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On Unequally Smooth Bivariate Quadratic Spline Spaces

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Abstract

In this paper we consider spaces of unequally smooth local bivariate quadratic splines, defined on criss-cross triangulations of a rectangular domain.
For such spaces we present some results on the dimension and on a local basis.
Finally an application to B-spline surface generation is provided.

Key words: bivariate spline approximation, unequally smooth bivariate spline space, B-spline basis
MSC 2000: 65D07; 41A15

1 Introduction

Aim of this paper is the investigation of bivariate quadratic spline spaces with less than maximum C^1 smoothness on criss-cross triangulations of a rectangular domain, with particular reference to their dimension and to the construction of a local basis. Indeed, in many practical applications, piecewise polynomial surfaces need to be connected by using different smoothness degrees and, in literature, tensor product spline surfaces of such a kind have already been investigated (see e.g. [1, 5]). In [2] the dimension and a B-spline basis for the space of all quadratic C^1 splines on a criss-cross triangulation are obtained. Since some supports of such B-splines are not completely contained in the rectangular domain, in [7] a new B-spline basis for such space is proposed, with all supports included in the domain.

The paper is organized as follows. In Section 2 we present some results on the dimension of the unequally smooth spline space and on the construction of a B-spline basis with different types of smoothness. In Section 3 an application to B-spline surface generation is presented.

2 Bases of unequally smooth bivariate quadratic spline spaces

Let $\Omega = [a, b] \times [c, d]$ be a rectangle decomposed into $(m + 1)(n + 1)$ subrectangles by two partitions

$$\bar{\xi} = \{\xi_i, \quad i = 0, \dots, m + 1\},$$
$$\bar{\eta} = \{\eta_j, \quad j = 0, \dots, n + 1\},$$

of the segments $[a, b] = [\xi_0, \xi_{m+1}]$ and $[c, d] = [\eta_0, \eta_{n+1}]$, respectively. Let $\mathcal{T}_{mn}$ be the criss-cross triangulation associated with the partition $\bar{\xi} \times \bar{\eta}$ of the domain Ω.

Given two sets $\bar{m}^\xi = \{m_i^\xi\}_{i=1}^m$, $\bar{m}^\eta = \{m_j^\eta\}_{j=1}^n$, with m_i^ξ, $m_j^\eta = 1, 2$ for all i, j, we set

$$M = 3 + \sum_{i=1}^m m_i^\xi, \quad N = 3 + \sum_{j=1}^n m_j^\eta \tag{1}$$

and let $\bar{u} = \{u_i\}_{i=-2}^M$, $\bar{v} = \{v_j\}_{j=-2}^N$ be the nondecreasing sequences of knots, obtained from $\bar{\xi}$ and $\bar{\eta}$ by the following two requirements:

(i) $u_{-2} = u_{-1} = u_0 = \xi_0 = a, \qquad b = \xi_{m+1} = u_{M-2} = u_{M-1} = u_M,$
$v_{-2} = v_{-1} = v_0 = \eta_0 = c, \qquad d = \eta_{n+1} = v_{N-2} = v_{N-1} = v_N;$

(ii) for $i = 1, \dots, m$, the number ξ_i occurs exactly m_i^ξ times in $\bar{u}$ and for $j = 1, \dots, n$, the number η_j occurs exactly m_j^η times in $\bar{v}$.

For $0 \le i \le M - 1$ and $0 \le j \le N - 1$, we set $h_i = u_i - u_{i-1}$, $k_j = v_j - v_{j-1}$ and $h_{-1} = h_M = k_{-1} = k_N = 0$. In the whole paper we use the following notations

$$\begin{aligned} \sigma_{i+1} &= \tfrac{h_{i+1}}{h_i + h_{i+1}}, \quad \sigma_i' = \tfrac{h_{i-1}}{h_{i-1} + h_i}, \\ \tau_{j+1} &= \tfrac{k_{j+1}}{k_j + k_{j+1}}, \quad \tau_j' = \tfrac{k_{j-1}}{k_{j-1} + k_j}. \end{aligned} \tag{2}$$

When in (2) we have $\frac{0}{0}$, we set the corresponding value equal to zero.

