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Research Article

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Price Reveal Auctions

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Abstract: A price reveal auction (PRA) is a descending price auction in which the current price of the item on sale is hidden. Buyers can privately observe the price only by paying a fee, and every time an agent does so, the price falls by a predetermined amount. We show that if the number of participants, n, is common knowledge, then in equilibrium a PRA replicates the outcome of a posted price mechanism. In particular, at most one buyer observes the price and the auction immediately finishes. In contrast, multiple entries can occur and profitability is enhanced when agents are uncertain about n. Under some conditions, a PRA may even yield higher expected revenues than standard auction formats.

Keywords: price reveal auctions, pay-per-bid auctions, endogenous price decrease
JEL Classification: D44, C72

1 Introduction

This paper analyzes a novel online selling mechanism, the price reveal auction (PRA). A PRA is a descending price auction in which the current price of the item on sale is not publicly observable. Each participant can privately observe the price by paying a fee, c. The agent is then given a limited amount of time (say, 10 sec) to decide if he wants to buy the good at the current price. If the agent buys the good, the auction ends. Otherwise, the price is decreased by a fixed amount, $\Delta < c$, and the auction continues.[1] In other words, in a PRA, the price is

1 PRAs were introduced in late 2009. Nowadays, they are taking place on a number of different websites in different countries, such as for instance http://u-wantit.com/, http://1250auctions.com/ (both in English), http://ambetion.com/ (in Spanish), and https://www.youbid.nl (in Dutch). Notice that the PRA mechanism is sometimes also labelled “scratch auction”, “express auction”, or “reverse auction”. Gallice and Sorrenti (2015) document that PRAs are typically used to sell electronic products (smartphones, laptops, digital cameras) and report an average

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hidden and falls by Δ every time a bidder observes it.[2] Therefore, and contrary to standard Dutch auction procedures, the price does not fall exogenously at a predetermined speed, but rather endogenously in response to bidders' behavior.

The goal of this paper is to study the PRA format from a theoretical point of view. We believe that this may be interesting for a number of reasons. First, a full-fledged analysis of the mechanism is still missing.[3] Indeed, we show that participants in a PRA face a rather unusual strategic situation with non-trivial equilibria. Second, the analysis allows us to assess some fundamental aspects of the mechanism. Does a PRA implement an efficient allocation? Is a PRA revenue equivalent to standard auction formats? These are relevant questions, since PRAs, together with similar mechanisms that we will shortly review, may potentially find applications also in non-commercial contexts. For instance, they may be used for charity purposes and/or fund-raising activities (see Morgan 2000 or Engers and McManus 2007).[4] Third, results that stem from the theoretical analysis can be tested against the data. It is relatively easy to collect field and/or experimental data about agents' actual behavior. The comparison between theoretical predictions and empirical results can then highlight if and how agents deviate from the rational paradigm and, perhaps, convey some more general lessons about individuals' actual behavior. Finally, PRAs are a specific example of a fast-growing wave of new forms of e-commerce. These mechanisms not only include auction-based formats but also other peculiar institutions, such as "pay what you want" mechanisms and platforms that match buyers and sellers in various markets (short-term property rentals, car sharing, etc.). A sound knowledge of existing mechanisms is thus needed to assess the peculiarities, risks, and prospects of those that will follow. In this respect, theoretical insights may become useful in advising market design and in protecting consumers.

We model a PRA as a sequential game with imperfect information. First, the seller sets the initial price. Then, a number of buyers with heterogeneous and private valuations decide if and when to observe the hidden price. A player that

market value of the auctioned item of approximately 443€, an average fee (c) of 1.07€, and an average price decrese (Δ) of 0.53€.

2 Notice that the decision to observe the price remains private. Therefore, no buyer can infer the current price by counting the number of times the price has been observed.

3 Initial attempts to formally analyze the PRA format are Gallice (2010) and Di Gaetano (2011). However, both papers relied on a number of simplifying assumptions that made the models considerably different with respect to the actual mechanism. In particular, Gallice (2010) assumed that the initial price was commonly known and only buyers with a valuation below that level could participate. Di Gaetano (2011) assumed instead that the starting price is a random variable that is not strategically chosen by the seller and that every buyer could observe the price only once.

4 Indeed, some charities are already using the PRA mechanism, e. g., http://www.greendealauction.co.uk/en/.

observes the price must also decide whether to buy the item or not. We characterize the perfect Bayesian equilibria of the game and discuss the different outcomes that can emerge on the equilibrium path. Given that observing the price is costly, in equilibrium an agent decides to do so only when he is confident that the hidden price has reached a "good" level, i. e., a level at which the agent is willing to purchase the item. Agents' behavior thus depends on their beliefs about the hidden price. Our analysis show how these beliefs, in turn, hinge on agents' information set and particularly on their knowledge about some "fundamentals" of the game. In this respect, we consider two different scenarios.

In the first scenario (which is in line with the standard approach in auction theory, e. g., Krishna 2002), we postulate that the only aspect about which agents are uncertain is the other buyers' type (i. e., their valuation of the good). In particular, we assume that the number of participants (n) and the valuation of the seller (v_S) are common knowledge. We show that in such a situation, uncertainty about the hidden price disappears. The intuition is that, since the optimal initial price is a deterministic function of n and v_S, common knowledge about these parameters leads to common knowledge about the initial price. This, in turn, implies that, in equilibrium, the hidden price is observed at most once by a single bidder. The auction then either immediately finishes (i. e., an agent observes the price and buys the item as soon as the auction opens) or reaches the final period with no player ever observing the price (i. e., the item remains unsold). We also show that, in such a context, a PRA replicates the same outcome that would emerge in a posted price mechanism (PPM). As such, a PRA yields lower expected revenues than standard auction formats and implements an allocation that is not necessarily efficient.

The second scenario more closely resembles real PRAs that take place on the Internet, as it assigns buyers a less precise information set. In particular, we consider the case in which buyers are uncertain about the number of participants. Since a different optimal initial price exists for every possible realization of n, uncertainty about the number of participants translates into uncertainty about the initial price. Agents' beliefs are thus described by a non-degenerate probability distribution and buyers choose whether to observe the price or not on the basis of their expected payoffs. In particular, a buyer may rationally decide to observe the price, discover a price that is above his willingness to pay, and thus refuse to buy the item. It follows that the price may fall even on the equilibrium path. The final allocation may again fail to be efficient but multiple entries enhance expected revenues. Indeed, we show that, under some quite specific conditions (high number of participants paired with highly dispersed beliefs), a PRA may dominate standard auction formats in terms of expected revenues.

1.1 PRAs Versus Other Pay-Per-Bid Auctions

PRAs share some similarities with two other recent on-line selling mechanisms: penny auctions and lowest unique bid auctions (LUBAs). The three formats are collectively referred to as pay-per-bid auctions.[5] This labelling indicates two distinguishing features of this family of mechanisms. First, these are formats in which the participants must pay the seller a fee every time they "move" (and these fees constitute the main source of revenues for the seller). Second, these mechanisms are, at least at first sight, reminiscent of standard auction formats. In this section, we briefly describe the functioning of penny auctions and LUBAs. We also discuss their main theoretical properties in light of our findings about PRAs. Finally, we report some figures about the actual performance of the three mechanisms.

In a penny auction, buyers observe a public countdown (say, 30 sec). Every time an agent bids, the current price of the item increases by a fixed amount (a penny) and the countdown restarts. The winner is the bidder who holds the winning bid when the countdown expires. Penny auctions have been studied extensively. Hinnosaar (2014) fully characterized the equilibria in a very general framework, whereas Augenblick (forthcoming) and Platt, Price, and Tappen (2013) analyzed more specific versions of the mechanism and later focused on an empirical analysis. A common result of the three models is that the support of possible final prices is highly dispersed. This follows from the fact that, in equilibrium, players use mixed strategies. Since all these papers assume that bidders have homogeneous valuations, efficiency of the final allocation and revenue equivalence with standard auction formats hold trivially; however, and in line with what we find in the case of PRAs, these two properties would not necessarily hold if one allows for heterogeneity in buyer types.[6]

In a LUBA, bidders place private bids and the winner is the agent who submits the lowest offer that is not matched by any other bid. A number of papers (Rapoport et al. 2009; Houba, van der Laan, and Veldhuizen 2011; Östling et al. 2011) investigate the theoretical properties of such a mechanism under the restriction that each player submits a single bid. Other papers (Eichberger and Vinogradov 2008; Gallice 2009; Scarsini, Solan, and Vieille (2010) allow instead

5 Notice, however, that in the literature, the term pay-per-bid auctions is sometimes synonymous with penny auctions.

6 From an empirical point of view, Augenblick (forthcoming), Hinnosaar (2014), and Platt, Price, and Tappen (2013) documented actual profits that are, on average, considerably larger than those that theory would predict. Augenblick (forthcoming) rationalized this inconsistency by assuming that bidders may suffer from the sunk-cost fallacy. Platt, Price, and Tappen (2013) postulated instead that bidders may be risk lovers rather than risk neutral. Finally, Gnutzmann (2014) provided an additional explanation based on prospect theory.

for multiple bids. A common finding of all these studies is that, in equilibrium, bidders randomize over the support of possible bids according to a decreasing probability distribution. Similarly to penny auctions, these studies show that a LUBA may fail to implement an efficient allocation when the bidders are heterogeneous. Gallice (2009) and Scarsini, Solan, and Vieille (2010) also investigate the profitability of the mechanism and show that if bidders are fully rational, a LUBA yields expected revenues that can at most match those that a standard auction format would raise.

Although penny auctions, LUBAs, and PRAs are relatively similar in terms of some key theoretical aspects of their equilibria, the differences in their actual performance are quite pronounced. In the case of penny auctions, Augenblick (forthcoming) and Hinnosaar (2014) report average profit margins of, respectively, 51% and 71%. Moreover, Platt, Price, and Tappen (2013) find that the final price amounts, on average, to 10% of the retail price, i. e., an average discount of 90%. Concerning LUBAs, Gallice (2009) reports a profit margin of 441% and an average discount of 99%. Finally, in the case of PRAs, Gallice and Sorrenti (2015) document a profit margin of 36% and an average discount of 37%. In practice, the PRA mechanism thus appears to be less rewarding for both the seller and the winning agent. In line with this finding, PRAs are nowadays less widespread than penny auctions or LUBAs.

2 The Model

We model the sale of a single indivisible item through a price reveal auction. There are $n+1$ risk-neutral players: a seller (indexed by s) and a finite set $N=\{1,\ldots,n\}$ of potential buyers. We will investigate two different scenarios: in the first one (Section 3.2), the number of participants, n, is common knowledge among all players. In the second one (Section 3.3), buyers are uncertain about n.

The seller's valuation of the good is given by $v_s \geq 0$, and this is commonly known.[7] Each buyer has a valuation v_i that is independently and identically distributed on the interval $[0, \bar{v}]$, according to the cumulative distribution function F, which is strictly increasing and continuously differentiable with density f and such that $\bar{v} > v_s$. In line with the standard independent private value assumption, each buyer knows his type v_i and all players know that every $v_{j\neq i}$ is drawn from F.

