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Original Citation:

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(Article begins on next page)

27 April 2024

UNIVERSITÀ DEGLI STUDI DI TORINO

This is an author version of the contribution published on:
Questa è la versione dell'autore dell'opera:

Maffia, A. & Sabena, C. (2015). Networking of theories as resource for classroom activities analysis: the emergence of multimodal semiotic chains. In C. Sabena, B. Di Paola (eds.), *Teaching and learning mathematics: resources and obstables, Proceedings of CIEAEM 67, Quaderni di ricerca didattica, 25-2* (pp. 405-417). Aosta, July 20-24, 2015. ISSN: 1592-4424.

The definitive version is available at:
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Networking of theories as resource for classroom activities analysis: the emergence of multimodal semiotic chains

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Abstract: This paper networks two different theoretical frameworks in order to describe teaching-learning processes in the mathematics classroom. Specifically, it considers the paradigm of multimodality (Arzarello et al., 2009), which addresses a wide variety of semiotic resources in the classroom context and analyses them with the Semiotic Bundle tool, and the Theory of Semiotic Mediation (Bartolini Bussi & Mariotti, 2008), which focuses on the role that signs play in the transition from situated activities realized using artefacts to culturally shared mathematics. Basing on empirical analysis from a case study in middle school, the role of gestures performed by the teacher during a mathematical discussion is investigated, and the new construct of "multimodal semiotic chain" is introduced.

Résumé: Cette article mises en réseau deux cadres théoriques différents dans le but de décrire les processus d'enseignement-apprentissage dans la classe de mathématiques. Plus précisément, il considère le paradigme de la multimodalité (Arzarello et al., 2009), qui traite d'une grande variété de ressources sémiotiques dans le contexte de la classe et les analyse avec l'outil Sémiotique Bundle, et la théorie de la Médiation Sémiotique (Bartolini Bussi & Mariotti, 2008), qui met l'accent sur le rôle que jouent les signes dans la transition entre les activités situées réalisé en utilisant des artefacts aux mathématiques culturellement partagées. Basant sur une étude de cas en collège, le rôle des gestes à l'enseignant guidant une discussion mathématique sont étudiées, et le nouveau concept de «chaîne sémiotique multimodal» est présenté.

Introduction

As several researchers have remarked, many theoretical approaches have flourished in mathematics education, and in last years reflections on how different theoretical approaches can be used together in the same study has lead to a new branch of research, called 'networking of theories' (Bikner & Prediger, 2014). Within the networking theories approach, researchers seek to combine/compare/contrast/integrate different theories (usually two) in order to focus on a same research problem and to carry out data analysis in order both to have a better understanding of the data, and to investigate and reflect at meta-level on the theories used.

In our study we network two semiotic approaches: the former points to the role of multimodality in mathematics teaching and learning and has lead to the Semiotic Bundle tool of analysis (Arzarello et al., 2009), whereas the latter frames the role of artefacts and their exploitation by the teacher within the Theory of Semiotic Mediation (Bartolini Bussi & Mariotti, 2008). As it will be discussed below, the two theories share some principles, and previous research has shown that their networking is a promising route in order to investigate the evolution of semiotic resources in the mathematics classroom processes (Maffei et al., 2009; Maffia & Mariotti, in press).

In this paper, we analyse a middle school mathematical discussion (as conceived by Bartolini Bussi, 1996) on the definition of altitude in triangles. Specifically, we will focus on the role of the teacher in managing the various semiotic resources during the collective construction of the definition. As a result of the data analysis —carried out through the double lens—we will show the emerging of a 'multimodal semiotic chain'.

Multimodality and the Semiotic Bundle tool of analysis

The perspective of multimodality in mathematics education has its roots in the psychological theories that emphasize the crucial role of the body in thinking and in knowledge development; in particular, it is related to the so-called embodied cognition perspective, but assigns also relevance to the social-cultural dimension, drawing on the work of Vygotskij and Vygotskian scholars.

The embodied cognition perspective is a stream in cognitive science that assigns the body a central role in shaping the mind. It has been brought to the fore in mathematics education by the provocative book *Where Mathematics Comes From* by Lakoff and Núñez (2000), and then applied by researchers in several studies (e.g., Nemirovsky 2003; Arzarello & Robutti 2008; Edwards 2009). This view has been supported by neuroscientific results[7], which Gallese and Lakoff (2005) interpret as indicating a *multimodal* character in the brain sensory-motor system These authors point out that "multimodal integration has been found in many different locations in the brain, and we believe that it is the norm. That is, sensory modalities like vision, touch, hearing, and so on are actually integrated with each other *and* with motor control and planning" (*ibid.*, p. 459).

