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Lead Time Dependent Product Deterioration in Manufacturing Systems with Serial, Assembly and Closed-Loop Layout

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In several manufacturing systems found in the automotive, food and semiconductor industries, product quality or value deterioration due to excessive residence time in the system are observed. This phenomenon generates defective or low value products, thus undermining the performance of these systems. In this paper, a method to compute the lead time distribution under a wide set of system architectures is proposed. The method is based on the analysis of the probability that a part enters the system in a certain position in the buffer and on the calculation of the distribution of the time to absorption of a Markov chain representing the states of the downstream portion of system. Then, considering a function that expresses how the product deteriorates with the residence time in the system, the main performance measures are derived. Numerical results show previously uninvestigated behaviors of manufacturing systems under lead time constraints and provide insights on the design of these complex systems. In particular, the method supports the optimal selection of buffer sizes to achieve the target effective throughput in these systems.

Key words: Manufacturing systems, Lead time distribution, Perishable products, Assembly, Loop systems

1. Introduction, Motivation and Objectives

Product quality and value deterioration due to excessive residence times, or lead times, during production is a significant phenomenon in several industries, including automotive, food manufacturing, semiconductor, electronics manufacturing and in polymer forming. For example, in automotive paint shops a car body that is affected by prolonged exposure to the air in the shop floor caused by excessive lead times between operations, is prone to particle contamination, leading to unacceptable quality of the output of the painting process. Moreover, food production is pervaded by strict requirements on hygiene and delivery precision requiring a maximum allowed storage time before packaging (Wang et al. 2014). If the production lead-time exceeds this limit, the product has to be considered as defective and cannot be delivered to the customer. In these systems, higher inventory increases the system throughput but also increases the production lead times, thus increasing the probability of producing defective items. Therefore, a relevant trade-off is generated between production logistics and quality performance that requires advanced system engineering methods to be profitably addressed (Colledani et al. 2014b), (Inman et al. 2013). The same situation is found in production systems where strict lead time constraints are imposed by the customers (Biller et al. 2013).

The first model considering this phenomenon is proposed in (Liberopoulos and Tsarouhas 2002). The installation of a properly sized in-process buffer led to a reduction in failure impact on product

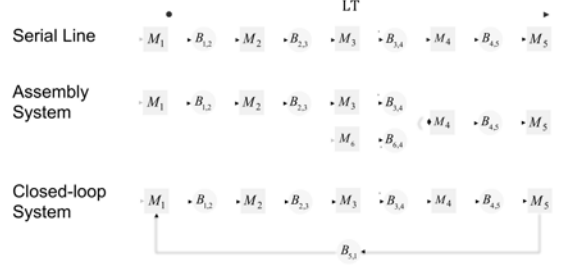


Figure 1 Examples of serial lines, assembly systems and closed-loop systems.

quality and an increase of the system efficiency in a croissant production line. In (Liberopoulos et al. 2007), the authors focused on the production rate of asynchronous production lines in which long failures cause the material under processing in the upstream machines to be scrapped by the system. Moreover, in (Biller et al. 2013) raw material release policies have been proposed to maximize the throughput under an average lead time constraint. In these contributions, the analysis is focused on the average lead time and the distribution of lead time is not taken into consideration. More recently, the calculation of the distribution of the residence time in manufacturing systems have attracted increasing attention. In (Shi and Gershwin 2012) a procedure to numerically compute the distribution of the lead time in two-machine lines with machines having one operational state and one down state has been proposed. In (Colledani et al. 2014a) the analysis of manufacturing systems under lead time dependent product deterioration has been proposed for two-machine lines with general Markovian machines. Finally, the performance of serial lines with product deterioration is analyzed by calculating the distribution of the residence time in Bernoulli lines in (Naebulharam and Zhang 2014). Although these contributions are important to highlight the relevance of the problem, they do not provide methods to predict and control production lead time distributions under realistic manufacturing system features.

