

Supplementary Information for:

“Preys’ exploitation of predators’ fear: when the caterpillar plays the Gruffalo”

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Contents

A1. Model assumptions and the meaning of adaptive cognitive biases.	2
A2. Computation of the error probabilities and mean decision times.	4
A3. The effect of hawk predation risk on the optimal decision strategy.	6

A1. Model assumptions and the meaning of adaptive cognitive biases

In our model, the decision maker (the bird) is uncertain about the true nature of the inspected animal, but it has a “complete knowledge” about such an uncertainty, in that the bird is assumed to make decisions as he knew the conditional probability density functions that the inspected animal is (or is not) either a caterpillar or a snake. At first sight, this might seem a strong as well as improbable assumption. However, as it has been first suggested by Castellano (2015), the bird does not need to have an internal representation of these probability distributions, but just a neural linear filter, which converts the sensory input into perceived evidence (see Figure S1). This filter might have been set through learning during an individual lifetime or genetically over evolutionary time.

For this reason, the model assumes that the bird has an unbiased perception of the nature of the perceived stimuli and that the mechanism that skews the response errors involves only the decision rule (the decision threshold). Indeed, the bird is more likely to confound a caterpillar for a snake than vice versa simply because it adopts different decision thresholds for prey and predator recognition. We call the asymmetry in the decision thresholds an ‘adaptive cognitive bias’, because it is ‘the mechanism that allows decision agents to adjust their decision rules to the internal and external conditions they are facing’ (Castellano 2015, but see Johnson et al. 2013 and Marshall et al. 2013 for a contentious interpretation of adaptive cognitive bias). This definition focuses on the mechanism of information processing and, in fact, our model makes specific assumptions about the way information is processed, but it makes no assumptions about the way information is perceived. The bird that mistakes a caterpillar for a snake may actually “see” a snake in the caterpillar and experience “irrational” fear, or it may perceive uncertainty and “rationally” decide that further assessment is not worthy, or it may behave as our stochastic decision variable in one of the thousands of simulations we carried out, which simply obeys to the rules implemented by the program without any perception of what is going on. Whatever the meaning (if any) that the decision maker assigns to the acquired information, the mechanism of information processing introduces a bias in decision making, which makes some behaviours more likely than others. We call this bias an adaptive cognitive bias.

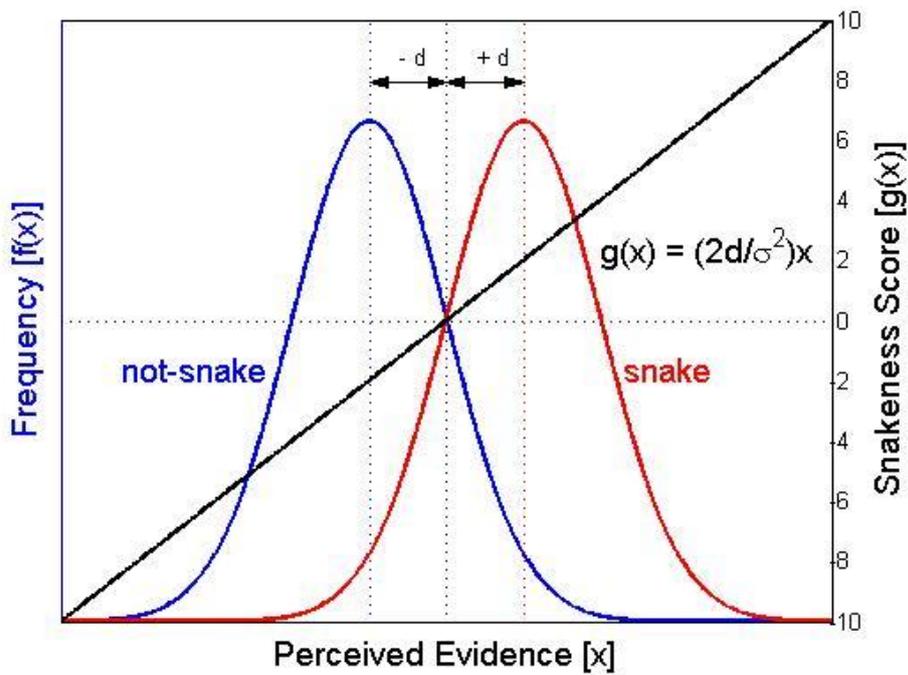
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Figure S1. The bird's Snake S-module converts sensory information into snakesness scores. When inspecting a prospective prey, the bird is assumed to collect several pieces of information, whose frequencies are distributed according to two normal curves, with similar variance and different mean: if the inspected animal is a snake, the pieces of evidence follow the red curve (i.e. $f_{snake}(x) = P(x|H^s)$), if it is not a snake, but either a caterpillar or a millipede, they distribute along the blue curve (i.e. $f_{not-snake}(x) = P(x|H^{c,m})$). When a single piece of evidence reaches the S-module, it is converted in a snakesness score by the linear filter $g(x)$, which is the log-ratio of the two normal curves.



