2em]
\dfrac{(w - ty^* - x)^{1-\alpha}}{1 - \alpha} \;\geq\; \dfrac{(1-p)\,(w - ty^*)^{1-\alpha}}{1 - \alpha} \\[2em]
\qquad\qquad\qquad + \dfrac{p\,[w - ty^* - (1+s)t\,(w - y^*)]^{1-\alpha}}{1 - \alpha}
\end{cases} \tag{9}
$$

where the reported income y^* and premium x are given and α and w are the unknowns. The first equation of system (9) links the taxpayer's true income w to y^* according to (5). The second equation is the weak preference condition for paying the threshold tax instead of reporting y^* and risking an audit. All pairs (α, w) solving system (9), characterize in terms of relative risk-aversion α and true income w the subset of taxpayers whose optimal reported income is y^* and who prefer to pay the threshold tax instead of optimally filing a tax report and risking audits.

2.2 Who pays the threshold tax?

By plugging the first equality in system (9) into the second inequality, we obtain a single inequality where the unknown is the sole variable α:

$$
\frac{1}{1-\alpha}\left(A - Br^{\frac{1}{\alpha}}\right)^{1-\alpha} \geq \frac{1}{1-\alpha}\left[1 - p + p\left(r^{\frac{1}{\alpha}}\right)^{1-\alpha}\right], \tag{10}
$$

where A, B and r are constants defined by

$$
A = 1 - \frac{[(1+s)\,t - 1]\,x}{(1+s)\,(1-t)\,ty^*} \tag{11}
$$

$$
B = \frac{x}{(1+s)\,(1-t)\,ty^*} \tag{12}
$$

$$
r = \frac{ps}{1-p}. \tag{13}
$$

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Since $(1+s)t > 1$, $A < 1$; moreover, $B > 0$ as long as $x > 0$. To study inequality (10), we need also $0 < x < (1-t)y^*$ to hold. That is, the premium x must be smaller than the disposable income when tax evasion is small, *i.e.*, y^* is very close to w. Note that this bound implies $0 < B < A < 1$, and thus also $0 < A - B < A - Br < 1$ holds. Therefore, inequality (10) - and thus system (9) - is equivalent to

$$\begin{cases} f(\alpha) \geq 0 & \text{if } 0 < \alpha \leq 1 \\ f(\alpha) \leq 0 & \text{if } \alpha \geq 1. \end{cases} \tag{14}$$

where $f : \mathbb{R}_{++} \to \mathbb{R}$ is defined by

$$f(\alpha) = \left(A - Br^{\frac{1}{\alpha}}\right)^{1-\alpha} - p\left(r^{\frac{1}{\alpha}}\right)^{1-\alpha} - (1-p). \tag{15}$$

Note that $f(\alpha)$ is well defined for all $\alpha > 0$ and is C^∞. Moreover, $f(1) = 0$. However, since the original system (9) is defined for $\alpha \neq 1$, it is readily understood from system (14) that a sufficient condition for a nonempty solution set, that is, a positive measure of taxpayers (described by a non trivial interval of values of relative risk aversion α) opting for the cutoff, is that f crosses the horizontal axis from above at $\alpha = 1$.

Function f defined in (15) does not allow for a closed form solution of (14). Hence, we shall characterize solutions of a slightly simplified system and under some further conditions. Specifically, we shall use a suitable lower bound $l < f$ for $0 < \alpha < 1$, while for $\alpha \geq 1$ we will be able to characterize solutions only for a subclass of models. However, we shall see that our technique covers the most meaningful cases.

We now decompose function $f(\alpha)$ in three parts in order to single out the most problematic ones and replace them with a suitable approximation. Let

$$\phi(\alpha) = \left(A - Br^{\frac{1}{\alpha}}\right)^{1-\alpha}$$

$$\varphi(\alpha) = -p\left(r^{\frac{1}{\alpha}}\right)^{1-\alpha}.$$

Clearly $f = \phi + \varphi - (1-p)$. Now define

$$\psi(\alpha) = 1 - \ln(A - Br)(\alpha - 1)$$

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and

$$l(\alpha) = \begin{cases} \psi(\alpha) + \varphi(\alpha) - (1-p) & \text{for } 0 < \alpha < 1 \\ f(\alpha) & \text{for } \alpha \geq 1. \end{cases}$$

We shall characterize solutions of the system

$$\begin{cases} l(\alpha) \geq 0 & \text{if } 0 < \alpha \leq 1 \\ l(\alpha) \leq 0 & \text{if } \alpha \geq 1. \end{cases} \tag{16}$$

The construction of function l is motivated as follows. Since function ϕ is not well behaved over $(0,1)$, being neither convex nor concave over the whole interval, we substitute it with the convex lower bound $(A - Br)^{1-\alpha}$, then we further lower it by taking its first order approximation centered on $\alpha = 1$ (Lemma 1 in the Appendix shows that $\psi(\alpha) < \phi(\alpha)$). Thus, l turns out to be a lower bound[9] for f on the interval $(0,1)$, with an improved (linearized) shape for component ϕ in f; while, by construction, $l = f$ for all $\alpha \geq 1$. Therefore, solutions of (16) are a subset of solutions of (14); in particular, some points in the "left-side" solution set of (14), a subset of interval $(0,1)$, are lost through our approximation. Note that, by construction, $l'(1) = f'(1) = -\ln(A - Br) + p \ln r$.