Research in psychology and psycholinguistics have also stressed the role of gestures in communicating and in thinking. McNeill (1992) defines gestures as "the movements of the hands and arms that we see when people talk" (p. 1). This approach comes from the analysis of conversational settings and has been widely adopted in research studies in psychology, in which gestures are viewed as distinct but inherently linked with speech utterances. Research in a number of disciplines (such as psychology, cognitive linguistics, and anthropology) is increasingly showing the importance of gestures not only in communication, but also in cognition (e.g., Goldin-Meadow 2003; McNeill, 2005). Also in mathematics education, gestures have been paid attention by several researchers, interested in cognitive and communicative aspects of mathematics teaching/learning (Arzarello et al., 2009; Edwards, 2009; Nemirovsky, 2003; Radford, 2003; Roth, 2001).

In particular, Arzarello and colleagues (Arzarello, 2006; Arzarello et al., 2009; Sabena et al., 2012), stress that *mathematics teaching and learning have a multimodal character* and involve different perceptuo-sensory-motor activities, including speaking, acting with artefacts, and gesturing. They highlight that the processes of teaching and learning are shaped by resources of different kinds as words (written or spoken), extra-linguistic ways of expression (gestures, gazes, …), written representations (drawings, symbols,…) or tools (from paper and pencil to technological devices).

They can be framed as "signs" within a Vygotskian perspective (Vygotskij, 1931/1978), forming a sort of *semiotic bundle* through which not only communication, but also cognitive processes evolve. The semiotic bundle is a dynamic structure including signs that are produced by students and teachers during the mathematical lessons. Specifically, it consists in:

> a *system of signs* […] that is produced by one or more interacting subjects and that evolves in time. Typically, a semiotic bundle is made of the signs that are produced by a student or by a group of students while solving a problem and/or discussing a mathematical question. Possibly the teacher too participates to this production and so the semiotic bundle may include also the signs produced by the teacher. (Arzarello et al. 2009, p. 100)

Differently from other semiotic approaches, the semiotic bundle construct allows us to theoretically

[7] These results concerns "mirror neurons" and "multimodal neurons": neurons firing when the subject performs an action, when he observes something, as well as when he imagines it (Gallese and Lakoff, 2005). Gallese and Lakoff use the notion of "multimodality" to describe a cognitive model in which there is not any central "brain engine" responsible for sense-making, controlling the different brain areas devoted to different sensorial modalities (which would occur if the brain behaved in a modular manner). Instead, there are multiple modalities that work together in an integrated way, overlapping with each other, such as vision, touch, and hearing, but also motor control and planning.

frame gestures and more generally all the bodily means of expression, as semiotic resources in learning processes. Furthermore, it allows at looking at their relationship with the traditionally studied semiotic systems. Such a notion appears adequate to analyse the multimodal character of learning, with respect to static and dynamic aspects, according to two kinds of analysis:

- *synchronic analysis:* focusing on the relationships between signs of a different nature (for example words and gestures) which are activated simultaneosly;
- *diachronic analysis:* focusing on the evolutions of signs as time goes by; for instance, a gesture may have a genetic function with respect to a certain diagram (Arzarello, 2006).

Through diachronic analysis, in gesture literature the phenomenon of 'catchment' has been identified (McNeill, 2005), to indicate the cases in which some gesture features are recurring in two or more (not necessarily consecutive) gestures. McNeill (*ibid.*) interprets catchments as indicating discourse cohesion, supported by the recurrence of consistent visuospatial imagery in the speaker's thinking. Using the semiotic bundle analysis, Arzarello and Sabena (2014) have pointed out the role of catchment with respect to logical aspects of mathematical arguments.
Synchronic and diachronic analyses have been also applied to focus on the semiotic resources activated by the *teacher* in classroom processes. In particular, 'semiotic games' have been described as those cases in which during teacher-students interaction, the teacher imitates a sign used by one or more students, and accompanies it with another kind of sign, in order to foster meaning evolution (Arzarello et al., 2009). The most typical case of semiotic game is when the teacher repeats a student's gesture, and accompanies it with proper mathematical words. Through a semiotic game, the teacher is not only referring to specific mathematical content, but he is also establishing, in an implicit way, that gestures are acceptable resources in the mathematics classroom.

