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Abstract

Kakwani and Lambert (1998) state three axioms which should be respected by an equitable tax system. Using the Atkinson-Plotnick-Kakwani re-ranking indexes of taxes, tax rates and post-tax incomes, calculated with respect to the ranking of pre-tax income distribution, they then propose a measurement system to evaluate the negative influences that axiom violations exert on the redistributive effect of taxes. In this paper we reconsider the way Kakwani and Lambert measure violations of their second axiom, which concerns the re-ranking of tax rates. We construct a non-negative index which is strictly faithful to Kakwani and Lambert's commands; we show that the Authors' measure does not exactly fit the statements made in their second axiom. Both Kakwani and Lambert's original measurement system and the modified one are then applied to Italian personal income tax in 2008. According to the modified measurement system, the average tax rate seems to play a smaller role than that suggested by the results gained using Kakwani and Lambert's original methodology.

JEL Codes: *C81, H23, H24*

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I. Introduction

Kakwani and Lambert (1998), hereafter KL, propose an approach for measuring inequity in taxation. They introduce three axioms which should be respected by an equitable tax system: (Axiom 1) tax should increase monotonically with respect to people's ability to pay; (Axiom 2) richer people should pay taxes at higher rates; (Axiom 3) no re-ranking should occur in people's living standards. A violation of Axiom 1 automatically entails a violation of Axiom 2, although not necessarily the other way round. Moreover, Axiom 3 can be violated only if Axiom 2 (and consequently Axiom 1) holds. In their article KL explain the extent to which axiom violations can be evaluated: in this paper we reconsider the issue of measuring Axiom 2 violations and introduce a different measuring index for these violations.

Let x_1, x_2, ...x_K the pre-tax income levels associated to K income units, who are paying t_1, t_2, ...t_K in tax. Both incomes and taxes can be expressed either in nominal values or in equivalent values. Moreover let $y_i = x_i - t_i$ and $a_i = t_i/x_i$ represent the disposable income and the tax rate, respectively, which result in unit i (i=1, 2,...,K), after having paid tax t_i.

KL (page 372) conclude that *"Axiom 1 is violated if and only if the ranking of income units by X and T differ. Axiom 2 is violated iff the rankings by X and by A of income units pairs {i, j} for which Axiom 1 holds differ [........]. Axiom 3 is in fact violated whenever the rankings of income units by X and by Y differ"*.

Let G_X, G_T, G_A and G_Y be the Gini coefficients for attributes X, T, A and Y, respectively. Let $C_{Z|X}$, the concentration coefficient for an attribute Z of income units (Z= T, A, Y). KL consider the Atkinson-Plotnick-Kakwani re-ranking indexes $R_{T|X} = \left(G_T - C_{T|X}\right)$, $R_{A|X} = \left(G_A - C_{A|X}\right)$ and $R_{Y|X} = \left(G_Y - C_{Y|X}\right)$. According to KL if $R_{T|X} > 0$, Axiom 1 is violated; analogously, if $R_{Y|X} > 0$ Axiom 3 is violated; regarding what concerns Axiom 2 one should *"check* $R_{A|X} - R_{T|X}$ *: if zero [positive], this suggests that Axiom 2 is upheld [violated]*" (KL, page 373). Even if $R_{A|X} - R_{T|X}$ could also be negative, the authors observe that it is always non-negative in extensive simulations, as "*the difference between the X- and A- rankings is, indeed, 'at least as extensive' as between the X- and T- rankings.*" Summarizing, according to KL measurement system:

Axiom 1 is violated whenever $R_{T|X} = G_T - C_{T|X} > 0$;

Axiom 2 is violated whenever $R_{A|X} - R_{T|X} = \left(G_A - C_{A|X}\right) - R_{T|X} > 0$; (1)

Axiom 3 is violated whenever $R_{Y|X} = G_Y - C_{Y|X} > 0$

In this paper we focus on measuring the extent of Axiom 2 violations differently from KL, who base their approach on concentration and Lorenz curves. We evaluate Gini and concentration coefficient by differences between attributes related to pairs of tax payers units. Applying exactly the Axiom 2 statement we elaborate a re-ranking index for tax rates which excludes all income unit pairs violating the Axiom 1 command.