3   The Main Result

The following result completely describes the solution set of our simplified model, system (16). Following the discussion in the previous sections, we need to restrict the range of the parameters of the model, as summarized below.

A. 1 *The sanction rate s and the tax rate t satisfy*

$$(1+s)\, t > 1.$$

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[9] A lower (as opposed to a higher) bound is the basis for prudentially assessing the government revenue under a cutoff policy.

A. 2 *The premium x satisfies*

$$0 < x < (1 - t) y^*.$$

Moreover, the following restriction is necessary in the proof of Proposition 1.

A. 3 *Probability of detection p and the sanction rate s satisfy*

$$r = \frac{ps}{1 - p} \le e^{-2}.$$

Assumption A.3 is purely technical. The idea behind the proof of Proposition 1, however, can be easily replicated through a symmetrical argument to obtain analogous results for the case $r > e^{-2}$.

Proposition 1 *Suppose Assumptions A.1, A.2 and A.3 hold true. Then the solution set $S \subseteq \mathbb{R}_{++}$ of system (16) has the following properties.*

i) *If A.3 holds with equality, then S is a nonempty interval[10]: $S = [\underline{\alpha}, \overline{\alpha}]$, with $0 \le \underline{\alpha} < 1 < \overline{\alpha} < +\infty$, if and only if*

$$x < \frac{(1 + s)(1 - t) t y^*}{(1 + s) t - (1 - e^{-2})} \left(1 - e^{-2p}\right). \tag{17}$$

ii) *If A.3 holds with strict inequality, a sufficient condition for S to be nonempty and of the form $S = [\underline{\alpha}, \overline{\alpha}]$ with $0 \le \underline{\alpha} < 1 < \overline{\alpha} < +\infty$, is the following:*

$$x \le \frac{(1 + s)(1 - t) t y^*}{(1 + s) t - (1 - e^{-2})} \left\{ 1 - \left[1 + p(\ln r)\left(\frac{\ln r}{2} + 1\right)\right]^{\frac{2}{2 + \ln r}} \right\} = \overline{x}. \tag{18}$$

iii) *If*

$$x > \frac{(1 + s)(1 - t) t y^*}{(1 + s) t - (1 - r)} \left[1 - \exp\left(\frac{4e^{-2}p}{r \ln r}\right)\right], \tag{19}$$

then S is empty.

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[10] To be precise $S = (0, \overline{\alpha}]$ whenever $\underline{\alpha} = 0$, since $\mathbb{E}u(\cdot)$ is not defined for $\alpha = 0$.

Proof. See the Appendix. ∎

Proposition 1 characterizes the solution set S of system (16) by focussing on the properties of the function l. If A.3 holds with equality, then $l(\alpha) = \psi(\alpha) + \varphi(\alpha) - (1 - p)$ turns out to be strictly concave over $(0, 1)$ and strictly convex over $[1, +\infty)$, and has a unique flex-point at $\alpha = 1$. In order to have a non-trivial solution of system (16) we need $l(\alpha)$ to cross the abscissa from above, or $l'(1)$ to be negative, which is the same as condition (17). If A.3 holds with strict inequality, the flex point of $l(\alpha)$ is at some value $c > 1$. We extend the same argument of case (i) by constructing a function $h(\alpha)$ that is as similar as much as possible to l but is better shaped than l over the interval $(1, c)$ where the required convexity property of l cannot be verified directly. Condition (18) now ensures that $h(\alpha)$ crosses the abscissa from above. Finally condition (19) characterizes the case in which l crosses the abscissa at $\alpha = 1$ from below and thus the solution set S is empty[11].

3.1 Discussion

Proposition 1 is only theoretically meaningful: it provides the intrinsic shape of the solution set of a model that approximates (14). It basically states that taxpayers who pay the threshold tax, if any, have a relative risk aversion coefficient belonging to some interval which contains 1. According to the length of this interval, the following three cases can occur.

1. If $x \to 0$ then the cutoff tend to be accepted by all the taxpayers. This case is degenerate as it would not provide extra revenue for the government.

2. If $\underline{\alpha} = 0$ and $\overline{\alpha} > 1$, relatively rich taxpayers with low risk aversion, *i.e.* with $\alpha \leq \overline{\alpha}$, pay the threshold tax, while relatively poor ones with high

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[11] Clearly, in this last case, in order to let assumption A. 2 to be satisfied,

$$\frac{(1 + s)(1 - t)\,ty^*}{(1 + s)\,t - (1 - r)} \left[1 - \exp\left(\frac{4e^{-2}p}{r \ln r}\right)\right] < (1 - t)\,y^*$$

must hold. We shall see in the example provided in the Section 4 that this is the case for reasonable values of all parameters.

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risk aversion, that is with $\alpha > \overline{\alpha}$, do not accept the cutoff. The latter prefer to submit their optimal report y^*, which, by (7), conceals only a relatively small income amount, and risk audits. This finding parallels a similar result about the effect of the cutoff rule under risk neutrality. In our case, the result can be explained as follows. As reported income y^* approaches true income w for very risk averse taxpayers, the expected sanction decreases, while the increase in risk premium under a CRRA utility function does not countervail the former effect.