In this paper, we propose an exact analytical method for the calculation of lead time distribution under a large set of different system architectures, including serial lines, closed-loop systems and assembly lines. Therefore, the method makes it possible to evaluate the performance of a system under token-based production control policies. Moreover, general Markovian machines are considered, thus enabling to model a wide set of machine behaviors. The lead time distribution is used to calculate the throughput of conforming parts in systems subject to product deterioration.

2. System Modeling

2.1. System Architectures

We consider manufacturing systems formed by K machines and L buffers under three different architectures, namely serial lines, closed-loop systems and assembly systems, as represented in Figure 1. For each system, the material flow is modeled as a discrete flow of parts and the Blocking Before Service (BBS) assumption is considered. Each buffer $B_{i,j}$, located between machine M_i and machine M_j , has finite capacity denoted as $N_{i,j}$. System topology considerations are given below.

- *Serial lines.* In serial lines, machines are connected in an open and linear architecture and the parts visit all the machines, from M_1 to M_K , before leaving the system as final products. The first machine M_1 is never starved and the last machine M_K is never blocked. The system architecture can be represented in form of a directed graph as the set of connections between generic stages i and j . For serial lines this set is $\Omega_{serial} = \{(i, j) | j = i + 1, \forall i = 1, \dots, K - 1\}$.

- *Closed-loop systems.* In closed-loop systems, machines are connected in a closed and linear architecture with a fixed and invariant population N_p which is the total number of parts circulating

in the system. Parts visit all the machines, from M_1 to M_K and go back from M_K to M_1 . The set of connections between generic stages M_i and M_j is expressed by the set $\Omega_{loop} = \Omega_{serial} \cup (K, 1)$, therefore there are K buffers in the system. It is worth to mention that under these modeling features, several token based production control policies, among which kanban, conwip and base stock can be analyzed.

- *Assembly systems.* Assembly systems are tree-structured networks in which input sub-components are assembled to form the final output product. For each machine M_i , the set of buffers from which M_i takes parts is denoted as Υ_i . If at least one buffer in the set Υ_i is empty, then machine M_i is starved. For assembly systems the set of possible connections between generic stages is $\Omega_{assembly} = \{(i, j) | 1 \leq i, j \leq K, i \in \Upsilon_j\}$.

2.2. Machine Behavior

The dynamics of each stage is modeled by a discrete-time and discrete-state Markov chain of general complexity. This setup allows one to analyze a wide set of different stage models within a unique framework. For example, stages with unreliable machines characterized by generally distributed up and down times and also stages with non-identical processing times can be considered within the same framework, thus making the proposed approach applicable to a wide set of real manufacturing systems. In detail, each stage M_k is represented by I_k states, and thus the state indicator α_k assumes values in $[1, \dots, I_k]$. The set containing all the states of M_k is called S_k . The dynamics of each stage in visiting its states is captured by the transition probability matrix λ_k , that is a square matrix of size I_k . Moreover, a quantity reward vector μ_k is considered, with I_k entries. While in the generic state i , M_k produces $\mu_{k,i}$ parts per time unit. Therefore, state i with $\mu_{k,i} = 1$ can be considered as an operational state for stage M_k , while state i with $\mu_{k,i} = 0$ is a down state for stage M_k . The set of operational states for M_i is denoted as U_i and the set of down states is denoted as D_i . Stages with the same features have been considered for the first time in (Gershwin and Fallah-Fini 2007), and, later, in (Colledani 2013).

2.3. Part Quality Deterioration

The quality of parts deteriorates with the time parts spend in a critical portion of the system, denoted by two integers, e and q with $1 \leq e < q \leq K$, and composed of those buffers that are between stages M_e and M_q , in the direction of the material flow. We will refer as lead time of a part to the time spent in the buffers between stages M_e and M_q . If $e = 1$ and $q = K$, then the time spent in the whole system is considered (see for example the serial line in Figure 1). Without loss of generality, we assume that the machines are indexed in such a way that the critical portion contains machines with consecutive indices. This means that the critical portion is composed of stages M_e, M_{e+1}, \dots, M_q and the involved buffers are $B_{e,e+1}, B_{e+1,e+2}, \dots, B_{q-1,q}$. The probability that a part is defective at the end of the line is a non-decreasing function of its lead time. The function $\gamma(h)$ indicates the probability that a part released by the system is defective given that it spent h time units in the critical portion of system. Defective parts are scrapped at the end of the line.