On the triangulation $\mathcal{T}_{mn}$ we can consider the spline space of all functions s, whose restriction to any triangular cell of $\mathcal{T}_{mn}$ is a polynomial in two variables of total degree two. The smoothness of s is related to the multiplicity of knots in $\bar{u}$ and $\bar{v}$ [4]. Indeed let m_i^ξ (m_j^η) be the multiplicity of ξ_i (η_j), then

$$m_i^\xi \ (m_j^\eta) \ + \ \text{degree of smoothness for } s \text{ crossing the line } u = \xi_i \quad (v = \eta_j) \\ = 2.$$

We call such space $\mathcal{S}_2^{\bar{\mu}}(\mathcal{T}_{mn})$. We can prove [4] that

$$\dim \mathcal{S}_2^{\bar{\mu}}(\mathcal{T}_{mn}) = 8 - mn + m + n + (2 + n) \sum_{i=1}^m m_i^\xi + (2 + m) \sum_{j=1}^n m_j^\eta. \tag{3}$$

Now we denote by

$$\mathcal{B}_{MN} = \{B_{ij}(u, v)\}_{(i,j) \in \mathcal{K}_{MN}}, \quad \mathcal{K}_{MN} = \{(i, j) : 0 \le i \le M - 1, 0 \le j \le N - 1\}, \tag{4}$$

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the collection of $M \cdot N$ quadratic B-splines defined in [4], that we know to span $\mathcal{S}_2^{\bar{\mu}}(\mathcal{T}_{mn})$. In $\mathcal{B}_{MN}$ we find different types of B-splines. There are $(M-2)(N-2)$ inner B-splines associated with the set of indices $\widehat{\mathcal{K}}_{MN} = \{(i,j) : 1 \le i \le M-2, 1 \le j \le N-2\}$, whose restrictions to the boundary $\partial\Omega$ of Ω are equal to zero.

To the latter, we add $2M+2N-4$ boundary B-splines, associated with

$$\widetilde{\mathcal{K}}_{MN} := \{(i,0),(i,N-1), 0 \le i \le M-1; (0,j),(M-1,j), 0 \le j \le N-1\},$$

whose restrictions to the boundary of Ω are univariate B-splines [7].

Any B_{ij} in $\mathcal{B}_{MN}$ is given in Bernstein-Bézier form. Its support is obtained from the one of the quadratic C^1 B-spline $\bar{B}_{ij}$, with octagonal support (Fig. 1) [2, 7], by conveniently setting h_i and/or k_j equal to zero in Fig. 1, when there are double (or triple) knots in its support. The B_{ij}'s BB-coefficients different from zero are computed by using Table 1, evaluating the corresponding ones related to the new support [3]. The symbol "O" denotes a zero BB-coefficient.

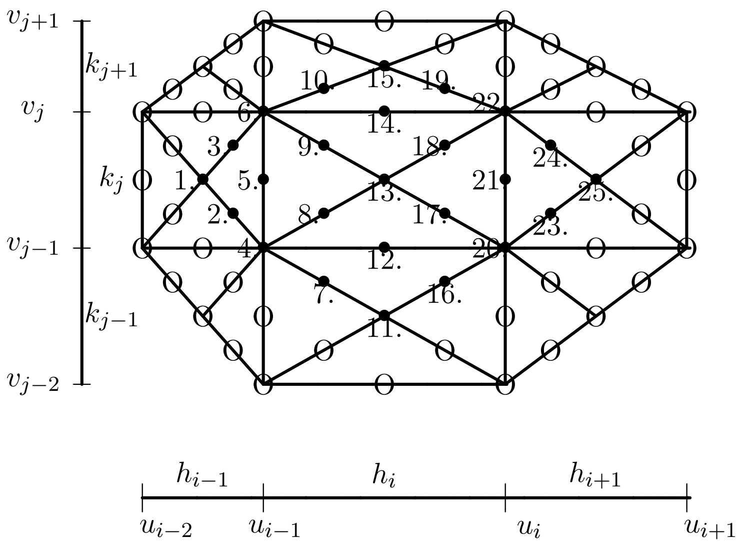

Figure 1: Support of the C^1 B-spline $\bar{B}_{ij}(u,v)$.