3. If $\underline{\alpha} > 0$ and $\overline{\alpha} > 1$, besides the reaction already discussed in the last point, the cutoff is refused also by taxpayers with a risk aversion coefficient below some lower bound $\underline{\alpha} < 1$, whose optimal income report y^*, again by (7), conceals a relatively large income amount. These taxpayers too prefer to risk audits instead of paying the threshold tax. Hence there are two groups of taxpayers who refuse the cutoff, one characterized by relatively high and the other by relatively low true income. This finding is new and does not correspond to the results reached under the assumption of risk neutrality. It deserves further investigation which will be tackled separately in the next Paragraph.

One might also consider the possibility of a refusal of the cutoff only by relatively rich and low risk averse taxpayers, that is a lower bound $\underline{\alpha}$ that remains strictly positive while $\overline{\alpha} \to \infty$. However, this is ruled out by the consideration that a premium $x \to 0$ is needed to ensure that taxpayers with risk aversion $\alpha \to \infty$ accept the cutoff; in this case also taxpayers with $\alpha \to 0$ would accept it.

3.2 The Reaction of Rich Taxpayers with Low Risk Aversion

The refusal of paying the threshold tax by rich taxpayers with low risk aversion arises under given conditions, as the following proposition shows. For simplicity, we consider only the general case (ii) of Proposition 1, that is when

assumption A.3 holds with strict inequality and the solution set $S = [\underline{\alpha}, \overline{\alpha}]$ is nonempty.

Proposition 2 *Under A.1 and A.2, assume that A.3 holds with strict inequality. Moreover, assume that $s \geq 1$. If the tax rate t is such that*

$$(1+s)\,t > \frac{1 - p\left(1 - e^{-2}\right) - \left[1 + p\left(\ln r\right)\left(\frac{\ln r}{2} + 1\right)\right]^{\frac{2}{2 + \ln r}}}{\left\{1 - p - \left[1 + p\left(\ln r\right)\left(\frac{\ln r}{2} + 1\right)\right]^{\frac{2}{2 + \ln r}}\right\}}, \qquad (20)$$

then there exist values of premium x such that $\underline{\alpha} > 0$. Specifically, $\underline{\alpha} > 0$ for all values of x satisfying

$$\frac{p\left(1 + s\right)\left(1 - t\right)ty^*}{\left(1 + s\right)t - 1} < x \leq \overline{x}, \qquad (21)$$

where $\overline{x}$ is defined in (18).

Proof. See the Appendix. ∎

Proposition 2 states that for a large enough sanction rate there exist (small) values of α - corresponding to high values of true income w - characterizing a group of taxpayers who do not pay the threshold tax.

Let us provide an intuitive explanation of the possible refusal of paying the threshold tax by rich taxpayers. For any optimal reported income y^*, there is an upper bound to the values that the true taxpayer's income can assume. Since $0 < r < 1$, by taking the limit in (7) for $\alpha \to 0^+$, one gets:

$$\lim_{\alpha \to 0^+} w\left(\alpha\right) = \frac{sty^*}{\left(1 + s\right)t - 1} = \overline{w} < +\infty. \qquad (22)$$

The upper bound $\overline{w}$ decreases in the sanction rate $(1 + s)\,t$. Hence, when the sanction rate is high, and thus $\overline{w}$ is low, it is possible that the requested x is too large relatively to the expected sanction plus the (about nil in relative terms) risk premium the agent is willing to pay, thus discouraging the payment of the threshold tax.

To summarize: since taxpayers with a CRRA utility function cannot bear ruin, even at low values of the risk aversion coefficient they might not bet very

large sums in tax evasion[12], while on the other hand their low risk aversion coefficient might imply the refusal of an insurance offer. Proposition 2 shows the conditions under which a high enough value of the sanction rate $(1 + s) t$ can originate this effect.

4   A Numerical Example

Let us study an example, with the following values of parameters: $t = 0.44$, $p = 0.01$, $s = 1.7$ and $y^* = 1000$. Assumptions A.1 and A.3 are verified since $(1 + s) t = 1.188 > 1$ and $r \simeq 0.017 < 0.135 \simeq e^{-2}$. In particular, condition A.3 holds with strict inequality and thus parts (ii) and (iii) of Proposition 1 will be relevant.

The upper bound for x in condition (18) turns out to be $\overline{x} \simeq 80$, which clearly satisfies assumption A.2. Therefore any fixed premium that satisfies $x \leq 80$ produces a nonempty interval $[\underline{\alpha}, \overline{\alpha}]$ of relative risk aversion coefficients characterizing agents who pay the threshold tax. For example, with $x = 50$, the interval has $\underline{\alpha} \simeq 0.357$ and $\overline{\alpha} \simeq 5.645$ as its extremes, as shown in figure 1 (a). These two values, through (7), correspond to a minimum true income $w \simeq 1426$ (corresponding to $\overline{\alpha} \simeq 5.64$) and a maximum true income $w \simeq 3946$ (corresponding to $\underline{\alpha} \simeq 0.64$), which imply an evasion (in terms of share of concealed income when y^* is reported) of around 30% and around 75% respectively.

The true income of taxpayers reporting $y^* = 1000$ lies in the interval $(1000, 3979)$. The lower value refers to full compliance, while the higher one to the maximal evasion compatible with limited liability, as a taxpayer with a true income of 3979 who reports 1000 would receive zero net income in case of detection.