A2. Computation of the error probabilities and mean decision times.

In this section we detail the procedure that we have used to compute the error probabilities α_c^s and α_c^m of mistaking a caterpillar by a snake or a millipede, the probabilities α_s^c and α_s^m of mistaking a snake by a caterpillar or a millipede, as well as the expected decision times t_s , t_c and t_m for snakes, caterpillars and millipedes. First of all, assuming that the number of samples is very high, we can replace the random walks in EQ. 1 (see main text) by two 2-dimensional diffusion processes in the (V_s, V_c) -plane [1,2], given by

$$dV^s(t) = \mu^s dt + \frac{2d}{\sigma} dW(t), \quad \text{and} \quad dV^c(t) = \mu^c dt + \frac{2d}{\sigma} dW(t),$$

where t is time, $V^s = (V_s^s, V_c^s)$ and $V^c = (V_s^c, V_c^c)$ are the vectors of accumulated evidence under the hypotheses H^s and H^c , respectively, i.e., when the signal is emitted by a snake or by a caterpillar, $\mu^s = (2d^2/\sigma^2, -2d^2/\sigma^2)$, $\mu^c = (-2d^2/\sigma^2, 2d^2/\sigma^2)$ are the drift vectors under the hypotheses H^s and H^c , respectively, and $W(t)$ is 2-dimensional Brownian motion. The assumption that there is no correlation between the snake and the caterpillar components of the signal, i.e., that the covariance matrices are diagonal, is not restrictive: writing R^s and R^c for the covariance matrices under the hypotheses H^s and H^c , respectively, yields

$$dV^s(t) = \mu^s dt + Z^s dW(t), \quad \text{and} \quad dV^c(t) = \mu^c dt + Z^c dW(t),$$

with $(Z^s)^2 = R^s$ and $(Z^c)^2 = R^c$. Hence, a correlation between the components of the signal would not affect the drift, but, according to the sign, would reduce or increase the fluctuations of the perceived signal and in turn, the perception error.

For each decision strategy $(a_s; -b_s; a_c; -b_c)$, the region of the decision plane lying within these thresholds is an unbounded elbow-shaped region: hence, in order to bound decision times, we assign an additional lower threshold l such that when either V_s or V_c are below l the bird concludes that the signal corresponds to a millipede (Fig. 1). Let

$$\begin{aligned} \Omega &= \{(V_s, V_c): -b_s \leq V_s \leq a_s, -b_c \leq V_c \leq a_c\} \\ &\cup \{(V_s, V_c): -b_s \leq V_s \leq a_s, -l \leq V_c \leq -b_c\} \\ &\cup \{(V_s, V_c): -l \leq V_s \leq -b_s, -b_c \leq V_c \leq a_c\} \end{aligned}$$

be the uncertainty domain, and

$$\Sigma = \{(V_s, V_c): V_s = a_s, -b_c \leq V_c \leq a_c\} \subset \partial\Omega,$$

$$\Gamma = \{(V_s, V_c): -b_s \leq V_s \leq a_s, V_c = a_c\} \subset \partial\Omega,$$

$$M = \{(V_s, V_c): V_s = -b_s, -l \leq V_c \leq -b_c\} \cup \{(V_s, V_c): -l \leq V_s \leq -b_s, V_c = -b_c\} \subset \partial\Omega$$