Since $\bar{u}$ and $\bar{v}$ can have multiple knots, then the B_{ij} smoothness changes and the B-spline support changes as well, because the number of triangular cells on which the function is nonzero is reduced. For example, in Fig. 2 we propose: (a) the graph of a B-spline B_{ij}, with the double knot $v_{j-1} = v_j$, (b) its support with its BB-coefficients different from zero, computed by setting $k_j = 0$ in Fig. 1 and Table 1. Analogously in Figs. 3÷6 we propose some other multiple knot B-splines. In Figs. 2(b)÷6(b) a thin line means that the B-spline is C^1 across it, while a thick line means that the function is continuous across it, but not C^1 and a dotted line means that the function has a jump across it.

All B_{ij}'s are non negative and form a partition of unity.

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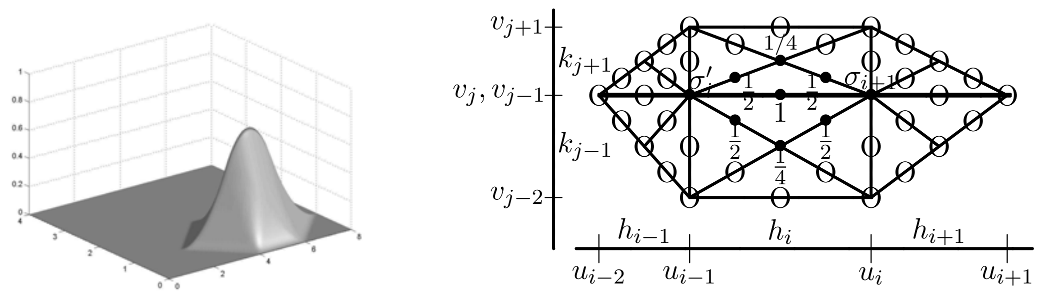
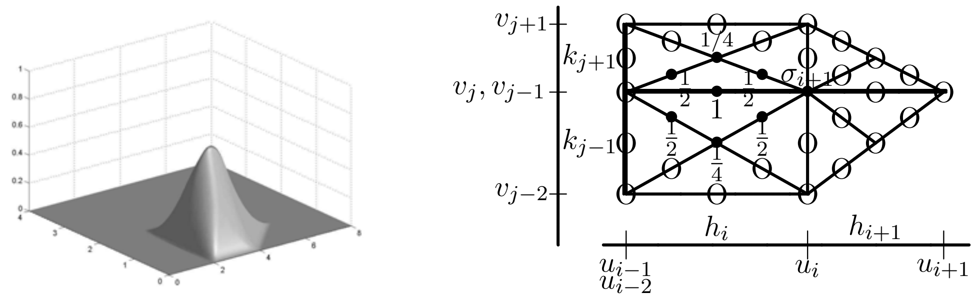
1. $\frac{\sigma_i'}{4}$,	2. $\frac{\sigma_i'}{2}$,	3. $\frac{\sigma_i'}{2}$,	4. $\sigma_i'\tau_j'$,	5. σ_i',
6. $\sigma_i'\tau_{j+1}$,	7. $\frac{\tau_j'}{2}$,	8. $\frac{\sigma_i'+\tau_j'}{2}$,	9. $\frac{\sigma_i'+\tau_{j+1}}{2}$,	10. $\frac{\tau_{j+1}}{2}$,
11. $\frac{\tau_j'}{4}$,	12. τ_j',	13. $\frac{\sigma_i'+\sigma_{i+1}+\tau_j'+\tau_{j+1}}{4}$,	14. τ_{j+1},	15. $\frac{\tau_{j+1}}{4}$,
16. $\frac{\tau_j'}{2}$,	17. $\frac{\sigma_{i+1}+\tau_j'}{2}$,	18. $\frac{\sigma_{i+1}+\tau_{j+1}}{2}$,	19. $\frac{\tau_{j+1}}{2}$,	20. $\sigma_{i+1}\tau_j'$,
21. σ_{i+1},	22. $\sigma_{i+1}\tau_{j+1}$,	23. $\frac{\sigma_{i+1}}{2}$,	24. $\frac{\sigma_{i+1}}{2}$,	25. $\frac{\sigma_{i+1}}{4}$,

Table 1: B-net of the C^1 B-spline $\bar{B}_{ij}(u,v)$.