2.4. Performance Measures

The main performance measures of interest for this set of systems are:

- Average total production rate of the system, denoted by E^{Tot} .
- Probability that the lead time, LT , is equal to the number of time units, h , i.e. $P(LT = h)$.
- Average effective production rate of conforming parts, E^{Eff} , which is given by:

$$E^{Eff} = E^{Tot} \sum_{h=1}^{\infty} P(LT = h) [1 - \gamma(h)] \quad (1)$$

- System yield, Y^{system} , i.e. fraction of conforming parts: (E^{Eff} / E^{Tot}) .
- Total average inventory of the system, WIP .

3. Lead Time Distribution

The (i, j) entry of a matrix, M , will be referred to as $M(i, j)$. Similarly, the i th entry of a vector, v , is denoted by $v(i)$.

3.1. Discrete time Markov chain of the system

The state of the system is given by a vector, $s = (a_1, \dots, a_K, b_1, \dots, b_L)$, that contains an entry for each machine, namely, the state of the machine, and an entry for each buffer, namely, the number of parts of the buffer. Considering every combination, there are $(\prod_{k=1}^K I_k)(\prod_{(i,j) \in \Omega} (N_{i,j} + 1))$ states. Some of these states are transient and have zero probability in steady state. For example, every state in which M_1 is up and its output buffer is empty is transient because while M_1 is operational its output buffer cannot get empty. A machine is blocked if it has an output buffer and the output buffer is full. A machine is starved if at least one of its input buffers is empty. The overall behavior of the system can be described by a Discrete Time Markov Chain (DTMC). We denote by R its transition probability matrix and by π its steady state probability vector, which can be obtained by standard techniques, (Stewart 2009). The set of states is denoted by \mathcal{S} .

3.2. Probability of the states when a part enters the critical portion of system

The first step is to calculate the probability of the states of the system at the moment when an arbitrary part enters the critical portion, i.e., when an arbitrary part is put into $B_{e,e+1}$. I.e., we need to determine

$$\pi_E(s) = Pr\{\text{system is in state } s | \text{part entered } B_{e,e+1}\}$$

To this end, we define a modified transition probability matrix, R_E , that contains the probability of those transitions only along which a part is put in $B_{e,e+1}$. Its entry $R_E(s, s')$ for two states, $s = (a_1, \dots, a_K, b_1, \dots, b_L)$ and $s' = (a'_1, \dots, a'_K, b'_1, \dots, b'_L)$, is given as

$$R_E(s, s') = \begin{cases} R(s, s') & \text{if } M_e \text{ is neither starved nor blocked in } s \text{ and it is up in } s' \\ 0 & \text{otherwise} \end{cases}$$

With the above quantities we have

$$\pi_E = \frac{\pi R_E}{\pi R_E \mathbb{1}}$$

where $\mathbb{1}$ is a column vector with all entries equal to 1. The denominator is also the total throughput of the system.