Theory of Semiotic Mediation

The Theory of Semiotic Mediation (TSM, Bartolini Bussi & Mariotti, 2008) has been developed for modelling the relationship between an artefact that is used in the mathematics classroom to solve a specific task and the mathematics underpinning the artefact itself. The word 'artefact' has to be interpreted, according to Rabardel (1995), as any material or symbolic object designed for a particular goal (f.i. manipulative, software, symbols) and has not to be confused with 'instrument', which is the mixed entity constituted both of the artefact and the utilization schemes developed by a user. Also linguistic expressions can be considered as artefacts. Maffei and Mariotti (2011) define as *discursive artefacts* those sentences which have the potentialities to lead to the evolution of signs. This definition designate also what Sfard (2001) calls 'templates', referring to already known and used sentences in which a new word is inserted.

The main object of analysis of TSM is the evolution of signs along time, from contingent to culturally determined ones, according to a kind of diachronic analysis as defined in the previous section. This transition can be described as an example of passage from spontaneous concepts to scientific ones (Vygotskij, 1987).

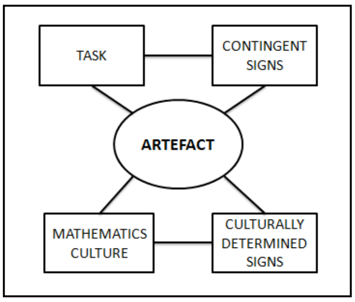


Fig.1. Artefact's polysemy

Specifically, according to TSM, each artefact used in the mathematics classroom can produce signs that can be conceived as *mathematical signs*. But, when the student uses the artefact to accomplish a task, she can be unaware of the mathematics that is "inside" the object and then the signs that she produces are contingent to the solution of a particular task with that particular artefact: they are called *artefact signs*. The set of relationships between the artefact and the task together with those between the artefact and the related mathematics constitutes the *semiotic potential* of the artefact itself. As the signs produced through the activity with the artefact can be interpreted differently by the student and the mathematics expert, we can say that the artefact is polysemic (Fig.1).

The substantial difference between artefact signs and mathematical ones is in the context to which they refer: artefact signs refer to the activity realized with the artefact itself, they may be related to what Radford (2003) calls *contextual generalization* or to what Sfard (2008) defines as *routine-driven* use. Let's give an example: if a student uses the word 'circumference' referring to all the closed lines that are produced using a compass, she is using a sign that appears as a mathematical one but that can be far away from the culturally shared meaning that is the set of points with the same distance from the centre.

According to TSM a crucial educational aim is to promote "the evolution of signs expressing the relationship between the artefact and tasks into signs expressing the relationship between artefact and knowledge" (Bartolini Bussi & Mariotti, 2008, p.753). A central role is played by the teacher. As the teacher has the awareness of the semiotic potential of the artefact, she can guide a *mathematical discussion* with the aim of prompting such evolution of signs. A mathematical discussion orchestrated by the teacher is intended as a "polyphony of articulated voices on a mathematical object that is one of the motives for the teaching-learning activity" (Bartolini Bussi, 1996, p. 6).

During the process of semiotic mediation, the teacher uses many different signs, specifically to turn artefact signs into mathematical ones. All the signs that are used with this objective are defined as *pivot signs*. An example of pivot signs can be the hybridization of a word or sentence belonging to the artefact domain (as reference to parts of a manipulative, a command in a software, …) with other words coming from the mathematical culture. The set of artefact and mathematical signs, together with the pivot signs that are used to relate them, is called *semiotic chain*. Such a construct has been introduced in literature by Walkerdine (e.g. 1990) and applied to mathematics education by different authors. For example Presmeg (2006) defines this process of chaining as a sequence of abstractions that is created preserving the relationships to everyday practices of the students, and Hall (2000) stresses how the teacher can create chains to develop mathematical concepts drawing on everyday situation. The role of the teacher as an expert who works in the zone of proximal development (Vygotskij, 1931/1978) of her pupils is central in TSM because she is conceived as the only agent in the classroom who can guide the process of semiotic mediation through the construction of semiotic chains.

Research question and methodology

In this paper, we focus our attention on the students-teacher interaction during mathematical discussions, in order to answer the following research question:

How does the teacher use different semiotic resources (gestures in particular) in the construction of the semiotic chain?

Data come from videos of two lessons in grade 6 aimed to define the height of the triangle. The teacher and the students knew that there was a camera in the classroom for research purpose, which was no more specified. Information about the teacher's intentions or about students' productions were collected through an informal preliminary interview and a follow-up interview to the teacher.