According to our empirical analysis, when measured by the alternative method here proposed, Axiom 2 violations seem to affect the implicit potential equity to a lower degree than by the original KL method.

The remainder of the paper is structured as follows. Section 2 summarizes the KL approach and presents our alternative measure for the extent of Axiom 2 violations. In Section 3 we apply the original KL measurement system and our modified version to the 2008 Italian personal income tax. Section 4 acts as our conclusion.

II. The Extent of Axiom Violations and the Redistributive Effect Decomposition

Starting from the Kakwani progressivity index $P = C_{T|X} - G_X$, KL write the redistributive effect of a tax system as $RE = G_X - G_Y = \tau P - R_{Y|X}$, where τ is the ratio between the total amount of the tax and the total amount of the after tax income; then they decompose the redistributive effect as[1]

$$RE = \tau\left[P + \left(R_{A|X} - R_{T|X}\right) + R_{T|X}\right] - S_1 - S_2 - S_3 \quad (2)$$

In Equation (2), $\tau\left[P + \left(R_{A|X} - R_{T|X}\right) + R_{T|X}\right]$ represents the potential redistributive effect, while $S_1 = \tau R_{T|X}$, $S_2 = \tau\left(R_{A|X} - R_{T|X}\right)$ and $S_3 = R_{Y|X}$ represent the loss in redistributive effect due to violations of Axiom 1, Axiom 2 and Axiom 3, respectively.

The KL approach in calculating re-ranking index is based on Lorenz and concentration curves. We shall now look at re-ranking indexes through attribute differences for pairs of income units.

For an attribute *Z*, the Gini coefficient can be calculated by the well known formula based on the average of absolute differences, which we express in the following form:

$$G_Z = \frac{1}{2\mu_Z N^2} \sum_{i=1}^{K} \sum_{j=1}^{K} \left(z_i - z_j\right) p_i p_j I_{i-j}^Z , \qquad I_{i-j}^Z = \begin{cases} 1: & z_i \geq z_j \\ -1: & z_i < z_j \end{cases} \quad (3)$$

where p_i and p_j are weights associated to z_i and z_j, respectively, $\sum_{i=1}^{K} p_i = N$, μ_Z is the average of *Z*, and I_{i-j}^Z is an indicator function.

[1] In Kakwani and Lambert (1998) Equation (2) is written as $RE = \tau\left[P + R_{A|X}\right] - S_1 - S_2 - S_3$: we prefer to keep distinct the role of the components $R_{A|X}$ and $R_{T|X}$ within the square brackets to make comparisons more immediate when we introduce Equation (11).

When income units are lined up by ascending order of *X*, the concentration coefficient of attribute *Z* can be written as (Vernizzi, 2009)

$$C_{Z|X} = \frac{1}{2\mu_Z N^2} \sum_{i=1}^{K}\sum_{j=1}^{K}\left(z_i - z_j\right) p_i p_j I_{i-j}^{Z|X} , \qquad I_{i-j}^{Z|X} = \begin{cases} 1: & x_i > x_j \\ -1: & x_i < x_j \\ I_{i-j}^{Z}: & x_i = x_j \end{cases} \tag{4}$$

Consequently the re-ranking index of *Z* with respect to *X* becomes:

$$R_{Z|X} = \frac{1}{2\mu_Z N^2} \sum_{i=1}^{K}\sum_{j=1}^{K}\left(z_i - z_j\right) p_i p_j \left(I_{i-j}^{Z} - I_{i-j}^{Z|X}\right) \tag{5}$$

According to KL (see their Equation (3)), the extent of violations concerning Axiom 1, 2 and 3 can be evaluated by

$$S_1 = \tau R_{T|X} = \frac{\tau}{2\mu_T N^2} \sum_{i=1}^{K}\sum_{j=1}^{K}\left(t_i - t_j\right) p_i p_j \left(I_{i-j}^{T} - I_{i-j}^{T|X}\right) \tag{6}$$

$$S_2 = \tau\left(R_{A|X} - R_{T|X}\right) = \frac{\tau}{2N^2} \sum_{i=1}^{K}\sum_{j=1}^{K}\left[\frac{a_i - a_j}{\mu_A}\left(I_{i-j}^{A} - I_{i-j}^{A|X}\right) - \frac{t_i - t_j}{\mu_T}\left(I_{i-j}^{T} - I_{i-j}^{T|X}\right)\right] p_i p_j \tag{7}$$

$$S_3 = R_{Y|X} = \frac{1}{2\mu_Y N^2} \sum_{i=1}^{K}\sum_{j=1}^{K}\left(y_i - y_j\right) p_i p_j \left(I_{i-j}^{Y} - I_{i-j}^{Y|X}\right) \tag{8}$$

where $\tau = \left(\mu_T / \mu_Y\right)$.