(a) (b)

Figure 2: A double knot quadratic C^0 B-spline B_{ij} with $v_{j-1} = v_j$ and its support.



(a) (b)

Figure 3: A double knot quadratic C^0 B-spline B_{ij} with $u_{i-2} = u_{i-1}$, $v_{j-1} = v_j$ and its support.

Since $\sharp\mathcal{B}_{MN} = M \cdot N$, from (3) and (1) it results that $\sharp\mathcal{B}_{MN} > \dim \mathcal{S}_2^{\bar{\mu}}(\mathcal{T}_{mn})$. Therefore the set $\mathcal{B}_{MN}$ is linearly dependent and we can prove [4] that the number of linearly independent B-splines in $\mathcal{B}_{MN}$ coincides with $\dim \mathcal{S}_2^{\bar{\mu}}(\mathcal{T}_{mn})$. Then we can

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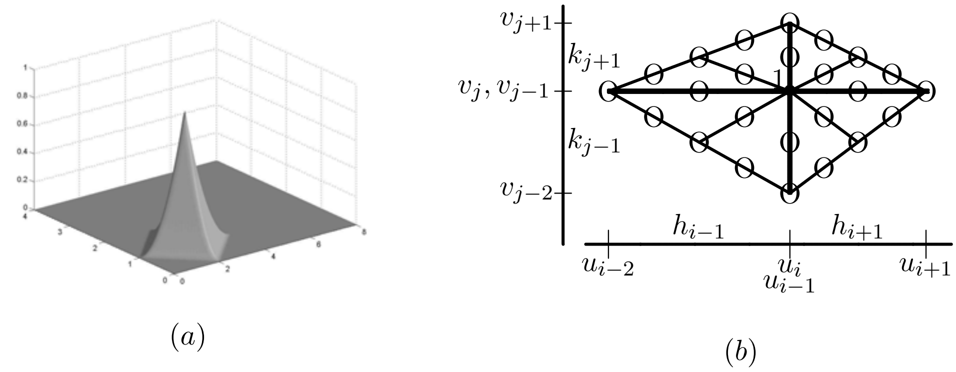

Figure 4: A double knot quadratic C^0 B-spline B_{ij} with $u_{i-1} = u_i$, $v_{j-1} = v_j$ and its support.

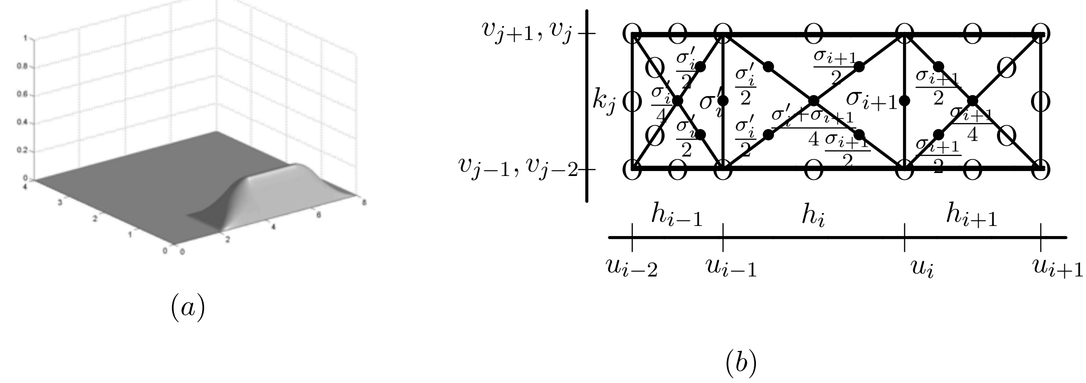

Figure 5: A double knot quadratic C^0 B-spline B_{ij} with $v_{j-2} = v_{j-1}$, $v_j = v_{j+1}$ and its support.

