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Abstract. We investigate the physical properties of the protoneutron stars in the framework of a relativistic mean-field theory based on nonextensive statistical mechanics, characterized by power-law distributions. We study the finite-temperature equation of state in β-stable matter at fixed entropy per baryon, in the absence and in the presence of hyperons and trapped neutrinos. We show that nonextensive power-law effects could play a crucial role in the structure and in the evolution of the protoneutron stars also for small deviations from the standard Boltzmann-Gibbs statistics.

1. Introduction

A protoneutron star (PNS) is formed in a stellar remnant after a successful core-collapse supernova explosion of a star with a mass smaller than about 20 solar masses and in the first seconds of its evolution it is a very hot (temperature of up to 50 MeV), lepton rich and β-stable object and a lepton concentration typical of the pre-supernova matter [1].

The essential microphysical ingredients that govern the macrophysical evolution of the PNS in the so-called Kelvin-Helmholtz epoch, during which the remnant changes from a hot and lepton-rich PNS to a cold and deleptonized neutron star, are the equation of state EOS of dense matter and its associated neutrino opacity. The diffused neutrinos are observed by terrestrial detectors and provide unique information on the supernova and the PNS. Nevertheless, the PNS is in quasi-stationary β-equilibrium state during its evolution because the time scale of the weak interaction is much shorter than the time scale of neutrino diffusion.

The knowledge of the nuclear EOS of dense matter at finite temperature plays a crucial role in the determination of the structure and in the macrophysical evolution of the PNS [2, 3]. The processes related to strong interaction should in principle be described by quantum chromodynamics. However, in the energy density range reached in the compact stars, strongly non-perturbative effects in the complex theory of QCD are not negligible [4]. In the absence of a converging method to approach QCD at finite density one often turns to effective and phenomenological model investigations. In the last years there is an increasing evidence that the nonextensive statistical mechanics, originally proposed by C. Tsallis and characterized by power-law quantum equilibrium distributions, can be considered as an appropriate physical and mathematical basis to deal with physical phenomena where strong dynamical correlations, long-range interactions, anomalous diffusion and microscopic memory effects take place [5, 6]. In this framework, several authors have outlined the relevance of nonextensive statistical mechanics effects in high energy physics and astrophysical problems [7, 8, 9, 10].

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In this work we limit ourselves to consider only small variation from the standard Boltzmann-Gibbs (BG) statistics (from $q = 0.97$ to $q = 1.03$). In particular, we concentrate our study in the stage immediately after the supernova explosion (called deleptonizazion era), when the PSN assumes the maximum heating and entropy per baryon ($s = 2$) and the presence of nonextensive effects may alter more sensibly the thermodynamical and mechanical proprieties of the PNS.

2. Nonextensive hadronic equation of state

The hadronic EOS is calculated in the framework of the nonextensive statistical mechanics introduced by Tsallis [5, 6]. The nonextensive statistics represents a physical mathematical tool in several physical fields and it is based on the following definition of q-deformed entropy functional

$$S_q[f] = \frac{1}{q-1}\left(1 - \int [f(\mathbf{x})]^q \, d\Omega\right), \left(\int f(\mathbf{x})\, d\Omega = 1\right), \tag{1}$$

where $f(\mathbf{x})$ stands for a normalized probability distribution, $\mathbf{x}$ and $d\Omega$ denoting, respectively, a generic point and the volume element in the corresponding phase space. The nonextensive statistics is, therefore, a generalization of the common BG statistical mechanics and for $q \to 1$ it reduces to the standard BG entropy. Furthermore, nonextensive statistical effects vanishes approaching to zero temperature.

In this context, the nonextensive statistics entails a sensible difference on the power-law particle distribution shape in the high energy region with respect to the standard statistics.

We are going to study the nonextensive hadronic EOS in the framework of a relativistic mean field theory in which baryons interact through the nuclear force mediated by the exchange of virtual isoscalar-scalar (σ), isoscalar-vector (ω) and isovector-vector (ρ) mesons fields [11, 12]. In our analysis we include all the baryon octet in order to reproduce the chemical composition of the PNS at high baryon chemical potential. We also take into account of leptons particle by fixing the lepton fraction $Y_L = Y_e + Y_{\nu_e} = (\rho_e + \rho_{\nu_e})/\rho_B$, where ρ_e, ρ_{ν_e} and ρ_B are the electron, neutrino and baryon number densities, respectively. This is because, in the first stage of PNS evolution, electrons and neutrinos are trapped inside the stellar matter and, therefore, the lepton number must be conserved until neutrinos escape out of the PNS [1].