Data analysis is carried out at a micro-analytical level, and combines tools from the two considered theories. Specifically, classroom discourses have been transcribed and coded looking at the evolution of signs both from the point of view of the context in which they are used (artefact, mathematical or pivot signs, according to the TSM) and the modality in which they are expressed (written symbols, words, gestures, using the Semiotic Bundle). Speech transcripts have been enriched with gestures images looking at the co-timing of words and gestures (or other semiotic resources). The resulting 'multimodal transcript' has then been analysed in detail, in order to seek for the elements that could provide the fabric of semiotic chains related to the meaning of height of a triangle. In the following section we summarize the classroom discussion with a selection of excerpts from the transcript, useful to show the results of the analysis (students' name have been changed, to preserve anonymity).

The classroom discussion

The discussion is aimed to introduce the definition of altitude in a triangle, drawing on the usage of the word '*altezza'* (altitude, height) in Italian colloquial language. Italian students usually are introduced to intuitive definitions of basic plane geometry concepts in primary school. Grade 6 is the first year of middle school, in which those concepts are re-thought in a more formal way.

Students are acquainted with *parallel* and *perpendicular* lines on the plane, and were also introduced to the definition of *distance* between point and line, as the segment line perpendicular to the line. Distance was also related to the idea of minimal path from a point to a line.

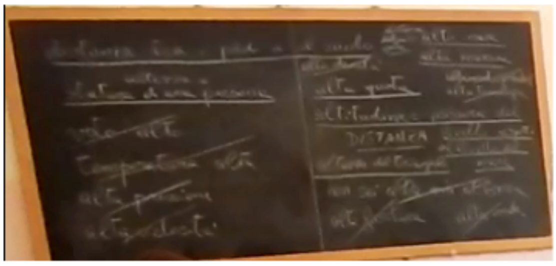
The emergence of a discursive artefact

The discussion begins with a brief brainstorming on the possible use of the term 'altezza' in ordinary language (see Fig. 2). The Italian word 'altezza' can be translated both as altitude and height. Figure 2 reports the different interpretations and associations that students relate to the words 'altezza' and 'alto' (high).

Among these proposed expressions, the teacher asks to select those that are more related to the geometrical context:

1 T: Now I ask you this: take your notebook and, from all these meanings that we have seen, from all these meanings, from all this expressions with the word 'high' that I wrote on the blackboard…can you write a list of those in which the word 'high' is used in a way that is similar to how we use it in geometry? […] Then you are going to tell me which one I have to underline and I delete the others.

Students write down their selection. Then they read their answers aloud, and the teacher underlines the chosen words at the blackboard (see Fig. 2):



Distance between feet and ground	High water
A person's tallness	Deep sea[2]
High temperature/pressure	High speed
High technology	High probability
Altitude: distance from sea level	High mark
High altitude	Triangle altitude
High density	High culture

Fig. 2. The classroom blackboard with the selected associations to the word 'altezza'.

2 Destiny: Distance between feet and ground.
3 T: Distance between feet and ground [*he underlines this expression at the blackboard*] Ok.
4 Sonia: Distance
5 T: Distance…?
6 Sonia: From sea level […]
7 T: Do we underline altitude too?
8 All together: Yes!
9 T: Distance between a point and sea level. So altitude [*Marco rises his hand*] Marika?
10 Marika: Deep sea.
11 T: Deep sea?
12 Marika: Because it is again a distance.
13 T: [*Fulvio rises his hand*] Fulvio?
14 Fulvio: High-water.
15 T: High-water. Why high-water?
16 Fulvio: It is the distance between the ground and how it is high…
17 Federico: Between the sea bed and the sea.

From this brief excerpt, we can notice that there is a kind of template sentence, which is repeated many times. It is "*the distance between ... and ...*". The teacher invites the students to identify it:

18 T: I am happy that you found so many many different ideas that can be used to express the idea of altitude. They all have something in common…

19 Federico: The distance. A person's tallness could be the distance between feet and head. Yes, the height (altezza)

A couple of months before, students were introduced to the definition of 'distance' between a point and a line, as the segment line that is perpendicular to the line and with one end-point in the point and the other one on the line. According to this definition, the altitude can be conceived as *the distance between a vertex and the opposite edge (the base)*. For this reason, the template "the distance between … and …" appears as a *discursive artefact* with the potential of leading to a mathematically consistent definition of altitude.

