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Effective string picture for confinement at finite temperature: theoretical predictions and high precision numerical results

M. Caselle[a],M. Hasenbusch[b] and M. Panero[a]

[a]Università di Torino and INFN, Sezione di Torino, Via P. Giuria 1, I–10125 Torino, Italy,
E-mail: caselle@to.infn.it, panero@to.infn.it

[b]NIC/DESY Zeuthen, Platanenallee 6, D–15738 Zeuthen, Germany,
E-mail: Martin.Hasenbusch@desy.de

The effective string picture of confinement is used to derive theoretical predictions for the interquark potential at finite temperature. At short distances, the leading string correction to the linear confining potential between a heavy quark-antiquark pair is the "Lüscher term". We assume a Nambu–Goto effective string action, and work out subleading contributions in an analytical way. We discuss the contribution given by a possible "boundary term" in the effective action, comparing these predictions with results from simulations of lattice $\mathbf{Z}_2$ gauge theory in three dimensions, obtained with an algorithm which exploits the duality of the $\mathbf{Z}_2$ gauge model with the Ising spin model.

1. EFFECTIVE STRING PICTURE

The effective string picture is expected to provide a good physical description of confining gauge theories in the low energy regime. Assuming that two confined color charges are joined by a thin flux tube that fluctuates like a vibrating string, and describing the string world sheet dynamics by an effective action S_{eff}, one can derive quantitative predictions about the potential between a confined quark–antiquark pair. In particular, for *massless* string fluctuations, the simplest choice is the Nambu–Goto string action: $S_{\text{eff}} = \sigma \cdot \mathcal{A}$, which is proportional to the area $\mathcal{A}$ of the world–sheet surface and σ is the *string tension*, appearing as a parameter of the effective theory.

We consider a 3D system with the extension $L_s^2 \times L$ with $L_s >> L$. With periodic boundary conditions employed (at least) in the short direction, the temperature T is proportional to $1/L$.

Taking into account leading quantum fluctuations, the result for the expectation value of the Polyakov loop correlator is given by

$$\langle P^\dagger(R)P(0)\rangle = \frac{e^{-\sigma RL+k}}{\eta\left(i\frac{L}{2R}\right)} , \qquad (1)$$

where η is Dedekind's function. The term associated with the minimal world sheet surface induces the exponential area-law fall off, and the consequent linear rise in the interquark potential $V(R)$. The first non-trivial contribution in S_{eff} results in the determinant of the Laplace operator, and the corresponding contribution to the interquark potential $V(R)$ — in a regime of distances shorter than $\frac{L}{2}$ — is the Lüscher term [1]:

$$V(R) = -\frac{1}{L}\ln\langle P^\dagger(R)P(0)\rangle \simeq \sigma R - \frac{\pi}{24R} . \qquad (2)$$

Further terms in the expansion of the Nambu–Goto action give rise to a contribution involving a combination of Eisenstein functions [3]:

$$-\frac{\pi^2}{1152\sigma R^3}\left[2E_4\left(i\frac{L}{2R}\right) - E_2^2\left(i\frac{L}{2R}\right)\right] . \qquad (3)$$

However, it is worth mentioning that, while Eq. (1) and Eq. (2) are generally accepted, on the other hand the further contribution to $V(R)$ expressed by Eq. (3) is still under debate.

A possible "boundary term" in the effective action, as proposed in ref. [2], can be treated by means of a perturbative expansion in b (a parameter proportional to the coefficient of such a term),

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which induces a leading order correction like [4]:

$$R \longrightarrow \frac{R}{\sqrt{1+\frac{2b}{R}}}\,, \tag{4}$$

with a short distance contribution to $V(R)$: $-\frac{b\pi}{24R^2}$.

2. $\mathbf{Z}_2$ LATTICE GAUGE THEORY

We run numerical simulations of $\mathbf{Z}_2$ lattice gauge theory in three space-time dimensions [4]. We chose this model because the effective string picture is believed to be independent of the underlying gauge group, and $\mathbf{Z}_2$ gauge group is interesting from the point of view of the *center relevance* in confinement. Moreover, the duality with the Ising spin model and a novel algorithm allow us to reach high precision results.

The pure 3D lattice gauge model is described in terms of $\sigma_{x,\mu} \in \mathbf{Z}_2$ variables defined on the lattice bonds. The dynamics is governed by the standard Wilson action, which enjoys $\mathbf{Z}_2$ gauge invariance. The partition function is expressed in terms of plaquette variables σ_p and reads:

$$Z(\beta) = \sum_{\text{c.}} e^{-\beta S} = \sum_{\text{c.}} \exp\left[+\beta \sum_p \sigma_p\right]\,. \tag{5}$$

There are different phases: a confined, strong coupling phase, with massive string fluctuations for $\beta < 0.47542(1)$ [5], a confined, *rough* phase, with massless string fluctuations (this is the regime we studied in our simulations) and a deconfined phase for $\beta > 0.7614134(2)$ [6].