3.3. Lead time distribution in serial and assembly systems

Consider a part that has just been put in $B_{e,e+1}$ by M_e . The lead time of this part cannot be affected by parts that are put in $B_{e,e+1}$ afterwards. Consequently, the state of machines and buffers that are upstream of $B_{e,e+1}$ (including M_e) do not affect the lead time of the considered part. The portion of the system that is downstream M_q can affect the lead time through backward propagation of blockages. The part that is neither upstream nor downstream, which can exist in case of assembly systems, can affect the lead time by forward propagation of starvation. Accordingly, in order to compute the lead time distribution, we need to consider the subsystem that contains those machines and buffers that are not upstream of $B_{e,e+1}$. We refer to this subsystem as S' . Accordingly, a state of S' is given by a vector $s = (a_{e+1}, \dots, a_K, b_f, \dots, b_L)$ where b_f refers to $B_{e,e+1}$. The set of states is denoted by \mathcal{S}_T and the transition probability matrix by R_T . Note that, since there is no machine that puts parts in $B_{e,e+1}$, all buffers in S' will eventually become empty. The initial probability vector, π_T , will be set in such a way that it reflects the steady state probabilities of the whole system at the moment when an arbitrary part enters $B_{e,e+1}$. Therefore, for $s = (a_{e+1}, \dots, a_K, b_f, \dots, b_L)$ we have

$$\pi_T(s) = \sum_{\forall s' = (a'_1, \dots, a'_K, b'_1, \dots, b'_L) \in \mathcal{S} \text{ such that } \forall i, e+1 \leq i \leq K: a'_i = a_i \wedge \forall i, f \leq i \leq L: b'_i = b_i} \pi_E(s')$$

Consider the part that is last in buffer $B_{e,e+1}$. Note that, due to starvation of the involved machines, it is not possible that a part enters one of the buffers of the critical portion after the considered part. Consequently, the lead time of this part ends when all buffers in the critical portion become empty and after that these buffers remain empty permanently. Denoting by $\pi_{T,n}$ the n -step transient probabilities of the DTMC started according to π_T and evolving according to R_T (i.e., $\pi_{T,n} = \pi_T R_T^n$) we have

$$P\{LT \leq n\} = \sum_{\forall s=(a_{e+1}, \dots, a_K, b_f, \dots, b_L) \in \mathcal{S}_T \text{ such that } \forall i, 0 \leq i \leq q-e-1: b_{f+i}=0} \pi_{T,n}(s)$$

i.e., the probability that the lead time is smaller or equal to n equals to the probability that S' reached in n step a state in which buffers $B_{e,e+1}, B_{e+1,e+2}, \dots, B_{q-1,q}$ are empty.

3.4. Lead time distribution in closed-loop systems

As before, consider a part that has just been put in $B_{e,e+1}$. In case of machines organized in a loop, parts that enter $B_{e,e+1}$ afterwards can affect the lead time of the considered part by backward propagation of blockings. This means that the whole system must be considered in order to evaluate the lead time in the critical portion. Moreover, the end of the lead time of the considered part cannot be identified based on testing that the buffers in the critical portion are empty. This implies that we need a model in which the position of the considered part is represented and thus allows us to identify the moment when the considered part leaves the critical portion. In order to have such a model, we will distinguish the parts that are in the critical portion when the considered part enters $B_{e,e+1}$, called inner parts (including the considered part), from those that are elsewhere, called outer parts. Moreover, when an inner part leaves the critical portion it will become an outer part. On the other hand, a part that enters the critical portion remains outer part. Note that this way every part becomes eventually outer part. Consequently, the lead time of the considered part ends when there are no inner parts in the model. The state of the model is described by a vector $s = (a_1, \dots, a_K, b_1, \dots, b_K, c_e, \dots, c_{q-1})$ where a_i is the state of M_i , b_i is the number of outer parts in $B_{i,i+1 \pmod K}$, and c_i is the number of inner parts in $B_{i,i+1 \pmod K}$. (The set of states is \mathcal{S}_T .) Note that there cannot be inner parts out of the critical portion. The initial probabilities of the model, π_T , will again be set to reflect the state of the system when a part enters $B_{e,e+1}$ and keeping in mind that parts inside and outside the critical portion are distinguished. For $s = (a_1, \dots, a_K, b_1, \dots, b_K, c_e, \dots, c_{q-1})$ we have