The appearance of gestures and their role in pivot signs

In order to reflect on the height of real objects, the teacher asks about the height of a mountain:

20 T: Can the height of a mountain be considered as the distance between something and something else? How can the height of a mountain be explained using the word 'distance'? [*The students are silent*] If it is true what some of you have said, that the word 'distance' has something to do with the word 'height'…

21 Christian: The height?

22 T: Using the word 'distance', how can you explain the idea of the height of a mountain?

23 Christian: The distance between… the top and…[*gesture pointing downwards, Fig. 3*]

24 T: Between the top [*pinching gesture as in Fig.4a*] Christian says…

25 Federico: And the base!

Fig. 3. Christian's gesture.

26 T: And the base [*gesture with open hand, palm upwards as in Fig.4b*]. So also in the height of a mountain, we can think at the highest point [*pinching gesture as in Fig. 4c*], like our height, the highest point on our head [*pinching gesture as in Fig. 4d*] and our base [*gesture with open hand, palm upwards as in Fig. 4e*] that are the feet. The height of the mountain [*pointing gesture as Fig 4f*] is the distance from the highest point of the mountain, with respect to the base of the mountain [*gesture as Fig 4g*], the ground [*gesture with open hand, palm downwards as Fig 4h*] where it rests. Now, if we think in this way at the height, the geometrical height, let's take a pencil and a ruler and I will let you apply this idea of height as distance, to the height of buildings. So, in a building, Alessandra, can you define the height using the word 'distance'?

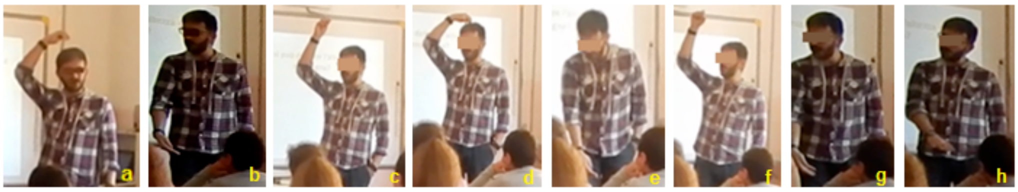


Fig. 4a-h: The "highest point gesture" and the "base gesture".

27 Alessandra: Yes.

28 T: How? In a building, the height is…

29 Alessandra: The distance between the highest point and the ground.

30 T: The distance between the highest point and another point that is on the ground [*while talking he repeats both the types of gestures as shown in Figg. 4f and 4h*].

The teacher projects now the images of three real building and a fictitious one, one by one (Fig. 5), and asks the students to draw their altitudes. In such a task, students are expected to draw a vertical line that connects the highest point of the (image of a) building to the ground, that is *the distance between the highest point and the ground*. As the ground is represented as horizontal, the drawn altitude will be *perpendicular* to the ground itself.

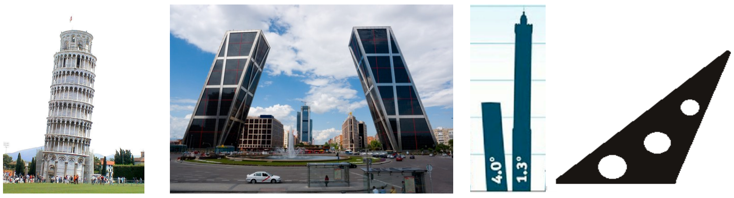


Fig. 5. The buildings projected on the multimedia whiteboard.

Students draw the shape of the building on their notebooks and then represent the height through line segments. While they are drawing, a student asks if the height of a building is bigger if there are underground floors. A discussion about this topic takes place. Some students think that it would be better to consider the distance between the highest point and the underground while the teacher suggests that, in many cases, the street-level is considered as a reference.

It is interesting to notice that while saying 'highest point' the teacher and the pupils always point to a upper position, over the head or upon the head: we refer to such a gesture as the "*highest point gesture*". Furthermore, the words 'ground' and 'underground' are always co-timed with a horizontal movement of the open hand on a lower level: we call it the "*base gesture*". For example, while talking about the height of buildings, Marika says:

31 Marika: The distance between the highest point [*gesture in Fig.6b*] and… [*gesture in Fig.6e*]

The girl is completing her sentence with a gesture (Fig. 6b) instead of words. Figure 6 reports some other examples of gestures from the teacher and from another student as well.

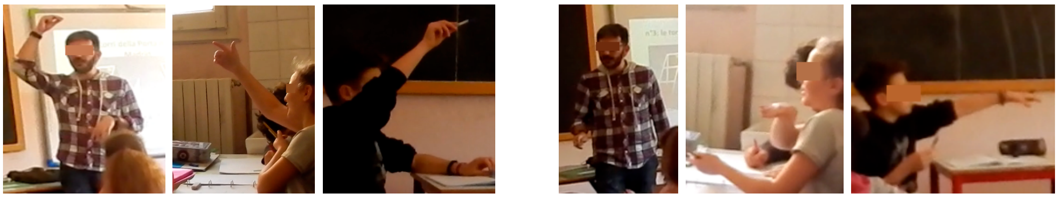


Fig. 6. Gesture catchment: Teacher and students repeating many times the "highest point gesture" and the "base gesture" during the discussion.