This model is *dual* with respect to the $\mathbf{Z}_2$ Ising spin model in 3D. The ratio of Polyakov-loop correlators $G(R)$ in the gauge model can be expressed as a ratio of partition functions $Z_{L\times R}$ in the spin model:

$$\frac{G(R)}{G(R+1)} = \frac{Z_{L\times R}}{Z_{L\times(R+1)}}\,. \tag{6}$$

The index $L \times R$ means that links perpendicular to an $L \times R$ plane have an antiferromagnetic coupling constant. The ratio of partition functions can be expressed as an expectation value in one of the two ensembles. However the corresponding observable shows extremely large fluctuations. This problem can be overcome by a factorization

$$\frac{Z_{L\times R}}{Z_{L\times(R+1)}} = \prod_{i=0}^{L-1} \frac{Z_{[L\times R]+i}}{Z_{[L\times R]+i+1}}\,, \tag{7}$$

where $Z_{[L\times R]+i+1}$ has just one more antiferromagnetic link than $Z_{[L\times R]+i}$. The expectation values corresponding to these ratios can be easily obtained. A similar method (*snake algorithm*) has been employed in refs. [7] to compute the 't Hooft loop in the $SU(2)$ gauge model.

Important features of our implementation are multi-level updating and a hierarchical organization of sublattices. The CPU time is roughly proportional to the inverse temperature L, and *independent of the distance R between the quark sources*, thus the algorithm is particularly useful for large interquark distances.

3. NUMERICAL RESULTS

Let $F(R,L)$ be the free energy of a heavy quark-antiquark pair at finite temperature: $G(R) = e^{-F(R,L)}$. We studied "quantum terms" in free energy differences, by measuring the quantity defined as:

$$Q(R,L) = F(R+1,L) - F(R,L) - \sigma L\,. \tag{8}$$

Fig. 1 shows that for large interquark distances (namely: $L < 2R$) our numerical results are in good agreement with the NLO prediction eq. (3) from Nambu–Goto string. It is important to note that the agreement between numerical results and the NLO prediction is not the result of a fitting procedure. A "purely classical" area law is definitely ruled out, and the LO term alone is not sufficient to describe the data. We also found that the coefficient for a possible "boundary term" in the effective action for this model is very small, suggesting that it is exactly zero, i.e. $b = 0$.

Fig. 2 — see [4] for details — shows the deviation of a quantity related to the free energy second derivative from the free string behaviour in the $L > 2R$ regime. The normalization is chosen so as to allow a meaningful comparison among different LGT's. Our $\mathbf{Z}_2$ results and $SU(2)$ data

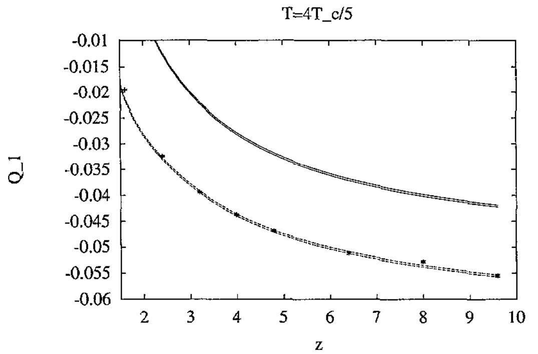

Figure 1. $Q(R, L)$ for $L = 10$ (i.e. $T = 4T_c/5$) at $\beta = 0.75180$. z is defined as: $z \equiv \frac{2R}{L}$. Solid curves correspond to the free bosonic string prediction, while dashed lines are the NLO Nambu–Goto correction. A pure area–law would be described by $Q = 0$.

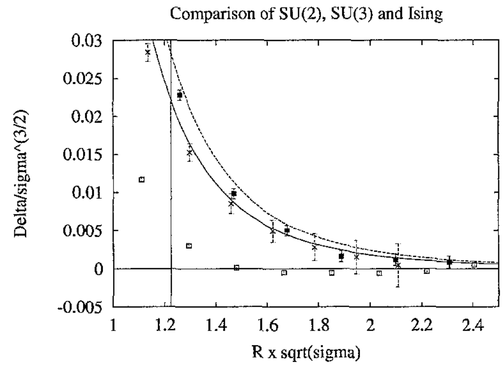

Figure 2. Behaviour of $SU(3)$ [2] (white squares), $SU(2)$ [8] (crosses), and $\mathbf{Z}_2$ [4] (black squares) gauge models at short distances.

are in good agreement: this may be a possible signature of the center relevance to confinement.

4. CONCLUSIONS

We tested the effective string predictions with precise numerical data for the finite temperature $\mathbf{Z}_2$ lattice gauge theory, using an algorithm that exploits the duality properties of the model. We explored a wide range of distances and detected next-to-leading order effects. Our numerical results seem to rule out a possible "boundary term" in the effective string action describing the present model. For large distances R and high temperatures (i.e. small L) we see an excellent agreement of the numerical data with the NLO prediction Eq. (3). In the regime of short distance and low temperature, we compared our results with different gauge theories, which have been studied by other authors. We focused onto the deviation of a well-suited combination of free energy differences with respect to the free string prediction. In particular, the agreement between our $\mathbf{Z}_2$ results and $SU(2)$ data might be interpreted as a possible signature of the fact that the center degrees of freedom play an important role in the confinement mechanism.

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