In view of inequality (20), to observe some rich and low risk-averse taxpayers who refuse to pay the threshold tax, the sanction rate $(1 + s) t$ must be such that $(1 + s) t \gtrsim 1.0467$, given the values of the other parameters shown above, a condition which is met in this example as $(1 + s) t = 1.188$.

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[12]That is, the concealed amount cannot encompass $\overline{w} - y^*$.

The lower bound for x expressed by condition (19) in (iii) of Proposition 1 is $\underline{x} \simeq 242$, hence any fixed premium that satisfies $242 < x < 560 = (1-t)\,y^*$ is in the range of assumption A.2 and produces an empty solution set. This means that function l is strictly increasing and crosses the x axis at the unique point $\alpha = 1$, as is shown in figure 1 (b) for $x = 292$.

For any value between $\overline{x} \simeq 80$ and $\underline{x} \simeq 242$ Proposition 1 cannot be applied and, in principle, nothing can be said. The sufficient condition (18) fails to detect the existence of a nonempty interval of values of relative risk aversion coefficients even if such an interval exists for some values of parameter $x \in [\overline{x}, \underline{x}]$. We have worked out, however, many numerical counterexamples which support the robustness of Proposition 1; these counterexamples show that Proposition 1 proves to be useless only in special cases where the possible interval is extremely tiny, a circumstance that does not seem very appealing for the government. Note also that the error on the lower bound $\underline{\alpha}$ introduced by considering the approximated model (16) in place of (14) affects only the side on the left of $\alpha = 1$ and seems negligible.

5   Classes

When the Tax Administration faces a taxpayer whose optimal reported income is y^*, thanks to (22), it can infer that the true taxpayer's income w belongs to the open interval $(y^*, \overline{w})$, which we define from now on as a *taxpayers' class.* In other words, in the sequel each taxpayer is identified by the tax Administration through her class $(y^*, \overline{w})$, or, equivalently, through her reported income y^*.

We do not tackle here the choice problem of designing an optimal threshold tax offer function specifying the cutoff assigned to each class, that the Tax Administration might select having in mind some social or governmental goal. We simply assume that some basic threshold tax value is chosen for the bottom class, while for the upper classes the threshold tax is proportionally adjusted with respect to the optimal reported income.

5.1 A Proportional Tax System

Let $V(y_0^*) = ty_0^* + x_0$ be the cutoff payment chosen by the Tax Administration for the bottom class represented by a reported income y_0^*. For values of reported income y^* larger than y_0^* we assume that the premium increases accordingly, that is,

$$x(y^*) = \frac{y^*}{y_0^*} x_0. \tag{23}$$

Thus, the threshold tax requested from a taxpayer whose optimal reported income is y^* is now

$$V(y^*) = ty^* + x(y^*) = ty^* + \frac{y^*}{y_0^*} x_0. \tag{24}$$

This approach not only seems coherent with a proportional tax system, but it allows for an important extension of Proposition 1 to all taxpayers' classes.

Proposition 3 *Under A.1, A.2 and A.3, suppose that x_0 verifies either (i) or (ii) or (iii) of Proposition 1. Then also the premium $x(y^*)$ defined in (23) satisfies (i) or (ii) or (iii) of Proposition 1 for any reported income y^*.*

Proof. It is immediately seen that inequalities (17), (18) and (19) remain unchanged if the threshold tax $V(y^*)$ is defined as in (24), as both sides of the inequalities vary proportionally to y^*. ∎

Proposition 4 characterizes the tax system under the previously described cutoff policy with reference to vertical equity.

Proposition 4 *The revenue collected by the Tax Administration from each taxpayer with risk aversion coefficient α, turns out to be proportional to her true income w, with a factor of proportionality that depends on α itself and is lower than the legislated tax rate t.*

Proof. We must establish the result separately for taxpayers accepting the cutoff and for those who refuse it.

For the former group, Proposition 3 ensures that the interval of α values is the same in each class. As $V(y^*)$ is proportional to y^*, while y^* is a constant share of w for each given α (recall (6) in Section 2), the assert is proven with reference to these taxpayers.

For each taxpayer who refuses the cutoff, the expected payment in taxes and fines is

$$ty^* + p(1+s)t(w - y^*). \tag{25}$$

By substituting (6) into (25) proportionality of the expected payment to w for each given α is readily seen.

The factor of proportionality is lower than the legislated tax rate for all the taxpayers, as, on the basis of the second inequality in (9), $u(w - ty^* - x) = \mathbb{E}u(y^*) > u(w - tw)$, and hence $w - ty^* - x > w - tw$, while $y^* < w$, that is, $ty^* + x < tw$. ∎

Proposition 4 states that, whenever income classes are considered, the actual tax system turns out to be vertically equitable in a mild sense, as there is a proportional relationship between expected total payments and true income, but only among taxpayers who have the same relative risk aversion. Since taxpayers with the same true income but different relative risk aversion pay different amounts, horizontal equity is at any rate violated. Moreover, specific equity problems are raised by the cutoff policy. On the one hand, taxpayers who accept the cutoff pay larger sums than under a regime without a cutoff, thus providing a larger contribution to government revenue. On the other hand, they gain from voluntarily insuring against controls, and thus in utility terms they are favored in comparison with other taxpayers who do not receive a suitable offer.