The repetition of similar gestures during the discussion is an example of *catchment*: this phenomenon is usually interpreted as providing structural cohesion to the co-occurring discussion (McNeill, 2005; Arzarello & Sabena, 2014). In our case, referring to TSM, such gestures can also be interpreted as carrying the germs of the expected *mathematical signs*. In fact, the highest point gesture is indicating an imaginary *point* while the base gesture consists of a linear movement of the open hand, as to represent a *line* or a *plane*. Teacher's hands anticipate the figural/graphical aspects of the geometrical terms that are crucial in the chosen definition of altitude. It seems here that the teacher, taking into account this didactical goal, is using this gestures as pivot signs to start a bridge between the expressions created to describe the heights of real objects, and the expected definition of altitude in a triangle. We can notice that the words 'point' and 'base', in this moment, refer to the to the "highest point" (line 26) and to the base of the body or of a mountain. They seems to be conceived as parts of real objects: in this sense they are still not mathematical signs but they have the potential to develop a semiotic chain. As the described gestures and these words appear often as co-timed, we can affirm that is the gesture-word couple to be used by the teacher as *pivot sign*.

Towards mathematical sign

At the end of the individual work, different solutions are shown at the blackboard. In particular it is noticed that sometimes the altitude can be external to the figure. Looking at students' productions, the teacher realizes that some pupils have drawn in a wrong way the altitude of the fictitious building (Fig.5d). The teacher draws Figure 5d on the blackboard and asks to Ester to draw the altitude. She draws a vertical line from the highest point downwards. Then the teacher asks:

32 T: How does it have to reach the ground? Rosi?

33 Rosi: Vertically.
34 T: Vertically! Why? How is the ground?
35 Many voices: Horizontal!
36 Destiny: It is flat.
37 T: It is flat, let's imagine it is flat and horizontal [*he does the "base gesture" with both his hands, Fig.7*] so in this case we have to reach it vertically.
38 Federico: Teacher, we have to reach it perpendicularly.
39 T: Wait a minute Federico.

Fig. 7. "Base gesture" with both hands

The teacher stops Federico. He prefers not to go directly toward the mathematical sign suggested by this student, because the lesson is going to finish in few minutes.

The discussion keeps going on in a second lesson. It starts from a new task, which consists in drawing the height of a boat while it is sailing a rough sea. Using the discursive artefact, the height of a boat is *the distance from the highest point and the boat's base* (that is the hull). The difference between this task and the building's one is in the position of the base, which is no more horizontal (as it was in the buildings in the previous lesson). Hence, the use of the discursive artefact within this task has the potential to trigger the discussion about the fact that the altitude is perpendicular to the base and not always vertical. Starting from different students' productions (Fig. 8), the teacher opens this discussion, which continues until pupils agree about the perpendicularity between altitude and base.

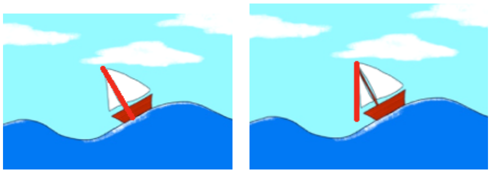


Fig. 8. Reproduction of the different students' productions discussed during the lesson.

Fig. 9a-b-c: Teacher's gestures during the second discussion.

40 T: So, this is one critical point, when we have to think about altitudes of objects without horizontal ground [*gesture in Fig. 9a*]. Because when the ground is horizontal there are no problems, you do the vertical line [*he draws an imaginary vertical line in the air, Fig. 9b*], you go downward [*he repeats the same gesture*] inside or outside the figure. But when the ground on which the figures lay is no more [*gesture in Fig. 9c*] horizontal, at that point we risk to make a wrong altitude. So, what can we write as a reminder to remember about this difficult point?
41 Marika: The perpendicular
42 T: So, I begin the sentence, then we see how we can conclude it. Let's keep in mind that the laying plane of an object can be… [*he begins to write it on the blackboard*]
43 Marika & Eleonora: atilt!
44 T: Atilt [*he keeps writing*]. In these cases, to draw the altitude of an object… Think about how you could complete this sentence to help your peers to avoid a future error. In these cases, to draw the altitude, we have to keep attention to…what? […]
45 Federico: The altitude has to be perpendicular [*he moves his hand vertically, Fig. 10a*] to the base [*"base gesture", Fig. 10b*]