7 We will later investigate (see Section 4) how the results of the model would change under the alternative assumption that v_s is unknown to potential buyers. In particular, we will show that buyers' uncertainty about v_s triggers similar effects as the ones caused by uncertainty about n.

Time is discrete and goes from $t = 0$ to $t = t_e$ where $t_e \in \{1, \ldots, T\}$ denotes the ending period and T is finite and common knowledge. In particular, $t_e \leq T$ if an agent buys the item whereas $t_e = T$ if the item remains unsold. At $t = 0$ the seller sets the price $p_0 \in [0, \bar{v}]$. The initial price p_0, as well as the current price p_t for any $t \in \{1, \ldots, t_e\}$, are unknown to potential buyers unless agents explicitly decide to observe it. The price remains constant whenever no one observes it. On the contrary, two things simultaneously and instantaneously happen as soon as an agent observes the price: first, the agent is charged the fee $c > 0$; second, the price decreases from p_{t-1} to $p_t = p_{t-1} - \Delta$ with $\Delta \in (0, c)$.[8]

At any period $t \in \{1, \ldots, t_e\}$, each agent $i \in N$ plays a sequence of two actions that we denote with $a_{i,t} = \left(a^1_{i,t}, a^2_{i,t}\right)$. Action $a^1_{i,t}$ is such that $a^1_{i,t} \in \{\emptyset, wto\}$, where $a^1_{i,t} = \emptyset$ indicates that the player remains inactive whereas $a^1_{i,t} = wto$ indicates that the player informs the seller that he is willing to observe the hidden price (and thus willing to pay the fee c). In line with the actual tie-breaking rule, we then assume that the seller randomly selects within the set of all players who played $a^1_{i,t} = wto$ a single agent to whom he discloses the price.[9] We denote this agent by $\hat{i}_t$. Once $\hat{i}_t$ privately observes $p_t \in [0, \bar{v}]$, he plays action $a^2_{\hat{i}_t, t} = \gamma(p_t)$ where $\gamma(\cdot) : [0, \bar{v}] \to \{0, 1\}$ indicates whether the agent buys the good ($\gamma(p_t) = 1$) or not ($\gamma(p_t) = 0$). All agents $i \neq \hat{i}_t$ necessarily play $a^2_{\hat{i}_t, t} = \emptyset$.

Finally, let $\eta_{i,t} \in \mathbb{N}_0$ be the number of times agent i observes the price (and thus pays the fee c) up to period t included. Players' final payoffs at period $t_e \in \{1, \ldots, T\}$ thus take the following form:[10]

$$u_i = \begin{cases} v_{\mathrm{i}} - p_{t_e} - c\eta_{i,\, t_e} & \text{if } i \text{ buys the good} \\ -c\eta_{i,\, t_e} & \text{otherwise} \end{cases} \qquad \text{for } i \in N$$

$$u_s = \begin{cases} p_{t_e} - v_s + c\sum_{i \in N} \eta_{i,\, t_e} & \text{if there exists an } i \text{ that buys the good} \\ c\sum_{i \in N} \eta_{i,\, t_e} & \text{otherwise} \end{cases}$$

8 Notice that an agent that observes p_t discovers a price that has already been decreased by Δ. In general, and consistent with the actual implementation of the mechanism (see the figures reported in footnote 1), we have in mind situations in which both c and Δ are "small". We stress from the outset that the condition $\Delta < c$ plays an important role in determining the equilibria of the game and the profitability of the mechanism. We will discuss in Section 4 the alternative assumption (which is never implemented in reality) $\Delta \geq c$.

9 From the FAQ section of the website bidster.com: "What happens if a participant has started to purchase the product and someone else tries to buy it? If anyone has started the purchase of the product, no other participants can scratch the auction. Only one participant can buy the product and stand as the winner of the auction."

10 We assume that players' discount factor equals 1 so that we write u_i (respectively, u_s) rather than u_{i,t_e} (respectively, u_{s,t_e}).

2.1 Buyers' Information Sets

With respect to a standard descending price auction, the PRA mechanism thus displays several peculiar features. Before moving to the proper analysis of the equilibria, it is therefore useful to discuss in more detail how these specific features affect buyers' information sets throughout the game. Clearly, an important characteristic of a PRA is that agents' actions (and thus histories of play) are unobservable. Assume agent i plays $a_{i,t} = (\emptyset, \emptyset)$. Then, from i's point of view, actions $a_{j,t} = (\emptyset, \emptyset)$ (agent $j \neq i$ remains inactive), $a_{j,t} = (wto, \emptyset)$ (agent j informs the seller that he is willing to observe the price but he loses in a tie), and $a_{j,t} = (wto, 0)$ (agent j observes the price but decides not to buy the good) are indistinguishable. Indeed, the three actions are observationally equivalent, as in all cases agent i only observes that the auction remains open. This implies that in general, at any period $t > 1$, agents do not know how many times (if any) the price was observed. The only (partial) exception to this rule arises because of the possibility of ties. In this respect, an agent that observes the price does not know if he was involved in a tie or not. As such, he cannot infer any new information about what the opponents played. On the contrary, an agent that loses in a tie learns that someone else is currently observing the price (notice however that the agent does not know how many players were tied together). If the agent then sees that the auction remains open, he infers that the rival who observed the price did not buy the item. As such, he realizes that the hidden price felt from p_t to $p_{t+1} = p_t - \Delta$. All these considerations will play an important role in determining the way buyers update their beliefs throughout the game. They also imply that buyers may hold heterogeneous beliefs as these depend on each agent's specific and private history of play.

3 The Equilibria of The Game

A PRA is thus an extensive-form game with imperfect information, as players do not always know p_t and do not observe their rivals' types and actions. As a solution concept, we apply the notion of symmetric perfect Bayesian equilibrium. As it is often the case in Bayesian games, multiple equilibria exist. Different systems of beliefs may in fact sustain different equilibria. We focus however on what we call the "profit maximizing trading equilibrium" (PMTE) of the game. This is an equilibrium in which the seller maximizes his expected profits, buyers hold consistent beliefs, and trading potentially (but not necessarily) occurs.[11] The criterion of profit maximization seems an appropriate approach to guide equilibrium selection. Notice in fact that the seller is the first player to move (he sets the initial price). Moreover, and as already mentioned, the goal of a PRA is not the implementation of an efficient allocation but rather the creation of profits for the seller. Before properly defining the PMTE (Proposition 1 for the case in which n is common knowledge, Proposition 3 for the case in which n is a random variable), we first discuss some of its characteristics and introduce some additional notations.

3.1 Some Preliminary Results

The rules of the game are such that buyers accumulate costs every time they observe the price. Therefore, an agent would ideally observe the hidden price only once, discover a price $p_t = p_{t-1} - \Delta$ that he likes, and buy the item. The decision to observe p_t thus depends on the beliefs the agent holds about the hidden price p_{t-1}. Let $H_{i,t}(p_{t-1})$ be a probability distribution that describes such beliefs and $h_{i,t}(p_{t-1})$ be the associated density function. Given that the agent internalizes the fact that if he observes the price at period t, he is charged c, and the price moves from p_{t-1} to $p_t = p_{t-1} - \Delta$, the expected payoff associated with the decision of observing the price can be formulated as follows:

$$(\pi_i | H_{i,t}(p_{t-1})) = \int_0^{\bar{v}} h_{i,t}(p_{t-1})(v_i - p_{t-1} + \Delta) q_{i,t}(p_t) dp_{t-1} - c \qquad [1]$$

where $q_{i,t}(p_t) \in \{0, 1\}$ indicates the outcome of the purchasing decision for any possible realization of the actual price $p_t = p_{t-1} - \Delta$. In particular, $q_{i,t}(p_t) = 1$ (respectively, $q_{i,t}(p_t) = 0$) if, conditional upon observing p_t, the agent decides to buy (resp., not to buy) the item. Formally:

$$q_{i,t}(p_t) = \begin{cases} 0 & \text{if } p_t > b(v_i) \\ 1 & \text{if } p_t \leq b(v_i) \end{cases} \qquad [2]$$

where $b(v_i)$ denotes the agent's gross willingness to pay (to be identified in equilibrium). Notice that the agent anticipates that at the moment of deciding whether to buy the item or not, the cost of the fee, c, is sunk and thus will not affect his decision any more.

11 We will also show the existence of a "no trading equilibrium" in which the item certainly remains unsold.

Given that agents are risk neutral and that the strategy of never observing the price leads to a null payoff, a player will observe the hidden price whenever $(\pi_i|H_{i,t}(p_{t-1}))>0$. When this is the case, the actual decision to buy the item or not will then depend on how the actual price that the agent discovers, p_t, compares with his gross willingness to pay, $b(v_i)$.

In summary, potential buyers should play according to the following behavioral rule: be willing to observe the price whenever the transaction is expected to be beneficial, and then, in case the player does indeed observe the price, buy the item whenever the *actual* realization of p_t is beneficial.[12] Formally, $a_{i,t}=\left(a^1_{i,t},a^2_{i,t}\right)$ where:

$$a^1_{i,t}=\begin{cases}\emptyset & \text{if } (\pi_i|H_{i,t}(p_{t-1}))\le 0\\ wto & \text{if } (\pi_i|H_{i,t}(p_{t-1}))>0\end{cases}\qquad \text{for any } i\in N$$

$$a^2_{i,t}=\begin{cases}\gamma(\cdot)\text{ with }\gamma\ (p_t)=0 & \text{if } p_t>b(v_i)\\ \gamma(\cdot)\text{ with }\gamma\ (p_t)=1 & \textit{if } p_t\le b(v_i).\end{cases}\qquad \textit{for } i=\hat{\imath}_t \text{ and } a^2_{i,t}=\emptyset \text{ for any } i\ne\hat{\imath}_t. \tag{3}$$

3.2 The Equilibria When *n* is Common Knowledge

Assume that the number of participants is common knowledge among all the players. Then, in equilibrium, a PRA attracts, at most, one price observation by a single bidder. Moreover, when this is the case, the auction immediately finishes.

The intuition for such a result is that perfect information about the number of participants leads to perfect information about the initial price that the seller sets. Players can solve the seller's problem and correctly pin down $p^\star_0$.[13] As such, uncertainty disappears and bidders' beliefs about the initial price, as well as about how this evolves over time, become deterministic. More formally, agents' beliefs are described by the degenerate distribution $H^\star_{i,t}(p_{t-1})$ with $h^\star_{i,t}\left(p^\star_{t-1}\right)=1$ for all $i\in N$.

12 Say that $c=2$, $\Delta=1$, $v_i=100$, $b(v_i)=100$, and the agent's beliefs about p_{t-1} are such that $h_{i,t}(90)=0.3$ and $h_{i,t}(110)=0.7$. Then, $q_{i,t}(89)=1$ and $q_{i,t}(109)=0$ such that $(\pi_i|H_{i,t}(p_{t-1}))=0.3(100-89)-2=1.3$. Agent i thus signals his willingness to observe the price. If given this opportunity, he will then buy the item if $p_t=89$ and not buy it if $p_t=109$.