5.2 Signals and Classes

In a more realistic scenario, the Tax Administration does not know each taxpayer's y^* before the actual submission of the report. In order to formulate the threshold tax offer independently of the report's availability, some correlated variables such as profession, age, geographical location, etc., might

be exploited. In the followings it is shown that the implementation of the cutoff policy under imperfect information, when the Tax Administration only receives a signal, can have the same effects already discussed for the case of perfect information about y^*.

Fix a given optimal reported income y^* (that is, consider a taxpayer belonging to the class $(y^*, \overline{w})$), and assume that the Tax Administration observes some signal z associated to y^* (that is, the signal produced by that specific taxpayer). Following Scotchmer [13], let us assume that the signal is uniformly distributed over the interval $[k_0 y^*, k_1 y^*]$ with density $h(z \mid y^*) = [(k_1 - k_0) y^*]^{-1}$, where k_0 and k_1 are multiplicative factors[13] such that $0 \le k_0 < k_1$.

Under this setting, we assume that the government is only able to put the taxpayer within a fork of optimal reported income, and the taxpayer is equally likely to be put in classes $k_0 y^*$, $k_1 y^*$, or in all intermediate classes. Since the threshold tax offer is adjusted to the optimal reported income as perceived by the Tax Administration, the taxpayer can receive a better or worse offer according to the class to which she is assigned; *e.g.,* she could refuse paying the threshold tax in class $k_1 y^*$ while accepting it if put in a lower class.

The expected payment of the taxpayer with risk aversion coefficient α and reporting the optimal income y^* depends on whether the amount requested for the cutoff by the Tax Administration satisfies the second inequality in system (9), or, in view of Proposition 1, on whether α belongs to the interval $[\underline{\alpha}, \overline{\alpha}]$. However, the interval $[\underline{\alpha}, \overline{\alpha}]$ depends on the amount $V(z)$ requested for the cutoff, which, in turn, depends on the signal z perceived by the Administration. Therefore it is useful to relabel the extremes of the interval as functions of $V(z)$: $\underline{\alpha}[V(z)]$ and $\overline{\alpha}[V(z)]$, which also stresses dependence of the interval $[\underline{\alpha}, \overline{\alpha}]$ on the signal z. Two cases are possible.

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[13]This assumption implies that the range of possible percentage mistake in assessing y^* is constant. Following Scotchmer [13], one could assume instead that the signal is distributed on the interval $[y^* - a, y^* + a]$, where $a > 0$ is some additive constant, which, however, implies that the percentage error shrinks as y^* increases.

Note that in our case, even if it is not relevant for the subsequent analysis, one might reasonably assume that $k_0 < 1$ and $k_1 > 1$; specifically, if $k_0 + k_1 = 2$ one gets $\mathbb{E}[z \mid y^*] = y^*$.

1. Recall that by (24) $V(z)$ is increasing in its argument and Proposition 1, roughly speaking, essentially states that the interval $[\underline{\alpha}, \overline{\alpha}]$ shrinks as $V(z)$ increases (specifically, as $x(z)$ increases). Therefore, since $z \geq k_0 y^*$, $[\underline{\alpha}, \overline{\alpha}]$ is the largest interval for the class y^* whenever $\underline{\alpha} \equiv \underline{\alpha}[V(k_0 y^*)]$ and $\overline{\alpha} \equiv \overline{\alpha}[V(k_0 y^*)]$. Thus, all taxpayers lying outside, *i.e.* characterized by $\alpha < \underline{\alpha}[V(k_0 y^*)]$ or $\alpha > \overline{\alpha}[V(k_0 y^*)]$, will reject the cutoff and pay

$$\mathbb{E}[T(\alpha, y^*)] = ty^* + p(1+s)t(w - y^*) \qquad (26)$$

to the Tax Administration for all admissible values of the signal $k_0 y^* \leq z \leq k_1 y^*$.

2. If $\underline{\alpha}[V(k_0 y^*)] \leq \alpha \leq \overline{\alpha}[V(k_0 y^*)]$, then there exist values $z \geq k_0 y^*$ such that the cutoff will be paid. Specifically, the cutoff will be paid for all signals z such that $\underline{\alpha}[V(z)] \leq \alpha \leq \overline{\alpha}[V(z)]$. By monotonicity of the interval $[\underline{\alpha}, \overline{\alpha}]$ with respect to z, there will be some z_{α, y^*} such that either $\alpha = \underline{\alpha}[V(z_{\alpha, y^*})]$ or $\alpha = \overline{\alpha}[V(z_{\alpha, y^*})]$, and the taxpayer (characterized by risk coefficient α) will accept the cutoff offer whenever $k_0 y^* \leq z \leq z_{\alpha, y^*}$, while she will reject it for all values $z_{\alpha, y^*} < z \leq k_1 y^*$. Hence, the expected payment is given by

$$\mathbb{E}[T(\alpha, y^*)] = \int_{k_0 y^*}^{z_{\alpha, y^*}} V(z)h(z \mid y^*)dz \qquad (27)$$

$$+ \frac{k_1 - z_{\alpha, y^*}}{k_1 - k_0}\left[ty^* + p(1+s)t(w - y^*)\right].$$

Thus, case 1 refers to taxpayers whose risk aversion coefficient implies the refusal of the cutoff even when the signal z is at its lowest level $k_0 y^*$ and the requested threshold tax is small. They report income as usual and pay taxes and fines that are proportional to their true income according to (25) in Proposition 4. Case 2 refers to taxpayers who instead are ready to accept the cutoff when the signal is $k_0 y^*$ and might confirm this choice also in upper classes. They shall however refuse the cutoff when the requested threshold

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tax becomes so high that they drop out from the risk aversion interval $[\underline{\alpha}, \overline{\alpha}]$ relevant for acceptance. Thus, for the latter group, the expected payment depends also on the probability $(k_1 - z_{\alpha,y^*})(k_1 - k_0)^{-1}$ of paying taxes and expected fines according to general rules.