The teacher insists on stressing the difference between vertical and perpendicular, but Federico interrupts him:

46 Federico: Teacher, but…what if […] there is a triangle which has the altitude outside [*he draws a vertical line in the air, as in Fig. 10a*] the laying base [*"base gesture" with one hand*] and then we have to draw the altitude in respect to the base [*"base gesture"*] of the triangle…to the laying base of the plane [*"base gesture" with two hands, Fig. 10c*] when the plane is waving. Where do we…

Fig. 10a-b-c: Federico's gestures speaking about the external altitude.

It is interesting to notice that, again, while formulating his question, Federico repeats the teacher's gestures co-timed with the same words (line 46): also gestures, besides words, are *shared*.

Then, the teacher introduces a new artefact that is already known by students: a Dynamic Geometry Software (DGS). He opens a file in which a acute-angled triangle, the line containing one of its base and the corresponding altitude (in red) are drawn. The teacher moves the vertex from which the altitude begins (point E in Fig. 11) and transforms the triangle with an obtuse angle (Fig. 11). He asks to pupils to describe this case and they say that the altitude is still perpendicular to the base but outside it.

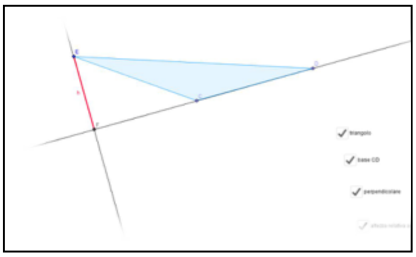


Fig. 11. The triangle in the DGS.

47 T: Pay attention! This is another very difficult point. It arrives perpendicular, and you say that it arrives perpendicular to the base. But, if I take away a thing from this drawing [*he makes invisible the line which the base belongs to*] Where does this altitude arrive to?
48 Federico: Perpendicular…to the extension of the altitude…ehm, of the base!
49 T: Ok. The altitude does not reach the base [*he points downward many times*] Why does it end outside the base in this case? Does it still arrive perpendicularly?
50 Mario: Of course!
51 T: To what?
52 Federico: To the base extension.
53 T: To the laying plane of the base [*he extends the gesture in Fig.9a opening his harms, Fig. 12*], so the extension of the base [*he repeats the gesture*]. So, in some case, the altitude starts from this famous highest point [*"highest point gesture"*] and it does not touch the base. Is it still perpendicular?
54 Some voices: Yes.
55 T: But…
56 Federico: It is external.
57 T: It is external and it does not touch the base [*"base gesture"*], it touches the extension [*he repeats the extension of the "base gesture"*] of the base. Let's write this.

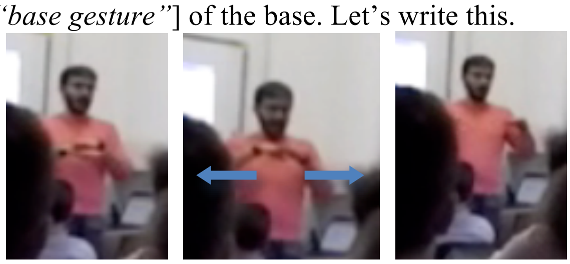


Fig. 12. Extension of the "base gesture" which correspond to the extension of the base.

As it can be noticed, the change in words (from 'base' to 'base extension') results in a gesture change (from gesture as in Fig. 9 to gesture as in Fig. 12), and in particular from a static to a dynamic gesture. The shared definition of altitude is evolving again. The rigorous mathematical definition will require another lesson, drawing on a task in which a the command "perpendicular" of the DGS will serve as an artefact to draw the *three* altitudes of a triangle.

Results and conclusion

At the beginning of the discussion, the teacher starts from the verbal expressions produced by the students to refer to the meaning of the word 'altezza' (altitude/height), and focuses their attention on the specific discursive artefact "*the distance between…and…*" (lines 18-30). The linguistic component of the semiotic bundle soon enriches with the gestural one. The first gesture is produced by Christian (line 24), in the attempt of completing the sentence asked by the teacher: the former part (referring to the top) is mentioned with the verbal resource, while the latter one (referring to the base) is expressed with the gesture of pointing downwards (Fig. 3).