13 Here and in what follows we use the asterisk * to indicate equilibrium prices, actions, and beliefs.

Indeed, consider the game from the point of view of a potential buyer. By knowing n, the agent is able to compute $p_0^\star$. He thus anticipates that if he observes the price in $t=1$, he will discover $p_1^\star = p_0^\star - \Delta$. As such, the agent has no reason to pay the fee c for learning a price that he already knows, unless he is willing to buy the item at $p_1^\star$. By symmetry, the agent also anticipates that all other players adopt the same behavior. Therefore, if at $t=2$ the auction is still open, it necessarily means that no one observed the price. As such, $p_1^\star = p_0^\star$. The same reasoning then applies to subsequent periods, such that on the equilibrium path the hidden price cannot fall. Each agent thus decides what to do by comparing his net willingness to pay $b^\star(v_i) - c$ with $p_1^\star = p_0^\star - \Delta$. In particular, whenever the agent finds the transaction beneficial, he will observe the price and buy the item as soon as he can, i. e., at $t=1$. Indeed, and exactly because the price cannot fall in equilibrium, by postponing the purchase an agent can only lose when preempted by a rival.

The fact that, in equilibrium, the price cannot fall obviously influences agents' willingness to pay and thus ultimately determines the profitability of the mechanism. In particular, we will show that agents' gross willingness to pay coincides with their private valuation (i. e., $b^\star(v_i) = v_i$). Perhaps more surprisingly, we will show that when n is common knowledge, a PRA essentially replicates the outcome of a posted price mechanism (PPM). As such, a PRA raises lower revenues with respect to standard auction formats.

Proposition 1: *In a "profit maximizing trading equilibrium" (PMTE) of a PRA in which n is common knowledge, the seller sets the initial price* $p_0^\star = \underset{p_0 \in [v_s + \Delta - c, \bar{v} + \Delta - c]}{\arg\max} \pi_s$ *where* π_s *(the seller's expected profits) is defined as follows:*

$$\pi_s = (1 - [F(p_0 - \Delta + c)]^n)(p_0 - \Delta + c - v_s) \qquad [4]$$

Each agent $i \in N$ *plays* $(a_{i,t}^\star)_{t=1}^{t_e}$ *where* $a_{i,t}^\star = (a_{i,t}^{\star 1}, a_{i,t}^{\star 2})$ *is such that:*

$$a_{i,t}^{\star 1} = \begin{cases} \emptyset \;\; \text{if} \left(\pi_i \mid H_{i,t}^\star(p_{t-1})\right) \le 0 \\ wto \;\; \text{if} \left(\pi_i \mid H_{i,t}^\star(p_{t-1})\right) > 0 \end{cases} \quad \text{for any } i \in N$$

$$a_{i,t}^{\star 2} = \begin{cases} \gamma(\cdot) \text{ with } \gamma\ (p_t) = 0 & \text{if } p_t > b^\star(v_i) \\ \gamma(\cdot) \text{ with } \gamma\ (p_t) = 1 & \text{if } p_t \le b^\star(v_i) \end{cases} \quad \text{for } i = \hat{i}_t \text{ and } a_{i,t}^{\star 2} = \emptyset \text{ for any } i \ne \hat{i}_t$$

and $(\pi_i | H_{i,t}^\star(p_{t-1}))$ *is as defined in [1],* $H_{i,t}^\star(p_{t-1})$ *is such that* $h_{i,t}^\star(p_{t-1} = p_0^\star) = 1$ *for any* $i \in N$ *and any* t, $b^\star(v_i) = v_i$ *for any* $i \in N$, *and* $t_e \in \{1, T\}$.[14]

14 A system of off-path beliefs that supports the equilibrium is the following:
– Agent $\hat{i}_t$: $H'_{\hat{i}_t,t}(p_t)$ with $h'_{\hat{i}_t,t}(p_t = p'_t) = 1$ where $p'_t \ne p_t^\star$ is the non equilibrium price that $\hat{i}_t$ observes. An agent that observes p'_t thus acknowledges the fact that his initial beliefs were wrong and updates them accordingly. Notice however that the agent cannot renege from

Proof: In the appendix. ■

Proposition 1 implies that, on the equilibrium path, only two situations can arise:
(1) An agent observes the price in $t = 1$, and immediately buys the item.
(2) No agent ever observes the price and the item remains unsold.

In the first case, the mechanism raises positive profits; in the second, it raises zero profits. Which of these two equilibrium outcomes occurs depends on how the actual realization of agents' types combines with the initial price, $p_0^\star$. When trade occurs (case 1), the allocation may fail to be efficient; in fact, whenever more than one agent is willing to observe the price in $t = 1$, the tie-breaking rule randomly selects the player to whom the price is actually disclosed. It follows that the agent who buys the good is not necessarily the one with the highest valuation.[15]

The following example illustrates the two possible equilibrium outcomes as well as the potential inefficiency of the final allocation:

Example 1: *Consider a price reveal auction with* $N = \{1, 2, 3\}$, $F \sim U[0, 150]$, $v_s = 0$, $c = 2$, *and* $\Delta = 1$. *The optimal initial price thus maximizes* $\pi_s = \left(1 - \left[\frac{1}{150}(p_0 + 1)\right]^3\right)(p_0 + 1)$. *Therefore,* $p_0^\star = 93.49$ *and* $\pi_s(p_0^\star) = 70.87$. *As such, every agent with valuation* $v_i > \underline{v} = 94.49$ *has a positive expected payoff* ($\underline{v}$ *solves* $\pi_i(p_0^\star) = \underline{v} - 93.49 + 1 - 2 = 0$). *Now consider three different cases concerning the actual realizations of agents' types*:

– *Case* $[a]$ *(trade with efficient allocation)*: $v_1 = 40$, $v_2 = 80$, *and* $v_3 = 130$. *Then,* $a_{i,1}^{\star 1} = \emptyset$ *for* $i \in \{1, 2\}$ *and* $a_{3,1}^{\star 1} = wto$. *Therefore,* $\hat{i}_1 = 3$ *and* $\gamma(p_1^\star) = 1$ *since* $p_1^\star = 92.49$. *The auction ends at* $t_e = 1$ *and payoffs are* $u_1 = 0$, $u_2 = 0$, $u_3 = 35.51$, *and* $u_s = 94.49$.

having played $a_{i,t}^{\star 1} = wto$. Notice also that action $a_{\hat{i}_t,t}^{\star 2}$ still disciplines this contingency: the agent is pragmatic and buys the item whenever $p_t \le b^\star(v_i)$.

– Generic agent i who played $a_{i,t-1}^{\star 1} = wto$ but lost in a tie: $H_{i,t}'(p_{t-1})$ with $h_{i,t}'(p_{t-1} > v_i) = 1$ for any t. The agent sees that the auction remains open and thus infers that $p_t > p_t^\star$. In particular, he loses confidence and assigns probability 1 to the event that the hidden price is higher than his valuation. In line with the equilibrium strategy, the agent then abstains from observing the price in any future period.
– Generic player i who played $a_{i,t-1}^{\star 1} = \emptyset$: $H_{i,t}'(p_{t-1})$ with $h_{i,t}'(p_{t-1} = p_{t-1}^\star) = 1$. The agent does not observe any deviation from the equilibrium path and thus his beliefs do not change.

15 In principle, the seller could use a truth-telling mechanism, such as discriminating among agents that are tied together, and thus allocate the good in an efficient way. However, such a mechanism seems difficult to implement in reality. Moreover, efficiency is not an objective that the seller pursues. In Section 3.3, we will actually see that, in cases of ties, a profit-maximizing seller may have incentives to give priority to agents with lower valuations.

- *Case $[b]$ (trade with inefficient allocation): as in case $[a]$ but with $v_2 = 100$ such that also agent 2 plays $a^{\star 1}_{2,1} = wto$. The auction ends at $t_e = 1$ and with probability $\frac{1}{2}$ leads to an inefficient allocation (i. e., $\hat{\imath}_1 = 2$ and $\gamma(p^\star_1) = 1$ such that the item goes to agent 2) with payoffs $u_1 = 0$, $u_2 = 5.51$, $u_3 = 0$, and $u_s = 94.49$.*
- *Case $[c]$ (no trade): as in case $[a]$ but with $v_3 = 90$. Then $\left(a^\star_{i,t}\right)^T_{t=1} = (\emptyset, \emptyset)$ for any $i \in N$. The auction ends at $t_e = T$, the item remains unsold, and payoffs are $u_i = 0$ for any $i \in N$ and $u_s = 0$.*

Going back to Proposition 1, an interesting feature of the equilibrium is that $b^\star(v_i) = v_i$, i. e., bidders' *gross* willingness to pay coincides with their valuation. The resulting *net* willingness to pay $(v_i - c)$ can thus be interpreted as the agent's valuation net of the cost of observing the price. As already mentioned, the intuition for such a result is that agents realize that, on the equilibrium path, the price cannot fall and thus decide what to do by comparing the payoff they would get if they observe the price and buy the item at $t = 1$ $(u_i = v_i - p^\star_0 + \Delta - c)$ with the payoff they would get if they do not buy at all $(u_i = 0)$.[16]

In equilibrium, agents' behavior is thus similar to the one they would adopt in a PPM (take it or leave it offer). Indeed, the following proposition shows that the two mechanisms are outcome equivalent.

Proposition 2: *The PMTE of a PRA with common knowledge about n and optimal initial price $p^\star_0$ is outcome equivalent to a PPM with n buyers and optimal price $p^\star_{PPM} = p^\star_0 - \Delta + c$. In particular, the two mechanisms yield the same expected revenues.*

Proof: Given any N, F, v_s, and $(v_1, \ldots, v_n)$, consider the functions $\pi_s = (1 - [F(x)]^n)(x - v_s)$ and $u_i = v_i - x$ for $i \in N$. Notice that when $x = p_0 - \Delta + c$, the function π_s captures the seller's expected profits in a PRA. Moreover, a buyer $i \in N$ is willing to observe the price and buy the item if and only if $u_i > 0$. Similarly, when $x = p_{PPM}$, π_s describes the seller's expected profits in a PPM and a buyer is willing to buy the item if and only if $u_i > 0$. As shown in the proof of Proposition 1, the function π_s is strictly convex such that it has a unique maximizer $x^\star$. Therefore, the optimal initial price in a PRA is such that $p^\star_0 - \Delta + c = x^\star$, whereas the optimal price in a PPM is such that $p^\star_{PPM} = x^\star$. It follows that $p^\star_{PPM} = p^\star_0 - \Delta + c$. Furthermore, $u_i(p^\star_0) = u_i(p^\star_{PPM})$ and $\pi_s(p^\star_0) = \pi_s(p^\star_{PPM})$, i. e., the two mechanisms yield the same expected revenues. ■

16 Clearly, such a result is also driven by the fact that $c > \Delta$, i. e., the private marginal cost of observing the price exceeds the marginal benefit. As such, no agent can gain by repeatedly observing the price with the goal of decreasing it on his own.

Two noticeable results directly stem from having proved that the PMTE of a PRA is outcome equivalent to a PPM. We present them in the following lemmas.