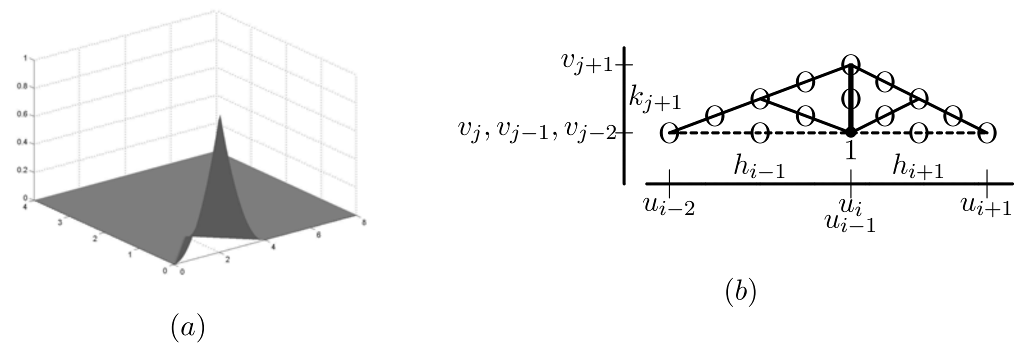

Figure 6: A triple knot quadratic B-spline B_{ij} with $u_{i-1} = u_i$, $v_{j-2} = v_{j-1} = v_j$ and its support.

conclude that the algebraic span of $\mathcal{B}_{MN}$ is all $\mathcal{S}_2^{\bar{\mu}}(\mathcal{T}_{mn})$.

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3 An application to surface generation

In this section we propose an application of the above obtained results to the construction of unequally smooth quadratic B-spline surfaces.

An unequally smooth B-spline surface can be obtained by taking a bidirectional net of control points $\mathbf{P}_{ij}$, two knot vectors $\bar{u}$ and $\bar{v}$ in the parametric domain Ω, as in Section 2, and assuming the B_{ij}'s (4) as blending functions. It has the following form

$$\mathbf{S}(u,v) = \sum_{(i,j)\in\mathcal{K}_{MN}} \mathbf{P}_{ij}\ B_{ij}(u,v), \quad (u,v)\in\Omega. \tag{5}$$

Here we assume $(s_i,t_j)\in\Omega$ as the pre-image of $\mathbf{P}_{ij}$, with $s_i = \frac{u_{i-1}+u_i}{2}$ and $t_j = \frac{v_{j-1}+v_j}{2}$.

We remark that in case of functional parametrization, $\mathbf{S}(u,v)$ is the spline function defined by the well known bivariate Schoenberg-Marsden operator (see e.g. [6, 9]), which is "variation diminishing" and reproduces bilinear functions.

Since the B-splines in $\mathcal{B}_{MN}$ are non negative and satisfy the property of unity partition, the surface (5) has both the convex hull property and the affine transformation invariance one.

Moreover $\mathbf{S}(u,v)$ has C^1 smoothness when both parameters $\bar{u}$ and $\bar{v}$ have no double knots. When both/either $\bar{u}$ and/or $\bar{v}$ have/has double knots, then the surface is only continuous at such knots [8].

Finally, from the B-spline locality property, the surface interpolates both the four points $\mathbf{P}_{00}$, $\mathbf{P}_{M-1,0}$, $\mathbf{P}_{0,N-1}$, $\mathbf{P}_{M-1,N-1}$ and the control points $\mathbf{P}_{ij}$ if both u_i and v_j occur at least twice in $\bar{u}$ and $\bar{v}$, respectively.

Example 1.