Even if the teacher is not looking at the student, he is maybe influenced by the use of the gesture resource, because we can see that his following utterance (line 26) is accompanied by two kinds of gestures, and specifically:

- the "highest point gesture", i.e. a gesture pointing in a upper position, over his head or materially upon his head (Fig. 4a-c-d-f), when verbally referring to the "highest point" of a mountain or of a person's body, and
- the "base gesture", i.e. a gesture pointing to a lower position, below his foot (Fig. 4e-b-g).

These two gestures, and specifically the two *spatial locations* of the gestures are repeated in the entire discussion, showing a catchment that provides cohesion to the discourse, while it is slowly evolving towards the scientific meaning of height of a geometric figure. At the beginning the gestures mediate specific aspects of the objects at stake (being them a mountain, a human body or a triangle) by means of iconic features, later they will be associated to the written representation of geometric figures: it is through this iconic features kept in the catchment that the figural aspects of the triangle are called into the scene in continuity with the reference to objects from everyday life or the human body. Through the figural aspects, the geometric properties of the figure are then brought to the fore and discussed.

With a detailed diachronic analysis, we can see that, although the spatial locations of the gestures are kept during the entire discussion, the base gesture evolves: at first it is made with the palm upwards (Fig. 4e) and it is co-timed with the word "base" referred to the human body (line 26: "our base that are the feet"): we symbolize it with ∪. Then, it is made with the palm downwards (Fig. 4h), referred to the base of a mountain or buildings: we represent with ∩ this gesture. Later the gesture is made with two hands, with reference to the "flat and horizontal ground" (line 37, symbolically ∩∩). After introducing the activity about the boat, the same gesture with two hands is repeated in a slanted inclination (Fig. 9c). Finally, the gesture with the two hands becomes a gesture made at the central part of the gesture space (just below the shoulders), with the two hands opening to stress the extension of the base of the triangle (Fig. 11, symbolized as). In the following table we sum up the diachronic analysis, correlating it with the synchronic analysis of words and gestures starting from the discursive artefact "*the distance between…and…*". Pointing gestures are represented with arrows according to the direction pointed at.

		Synchronically
Diachronically	24	Christian: The distance between **the top** and ↓
	26	T: [..] the distance from **the highest point of the mountain**, with respect to **the base of the mountain** ∪
	30	T: The distance between **the highest point** ↑and **another point that is on the ground** ∩
	31	Marika: The distance between **the highest point** ↑and ∩
	52-56	T: the altitude starts from **this famous highest point** ↑ […] It is external and it does not touch **the base** ∩∩, it touches **the extension of the base**

Table 1. Schematic representation of the multimodal semiotic chain.

Through the detailed analysis with the Semiotic Bundle we are therefore able to describe the evolution of the signs used by the teacher to guide the discussion from the set of everyday meanings that the students associate to the word "height" (the starting point of the discussion) to the scientific concept (Vygotskij, 1987) that is chosen as goal for the lessons, namely height of a triangle as distance from a vertex to the opposite side, which is called base. This evolution forms a chain of multimodal signs that is schematically illustrated in Table 1. Analysing within a multimodal perspective the evolution of the discursive sign during the discussion, we have been able to identify a semiotic chain including different kinds of semiotic resources: we call it "*multimodal semiotic chain*".

Although we schematically illustrated it in a linear way (Tab. 1), we are not claiming that signs evolution occurs linearly in the classroom activities, nor in the meaning evolution for the students. An extensive analysis of these lessons and of the following ones would show that, on the opposite, the route for mathematical meaning productions in students is complex and not reducible to any linear path. However, the metaphor of the chain underlines that the various parts of the evolution are connected each other, and the analysis in multimodal sense may shed light on how they are connected. In our study, the initial artefact has a discursive nature, and the main semiotic resources are words and gestures (being the reference to written productions on the background). In this context, we showed that gestures may provide a glue to link the various signs within the semiotic chain. This result not only is coherent with previous results in gesture studies (see those on catchment in McNeill, 2005), but also specifies a specific function that gestures may have in the field of mathematics education.

Prompted by this result, many new questions arise. We point out three questions that we feel as most urgent. The first one is about the influence of the semiotic resources of the chain *on students*: this theme is only touched but not faced in this paper. The second one, of more theoretical nature, is about the nature of the semiotic chains when the starting point is a concrete artefact (as a manipulative or a software), as it is more typical in TSM's applications: can we still speak of multimodal semiotic chains? And what is the role of embodied resources therein? The third issue concerns the intentional use of multimodal semiotic chains by teachers. Future research is needed to answer these and possibly further emerging questions.

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