$$\pi_T(s) = \pi_E(s')$$

with $s' \in \mathcal{S}$ and $s' = (a_1, \dots, a_K, b_1, \dots, b_{e-1}, c_e, \dots, c_{q-1}, b_q, \dots, b_K)$. The transition probability matrix, R_T , has to take into account both classes of parts. The formal definition of R_T is not provided in this paper, due to space limitations. We make however two remarks. R_T must be such that if an inner part moves from $B_{q-1,q}$ to $B_{q,q+1}$ (i.e., it leaves the critical portion) then it becomes an outer part (i.e., the number of inner parts is decremented and the number of outer parts is incremented). In the case when there are both inner and outer parts in a buffer (it can happen to buffers in the critical portion) then the inner parts leave the buffer first, because they entered the buffer first. Based on the n -step transient probabilities, $\pi_{T,n} = \pi_T R_T^n$, we have

$$P\{LT \leq n\} = \sum_{\forall s=(a_1, \dots, a_K, b_1, \dots, b_K, c_e, \dots, c_{q-1}) \in \mathcal{S}_T \text{ such that } \forall i, e \leq i \leq q-1: c_i=0} \pi_{T,n}(s)$$

i.e., the probability that the lead time is smaller or equal to n equals to the probability that the system reached in n step a state in which there are no inner parts.

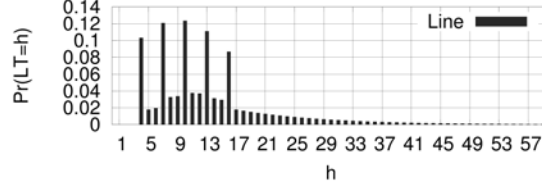


Figure 2 Lead Time distribution for a 5 machine line with identical machines and buffers, $p = 0.01$, $r = 0.1$, $N = 5$.

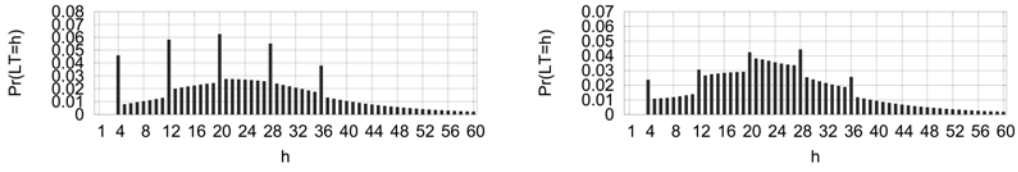


Figure 3 Lead time distribution: other cases with 5 machine lines.

4. Numerical Results

In this section, the proposed method is used to gather insights on the distribution of the lead time and the effective throughput behavior under different system architectures and parameters.

4.1. Multi-stage Serial Lines

In the first experiment, a serial production line formed of 5 identical machines and 4 buffers is considered. Machines are characterized by one operational state and one down state with failure probability $p = 0.01$ and repair probability $r = 0.1$. Buffers have identical size equal to 5. The distribution of the lead time between the first and the last machines ($e = 1$ and $q = 5$) for this system is shown in Figure 2.

As it can be noticed *the lead time distribution is multi-modal with a number of peaks equal to the number of production stages* included in the portion of line where the lead time is calculated, i.e. $q - e + 1$. The peaks appear for the following reasons. The first peak corresponds to the situation where the system is empty and the parts, after being processed by stage M_e , cross the remaining $q - e$ stages without waiting in the intermediate buffers. The j -th peak, with $j = 2, \dots, q - e + 1$, corresponds to the situation where stage M_{e+j-1} failed for sufficiently long time to fill up all the upstream buffers and empty all the downstream buffers. After the stage is recovered from the failure, an incoming part has to wait $N_{k,k+1} - 1$ time units in each upstream buffer, with $e \leq k < e + j - 1$, and 1 time unit in each of the $q - e - j + 1$ downstream stages, if no other failure occurs. *The j -th peak, with $j = 1, \dots, q - e + 1$, is located in correspondence to the following number of time units, h_j :*

$$h_j = q - e + 1 - j + \sum_{i=e}^{j+e-2} (N_{i,i+1} - 1) \quad (2)$$

Other examples are reported in Figure 3, respectively for a 5 machine line with the same machine parameters of the previous experiment and identical buffers of size 10, and for a 5 machine line with identical machines with parameters $p = 0.05$ and $r = 0.5$ and identical buffers of size 10. As it can be noticed, the curves confirm the previous statement.