We consider a test surface, given by the following functional parametrization:

$$\begin{cases} x = u \\ y = v \\ z = f(u,v) \end{cases},$$

with

$$f(u,v) = \begin{cases} \mid u \mid v & \text{if } uv > 0 \\ 0 & \text{elsewhere} \end{cases}.$$

We assume $\Omega = [-1,1]\times[-1,1]$ as parameter domain and $m = n = 5$. Moreover we set $\bar{\xi} = \{-1,-0.5,-0.25,0,0.25,0.5,1\}$ and $\bar{\eta} = \bar{\xi}$. We choose $\bar{m}^{\xi} = \{1,1,2,1,1\}$ and $\bar{m}^{\eta} = \bar{m}^{\xi}$. Therefore we have $M = N = 9$ and

$$\bar{u} = \{-1,-1,-1,-0.5,-0.25,0,0,0.25,0.5,1,1,1\}, \quad \bar{v} = \bar{u}.$$

In this case $\mathbf{P}_{ij} = f(s_i,t_j)$. The graph of the corresponding surface (5) is reported in Fig. 7(a). It is obtained by evaluating $\mathbf{S}$ on a 55×55 uniform rectangular grid of points in the domain Ω. In Fig. 7(b) we present the quadratic C^1 B-spline surface, obtained if all knots in $\bar{u}$ and $\bar{v}$, inside Ω, are assumed simple.

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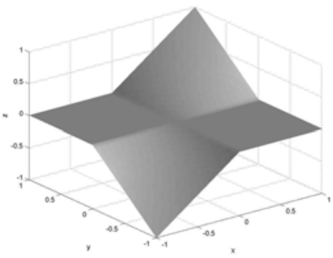
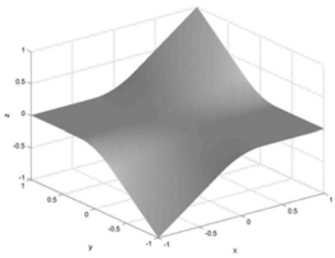


(a)



(b)

Figure 7: $\mathbf{S}$ with double (a) and simple (b) knots at $\xi_3 = \eta_3 = 0$.

We remark how the presence of double knots allows to well simulate a discontinuity of the first partial derivatives across the lines $u = 0$ and $v = 0$.

Example 2.

We want to reconstruct the spinning top in Fig. 8 by a non uniform quadratic B-spline surface (5).

Figure 8: A spinning top.

In order to do it we consider the following control points

$$\mathbf{P}_{00} = \mathbf{P}_{10} = \mathbf{P}_{20} = \mathbf{P}_{30} = \mathbf{P}_{40} = \mathbf{P}_{50} = (0, 0, 0),$$

$$\begin{array}{lll} \mathbf{P}_{01} = (0, \frac{1}{2}, \frac{1}{2}), & \mathbf{P}_{11} = (\frac{1}{2}, \frac{1}{2}, \frac{1}{2}), & \mathbf{P}_{21} = (\frac{1}{2}, -\frac{1}{2}, \frac{1}{2}), \\ \mathbf{P}_{31} = (-\frac{1}{2}, -\frac{1}{2}, \frac{1}{2}), & \mathbf{P}_{41} = (-\frac{1}{2}, \frac{1}{2}, \frac{1}{2}), & \mathbf{P}_{51} = \mathbf{P}_{01}, \end{array}$$

$$\begin{array}{lll} \mathbf{P}_{02} = (0, \frac{3}{4}, \frac{7}{12}), & \mathbf{P}_{12} = (\frac{3}{4}, \frac{3}{4}, \frac{7}{12}), & \mathbf{P}_{22} = (\frac{3}{4}, -\frac{3}{4}, \frac{7}{12}), \\ \mathbf{P}_{32} = (-\frac{3}{4}, -\frac{3}{4}, \frac{7}{12}), & \mathbf{P}_{42} = (-\frac{3}{4}, \frac{3}{4}, \frac{7}{12}), & \mathbf{P}_{52} = \mathbf{P}_{02}, \end{array}$$

$$\begin{array}{lll} \mathbf{P}_{03} = (0, \frac{13}{10}, \frac{5}{6}), & \mathbf{P}_{13} = (\frac{13}{10}, \frac{13}{10}, \frac{5}{6}), & \mathbf{P}_{23} = (\frac{13}{10}, -\frac{13}{10}, \frac{5}{6}), \\ \mathbf{P}_{33} = (-\frac{13}{10}, -\frac{13}{10}, \frac{5}{6}), & \mathbf{P}_{43} = (-\frac{13}{10}, \frac{13}{10}, \frac{5}{6}), & \mathbf{P}_{53} = \mathbf{P}_{03}, \end{array}$$