Lemma 1: *In the PMTE of a PRA, the optimal initial price and the expected revenues are strictly increasing in n.*

Proof: In a PPM the optimal price as well as the seller's expected revenues are strictly increasing in n (see Blumrosen and Holenstein 2008). By Proposition 2, the same relationships thus holds also in the PMTE of a PRA. ■

Figure 1 illustrates the results of Lemma 1 in the context of a PRA with $F \sim U[0, 150]$, $v_s = 0$, $c = 2$, $\Delta = 1$, and where the number of participants is given by $n \in \{3, 30, 300\}$.[17]

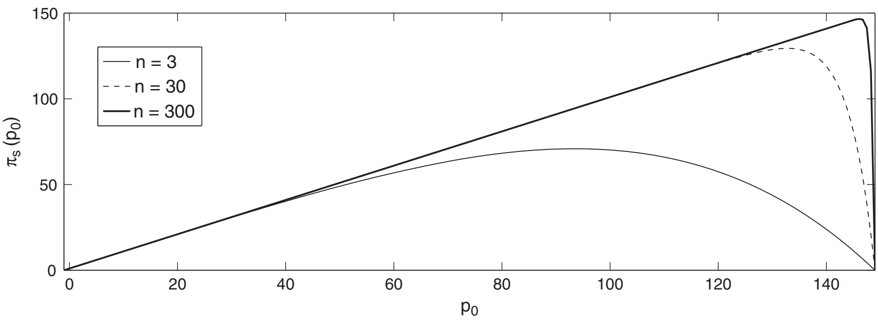

Figure 1 : Seller's expected revenues.

Lemma 2: *The PMTE of a PRA with common knowledge about n yields lower expected revenues than standard auction formats.*

Proof: A PPM with optimal price $p^{\star}_{PPM}$ is revenue dominated by a standard auction format with reserve price $r = p^{\star}_{PPM}$. A fortiori, a PPM is thus dominated by an auction in which the seller sets the reserve price optimally (see Myerson 1981, and Wang 1993). Because of Proposition 2, it then follows that a PRA yields lower expected revenues than standard auction formats. ■

The following example illustrates the result of Lemma 2 in a context that replicates the scenario introduced in Example 1.

17 Since we set $v_s = 0$, expected revenues coincide with expected profits. The lowest curve ($n = 3$) depicts the scenario introduced in Example 1.

Example 2: *Let* $N = \{1, 2, 3\}$, $F \sim U[0, 150]$, $v_s = 0$, *and compare in terms of expected revenues a PRA, a PPM, a first price auction (FPA), and a second price auction (SPA). In a PRA (as in Example 1 we set* $c = 2$ *and* $\Delta = 1$*), the seller sets* $p_0^\star = 93.49$ *such that* $\pi_{PRA} = 70.87$. *In a PPM, the seller sets* $p_{PPM}^\star = 94.49$ *such that, coherently with Proposition 2,* $\pi_{PPM} = 70.87$. *In a FPA (respectively, SPA), the optimal bid is given by* $b_{FPA}^\star(v_i) = \frac{2}{3}v_i$ *(respectively,* $b_{SPA}^\star(v_i) = v_i$*). Therefore, with reserve price* $r = 94.49$, *expected revenues are given by* $\pi_{FPA} = \pi_{SPA} = 77.06$. *With the optimal reserve price* $r^\star = 75$ (see Krishna 2002), expected revenues further increase to $\pi_{FPA} = \pi_{SPA} = 81.25$. *Revenue equivalence among the four mechanisms thus does not hold and, in particular,* $\pi_{PRA} = \pi_{PPM} < \pi_{FPA} = \pi_{SPA}$.

Notice that as the number of participants tends toward infinity, the expected revenues of both a PRA and a standard auction tend toward $\bar{v}$. In particular, the revenue gap between a PRA and a standard auction shrinks as the level of participation increases. Such a relationship will play an important role when we will discuss the expected profitability of a PRA with uncertainty about n.

Finally, as already mentioned, the PRA mechanism also features a "no trading equilibrium". In this equilibrium the seller sets an initial price $p_0^\star \in [\bar{v} - c + \Delta, \bar{v}]$ (i. e., a price such that $p_1^\star \geq v_i - c$ for any $i \in N$) and, consistent with the seller's choice, all agents play $a_{i,t}^\star = (\emptyset, \emptyset)$ at any t. The game ends at $t_e = T$, the item remains unsold and final payoffs are $u_i = 0$ for any $i \in N$ and $u_s = 0$. The "no trading equilibrium" is Pareto dominated (at least in expectations) by the PMTE defined in Proposition 1 where, when trade actually occurs, two players (the seller and the buyer) realize a positive payoff. As already mentioned, given that the seller is the agent who sets up the game, and that his goal is to obtain profits, the PMTE appears to be more relevant.

3.3 The Equilibria When *n* is Not Common Knowledge

Contrary to the previous section, assume now that the number of potential participants n is not common knowledge among all the players. More precisely, the seller still knows n with certainty whereas the buyers do not. This different information structure is certainly more appropriate to capture the features of PRAs that take place over the Internet, where indeed buyers do not know the number of rivals. Notice however that, in order to participate, agents must first register on the seller's website. As such, while somehow stark (registration does not necessarily imply participation), the assumption that the seller knows n with certainty seems justifiable.[18]

18 We will relax this assumption and investigate what would happen in a situation in which all the players (i. e., both the seller and the buyers) are equally uninformed in Section 4.

This information asymmetry has important implications for what concerns possible equilibrium outcomes and the profitability of the mechanism. Since the optimal initial price that the seller sets is a function of the number of participants, uncertainty about n leads in fact to uncertainty about $p_0^\star$. To see this point, assume that, from the point of view of each agent $i \in N$, the number of participants is a random variable $\tilde{N}$. Let $g(\cdot)$ be the non-degenerate probability mass function of $\tilde{N}$ and $S_{\tilde{N}} = \{\tilde{n}_{\min}, \ldots, \tilde{n}_{\max}\}$ with $\tilde{n}_{\min} \geq 1$ its support. Agents' beliefs about $p_0^\star$ are then captured by the distribution $H_{i,1}(p_0)$, which is defined over the support $S_{p_0^\star} = \{p_0^\star(\tilde{n}_{\min}), \ldots, p_0^\star(\tilde{n}_{\max})\}$ and assigns the probability mass function $h_{i,1}(p_0^\star(\tilde{n})) = g(\tilde{n})$ for every $\tilde{n} \in S_{\tilde{N}}$, where $p_0^\star(\tilde{n})$ is the optimal initial price the seller would set if the actual number of participants was $n = \tilde{n}$ (indeed, we will show that in equilibrium the seller faces the same problem, and thus adopts the same behavior, as the ones described in Section 3.2). Uncertainty about $p_0^\star$, in turn, implies uncertainty about the current price $p_t^\star$ at any $t \geq 1$. In such an environment, agents still play according to the behavioral rule defined in eq. [3], but now action $a^2_{\hat{i}_t, t} = \gamma(\cdot)$ with $\gamma(p_t^\star) = 0$ can be observed even on the equilibrium path. Indeed, it may well be the case that an agent rationally decides to observe the price, discovers an actual level that would instead lead to a negative payoff, and thus decides not to buy the item. This implies that, in equilibrium, the price may actually fall.

The following proposition formalizes these results:

Proposition 3: *In a "profit maximizing trading equilibrium" (PMTE) of a PRA in which n is uncertain, agents believe the initial price to be distributed over $S_{p_0^\star} = \{p_0^\star(\tilde{n}_{\min}), \ldots, p_0^\star(\tilde{n}_{\max})\}$ according to the distribution $H^\star_{i,1}(p_0)$ that assigns probability mass function $h_{i,1}(p_0^\star(\tilde{n})) = g(\tilde{n})$ for any $\tilde{n} \in \{\tilde{n}_{\min}, \ldots, \tilde{n}_{\max}\}$. Each $p_0^\star(\tilde{n})$ is defined as $p_0^\star(\tilde{n}) = \underset{p_0 \in [v_s + \Delta - c,\, \bar{v} + \Delta - c]}{\arg\max} \pi_s(\tilde{n})$ where:*

$$\pi_s(\tilde{n}) = \left(1 - [F(p_0 - \Delta + c)]^{\tilde{n}}\right)(p_0 - \Delta + c - v_s)$$

Each agent $i \in N$ plays $(a^\star_{i,t})_{t=1}^{t_e}$ where $a^\star_{i,t} = (a^{\star 1}_{i,t}, a^{\star 2}_{i,t})$ is such that:

$$a^{\star 1}_{i,t} = \begin{cases} \emptyset & \text{if } (\pi_i | H^\star_{i,t}(p_{t-1})) \leq 0 \\ wto & \text{if } (\pi_i | H^\star_{i,t}(p_{t-1})) > 0 \end{cases} \qquad \textit{for any } i \in N$$

$$a^{\star 2}_{i,t} = \begin{cases} \gamma(\cdot) & \text{with } \gamma(p_t) = 0 \ \text{ if } p_t > b^\star(v_i) \\ \gamma(\cdot) & \text{with } \gamma(p_t) = 1 \ \text{ if } p_t \leq b^\star(v_i) \end{cases} \qquad \textit{for } i = \hat{i}_t \textit{ and } a^{\star 2}_{i,t} = \emptyset \textit{ for any } i \neq \hat{i}_t$$

and $(\pi_i | H_{i,t}(p_{t-1}))$ *is as in [1], agents' beliefs* $H^{\star}_{i,t}(p_{t-1})$ *evolve from* $H^{\star}_{i,1}(p_0)$ *according to Bayes' rule,* $b^{*}(v_i) = v_i$ *for any* $i \in N$*, and* $t_e \in \{1, ..., T\}$*.*[19]

Proof: In the appendix. ■

The equilibrium is now characterized by three possible (classes of) outcomes:
(1) An agent observes the price in $t = 1$, and immediately buys the item.
(2) No agent ever observes the price and the item remains unsold.
(3) More than one agent observes the price and the auction ends at $t_e \in \{2, ..., T\}$.