In the field equations appear the baryon density and the baryon scalar density. In the framework of the nonextensive statistical mechanics, they are given by [9]

$$\rho_B = 2\sum_{i=B}\int \frac{\mathrm{d}^3k}{(2\pi)^3}\, n_i(k)\,, \tag{2}$$

$$\rho_S = 2\sum_{i=B}\int \frac{\mathrm{d}^3k}{(2\pi)^3}\,\frac{M_i^*}{E_i^*}\, n_i^q(k)\,, \tag{3}$$

where $n_i(k)$ is the q-deformed fermion particle distribution function. For $q > 1$ and $\beta(E_i^* - |\mu_i^*|) > 0$, we have, for example,

$$n_i(k) = \frac{1}{[1 + (q-1)\,\beta(E_i^*(k) - \mu_i^*)]^{1/(q-1)} + 1}\,. \tag{4}$$

The nucleon effective energy is defined as $E_i{}^*(k) = \sqrt{k^2 + M_i{}^{*2}}$, where $M_i{}^* = M_i - g_{\sigma B}\sigma$. The effective chemical potentials μ_i^* are given in terms of the meson fields as follows

$$\mu_i^* = \mu_i - g_{\omega B}\omega - \tau_{3iB}\, g_{\rho B}\,\rho\,, \tag{5}$$

where μ_i are the thermodynamical chemical potentials ($\mu_i = \partial\epsilon/\partial\rho_i$).

The further conditions, required for the β-stable chemical equilibrium and charge neutrality, can be written as

$$\mu_\Lambda = \mu_{\Sigma^0} = \mu_{\Xi^0} = \mu_n \,, \tag{6}$$

$$\mu_{\Sigma^-} = \mu_{\Xi^-} = \mu_n + \mu_e \,, \tag{7}$$

$$\mu_p = \mu_{\Sigma^+} = \mu_n - \mu_e \,; \tag{8}$$

$$\rho_p + \rho_{\Sigma^+} - \rho_{\Sigma^-} - \rho_{\Xi^-} - \rho_e = 0 \,. \tag{9}$$

In the case of trapped neutrinos, the new equalities are obtained by the replacement of $\mu_e \to \mu_e - \mu_{\nu_e}$. The total entropy per baryon is calculated using $s = (S_B + S_l)/(T\rho_B)$, where $S_B = P_B + \epsilon_B - \sum_{i=B} \mu_i \rho_i$ and $S_l = P_l + \epsilon_l - \sum_{i=l} \mu_i \rho_i$, and the sums are extended over all the baryons and leptons species.

The numerical evaluation of the above thermodynamical quantities can be performed if the meson-nucleon and meson-hyperon coupling constants are known. Concerning the meson-nucleon coupling constants ($g_{\sigma N}$, $g_{\omega N}$, $g_{\rho N}$), they are determined to reproduce properties of equilibrium nuclear matter such as the saturation densities, the binding energy, the symmetric energy coefficient, the compression modulus, and the effective Dirac mass at saturation. Here and in the following, we focus our investigation by considering the so-called GM3 parameter set [11]. The implementation of hyperon degrees of freedom comes from determination of the corresponding meson-hyperon coupling constants that have been fitted to hypernuclear properties and their specific values are taken from Ref.s [13, 14] for the GM3 parameter set.

In the figures, we report some of the most important effects in the structure and in the thermodynamical properties of the PNS in the framework of nonextensive statistical mechanics, during the maximum heating phase ($s = 2$ and $Y_\nu = 0$).