The impact of the buffer size on the effective throughput is considered next. The first 5 machine line of the previous experiment is taken into account and the sizes of buffers $B_{1,2}$, $B_{2,3}$ and $B_{3,4}$

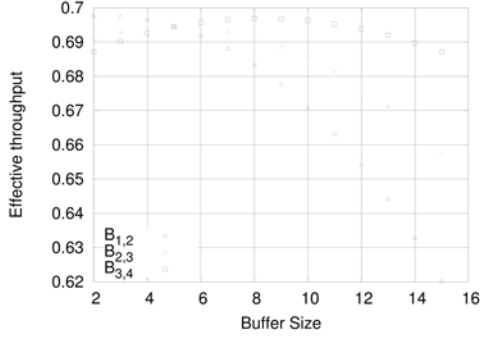


Figure 4 Effective throughput curve as a function of the buffer size in serial lines.

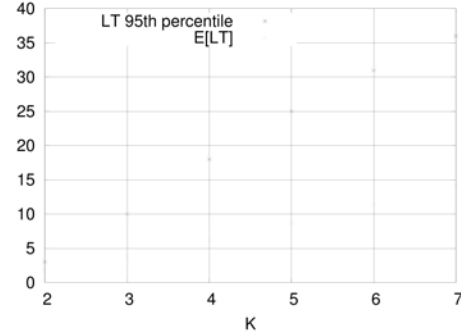


Figure 5 Mean and 95th percentile of the lead time as a function of the number of identical machines in the serial line, K .

are increased independently from 2 to 15. The effective throughput is calculated by considering a quality deterioration function $\gamma(h)$ which assumes value zero for $h = 1, \dots, h^*$ and value 1 for $h > h^*$, with $h^* = 35$. In other words, if the parts spend more than 35 time units in the system, they turn into defective. The resulting effective throughput curves are shown in Figure 4.

As it can be noticed, although the line is composed of identical machines, each buffer has a different effect on the effective throughput. In particular, *increasing the first buffer in the line has a detrimental effect on the effective throughput, while the effective throughput curve is concave if $N_{3,4}$ is increased*, presenting a maximum corresponding to a buffer capacity equal to 8. This behavior is due to the fact that increasing the size of the buffer $B_{3,4}$ has a positive impact on the total throughput of the system, but also a negative impact on the distribution of the lead time that parts spend in the system. When the first effect is more important than the second, i.e. for small buffers, the effective throughput curve increases with the buffer size. However, when the second phenomenon is more important, the probability that the lead time exceeds the threshold gets considerably higher and the yield reduction causes a drop in the effective throughput. The positive impact on the total throughput is less relevant than the negative impact on the yield if $B_{1,2}$ is increased. Indeed, according to equation 2, the capacity of the first buffer moves all the peaks in the lead time distribution to higher lead time values, thus considerably affecting the probability of exceeding the fixed lead time limit. This phenomenon leads to a very interesting consideration. It is well known that, since the line has identical machines, increasing the size of $B_{2,3}$ and the size of $B_{3,4}$ is equivalent in terms of total throughput. However, they are not equivalent in terms of lead time distribution and, thus, in terms of effective throughput. *In lines with identical machines and lead time dependent quality deterioration, large buffers should be located in the last stages of the system* as this results to be beneficial in terms of effective throughput. The line reversibility principle is indeed not verified in terms of effective throughput.

Furthermore, we analyzed the effect of the number of machines in the line, K , on the average lead time and the 95th percentile of the lead time, for lines with identical machines with $p = 0.01$ and $r = 0.1$. The results are shown in Figure 5. As it can be noticed, both the mean and the 95th percentile of the lead time increase almost linearly with the line length. However, the 95th percentile shows a higher slope. This experiment shows that in systems with severe constraint on the lead time the average lead time itself cannot be considered in the prediction of the system performance, since the tail of the distribution is much more sensitive to the line length. The entire lead time distribution should be considered for a reliable prediction of the system performance.