be the acceptance thresholds for snake, caterpillar and millipede, respectively. Finally, for $X \in \Omega$, let $\tau_{s, \partial\Omega}^X = \inf\{t: V^s(t) \in \partial\Omega, V^s(0) = X\}$ and $\tau_{c, \partial\Omega}^X = \inf\{t: V^c(t) \in \partial\Omega, V^c(0) = X\}$ be the first exit times of each process from $\partial\Omega$. The first exit time is a random variable that depends on the realization of the stochastic process $V(t)$ starting at X . The probability that a specific decision is taken for a signal with

initial value X is the probability that the first exit of the diffusion process from Ω occurs at the corresponding threshold boundary. More precisely, the functions

$$\varphi_s^c(X) = \text{Prob}[V^s(\tau_{s,\partial\Omega}^X) \in \Gamma] \quad \text{and} \quad \varphi_s^m(X) = \text{Prob}[V^s(\tau_{s,\partial\Omega}^X) \in M]$$

are the probabilities that a snake with initial signal X is mistaken by a caterpillar or a millipede respectively. Analogous definitions hold for the error probabilities of a caterpillar signal, $\varphi_c^s(X)$, $\varphi_c^m(X)$. Classical results guarantee that, for instance, the probability $\varphi_s^c(X)$ is the solution of the boundary value problem

$$\begin{cases} \frac{1}{2} \frac{4d^2}{\sigma^2} \Delta \varphi_s^c + \mu_c \cdot \nabla \varphi_s^c = 0 & \text{in } \Omega \\ \varphi_s^c = 1 & \text{on } \Gamma \\ \varphi_s^c = 0 & \text{on } \partial\Omega - \Gamma \end{cases}$$

similar formulas holding for the other probabilities. Solving the PDEs by finite differences methods allows to determine numerically the error probabilities. Knowing these, we can compute the error probabilities as

$$\alpha_s^c = E(\varphi_s^c | H^s), \quad \alpha_s^m = E(\varphi_s^m | H^s), \quad \alpha_c^s = E(\varphi_c^s | H^c), \quad \alpha_c^m = E(\varphi_c^m | H^c),$$

where $E(\cdot | H)$ denotes conditional expectation.

As to the decision times, these can be identified to the expected first exit times from Ω , i.e., letting

$$\psi_s(X) = E^X[\tau_{s,\partial\Omega}^X], \quad \psi_c(X) = E^X[\tau_{c,\partial\Omega}^X], \quad \psi_m(X) = E^X[\tau_{m,\partial\Omega}^X],$$

for the mean decision time for a snake, caterpillar or millipede signal starting at X , these are solutions of boundary value problems of the kind

$$\begin{cases} \frac{1}{2} \frac{4d^2}{\sigma^2} \Delta \psi_s + \mu_c \cdot \nabla \psi_s = -1 & \text{in } \partial\Omega \\ \psi_s = 0 & \text{on } \partial\Omega \end{cases}$$

similar formulas holding for $\psi_c(X)$, $\psi_m(X)$. The solutions can again be determined numerically, which allows to compute the expected decision times as

$$t_s = E(\psi_s | H^s), \quad t_c = E(\psi_c | H^c), \quad t_m = E(\psi_m | H^m).$$

References.

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A3. The effect of hawk predation risk on the optimal decision strategy.

Here we analyse the effects of hawk predation, by keeping snake predation constant at $\vartheta_s = 0.1$. When the hawk predation risk is very low, the bird is not constrained by the time spent searching for caterpillars and he minimizes snake predation by keeping a very low snake top threshold (filled circles) and a very high caterpillar top threshold (open circles) (as shown in panel *a*). This decisional strategy minimizes the probability of false-negatives in snake recognition (panel *b*), but exposes the bird to a high rate of false-negatives in caterpillar recognition and, consequently, it increases the expected number of inspected animals (panel *c*). As the hawk-predation risk increases, the difference between the snake and the caterpillar top thresholds decrease, causing a slight increase in snake false-negatives and a marked decrease in caterpillar false-negatives, which result in a reduction of the number of prospective preys inspected by the bird. In these simulations, the parameters are the following:

Figure S2. The effect of the hawk predation risk.