Proposition 5 *Let the (proportional) class system be defined as in Section 5, with the threshold tax defined, according to (24), as*

$$V(z) = \left(t + \frac{x_0}{y_0^*}\right) z. \tag{28}$$

Consider all taxpayers with risk coefficient α and optimal reported income $y^* \geq y_0^*$. Assume that the signal z associated to y^* is uniformly distributed over the interval $[k_0 y^*, k_1 y^*]$ with density $h(z \mid y^*) = [(k_1 - k_0) y^*]^{-1}$, $0 \leq k_0 < k_1$. Then, for each fixed α, the expected total payment of such taxpayers is proportional to y^*, and thus, thanks to (7), to the true income w.

Proof. Proportionality is obviously verified for the first case, i.e. when $\alpha < \underline{\alpha}[V(k_0 y^*)]$ or $\alpha > \overline{\alpha}[V(k_0 y^*)]$ and the expected payment is (26).

To see that also in the second case proportionality holds, note that, by Proposition 3 since the class system is proportional, with no loss of generality the value z_{α,y^*} may be written in product form: $z_{\alpha,y^*} = z_\alpha y^*$. Hence, by using (28), the integral in (27) is

$$\int_{k_0 y^*}^{z_\alpha y^*} V(z) h(z \mid y^*) dz = \frac{1}{2} \frac{(z_\alpha^2 - k_0^2)(t + x_0/y_0^*)}{k_1 - k_0} y^*,$$

and the proof is complete. ∎

The last result states that, under the assumption of uniform distribution for the signals z supported on intervals which are spreading as y^* increases, the mild vertical equity of our tax system is being preserved also in a more general framework including classes and asymmetric information. Since the signal distribution is the same at each y^* level, all taxpayers run the same proportional risk of gaining or losing as a consequence of mistakes in the class assignment.

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6   Conclusions

In this paper we have shown that a cutoff policy *cum* taxpayers' classes, when taxpayers are risk averse and have a CRRA utility functions, is compatible in a mild sense with vertical equity: taxpayers' expected payments are proportional to true income, for each given value of the coefficient of relative risk aversion. Horizontal equity, however, is violated, as taxpayers with equal income (and different risk aversion) pay different amounts. When the CRRA utility function is considered, these features would characterize also a proportional tax system without a cutoff policy. However, there are specific equity problems introduced by the cutoff. Within each class, only some taxpayers accept paying the threshold tax. While they make larger payments than those who refuse the cutoff, they also benefit from the possibility of insuring against the risk of audits. Those who refuse the cutoff are thus in a less favorable position, as they do not receive a suitable insurance offer. From the efficiency point of view, taxpayers who pay the threshold tax provide an extra source of income for the government, while less resources are needed for running audits. Summing-up, the policy involves a specific equity-efficiency trade-off.

The literature about the cutoff rule under risk neutrality has been largely concerned with the regressive bias of this policy and with the ways of correcting it, in particular through the introduction of taxpayers' classes. In our model the regressive bias need not exist as the taxpayers who accept paying the threshold tax might have intermediate characteristics in terms of income and relative risk aversion.

Unlike in models that assume risk neutrality, the cutoff policy when taxpayers are risk averse does not necessarily imply a full separation of types (relatively poor taxpayers who report as usual and relatively rich ones who pay the threshold tax). Indeed, we have highlighted cases in which also relatively rich taxpayers refuse the cutoff. Self revelation of types through the cutoff policy, as is well known, is problematic, as it implies the temptation for the Tax Administration of reneging its promises *ex post*. This problem does not arise in our model.

Admittedly, our paper is limited in scope, as we do not discuss how to shape an optimal cutoff policy, but only which reactions would arise given that a simple one is used. Our specific contribution pertains to determining the pattern of the reaction of taxpayers under the widely used CRRA utility function. Results could be used for describing the cutoff policy as part of a standard welfare maximization problem with taxation. This more general approach, which could provide further insight for assessing the equity and efficiency implications of the cutoff policy under risk aversion, is left for future research.

Appendix: Proofs

The proof of Proposition 1 will be accomplished through several steps. First we need two preliminary lemmas.

Lemma 1 $\psi(\alpha) < \phi(\alpha)$ *for all* $0 < \alpha < 1$.