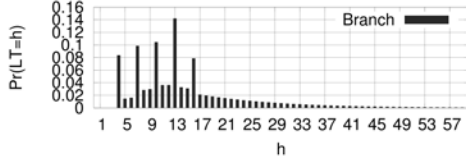


Figure 6 Lead Time Distribution for the upper branch of the assembly line in Figure 1.

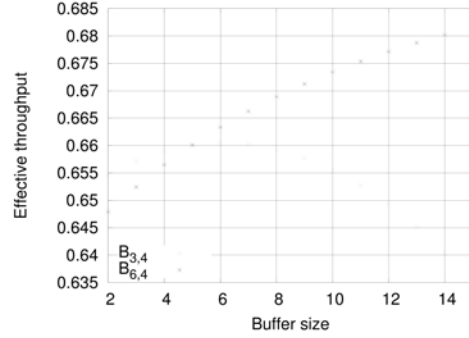


Figure 7 Effective throughput curve as a function of the buffer size in assembly systems.

4.2. Assembly Systems

In this experiment, the assembly system represented in Figure 1 is considered. The upper branch is formed by 5 machines that are identical to the machines of the line considered in the first experiment of the previous section. The lead time distribution of the components crossing the upper branch is reported in Figure 6.

As it can be noticed, the probability peaks are observed for the same time units of the serial line case. However, *the lower branch affects the lead time of parts crossing the upper branch*, due to the fact that the buffer $B_{6,4}$ can be empty, thus causing starvation at the assembly machine M_4 . As a consequence, the most probable peak is observed at $h = 10$ for the serial line while it is moved to $h = 13$ for the assembly line. This peak corresponds to the situation where M_4 is either down or starved (buffer $B_{6,4}$ is empty) and then it gets recovered.

Another interesting behavior can be observed by investigating the effective throughput curve as a function of the buffer size. In this analysis, two buffer sizes are independently increased, namely buffer $B_{3,4}$ in the upper branch of the assembly system and buffer $B_{6,4}$ in the lower branch. The effective throughput is calculated by considering the same quality deterioration function used for the experiment in section 4.1, for the branch with $e = 1$ and $q = 5$. The resulting effective throughput curves are shown in Figure 7.

As it can be noticed, the effective throughput curve is a concave function of $N_{3,4}$, presenting a maximum corresponding to a buffer capacity equal to 6. This behavior is due to the same phenomenon explained in the previous section, since the buffer $B_{3,4}$ is part of the branch considered for the calculation of the lead time. However, this phenomenon is not visible in the curve reporting the effective throughput as a function of the buffer $B_{6,4}$. Indeed, increasing $N_{6,4}$ is beneficial both in terms of total throughput and of system yield. *The lead time calculated in the upper branch of the system is, in fact, reduced while increasing $N_{6,4}$, as the probability of starvation of M_4 is reduced.* This causes a significant positive impact on the effective throughput of the system. A counter effect is the increase of the lead time of the lower branch of the assembly system, which is not shown here due to space limitation. This experiment proves that, in assembly systems, the effect of buffers on the lead time distribution and on the effective throughput is driven by complex dynamic phenomena which require the availability of a modeling tool to be properly addressed during the system design phase.

4.3. Closed-loop Systems

The last set of experiments is related to closed-loop systems. The considered system is obtained by adding one buffer, namely $B_{5,1}$, to the five machine serial line considered in the first experiment

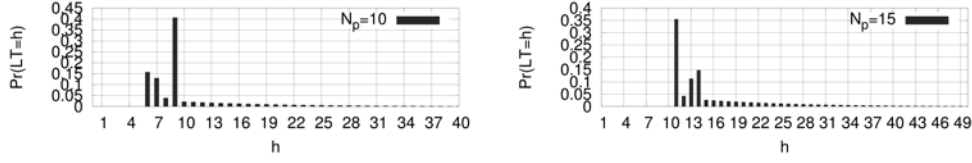


Figure 8 Lead time distribution: closed loop systems. $N_p = 10$ and $N_p = 15$.

h^*	E_{Line}^{Eff}	E_{Conwip}^{Eff}	Difference
35	0.6944	0.699	+0.7%
25	0.6503	0.674	+3.7%

Table 1 Comparison between effective throughput of serial and conwip controlled lines.