The first two outcomes are analogous to the ones that characterize the PMTE of a PRA in which n is common knowledge; the third one is instead specific of a PRA where there is uncertainty about the number of participants. In this third equilibrium outcome, the entry of multiple bidders and repeated observations of the price can occur. The following example illustrates such a possibility:

Example 3: *Consider a PRA where* $F \sim U[0, 150]$, $v_s = 0$, $c = 2$, *and* $\Delta = 1$. *Let* $S_{\tilde{N}} = \{1, ..., 5\}$ *with* $g(\tilde{n}) = 0.2$ *for every* $\tilde{n} \in \{1, ..., 5\}$. *The actual number of participants is* $n = 4$ *with* $v_1 = 50$, $v_2 = 88$, $v_3 = 93$, *and* $v_4 = 130$. *In equilibrium, the game can thus unfold in the following way*:

- *period* $t = 1$: *agents' initial beliefs are described by the distribution* $H^{\star}_{i,1}(p_0)$. *More precisely, all agents expect the initial price to be distributed over the support*:

$$S_{p_0^{\star}} = \{74,\ 85.6,\ 93.49,\ 99.31,\ 103.82\}$$

with probabilities $(0.2, 0.2, 0.2, 0.2, 0.2)$. *As such, the choice of observing the price leads to a positive expected payoff for any* $v_i > \underline{v}_{t=1} = 83$ *where* $\underline{v}_{t=1}$ *solves* $(\pi_i | H^{\star}_{i,1}(p_0)) = 0$. *Buyer 1 thus remains inactive whereas buyers 2, 3, and 4 are willing to observe the price. With probability* $\frac{1}{3}$, *buyer 2 is chosen. He observes* $p_1^{\star} = 92.49$, *such that* $p_1^{\star} > v_1$ *and thus does not buy the item.*
- *period* $t = 2$: *agent 1's beliefs do not change (i. e.,* $H^{\star}_{1,2}(p_1) = H^{\star}_{1,1}(p_0)$*) such that the agent remains inactive. Agent 2's beliefs are instead given by* $h^{\star}_{1,2}(p_1^{\star} = 92.49) = 1$; *the agent thus also plays* $a^{\star}_{1,2} = (\emptyset, \emptyset)$. *Agents 3 and 4 instead*

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19 A system of off-path beliefs that supports the equilibrium is analogous to the one already described in the case of Proposition 1 (see footnote 14). In particular:
- Agent $\hat{i}_t$: $H'_{\hat{i}_t,t}(p_t)$ with $h'_{\hat{i}_t,t}(p_t = p'_t(\tilde{n})) = 1$ where $p'_t(\tilde{n}) \neq p^{\star}_t(\tilde{n})$ is the non equilibrium price that $\hat{i}_t$ observes.
- Generic agent i who played $a^{*1}_{i,t-1} = wto$ but lost in a tie: $H'_{i,t}(p_{t-1})$ with $h'_{i,t}(p_{t-1} > v_i) = 1$ for any t.
- Generic player i who played $a^{*1}_{i,t-1} = \emptyset$: $H'_{i,t}(p_{t-1}) = H^{\star}_{i,t}(p_{t-1})$.

observe that the auction is still open. Therefore, they update their beliefs to $H^\star_{i,2}(p_1)$ *with* $i \in \{3,4\}$ *and expect* $p^\star_1$ *to be distributed over the support:*

$$S_{p^\star_1} = \{73,\ 84.6,\ 92.49,\ 98.31,\ 102.82\}$$

with probabilities $(0, 0.02, 0.153, 0.3, 0.527)$.[20] *Given* $H^\star_{i,2}(p_1)$, *it is worthwhile for a player to observe the price if and only if* $v_i > \underline{v}_{t=2} = 99.07$, *where* $\underline{v}_{t=2}$ *solves* $\left(\pi_i | H^\star_{i,2}(p_1)\right) = 0$. *Therefore, agent 3 now decides to remain inactive whereas agent 4 observes the price, discovers* $p^\star_2 = 91.49$ *such that* $p^\star_2 < v_4$, *and thus buys the item. The auction closes and payoffs are* $u_1 = 0$, $u_2 = -2$, $u_3 = 0$, $u_4 = 36.51$, *and* $u_s = 95.49$.

Example 3 shows which forces drive the Bayesian updating of agents' beliefs. On the one hand, learning that a rival observed the price and did not buy the item brings good news, as this implies that the price fell by Δ. As such, the support of possible prices shifts to the left. On the other hand, the fact that the rival who observed the price did not buy the item means that the actual price that he discovered was higher than his valuation, where the latter was certainly above the entry threshold $\underline{v}_{t=1}$. As such, the probability distribution becomes more skewed towards high values. For standard values of the parameters (small Δ, see footnote 1), the latter effect dominates such that the expected price, as well as the threshold $\underline{v}_t$ that makes the choice to observe the price worthwhile, weakly increases over time. Figure 2 below illustrates the effects of these two conflicting forces in the context of Example 3.

The way agents update their beliefs about the hidden price implies that, in equilibrium, two further results hold. First, an agent that is willing to observe the price in a given period but does not get the opportunity to do so may decide not to observe the price in subsequent periods, because the threshold $\underline{v}_t$ that solves condition $\left(\pi_i | H^\star_{i,t}(p_{t-1})\right) = 0$ increases over time. As such, a player may have a valuation $\underline{v}_t < v_i < \underline{v}_{t'}$ with $t' > t$ (this is the case of agent 3 in Example 3).

20 At $t = 2$, agents 3 and 4 assign probability 1 to the event that the valuation of the agent that observed the price (i. e., agent 2) is such that $v_2 \in [83, 102.8]$. The lower bound follows from the fact that the agent observed the price. The upper bound follows from the fact that the agent did not buy the item. Given the support $S_{p^\star_1}$, it thus immediately follows that $h^\star_{i,2}(73) = 0$. Then, under which conditions would agent 2 have refused to buy the item if the price that he discovered was 84.6? By expression [3], this would have happened if $v_2 \in [83, 84.6]$, an event with probability $\frac{F(84.6) - F(83)}{F(102.8) - F(83)} = 0.008$; however, if $v_2 \in [83, 84.6]$, then agent 2 would have refused to buy the item even if he discovered a price $p^\star_1 \in \{92.49, 98.31, 102.82\}$. As such, $h^\star_{i,2}(84.6) = \frac{0.08}{4} = 0.02$. Similarly, agent 2 would have refused to buy the item if $p^\star_1 = 92.49$ if (as before) $v_2 \in [83, 84.6]$, as well as if $v_2 \in [84.6, 92.49]$. The latter event has probability $\frac{F(92.49) - F(84.6)}{F(102.8) - F(83)} = 0.398$; however, a valuation $v_2 \in [84.6, 92.49]$ would have led agent 2 to refuse to buy the item even if $p^\star_1 \in \{98.31, 102.82\}$. As such, $h^\star_{i,2}(92.49) = 0.02 + \frac{0.398}{3} = 0.153$. Similar computations lead to define the distribution $(0, 0.02, 0.153, 0.3, 0.527)$.

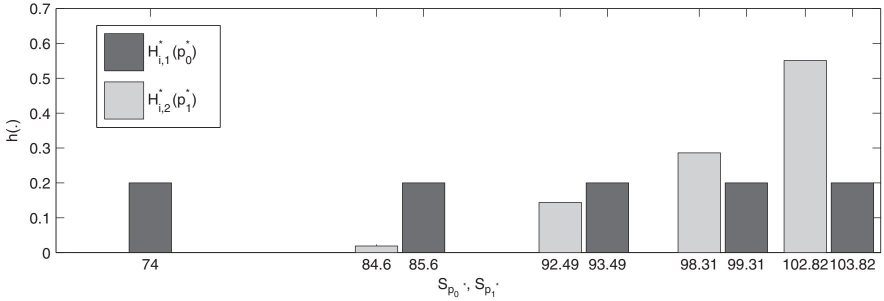

Figure 2: Updating of agents' beliefs.

Second, no agent that is unwilling to observe the price in a given period will decide to observe the price in subsequent periods. The intuition is the following. A player plays $a_{i,t}^{*1} = \emptyset$ if his expected payoff from observing the price is negative, i. e., if $v_i < \underline{v}_t$. Now suppose that in period $t' > t$, agent i observes that the auction is still open. The player realizes that two paths may have occurred: (1) No rival observed the price in any of the previous periods. If this was the case, then i's beliefs would not change and thus the expected payoff of playing $a_{i,t'}^{*1} \neq \emptyset$ would remain negative as it was in t. (2) Some other player observed the price in some of the previous periods. However, if this was the case, the agents that observed the price necessarily refused to buy the item, otherwise the auction would have closed. Moreover, if a player $j \in N$ observed the price in period $t'' \in \{t, ..., t'\}$, then it must be the case that $v_j > \underline{v}_{t''}$, which in turn implies $v_j > v_i$ given that $\underline{v}_{t''} \geq \underline{v}_t$. The fact that j did not buy the item thus indicates that, a fortiori, agent i would find the transaction unattractive.

A noticeable feature of the equilibrium is that it remains optimal for the agents to play $b^*(v_i) = v_i$. In other words, and despite the fact that, in equilibrium, the price may actually fall, agents' gross willingness to pay still coincides with their valuation. The reason is that in a PRA, the price falls endogenously in response to agents' behavior, rather than exogenously, as it does in a standard Dutch auction. Indeed, the probability that the price falls (i. e., the probability that one or more rivals observe the price and then do not buy the item) is too low to justify a strategy $b'(v_i) < b^*(v_i) = v_i$. We discuss this result in more detail in the proof of Proposition 3; however, as an informative example, consider a hypothetical PRA in which there are n participants but only agent i has a valuation $v_i > \underline{v}_{t=1}$. Assume moreover that the initial price is such that $b'(v_i) - c < p_1^*(n) \leq v_i - c$. If the price was falling exogenously, the current price $p_t^*(n)$ would eventually reach a level $p_t^*(n) \leq b'(v_i) - c$. Agent i would then enjoy a more substantial surplus by playing $b'(v_i)$ rather than $b^*(v_i)$. However, in a PRA, the price remains stuck at the initial level and thus reaches $b'(v_i)$ with zero probability. Therefore, $b^\star(v_i)$ dominates $b'(v_i)$, as the former leads to $u_i = v_i - p_1^\star(n) - c$ with $u_i \geq 0$, whereas the latter leads to $u_i' = 0$.

We conclude the analysis of the mechanism by studying its profitability. The fact that, in equilibrium, multiple entries can occur benefits the seller, as revenues increase by $(c - \Delta) > 0$ every time an agent observes the price. Indeed, the following Lemma shows that a PRA with uncertainty about the number of participants raises higher expected revenues than a PRA in which n is common knowledge.[21]

Lemma 3: *Consider a PRA with n participants. Let $PRA(n)$ denote the case in which n is common knowledge and $PRA(\tilde{n})$ the case in which, from the buyers' point of view, the number of participants is a random variable with mean n. Then, $\pi_{PRA(\tilde{n})} > \pi_{PRA(n)}$, i. e., $PRA(\tilde{n})$ yields higher expected revenues than $PRA(n)$.*