Concerning Axiom 2, if we consider that according to KL (page 378) *"the scope of the axiom is confined to those income pairs {i, j} for which Axiom 1 holds"*, the following recast index is a straight application of KL's indication:

$$S_2^{*} = \tau R_{A|X}^{*} = \frac{\tau}{2N^2} \sum_{i=1}^{K}\sum_{j=1}^{K}\frac{a_i - a_j}{\mu_A} p_i p_j \left[\left(I_{i-j}^{A} - I_{i-j}^{A|X}\right) - \left(I_{i-j}^{T} - I_{i-j}^{T|X}\right)\right] \tag{9}$$

Equation (9) derives from the general re-ranking formula (5); however in (9) tax rate re-rankings occurring together with tax re-ranking are not considered. It is immediately confirmed that $R_{A|X}^{*} \geq 0$, as $\left(I_{i-j}^{T} - I_{i-j}^{T|X}\right)$ is either equal to $\left(I_{i-j}^{A} - I_{i-j}^{A|X}\right)$ or it is zero.

So we have two alternatives to check whenever axiom 2 is violated. If we want to observe literally the KL command, we should adopt $\tau R^{*}_{A|X}$, defined by Equation (9).

If we want to adopt on the other hand the KL measure, we should be aware that expression (7) does not involve only income units pairs $\{i, j\}$ for which Axiom 1 holds and for which the rankings by X and by A differ; in fact the difference between S_2 and S_2^* is not necessarily equal to zero, as we can easily observe by investigating the expression

$$S_2 - S_2^* = \left\{ \frac{\tau}{2N^2} \sum_{i=1}^{K} \sum_{j=1}^{K} \left(\frac{a_i - a_j}{\mu_A} - \frac{t_i - t_j}{\mu_T} \right) p_i p_j \left(I^{T}_{i-j} - I^{T|X}_{i-j} \right) \right\} \tag{10}$$

According to our simulations the value of the expression in Equation (10) is positive and has the roughly the same magnitude as that in Equation (9).
If (9) substitutes (7), (2) can be rewritten as

$$RE = \tau \left[P + R^{*}_{A|X} + R_{T|X} \right] - S_1 - S_2^* - S_3 \tag{11}$$

Now, $\tau \left[P + R^{*}_{A|X} + R_{T|X} \right]$ measures potential equity.

In the next Section we investigate how different S_2 and S_2^* are as represented in the Italian personal tax system.

III. Data and results

Data

For input data, we make use of the Bank of Italy’s Survey on Household Income and Wealth (hereafter SHIW) published in 2010. It contains information on household post-tax income and wealth in the year 2008, covering 7,977 households, and 19,907 individuals. The sample is representative of the Italian population, composed of about 24 million households and 60 million individuals. For further details on the sample selection and aggregate statistics see Brandolini (1999) and Bank of Italy (2010).

This data base was used to obtain gross and net incomes according to the Italian Personal Income Tax.[2] Once gross and net incomes have been simulated, we considered both individual tax payers and households, the latter as aggregated income units. When considering households, equivalent components have been obtained by the Cutler and Katz Scale (*CS*), defined as:

$$CS = \left(N_A + 0.33N_C\right)^{0.68}$$

where N_A and N_C are, respectively, the number of adults and children (individual within the household aged 17 or less) within each household. The value for the coefficient associated to children (.33) and the value of the exponent (.68) have been chosen in order to minimize the re-ranking of after tax incomes.[3]

Results

Table 1 reports the redistributive effect decomposition for individual earners which are estimated by adopting both the KL methodology and the one discussed here. Table 2 reports the corresponding statistics obtained when considering households.