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$$\begin{array}{lll} \mathbf{P}_{04} = (0,1,1), & \mathbf{P}_{14} = (1,1,1), & \mathbf{P}_{24} = (1,-1,1), \\ \mathbf{P}_{34} = (-1,-1,1), & \mathbf{P}_{44} = (-1,1,1) & \mathbf{P}_{54} = \mathbf{P}_{04}, \end{array}$$

$$\begin{array}{lll} \mathbf{P}_{05} = (0,\frac{1}{2},1), & \mathbf{P}_{15} = (\frac{1}{2},\frac{1}{2},1), & \mathbf{P}_{25} = (\frac{1}{2},-\frac{1}{2},1), \\ \mathbf{P}_{35} = (-\frac{1}{2},-\frac{1}{2},1), & \mathbf{P}_{45} = (-\frac{1}{2},\frac{1}{2},1) & \mathbf{P}_{55} = \mathbf{P}_{05}, \end{array}$$

$$\begin{array}{lll} \mathbf{P}_{06} = (0,\frac{1}{8},1), & \mathbf{P}_{16} = (\frac{1}{8},\frac{1}{8},1), & \mathbf{P}_{26} = (\frac{1}{8},-\frac{1}{8},1), \\ \mathbf{P}_{36} = (-\frac{1}{8},-\frac{1}{8},1), & \mathbf{P}_{46} = (-\frac{1}{8},\frac{1}{8},1) & \mathbf{P}_{56} = \mathbf{P}_{06}, \end{array}$$

$$\begin{array}{lll} \mathbf{P}_{07} = (0,\frac{1}{8},\frac{3}{2}), & \mathbf{P}_{17} = (\frac{1}{8},\frac{1}{8},\frac{3}{2}), & \mathbf{P}_{27} = (\frac{1}{8},-\frac{1}{8},\frac{3}{2}), \\ \mathbf{P}_{37} = (-\frac{1}{8},-\frac{1}{8},\frac{3}{2}), & \mathbf{P}_{47} = (-\frac{1}{8},\frac{1}{8},\frac{3}{2}) & \mathbf{P}_{57} = \mathbf{P}_{07}, \end{array}$$

$$\begin{array}{lll} \mathbf{P}_{08} = (0,\frac{1}{8},2), & \mathbf{P}_{18} = (\frac{1}{8},\frac{1}{8},2), & \mathbf{P}_{28} = (\frac{1}{8},-\frac{1}{8},2), \\ \mathbf{P}_{38} = (-\frac{1}{8},-\frac{1}{8},2), & \mathbf{P}_{48} = (-\frac{1}{8},\frac{1}{8},2) & \mathbf{P}_{58} = \mathbf{P}_{08}, \end{array}$$

$$\mathbf{P}_{09} = \mathbf{P}_{19} = \mathbf{P}_{29} = \mathbf{P}_{39} = \mathbf{P}_{49} = \mathbf{P}_{59} = (0,0,2),$$

defining the control net in Fig. 9. Here $M = 6$ and $N = 10$.

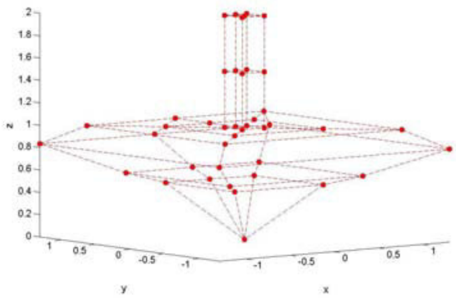

Figure 9: The control net corresponding to $\{\mathbf{P}_{ij}\}_{(i,j)\in\mathcal{K}_{6,10}}$.