    Proof. Since $r < 1$ and assumption A.2 implies $B < A$,

$$(A - Br)^{1-\alpha} < \left(A - Br^{\frac{1}{\alpha}}\right)^{1-\alpha} = \phi(\alpha) \tag{29}$$

for all $0 < \alpha < 1$. Since $(A - Br)^{1-\alpha}$ is strictly convex for all $\alpha > 0$, by the superdifferentiability property the following is true:

$$\psi(\alpha) = 1 - \ln(A - Br)(\alpha - 1) < (A - Br)^{1-\alpha}$$

for all $0 < \alpha < 1$, which, coupled with (29), proves the assert. ■

Lemma 2 *Under A.1, A.2 and A.3, the function* $\phi(\alpha) = \left(A - Br^{\frac{1}{\alpha}}\right)^{1-\alpha}$ *is strictly convex for* $\alpha \geq 1$, *while the function* $\varphi(\alpha) = -p\left(r^{\frac{1}{\alpha}}\right)^{1-\alpha}$ *is strictly concave for* $0 < \alpha \leq -(1/2)\ln r$ *and is strictly convex for* $\alpha \geq -(1/2)\ln r$.

    Proof. A tedious direct computation of the second derivatives of both ϕ and φ gives the result. ■

Proof of Proposition 1. *Part (i).* Since equality in A.3 is equivalent to $-(1/2)\ln r = 1$, by Lemma 2 function $l(\alpha) = \psi(\alpha) + \varphi(\alpha) - (1-p)$ turns out to be strictly concave over $(0,1)$ and strictly convex over $[1, +\infty)$. This is true since l is the sum of a constant, and functions ψ and φ, which are linear and strictly concave respectively over $(0,1)$, and both strictly convex over $[1, +\infty)$. In other words, l has a unique flex-point at $\alpha = 1$. Moreover, since $l(1) = 0$, S is non-empty and has the form $S = [\underline{\alpha}, \overline{\alpha}]$ if and only if its derivative is strictly negative in $\alpha = 1$, that is,

$$l'(1) = -\ln(A - Br) + p\ln r < 0$$

which, after some algebra, is the same as condition (17). Note also that $\overline{\alpha} < +\infty$ since $l(\alpha) \to +\infty$ as $\alpha \to +\infty$.

Part (ii). Strict inequality in A.3 is equivalent to

$$-\frac{\ln r}{2} > 1. \tag{30}$$

To simplify notation, let $c = -(1/2)\ln r > 1$. In this case we extend the argument above by constructing a function $h(\alpha)$ that is as similar to l as possible but is better shaped than l over the interval $(1, c)$ where the required convexity property of l cannot be verified directly. Define

$$\chi(\alpha) = 1 + \frac{\phi(c) - 1}{c - 1}(\alpha - 1)$$

and

$$h(\alpha) = \begin{cases} l(\alpha) & \text{for } 0 < \alpha \leq 1 \text{ and } \alpha \geq c \\ \chi(\alpha) + \varphi(\alpha) - (1-p) & \text{for } 1 < \alpha < c. \end{cases}$$

As in the construction of function l, where we substituted the badly shaped function ϕ with a linear one, ψ, over $(0,1)$, function h constitutes an improvement of function l again by linearizing ϕ, which, by Lemma 2, is convex for all $\alpha \geq 1$. As a result, h turns out to be strictly concave over $(1, c)$, being the sum of a constant, a linear and a concave function.

Function $h(\alpha)$ turns out to be the same as $l(\alpha)$ outside the interval $(1, c)$, where the same argument of Part (i) applies. Specifically, h is strictly concave

over $(0, 1)$ and $l(\alpha) \geq 0$ has a non-empty interval $[\underline{\alpha}, 1)$ as the solution as long as $l'_-(1) = l'(1) < 0$; while h is strictly convex over $(c, +\infty)$. Inside interval $(1, c)$, we have seen that $h(\alpha) = \chi(\alpha) + \varphi(\alpha) - (1 - p)$ is strictly concave. Moreover, since h is obtained by replacing the strictly convex function ϕ with the segment joining two points of its graph, $h(\alpha) > l(\alpha)$ holds true for all $\alpha \in (1, c)$, while $h(1) = l(1)$ and $h(c) = l(c)$. Note that h is not differentiable at points $\alpha = 1$ and $\alpha = c$, where is only left and right-differentiable, while $l'(1)$ exists.

Hence, $h'_+(1) \leq 0 \implies h(\alpha) < 0 \implies l(\alpha) < 0$ for all $\alpha \in (1, c]$. Furthermore, $l(c) < 0$ plus its convexity over $(c, +\infty)$ implies $l(\alpha) \leq 0$ for all $\alpha \in [c, \overline{\alpha}]$, where $c < \overline{\alpha} < +\infty$. To conclude, $h'_+(1) \leq 0 \implies l(\alpha) \leq 0$ for all $\alpha \in (1, \overline{\alpha}]$, while, on the other side, $h'_+(1) \leq 0 \implies l'_+(1) = l'(1) < 0$, thus also establishing the non-emptiness of the interval $[\underline{\alpha}, 1)$. A direct computation shows that condition $h'_+(1) \leq 0$ is equivalent to condition (18), and the proof is complete. Note that again, under our construction, we obtain a function h with a unique flex point at $\alpha = c$.