Focusing on individual taxpayers, the Gini coefficient for the pre-tax income distribution is 0.43398, whilst that for the post-tax income distribution is 0.37833. It follows that *RE* is equal to 0.05566. According to KL's original equation (expression 2), the potential redistributive effect $\tau\left[P + \left(R_{A|X} - R_{T|X}\right) + R_{T|X}\right]$ is 126.02% of *RE*, whilst according to our modified equation (11) it is 113.24. The difference between these two

[2] We used an updated version of the microsimulation model described in Pellegrino et al. (2011). See Pellegrino and Vernizzi (2010) for further details concerning the Italian personal income tax structure.

values is due to the lower value we obtain for the loss depending on Axiom 2 violations: S_2^*, defined in equation (9), which is 7.50% of *RE*, while S_2 calculated according to KL is much higher, being 20.27% of *RE*.

When considering aggregated households similar proportions between S_2 and S_2^* are observed: the potential redistributive effect is 114.82% of *RE* according to KL definition and 108.89 according to equation (11). Again, the difference is due to the lower value we obtain for the extent of Axiom 2 violations: S_2^* is 3.68, while S_2 is 9.60% of *RE*.[4]

The issue of investigating under what conditions S_2 can be a reasonable approximation of S_2^* remains open. However this is not our immediate aim: rather, we desire to point out that, concerning the extent of Axiom 2 violations, S_2^* is a fully coherent measure with KL commands, whilst S_2 is not. By using simulations and resampling techniques (such as bootstrap) it could be possible to check under which conditions $S_2 - S_2^*$ happens to be not significantly different from zero.[5] However, in our opinion, at least in the situation considered in this note, the differences between S_2 and S_2^* appear quite evident.

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[3] See van de Ven and Creedy (2005) for an exhaustive discussion about this approach in estimating equivalence scale parameters.

[4] For what concerns households, we did not limit our analysis to the scale reported above: in all the simulations we performed, by applying different Cutler-Katz equivalence scales and the modified OECD scale, the ratios between S_2 and S_2^* are similar to those which result from Table 1 and 2.

IV. Concluding remarks

By applying exactly the indications stated by Kakwani and Lambert (1998), in this paper we specify an alternative index to evaluate violations concerning the progressive command (KL Axiom 2) in a tax system. This alternative index applies the Axiom exactly as Kakwani and Lambert suggest, as it strictly focuses only on income units pairs which respect the minimal progression command (KL Axiom 1), and evaluates re-ranking index formulae using the differences between income unit pairs.

Our simulations give evidence that when applying the alternative method suggested here, Axiom 2 violations seem to produce much less of an effect than that which is estimated by using the original KL methodology: the extent of Axiom 2 violations measured using the approach we suggest is found to be more in line with the extent of Axiom 1 violations.

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[5] In any case when performing similar tests, one should always remind the criticisms brilliantly illustrated by McCloskey (1985).

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Table 1. *RE* decomposition for taxpayers

Decomposition		Pre-tax income	Post-tax income	*RE*	Potential equity	Axiom 1	Axiom 2	Axiom 3	Total Axioms
Kakwani and Lambert		0.43398	0.37833	0.05566	0.07014	0.00237	0.01128	0.00083	0.01448
		-	-	100.00	126.02	4.26	20.27	1.48	26.02
Modified		0.43398	0.37833	0.05566	0.06303	0.00237	0.00417	0.00083	0.00737
		-	-	100.00	113.24	4.26	7.50	1.48	13.24

Source: Our elaborations on SHIW.

Table 2. *RE* decomposition for households

Decomposition		Pre-tax income	Post-tax income	*RE*	Potential equity	Axiom 1	Axiom 2	Axiom 3	Total Axioms
Kakwani and Lambert		0.39793	0.34506	0.05287	0.06070	0.00195	0.00508	0.00081	0.00783
		-	-	100.00	114.82	3.68	9.60	1.53	14.82
Modified		0.39793	0.34506	0.05287	0.05757	0.00195	0.00195	0.00081	0.00470
		-	-	100.00	108.89	3.68	3.68	1.53	8.89

Source: Our elaborations on SHIW.