Proof: In $PRA(n)$ the optimal initial price $p_0^\star(n)$ is as defined in Proposition 1. Given $p_0^\star(n)$, let $\underline{v}(n)$ solve $u_i\big(p_0^\star(n)\big) = \underline{v}(n) - p_0^\star(n) + \Delta - c = 0$. Now consider $PRA(\tilde{n})$ where by construction $\sum_{\tilde{n}_{\min}}^{\tilde{n}_{\max}} \tilde{n}g(\tilde{n}) = n$. Let $v_i = \underline{v}(n)$ and evaluate $\big(\pi_i | H_{i,1}^\star(p_0)\big)$ where $H_{i,1}^\star(p_0)$ is as defined in Proposition 3. By Lemma 1, $p_0^\star(\tilde{n}) < p_0^\star(n)$ for any $\tilde{n} < n$ and $p_0^\star(\tilde{n}) > p_0^\star(n)$ for any $\tilde{n} > n$. Then, $u_i\big(p_0^\star(\tilde{n})\big) = -c$ whenever $\tilde{n} > n$ while $u_i\big(p_0^\star(\tilde{n})\big) = \underline{v}(n) - p_0^\star(\tilde{n}) + \Delta - c > 0$ whenever $\tilde{n} < n$. It follows that $(\pi_i | H_{i,1}(p_0)) = \sum_{\tilde{n}_{\min}}^{\tilde{n}_{\max}} g(\tilde{n}) u_i\big(p_0^\star(\tilde{n})\big) > 0$. Given that $\big(\pi_i | H_{i,1}^\star(p_0)\big)$ is strictly increasing in v_i, it then must be the case that $\big(\pi_i | H_{i,1}^\star(p_0)\big) = 0$ if and only if agent i has a valuation $\underline{v}(\tilde{n}) < \underline{v}(n)$. In other words, the valuation that makes an agent indifferent between observing the price or not is lower when the number of participants is uncertain. Now consider any profile of valuations $(v_1, \ldots, v_n)$ where without loss of generality $v_1 < \ldots < v_n$. Three situations may occur: (a) $v_n < \underline{v}(\tilde{n}) < \underline{v}(n)$ and thus $u_{s|PRA(\tilde{n})} = u_{s|PRA(n)} = 0$; (b) $\underline{v}(\tilde{n}) < v_n < \underline{v}(n)$ and thus $u_{s|PRA(\tilde{n})} > 0$ and $u_{s|PRA(n)} = 0$; (c) $\underline{v}(\tilde{n}) < v(n) < \underline{v}_n$ and thus $u_{s|PRA(\tilde{n})} > 0$, $u_{s|PRA(n)} > 0$, and $u_{s|PRA(\tilde{n})} \geq u_{s|PRA(n)}$ as multiple observations can occur in $PRA(\tilde{n})$. Therefore, $PRA(\tilde{n})$ raises actual revenues that are weakly larger than those that $PRA(n)$ raises. It follows that $\pi_{PRA(\tilde{n})} > \pi_{PRA(n)}$, i. e., $PRA(\tilde{n})$ yields larger expected revenues than $PRA(n)$.

The lower bound on the revenues that a PRA with uncertainty about n can raise is thus given by $u_{s|PRA(n)} = p_0^\star(n) + c - \Delta$. As for the upper bound, notice that in every round in which tied players update their beliefs, the probability of at least one of the possible prices (namely, the lowest one) drops to zero. Given that the number of possible initial prices coincides with the number of possible realizations of the random variable $\tilde{N}$, it follows that revenues can at most reach the

21 Such a result is in line with the literature that shows how the strategic obfuscation of some aspects of the allocation procedure can benefit the seller (e. g., Gabaix and Laibson 2006).

level $u_{s|PRA(\tilde{n})} = p_0^\star(n) + |\tilde{N}_n|(c - \Delta)$, where $\tilde{N}_n = \{\tilde{n}_{\min}, ..., n\}$ is the set of possible realizations of $\tilde{N}$ that are smaller or equal than the actual number of buyers n.

This latter result implies that, in some (highly specific) situations, a PRA with uncertainty about n may revenue dominate standard auction formats. Necessary conditions for this to happen are the following:

(1) Buyers' uncertainty about n is highly pronounced (i. e., the distribution of $\tilde{N}$ has a large variance),

(2) The true number of participants (i. e., the actual realization of $\tilde{N}$) is very large.

Indeed, as the number of participants gets larger, the revenue gap between a PRA where n is common knowledge and a standard auction shrinks (see Section 3.2). At the same time, the possibility that there are multiple players that are simultaneously willing to observe the price increases. However, if the distribution of $\tilde{N}$ has a high variance, so does the distribution that captures agents' beliefs about the hidden price. It follows that the chances that a player who observes the price refuses to buy the item become non-negligible. As such, the seller may collect multiple fees and the accrual of these fees may more than compensate for the fact that the expected selling price in a PRA is lower than in a standard auction. Conditional on the occurrence of the two above-mentioned necessary conditions, the expected revenues of $PRA(\tilde{n})$ may even exceed agents' maximal valuation $\bar{v}$. The following example illustrates this possibility. The example also highlights how the ranking of the different mechanisms depends on both the specific characteristics of the distribution of $\tilde{N}$ and the actual realization of n.

Example 4: *Consider two versions of a PRA with* $F \sim U[0, 150]$, $v_s = 0$, $c = 2$, *and* $\Delta = 1$.

- *In the first version, agents believe that the random variable* $\tilde{N}$ *is distributed over the support* $S_{\tilde{N}} = \{1, 2, 3, 4, 5\}$ *with* $g(\tilde{n}) = 0.2$ *for every* $\tilde{n} \in S_{\tilde{N}}$. *The actual number of participants is* $n = 3$. *Therefore (see examples 1, 2, and 3),* $\pi_{PRA(n)} = 70.87$, $\pi_{PRA(\tilde{n})} = 72.05$, *and* $\pi_{FPA} = \pi_{SPA} = 81.25$. *It follows that* $\bar{v} > \pi_{FPA} = \pi_{SPA} > \pi_{PRA(\tilde{n})} > \pi_{PRA(n)}$.
- *In the second version, agents instead believe that* $\tilde{N}$ *is distributed over* $S_{\tilde{N}} = \{3, 5, 10, 30, 2999\}$ *with* $g(\tilde{n}) = 0.2$ *for every* $\tilde{n} \in S_{\tilde{N}}$. *The actual number of participants is* $n = 2999$. *In such a case,* $\pi_{PRA(n)} = 149.55$, $\pi_{PRA(\tilde{n})} = 151.26$, *and* $\pi_{FPA} = \pi_{SPA} = 149.9$. *It follows that* $\pi_{PRA(\tilde{n})} > \bar{v} > \pi_{FPA} = \pi_{SPA} > \pi_{PRA(n)}$.

4 Robustness of the Equilibria

In this section, we investigate the robustness of the equilibria identified in Propositions 1 and 3 to a number of generalizations:

4.1 Random Arrival of Buyers

Assume a new buyer j becomes aware of the auction at a certain period $t' > 1$. If $t' > t_e$, then the auction has already closed and nothing changes in the equilibrium strategies played by the incumbent players. However, propositions 1 and 3 continue to hold even if $t' \le t_e$. Consider first the case in which n is common knowledge: player j (the $(n+1)th$ agent) observes that the auction is still open at t', correctly sets $p^\star_{t'-1} = p^\star_0$ where $p^\star_0$ is as defined in Proposition 1, and then plays accordingly; he observes the price and buys the item in t' if $\left(\pi_j | p^\star_{t'-1}\right) > 0$, whereas he remains idle and plays $a_{j,t} = (\emptyset, \emptyset)$ for any $t \ge t'$ otherwise. As such, the arrival of new participants can change t_e, but neither modifies the structure of the equilibrium nor makes the mechanism more profitable. Similarly, if n is a random variable, player j computes $\left(\pi_j | H^\star_{j,t'}(p_{t'-1})\right)$ as defined in Proposition 3 and then plays according to equilibrium. Notice that, in this case, the arrival of a new agent can modify the profitability of the mechanism. More precisely, the seller's profits can be enhanced (the new entrant observes the price and decides not to buy the item), penalized (the entrant observes the price and buys the item, pre-empting future observations by other players), or unaffected (the entrant does not observe the price).

4.2 Strategic Choice of c and Δ

Assume that at $t = 0$ the seller chooses not only the initial price $p_0 \in [0, \bar{v}]$ but also the amount of the fee c and the size of the price decrease Δ. As far as the condition $\Delta < c$ continues to hold, the qualitative features of the equilibria do not change; however, the choice of c and Δ clearly affects expected revenues. In particular, the seller must balance two conflicting forces. On one hand, a higher c (holding fixed Δ) decreases $(\pi_i | H_{i,t}(p_{t-1}))$ (see expression 1) and thus makes the equilibrium outcome in which the item remains unsold more likely. On the other hand, a higher c increases the marginal revenue $(c - \Delta)$ of each price observation. The effects of Δ (holding fixed c) are symmetric. A higher Δ favors participation but impairs the term $(c - \Delta)$. Players' incentives and the equilibria of the game radically change if one instead allows for the possibility that $\Delta > c$. If this is the case, the private marginal benefit of observing the price exceeds its cost. Then, an agent who happens to be the unique participant could drive $p^\star_t$ down to zero by observing the price $\frac{p^\star_0}{\Delta}$ times. This strategy costs an amount $\frac{p^\star_0}{\Delta} c$ and thus ensures positive profits as far as $\frac{p^\star_0}{\Delta} c < v_i$. With multiple participants, the game becomes more complicated, as obvious pre-emption issues arise. We conjecture that, in equilibrium, agents would buy the item whenever they discover a price $p_t^{\star} \le b^{\star}(v_i)$ where $b^{\star}(v_i)$ optimally solves the trade-off between the payoff the agent gets in case he wins and the risk of being preempted by a rival. As such, $b^{\star}(v_i) < v_i$ and agents' behavior would thus become similar to the one that characterizes a standard descending price auction. Which of the two versions of the mechanism ($\Delta < c$ vs. $\Delta > c$) yield higher revenues is a question that can then also be assessed through an experiment.

4.3 Different Information Structure

Section 3.2 defined the equilibrium of a PRA when the number of participants is common knowledge. Section 3.3 postulated instead that the seller knows n with certainty, whereas buyers do not. What would happen in a situation in which it is common knowledge that all the players are equally uninformed about n? To answer this question, let the non-degenerate probability mass function $g(\cdot)$ now describe the beliefs about the number of participants of both the buyers and the seller. As in Section 3.3, $S_{\tilde{N}} = \{\tilde{n}_{\min}, \dots, \tilde{n}_{\max}\}$ with $\tilde{n}_{\min} \ge 1$ denotes the support of $g(\cdot)$. The seller would then set the price that maximizes his expected profits

$$\pi_s = \left(1 - [F(p_0 - \Delta + c)]^{\bar{n}}\right)(p_0 - \Delta + c - v_s)$$

where $\bar{n} = \sum_{\tilde{n}=\tilde{n}_{\min}}^{\tilde{n}_{\max}} \tilde{n} \cdot g(\tilde{n})$ is the expected value of the random variable $\tilde{N}$. Buyers then correctly anticipate the price $p_0^{\star}(\bar{n})$ and thus behave as described in Proposition 1.

4.4 Other Sources of Uncertainty

We showed that the lack of common knowledge about the initial price $p_0^{\star}$ is a necessary condition to potentially trigger multiple price observations and thus enhance expected revenues. In the model, buyers' uncertainty about $p_0^{\star}$ originated from their uncertainty about the number of participants. However, uncertainty about the seller's valuation (v_s) would trigger similar dynamics, as different valuations lead to different initial prices (see expression 4).[22] Buyers would then still behave as discussed in Section 3.3. First, they use the distribution of v_s to compute the distribution of $p_0^{\star}$. Then, they are willing to observe the price if and only if such an action leads to a positive expected payoff. Finally,

22 I thank an anonymous referee for having suggested this point.

they buy the item if and only if such an action leads to a positive actual payoff. Notice that, in addition to the lack of perfect information about n and v_s, the PRA mechanism also embeds a further source of uncertainty. This stems from the possibility of ties: an agent that is willing to observe the price (and thus in principle willing to buy the item) faces the risk of not being able to do so because the seller may assign this right to another buyer. In this respect, this kind of uncertainty seems thus similar to the risk of incurring into rationing that characterizes any allocation procedure (e. g., a situation of excess demand in a posted price mechanism). Moreover, the payoffs consequences of losing in a tie are null as a buyer pays the fee c if and only if he actually observes the price. Because of these reasons, we conjecture that the effects of this source of uncertainty on agents' behavior should be minimal.