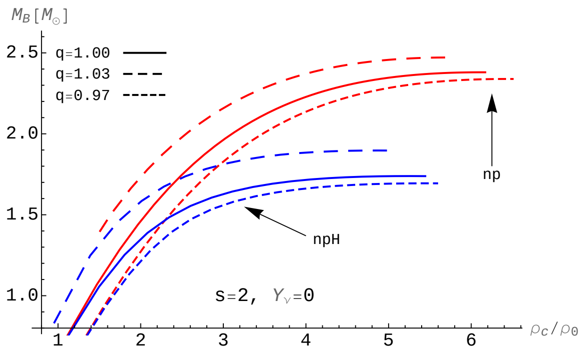

Figure 1. Maximum baryonic mass M_B in units of solar mass $M_\odot$ as a function of the central baryon density ρ_c (in units of the nuclear saturation density ρ_0) for different values of q and for nucleons and hyperons stars.

In Fig. 1, we show the variation of the maximum baryonic mass as a function of the central baryon density ρ_c, for different values of q, both for nucleonic (np) and hyperonic (npH) stars. We observe a remarkable increase (reduction) of the maximum baryonic mass when $q > 1$ ($q < 1$). In Fig. 2, we can see that the corresponding mass-radius relation is also sensibly modified in presence of a nonextensive statistics.

The above behaviors are substantially due to the softening of the EOS in presence sub-extensive statistics ($q < 1$), together with a remarkable reduction of the maximum temperature both for nucleonic and hyperonic PNS, as reported in Fig. 3. Contrariwise, when $q > 1$, we observe an increase of the maximum temperature. These remarkable differences in the stellar temperature have important consequences in the PNS evolution and, consequently, in the cooling of the PNS, making it longer when $q > 1$, and shorter when $q < 1$, with important astrophysical implications. All of these effects are also present when neutrinos are trapped in the PNS ($s = 1$ and $Y_L = 0.4$), however, due to the lower temperature achieved in this phase, they are less pronounced.

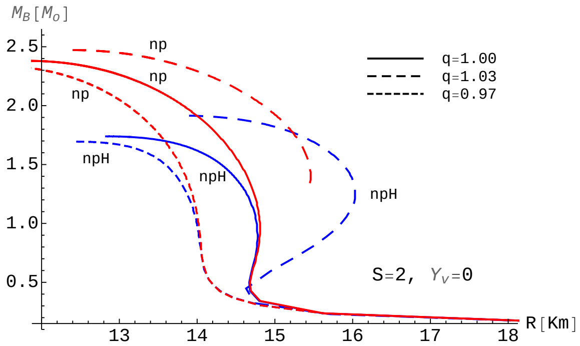

Figure 2. Stellar radius profile, as a function of the central baryon density ρ_c for different values of q and for nucleons and hyperons stars.

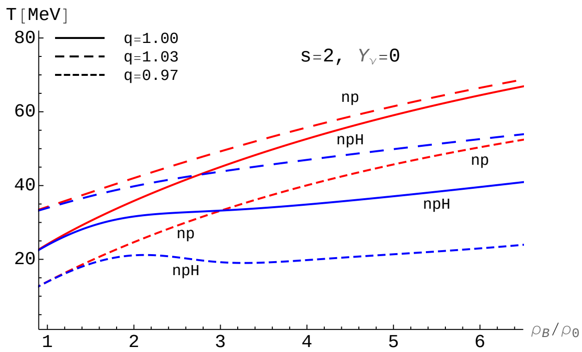

Figure 3. Temperature as a function of the baryon density for nucleonic and hyperonic PNS, for different values of q.

In conclusion, we have investigated the properties of PNS at different stages during its evolution. We have shown that the nonextensive statistical mechanics, characterized by power-law quantum distributions, can play a crucial role on the physical properties of the PNS and can be considered as an effective mathematical basis to investigate the complex structure and evolution of the PNS. From a phenomenological point of view, we have considered the nonextensive index q as a free parameter, even if, in principle, it should depend on the physical conditions inside the PNS, on the fluctuation of the temperature and be related to microscopic quantities (such as, for example, the mean interparticle interaction length).

We have shown that, in the presence of super-extensive statistical effects ($q > 1$) and hyperon degrees of freedom, the realization of a metastable phase is favored, with an enhancement of a possible black-hole formation after the deleptonization era. The variation of the maximum temperature, together with a significant variation of the maximum mass and stellar radius, could be considered as a phenomenological evidence of nonextensive statistical effects in PNS.

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