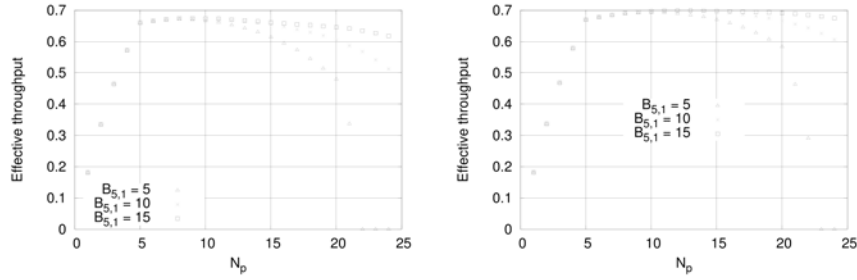


Figure 9 Effective throughput as a function of the population N_p , for three different sizes of $B_{5,1}$, for $h^* = 35$ (left) and $h^* = 25$ (right).

of section 4.1. The capacity of this buffer, $N_{5,1}$ is set to 5. The lead time distributions with $e = 1$ and $q = 5$ for two different population levels, $N_p = 10$ and $N_p = 15$ are shown in Figure 8. As it can be observed, *the distribution of the lead time is considerably different from the case of serial and assembly lines*. Indeed, if $N_{5,1} < N_p$ a part entering the first buffer will never see the *critical portion* of system empty, but at least $N_p - N_{5,1} + 1$ time units are needed before the part can leave the *critical portion* of system.

The effective throughput as a function of the loop population for three different values of the size of the buffer $B_{5,1}$ is reported in Figure 9. The same quality deterioration function as the previous experiments is considered, with two values of h^* , namely 25 and 35. Both figures show that, differently from the total throughput curves, *the effective throughput curves are not symmetric with the loop population N_p* . Indeed, small populations are preferable to large populations since the lead time reduces and it gets less probable to exceed the lead time limit.

An interesting consideration can be drawn by comparing the effective throughput of the serial line with identical buffers equal to 5 and the throughput of the corresponding loop system, with the same buffer sizes in the critical portion of system. Indeed, this second configuration corresponds to a serial line where the conwip token-based production control policy is applied. The serial line effective throughput is compared to the optimized conwip line effective throughput in Table 1, for both h^* equal to 25 and 35. The results show that *the conwip line outperforms the serial line both for less critical and more critical lead time constraints*, although the main benefit is found in the second condition.

5. Conclusions and Future Research

This paper has presented a general methodology to compute the lead time distribution in manufacturing systems where machines undergo general Markovian behavior and are connected as serial lines, assembly systems and closed-loop systems. The lead time distribution is useful to compute the throughput of conforming products, namely effective throughput, in systems processing parts which are subject to lead time dependent product deterioration or in systems with severe lead time constraints due to strict customer due date performance requirements. Several previously unknown system behaviors have been demonstrated and discussed showing the important effect of buffers on the throughput of conforming products for this set of systems. The proposed theory and methodology pave the way to the formulation and solution of new important system design problems. For example, the problem of designing the buffers in a manufacturing system under specific lead time constraints imposed by the product deterioration or by the market due-dates could be solved with the approach presented in this paper. Moreover, new production planning methods that exploit the availability of an exact prediction of the lead time distribution could be developed, thus enabling manufacturers to benefit increased robustness in production planning. Finally, new lead time oriented production control and part release policies can be formulated and solved with the proposed approach, aiming at limiting the lead time in the system.

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