4.5 Risk Attitudes of the Buyers

Since the PRA mechanism features some uncertainty (see the paragraph above), buyers' risk attitudes matter. In particular, and given the peculiar nature of a PRA and the self-selection of the participants, it is conceivable that some buyers are risk lovers rather than risk neutral. Clearly, a risk-loving agent can be more inclined to observe the hidden price and this may enhance the mechanism's expected profitability. More formally, if the agent is risk lover, the threshold valuation $\underline{v}'$ that solves the condition $\left(\pi_i | H^{\star}_{i,t}(p_{t-1})\right) = 0$ will be such that $\underline{v}'_t < v_t$. It is then possible that $\underline{v}'_t < v_i < \underline{v}_t$, i. e., a risk-neutral agent with valuation v_i would not observe the price whereas his risk-loving counterpart would. Related to this point, notice that in principle the seller could exploit the fact that some buyers are risk lovers by artificially increasing the level of uncertainty in the game. Consider for instance the case in which all the parameters of the game are common knowledge. The seller might then announce that the realization of the initial price $p^{\star}_0$ will be drawn from a commonly known distribution. However, in the current design of the mechanism and in the absence of delegation, the seller lacks commitment power such that his announcement would not be credible. Indeed, the seller would have an incentive to select the unique initial price that maximizes his expected payoff rather than honestly rely on the announced probability distribution.

4.6 Behavioral Biases of the Buyers

Similarly to other pay-per-bid auction formats, a PRA can trigger and exploit a number of behavioral biases of the participants. Clearly, the mechanism tickles participants' curiosity. As such, biases such as wishful thinking, optimism, and over confidence, may lead players to hold biased beliefs $\tilde{H}_{i,t}(p_{t-1})$ that erroneously lead to more favorable expected outcomes, i. e., $\left(\pi_i|\tilde{H}_{i,t}(p_{t-1})\right) > \left(\pi_i|H^{\star}_{i,t}(p_{t-1})\right)$. Behavioral agents may thus decide to observe the hidden price more easily and/or more often than what equilibrium analysis prescribes. Moreover, an agent that observes the price but then decides not to buy the item may fall victim to the sunk-cost fallacy (Shubik 1971) and thus invest additional money in the hope of eventually finding a good deal and recovering the sunk costs.

5 Conclusions

We studied the theoretical properties of a specific example of pay-per-bid mechanisms, the so-called price reveal auction. We showed that when there is common knowledge about the number of participants, the format basically replicates the outcome that would emerge in a posted price mechanism. In particular, in equilibrium, a PRA attracts at most one active buyer and raises lower expected revenues than a standard auction. We then investigated a situation in which buyers are uncertain about the number of participants and showed that in such a case a PRA can trigger multiple price observations even on the equilibrium path. This enhances expected revenues up to the point that, under some (highly) specific conditions, a PRA may revenue dominate standard auction formats.

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Appendix

Proof of Proposition 1

We divide the proof in a number of steps.

(1) The choice of the optimal initial price $p_0^{\star}$.

Conditional on bidders' subsequent behavior, the optimal initial price $p_0^{\star}$ maximizes the seller's expected profits $\pi_s(p_0)$. These are given by:

$$\pi_s(p_0) = \left(1 - [F(\underline{v}(p_0))]^n\right)(p_0 - \Delta + c - v_s) \quad [5]$$

The first term on the RHS of eq. [5] is the probability that there exists at least one bidder whose valuation v_i is such that $v_i > \underline{v}(p_0)$ where $\underline{v}(p_0)$ is the valuation of an hypothetical agent who is indifferent between buying the good at $t = 1$ at price $p_1 = p_0 - \Delta$ and not entering the auction. More formally, $b^\star(\underline{v}(p_0)) - c = p_0 - \Delta$ where $b^\star(\underline{v}(p_0))$ is the agent's optimal bid. Therefore, $\underline{v}(p_0) = b^{\star-1}(p_0 - \Delta + c)$. The second term on the RHS of eq. [5] is the payoff the seller realizes if he sells the good at price $p_1 = p_0 - \Delta$ (i. e., to the first agent who observes the price). The optimal initial price is thus given by:

$$p_0^\star = \underset{p_0 \in [v_s + \Delta - c,\, b^\star(\bar{v}) + \Delta - c]}{\arg\max} \pi_s(p_0) \tag{6}$$

(2) Existence of $p_0^\star$.
The lower bound of the interval $\left[v_s + \Delta - c, b^\star(\bar{v}) + \Delta - c\right]$ ensures that the payoff is non-negative while the upper bound is required for the probability to be well-defined. The function [5] is continuous and the interval, which we will show that in equilibrium always exists, is closed and bounded. By the extreme value theorem, it follows that $p_0^\star$ certainly exists.

(3) Uniqueness of $p_0^\star$.
Define $\delta(p_0) = (1 - [F(\underline{v}(p_0))]^n)$ and $\mu(p_0) = (p_0 - \Delta + c - v_s)$ such that eq. [5] can be expressed as $\pi_s(p_0) = \delta(p_0)\mu(p_0)$. The functions $\delta(p_0)$ and $\mu(p_0)$ are concave (in particular $\delta''(p_0) < 0$ for any $n \geq 2$ and $\mu''(p_0) = 0$). Now consider the interval $[\alpha, \beta]$ with $\alpha = v_s + \Delta - c$ and $\beta = b^\star(\bar{v}) + \Delta - c$. The following conditions hold:

(i) $\delta(p_0)$ and $\mu(p_0)$ are nonnegative on $[\alpha, \beta]$;
(ii) $\delta(\alpha)\mu(\alpha) = \delta(\beta)\mu(\beta) = 0$;
(iii) $\delta'(p_0)\mu'(p_0) = -n[F(\underline{v}(p_0))]^{n-1} f(\underline{v}(p_0))\underline{v}'(p_0) < 0$ on (α, β).

Because of (ii) and Rolle's theorem, there certainly exists a $\hat{p}_0 \in [\alpha, \beta]$ such that $\pi_s'(\hat{p}_0) = 0$. Now evaluate the second derivative of $\pi_s(p_0)$ at $p_0 = \hat{p}_0$. This is given by $\pi_s''(\hat{p}_0) = \delta''(\hat{p}_0)\mu(\hat{p}_0) + 2\delta'(\hat{p}_0)\mu'(\hat{p}_0) + \delta(\hat{p}_0)\mu''(\hat{p}_0)$. Condition i), paired with the concavity of $\delta(p_0)$ and $\mu(p_0)$, implies that the first and the third terms of $\pi_s''(\hat{p}_0)$ are non positive. Condition iii) implies that the second term is strictly negative. Then, $\pi_s''(\hat{p}_0) < 0$ such that $\hat{p}_0$ is a maximum, i. e., $\hat{p}_0 = p_0^\star$. Moreover, and given that $\pi_s''(p_0) < 0$ for every $p_0 \in (\alpha, \beta)$, the function $\pi_s(p_0)$ is strictly concave and therefore $p_0^\star$ is unique.

(4) Buyers' behavior.
Given that agents know $p_0^\star$, their initial beliefs are captured by the (degenerate) distribution $H_{i,1}^\star(p_0)$ with $h_{i,1}^\star(p_0^\star) = 1$ for every $i \in N$. Buyers' behavior is then optimal given these beliefs and the latter are consistent. In particular, an agent i for whom $(\pi_i | H_{i,1}^\star(p_{t-1})) > 0$ plays first $a_{i,1}^{\star 1} = wto$ and then, in case $i = \hat{i}_t$,

$a_{i,1}^{\star 2} = \gamma(\cdot)$ with $\gamma(p_1^\star) = 1$. On the contrary, an agent for whom $(\pi_i | H_{i,1}^\star(p_{t-1})) \le 0$ plays $a_{i,1}^\star = (\emptyset, \emptyset)$. The latter remains an equilibrium for every t given that $\Delta < c$ and therefore the private benefits of observing p_t are smaller than the cost. If the auction is still open at generic period $\tilde{t} > 1$, bidders correctly infer that $a_{i,t}^\star = (\emptyset, \emptyset)$ for any i and any $t \in \{1, \ldots, \tilde{t} - 1\}$. Therefore, $H_{i,\tilde{t}}^\star(p_{t-1})$ is such that $h_{i,t}^\star(p_{t-1} = p_0^\star) = 1$ and thus $a_{i,\tilde{t}}^\star = (\emptyset, \emptyset)$ for any i.

Finally, given that $u_i = 0$ is the payoff of a bidder who plays $(a_{i,t}^\star)_{t=1}^T$ with $a_{i,t}^\star = (\emptyset, \emptyset)$ at any t and that on the equilibrium path $p_{t-1}^\star = p_0^\star$ for any t, it follows that $b^\star(v_i) = v_i$. Every $b'(v_i) \ne v_i$ is in fact (weakly) dominated. First, let $b'(v_i) > v_i$: strategies $b'(v_i)$ and $b^\star(v_i)$ lead to the same payoff unless $b'(v_i) > p_0^\star + c - \Delta > b^\star(v_i)$. If this is the case, a bidder who plays $b^\star(v_i)$ does not enter the auction and gets $u_i = 0$, while a bidder who plays $b'(v_i)$ observes $p_0^\star$ and then either gets $u_i' = v_i - p_0^\star + \Delta - c < 0$ (the agent buys the item) or $u_i'' = -c$ (the agent does not buy the item). Alternatively, let $b'(v_i) < v_i$: strategies $b'(v_i)$ and $b^\star(v_i)$ lead to the same payoff unless $b^\star(v_i) > p_0^\star + c - \Delta > b'(v_i)$. If this is the case, a bidder who plays $b^\star(v_i)$ observes $p_0^\star$, buys the item, and gets $u_i = v_i - p_0^\star + \Delta - c > 0$ while a bidder who plays $b'(v_i)$ does not enter the auction and gets $u_i' = 0$.

By substituting $\underline{v}(p_0) = b^{\star -1}(p_0 - \Delta + c)$ and $b^{\star^{-1}}(v_i) = v_i$ (the latter follows from $b^\star(v_i) = v_i$) in eq. [5], one gets:

$$p_0^\star = \underset{p_0 \in [v_s + \Delta - c, \bar{v} + \Delta - c]}{\arg\max} (1 - [F(p_0 - \Delta + c)]^n)(p_0 - \Delta + c - v_s)$$

and the interval for p_0 always exists given that $\bar{v} \ge v_s$. ■

Proof of Proposition 3

The proof is similar to the proof of Proposition 1. The main differences concern buyers' initial beliefs about the hidden price and how these evolve over time. We thus focus on these aspects. Again, we divide the proof in a number of steps.