Then, to well model our object, we assume $\bar{u} = \{0,0,0,1,2,3,4,4,4\}$ and $\bar{v} = \{0,0,0,1,2,3,3,4,4,5,6,6,6\}$. The graph of the B-spline surface of type (5) is reported in Fig. 10(a), while in Fig. 10(b) the corresponding criss-cross triangulation of the parameter domain is given.

In Fig. 11 we present the quadratic C^1 B-spline surface based on the same control points and obtained if all knots in $\bar{u}$ and $\bar{v}$, inside Ω, are assumed simple, i.e.

$$\bar{u} = \{0,0,0,1,2,3,4,4,4\}, \quad \bar{v} = \{0,0,0,1,2,3,4,5,6,7,8,8,8\}.$$

In Fig. 12(a) and (b) the effects of multiple knots are emphasized. We remark that in such a way we can better model the real object.

The construction of the B-spline basis and the B-spline surfaces has been realized by Matlab codes.

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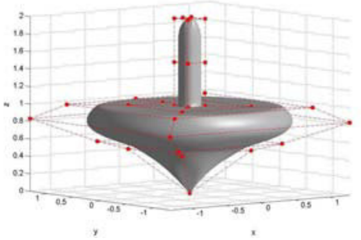
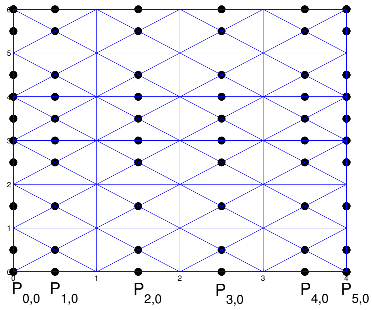
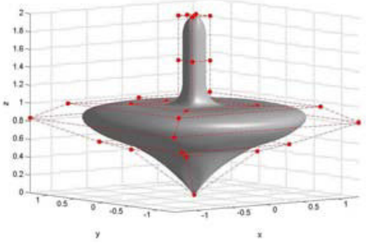


(a)

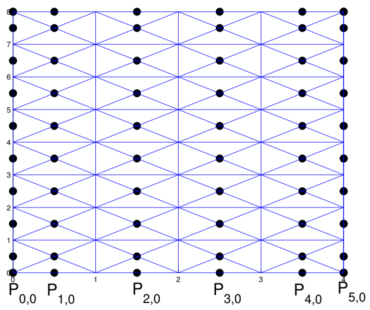


(b)

Figure 10: The surface $\mathbf{S}(u,v)$ with double knots in $\bar{v}$ and its parameter domain.

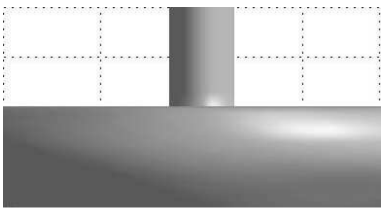


(a)

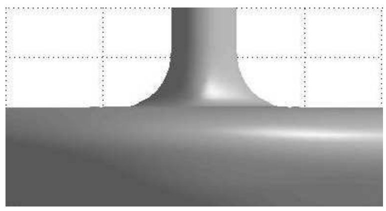


(b)

Figure 11: The surface $\mathbf{S}(u,v)$ with simple knots inside Ω and its parameter domain.



(a)



(b)

Figure 12: In (a) zoom of Fig. 10(a) and in (b) zoom of Fig. 11(a).

4 Conclusions

In this paper we have presented some results on the dimension of the unequally smooth spline space $\mathcal{S}_2^{\bar{\mu}}(\mathcal{T}_{mn})$ and on the construction of a B-spline basis with different types of smoothness.

We plan to use these results in the construction of blending functions for multiple knot NURBS surfaces with a criss-cross triangulation as parameter domain. Moreover such results could be also applied in reverse-engineering techniques, by using surfaces based on spline operators reproducing higher degree polynomial spaces [6, 9].

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