Part (iii). By construction, $\psi'(\alpha) = \phi'(1) = -\ln(A - Br)$ over $(0, 1]$. By Lemma 2,

$$\phi'(\alpha) \geq \phi'(1) = -\ln(A - Br) \qquad \forall \alpha \geq 1 \quad \text{and}$$

$$\varphi'(\alpha) \geq \varphi'\left(-\frac{\ln r}{2}\right) = \frac{4e^{-2}p}{r \ln r} \qquad \forall \alpha > 0.$$

Therefore,

$$l'(\alpha) \geq \phi'(1) + \varphi'\left(-\frac{\ln r}{2}\right) = -\ln(A - Br) + \frac{4e^{-2}p}{r \ln r}$$

and $l'(\alpha) > 0$ if $-\ln(A - Br) + \left(4e^{-2}p\right)(r \ln r)^{-1} > 0$, which is equivalent to (19); hence, (19) $\implies l'(\alpha) > 0$ for all $\alpha > 0$. Since $l(1) = 0$, $l' > 0$ means that l "crosses" level zero increasingly at $\alpha = 1$, and system (16) has empty solution set, as was to be shown. ∎

The proof showed how A.3 forces the unique flex-point of function φ to lie to the right of $\alpha = 1$. Clearly, it is possible to reproduce a similar technique for the case $r > e^{-2}$.

Proof of Proposition 2. In order to have $\underline{\alpha} > 0$ the following condition on function $f(\alpha)$ defined in (15) must hold:

$$\lim_{\alpha \to 0^+} f(\alpha) = A - (1 - p) < 0,$$

that is, function $f(\alpha)$ must have a negative intercept. By substituting A as in (11), this proves equivalent to:

$$x > \frac{p(1 + s)(1 - t)ty^*}{(1 + s)t - 1}$$

However, the above inequality, to be meaningful, requires a nonempty solution set S, that is, (18) of Proposition 1 must be met as well. Thus, we need to find values for the tax rate t such that

$$\frac{p(1 + s)(1 - t)ty^*}{(1 + s)t - 1} < \overline{x},$$

where $\overline{x}$ is defined in (18).

By plugging $\overline{x}$ as in (18) into the above inequality, we get

$$g(1 + s)t > 1 - p\left(1 - e^{-2}\right) - \left[1 + p(\ln r)\left(\frac{\ln r}{2} + 1\right)\right]^{\frac{2}{2 + \ln r}} \tag{31}$$

where

$$g = 1 - p - \left[1 + p(\ln r)\left(\frac{\ln r}{2} + 1\right)\right]^{\frac{2}{2 + \ln r}}.$$

To study the sign of g, it is convenient to rewrite it in terms of parameters r and s rather than p and r by using (13), which yields

$$p = \frac{r}{s + r}.$$

Graphical inspection of the function

$$g(r, s) = 1 - \frac{r}{s + r} - \left[1 + \frac{r(\ln r)}{s + r}\left(\frac{\ln r}{2} + 1\right)\right]^{\frac{2}{2 + \ln r}}$$

shows[14] that $g(r, s) > 0$ for $0 < r < e^{-2}$ and $s \geq 1$. Thus, dividing by $g \equiv g(r, s) > 0$ in (31), yields (20). ∎

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[14] Clearly, inequality $g(r, s) > 0$ does not allow for a closed form solution, thus we have checked it directly by plotting the graph of $g(r, s)$ through Maple 7 for the relevant values of parameters r and s.

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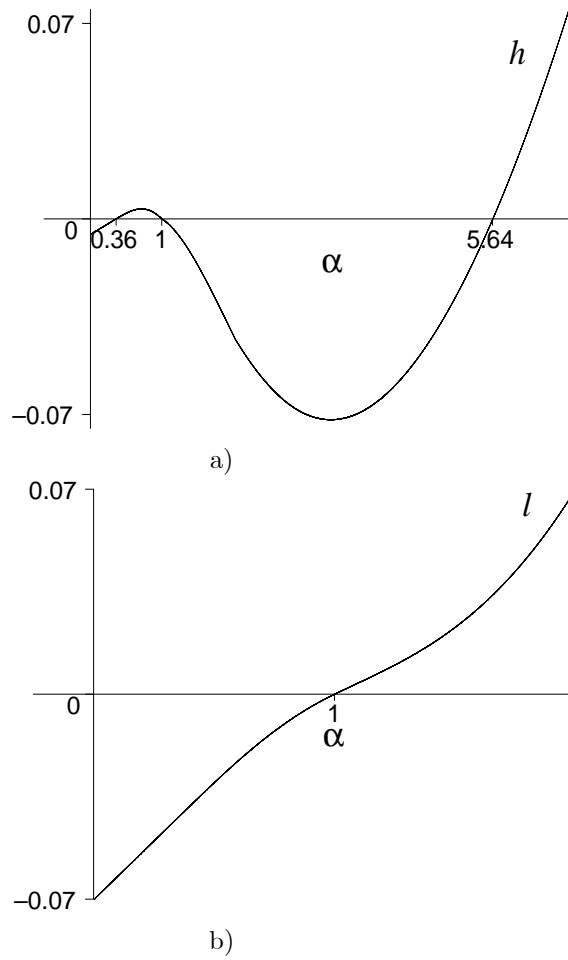
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Figures

Figure 1: illustration of Proposition 1: a) plot of the function h (approximating the function f) for $x = 50$: the solution set is the interval $[0.36, 5.64]$; b) plot of the approximating function l for $x = 250$: a case of empty solution set.