(1) Buyers' initial beliefs.
Buyers do not know n and thus cannot infer $p_0^\star$. However, by knowing that the random variable $\tilde{N}$ is distributed over the support $S_{\tilde{N}} = \{\tilde{n}_{\min}, \ldots, \tilde{n}_{\max}\}$ with probability mass function $g(\cdot)$, they can compute the distribution of p_0. In particular, players compute $p_0^\star(\tilde{n})$ for every $\tilde{n} \in S_{\tilde{N}}$ where each $p_0^\star(\tilde{n})$ (i. e., the optimal initial price that the seller would set if the actual number of participants was $\tilde{n}$) is given by:

$$p_0^\star(\tilde{n}) = \underset{p_0 \in [v_s + \Delta - c, \bar{v} + \Delta - c]}{\arg\max} \left(1 - [F(p_0 - \Delta + c)]^{\tilde{n}}\right)(p_0 - \Delta + c - v_s)$$

Buyers' initial beliefs are then captured by the distribution $H^{\star}_{i,1}(p_0)$ that assigns probability mass function $h^{\star}_{i,1}\left(p^{\star}_0(\tilde{n})\right) = g(\tilde{n})$ for every $\tilde{n} \in S_{\tilde{N}}$.

(2) Optimality of $b^{\star}(v_i) = v_i$.

We show that any $b^{'}(v_i) \neq v_i$ is weakly dominated by $b^{\star}(v_i) = v_i$. Notice that the choice of one's own gross willingness to pay $b(v_i)$ influences two aspects of agent's behavior. First, it affects the decision to observe the hidden price (see expressions 1 and 2 in the main text). Second, it determines the purchasing decision (see expression 3).

(2a) Consider first any $b^{'}(v_i) > b^{\star}(v_i) = v_i$. Strategies $b^{'}(v_i)$ and $b^{\star}(v_i)$ lead to the same payoff unless there exists a price $p^{\star\star}_0(\tilde{n}) \in \{p^{\star}_0(\tilde{n}_{\min}), ..., p^{\star}_0(\tilde{n}_{\max})\}$ such that $b^{'}(v_i) - c > p^{\star\star}_0(\tilde{n}) - \Delta > b^{\star}(v_i) - c$. If this is the case, $q_{i,1}\left(p^{\star\star}_1(\tilde{n})|b^{'}(v_i)\right) = 1$ while $q_{i,1}\left(p^{\star\star}_1(\tilde{n})|b^{\star}(v_i)\right) = 0$. Given that by assumption $v_i - p^{\star\star}_0(\tilde{n}) + \Delta - c < 0$, $b^{'}(v_i)$ lowers $(\pi_i|H_{i,t}(p_{t-1}))$ such that a player may decide not to observe the price even if such a choice is worthwhile in expected terms. Moreover, say that $b^{'}(v_i)$ and $b^{\star}(v_i)$ are such that the agent decides to observe the price in both cases. Then, with probability $g(\tilde{n}) > 0$, the price that the agent discovers is $p^{\star\star}_1(\tilde{n}) = p^{\star\star}_0(\tilde{n}) - \Delta$. An agent that sets $b^{'}(v_i)$ buys the item even if this leads to $u^{'}_i < -c$. On the contrary, an agent that sets $b^{\star}(v_i)$ does not buy the item and thus gets $u^{\star}_i = -c$. Therefore, $u^{\star}_i > u^{'}_i$.

(2b) Consider now any $b^{'}(v_i) < b^{\star}(v_i) = v_i$ and assume by now that in equilibrium the price cannot fall. Then, a similar argument as the one discussed in (2a) applies. Strategies $b^{'}(v_i)$ and $b^{\star}(v_i)$ lead to the same payoff unless there exists a price $p^{\star\star}_0(\tilde{n})$ such that $b^{'}(v_i) - c < p^{\star\star}_0(\tilde{n}) - \Delta < b^{\star}(v_i) - c$. If this is the case then $b^{'}(v_i)$ may lead an agent to decide to observe the price even if the expected payoff associated with such a choice is negative. Moreover, in case both $b^{\star}(v_i)$ and $b^{'}(v_i)$ lead the agent to observe the price, with probability $g(\tilde{n}) > 0$ the latter happens to be $p^{\star\star}_1(\tilde{n}) = p^{\star\star}_0(\tilde{n}) - \Delta$. Then, an agent that sets $b^{'}(v_i)$ does not buy the item and gets $u^{'}_i = -c$ whereas an agent that sets $b^{\star}(v_i)$ buys the item and gets $u^{\star}_i = v_i - p^{\star\star}_1(\tilde{n}) - c$. Therefore, $u^{\star}_i > u^{'}_i$.

However, the peculiarity of a PRA in which buyers are uncertain about the number of participants is that in equilibrium the price may fall. Therefore, even if $p^{\star\star}_0(\tilde{n})$ is such that the condition $b^{'}(v_i) - c < p^{\star\star}_0(\tilde{n}) - \Delta < b^{\star}(v_i) - c$ initially holds, the current price $p^{\star\star}_t(\tilde{n})$ may eventually reach a level $p^{\star\star}_t(\tilde{n}) < b^{'}(v_i) - c$. If such an event occurs, an agent who plays $b^{'}(v_i)$ could in principle buy the item in period t and thus get $u^{'}_i = v_i - p^{\star\star}_t(\tilde{n}) - c$.[23] An agent who instead plays $b^{\star}(v_i)$ buys the item in $t = 1$ with $u^{\star}_i = v_i - p^{\star\star}_1(\tilde{n}) - c$.

23 Notice that we are still assuming that agent observes the price, and thus pays the fee, only once. The proof that follows holds a fortiori in case the agent observes the price and pays the fee multiple times.

Given that $p_t^{**}(\tilde{n}) \le p_0^{**}(\tilde{n})$, condition $u_i' > u_i^*$ may occur. In what follows, we evaluate the marginal benefits and marginal costs associated with agent i's decision of playing $b'(v_i)$ rather than $b^*(v_i)$. We show that the probability that the hidden price reaches a level $p_t^{**}(\tilde{n}) < b'(v_i) - c$ is too low such that in expectations $b^*(v_i)$ dominates $b'(v_i)$. The rules of the PRA mechanism imply that a necessary condition for the price to fall and eventually reach the level $p_t^{**}(\tilde{n})$ is that some players observe the price and then refuse to buy the item. Consider thus the situation of generic buyer i who evaluates the pros and cons of postponing his decision to observe the price in the hope that this will fall. If agent i remains inactive (i. e., he plays $a_{i,1} = (\emptyset, \emptyset)$), then, from his point of view, the following events may occur:

(a) No agent $j \neq i$ observes the price. The event has probability

$$\sum_{\tilde{n}_{\min}}^{\tilde{n}_{\max}} g(\tilde{n}) F(\underline{v}_{t=1})^{\tilde{n}-1}$$

(b) An agent $j \neq i$ observes the price. The event has probability

$$1 - \sum_{\tilde{n}_{\min}}^{\tilde{n}_{\max}} g(\tilde{n}) F(\underline{v}_{t=1})^{\tilde{n}-1}$$

In case event (b) occurs then the following may happen:

(b1) Agent j buys the item. The event has probability

$$\sum_{\tilde{n}_{\min}}^{\tilde{n}_{\max}} g(\tilde{n}) \min\left\{\frac{F(\bar{v}) - F\left(p_0^*(\tilde{n})\right)}{F(\bar{v}) - F(\underline{v}_{t=1})}, 1\right\}$$

(b2) Agent j does not buy the item. The event has probability

$$1 - \sum_{\tilde{n}_{\min}}^{\tilde{n}_{\max}} g(\tilde{n}) \min\left\{\frac{F(\bar{v}) - F\left(p_0^*(\tilde{n})\right)}{F(\bar{v}) - F(\underline{v}_{t=1})}, 1\right\}$$

The only favorable event for agent i is (b2) where indeed the price falls by Δ and the auction remains open. The expected marginal benefit from not observing the price is thus given by:

$$\left(1 - \sum_{\tilde{n}_{\min}}^{\tilde{n}_{\max}} g(\tilde{n}) F(\underline{v}_{t=1})^{\tilde{n}-1}\right)\left(1 - \sum_{\tilde{n}_{\min}}^{\tilde{n}_{\max}} g(\tilde{n}) \min\left\{\frac{F(\bar{v}) - F\left(p_0^*(\tilde{n})\right)}{F(\bar{v}) - F(\underline{v}_{t=1})}, 1\right\}\right) \cdot \Delta \qquad [7]$$

The expected marginal cost (foregone positive payoff) is instead given by:

$$\left(\sum_{\tilde{n}_{\min}}^{\tilde{n}_{\max}} g(\tilde{n})F(\underline{v}_{t=1})^{\tilde{n}-1} + X \cdot Y\right) \cdot \left(v_i - p_0^{\star}(\tilde{n}) + \Delta - c\right) \qquad [8]$$

where

$$X = \left(1 - \sum_{\tilde{n}_{\min}}^{\tilde{n}_{\max}} g(\tilde{n})F(\underline{v}_{t=1})^{\tilde{n}-1}\right)$$

and

$$Y = \left(\sum_{\tilde{n}_{\min}}^{\tilde{n}_{\max}} g(\tilde{n}) \min\left\{\frac{F(\bar{v}) - F\left(p_0^{\star}(\tilde{n})\right)}{F(\bar{v}) - F(\underline{v}_{t=1})}, 1\right\}\right).$$

Given that:

(a) $\sum_{\tilde{n}_{\min}}^{\tilde{n}_{\max}} g(\tilde{n})F(\underline{v}_{t=1})^{\tilde{n}-1} > 0,$

(b) $\sum_{\tilde{n}_{\min}}^{\tilde{n}_{\max}} g(\tilde{n}) \min\left\{\frac{F(\bar{v}) - F\left(p_0^{\star}(\tilde{n})\right)}{F(\bar{v}) - F\left(\underline{v}_{t=1}\right)}, 1\right\} > 1 - \sum_{\tilde{n}_{\min}}^{\tilde{n}_{\max}} g(\tilde{n}) \min\left\{\frac{F(\bar{v}) - F\left(p_0^{\star}(\tilde{n})\right)}{F(\bar{v}) - F\left(\underline{v}_{t=1}\right)}, 1\right\}$
(in expectations an agent that observes the price is more likely to buy the item rather than not), and

(c) $\left(v_i - p_0^{\star}(\tilde{n}) + \Delta - c\right) > \Delta$ (by assumption of case 3b), it follows that $[8] > [7]$. Therefore, the deviation to $b'(v_i) < b^{\star}(v_i)$ is not profitable and in equilibrium $b^{\star}(v_i) = v_i$.

(3) The choice of the optimal initial price $p_0^{\star}$.

The seller knows n with certainty. Moreover, in equilibrium agents still play $b^{\star}(v_i) = v_i$ (see point 2 above). The seller's problem is thus analogous to the one described in Proposition 1: to choose the price that maximizes expected profits given that there are n participants who play $b^{\star}(v_i) = v_i$ and whose private valuations are independently drawn from F. The optimal initial price is thus given by:

$$p_0^{\star} = \underset{p_0 \in [v_s + \Delta - c, \bar{v} + \Delta - c]}{\arg\max} \left(1 - [F(p_0 - \Delta + c)]^n\right)(p_0 - \Delta + c - v_s)$$

The price $p_0^{\star}$ exists and is unique (see the proof of Proposition 1). ■

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