Measuring Representativeness in Different Electoral Systems, Using Italian and Dutch Data

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Abstract

I use power indices to assess the level of representative efficiency of different electoral systems. A representative democracy should "give voice" to as many people's preferences as possible. This paper evaluates how close a given electoral system is to mirroring the distribution of votes cast in an election. To this goal both power indices and differences between shares of votes received and seats gained are used. The distribution of power is also compared with both the share of seats of parties in the assembly and the share of votes obtained in the election. The results show that proportional systems and runoff majority are more efficient in terms of representativeness than first-past-the-post methods. Moreover, as the total number of seats in a parliament decreases, representative efficiency tends to increase. The analysis is conducted through simulations using conditional and real (Dutch and Italian) data.

Keywords: simulations; representative efficiency; electoral systems; power indices

1. Introduction

Representativeness is perhaps the most evident characteristic of a democratic system. But actually how representative are democracies? Do the current electoral rules ensure that the largest possible share of the population is represented in parliament, or do these rules exclude some from being represented? And if yes, how many? In this study I will provide an initial answer to these questions. By means of simple indices of representative efficiency, I will show that some currently used electoral systems (such as first-past-the-post and runoff majority) leave large shares of votes unrepresented (or underrepresented) in parliament. While some of these systems may be adopted to reduce the number of parties sitting in the legislative assembly, and to ensure higher levels of governability, the costs in terms of representativeness are often not considered. However, any electoral rule and any reform in this field should account for the effects in terms of representativeness.

The issue of how power is distributed between parties in a parliament has been widely studied and has produced several power indices. These measure "each [party's] relative influence over decision making in the sense of its ability to use its vote to change a coalition of others from one which is losing to one which satisfies the majority requirement and wins".[1] Several of these indices quantify power *a priori*, as "they calculate voting power under a given electoral system without reference to past or anticipated voting patterns in any particular election".[2] Others allow for assessing power after establishing government coalitions: these are more realistic than the former indices, but

[1] Leech (2002), p. 2.
[2] Gelman et al. (2004), p. 658.

they rely on assumptions that can be violated in reality (e.g. minimal winning coalitions, changes in political alliances, defections of party members, etc.).

Power indices have so far been used for two main purposes: to assess the power of voters, either electors, parties, or country members of international organisations (Tsebelis and Garrett, 1996; Bilbao et al., 2002; Napel and Widgrén, 2006 and Le Breton et al., 2012), or to evaluate the difference between the power of a party or of a country on one side, and its share of votes and/or of seats under different electoral systems on the other (Algaba et al., 2003; Ansolabehere et al., 2005; Snyder et al., 2005 and Mc Gann et al., 2009). The majority of these studies show that (depending on the type of assembly considered, the electoral system and the voting mechanism), parties (or, more generally, agents) enjoy more or less power than that represented by their share of votes or their weighting in the assembly. However, while some agents gain power with respect to their shares of votes and/or seats, others lose: as all the power must be allocated to the parties sitting in the assembly, the sum of the gains has to balance the sum of the losses.

The basis of representative democracy is to give voice to the most people's preferences as possible, through representation in a parliament. Ideally, then, a representative democracy should be characterised by a parliament that mirrors the preferences of the electorate on the possible policies. In other words: if 5% of the electors prefer proposal A to proposal B while 95% prefer B to A, then the former should obtain 5% and the latter 95% of the votes in the parliament. Alternatively the party which represents that 5% of the electors should have 5% of the power in the parliament. Of course the two options are not equivalent. The power indices show that the share of power and that of the votes almost never coincide, nor do power and share of seats of a given party. However, the formal number of seats could be deemed less relevant than the actual power enjoyed by a party, as what matters is the real power rather than the number

of seats. In other words, one may claim that a preference should be represented by a share of power equal to the share of the population supporting it, under the assumption that all individuals are equal. In this paper I define strongly efficient in terms of representativeness as a situation in which the power of a party is equal to the share of votes (i.e. the share of electors who voted for it) obtained in an election. I define weakly efficient in terms of representativeness as the case in which the power of each party is equal to the respective share of seats in the parliament. Under the assumption that all electors are equal, 5% of the electorate holds 5% of power within the population. Hence strong efficiency requires the power of the representatives to mirror the power of their electors. However weak efficiency is also important, as several electoral systems are designed to avoid perfect proportionality (which is almost indispensable to meet strong efficiency). Consequently under these systems strong efficiency of representativeness is rather a chimera than a possible outcome. In such situations, the power of a party is more meaningfully compared to the share of seats it gains through the electoral mechanism. While the correspondence between the share of votes received and that of seats gained is not perfect, it is preserved to some extent in all the plurality and majority systems[3].

The extent to which a parliament is efficient in the sense discussed above depends on how much the real power of each party differs from its share of votes or of seats. The goal of this paper is to apply some of the most used power indices to both conditional and real electoral data to assess: 1) which electoral system (if any) is more efficient and 2) the sensitivity of efficiency to the number of components of a parliament. Conditional and real data are used to simulate[4] parliaments elected under different systems, keeping the

[3] They are all designed so that if party A gets more votes than party B, then the former is assigned more seats than the latter.

[4] Using ALEX4.2, a program for the simulation of electoral systems, developed at the Università del Piemonte Orientale and freely downloadable at http://alex.unipmn.it/Ita/Alex_Software.php. This program has already been

preferences of the electors constant. The results of the paper lead to mixed conclusions: on the one hand majority systems are in general less efficient in terms of representativeness than proportional systems; however the introduction of a threshold in the latter has no unidirectional effects, in the sense that efficiency with respect to pure proportionality may either increase or decrease, depending on the number of parties and the total available seats.

I would like to stress that the indices used, especially *a priori* indices, have some limitations, since the parties may enjoy a real power different from that indicated by an *a priori* index. While some scholars believe that this is a major shortcoming (see for instance Garrett and Tsebelis, 1999; Muto, 1999 and Gelman et al., 2004), others (Holler, Widgrén and Turnovec, 2008) claim that the indices are actually "based on the fundamental idea that voting power is the ability to have influence in voting".[5] Besides their limits, therefore, they are of interest for political science because they can be considered as a benchmark for the actual power enjoyed by parties.[6] In the words of Felsenthal and Machover (2004): "The main purpose of measuring a priori voting power is not descriptive but prescriptive; not empirical but normative. It is indispensable in the proper constitutional design and assessment of decision rules."[7]

2. Data and Methodology

used to generate data supporting other papers (see for example Migheli and Ortona, 2011) and (an older version of) it is described in Bissey et al. (2004).

[5] Leech (forthcoming), p. (2).

[6] While the debate on the topic is wide and several scholars have written on it, the focus of the paper is far from this discussion, and therefore I would like to limit the space devoted to this problem in the paper. The reader interested in this topic will find interesting formal treatments in Aleskerov (2007) and Freixas et al. (2012) and empirical investigations in Felsenthal and Machover (2004).

[7] Felsenthal and Machover (2004), p. 15.

As already mentioned in the previous section, the data used for simulating the parliaments are either conditional or real. In particular, the conditional data come from a question contained in the *European Value Survey* about the political orientation of the interviewees, who are asked to indicate their political orientation on a ten-digit scale, from 1 (extreme left) to 10 (extreme right). For the purposes of this paper, each digit is initially assumed to be a different party (see Appendix B for the input data). The reason for including these artificial data is that it allows us to study the effect of a reduction in the number of seats, given the same distribution of preferences, without the possible effect that pre-electoral coalitions may have in reality[8]. In other words: when an elector casts the ballot in the box, s/he is influenced by the pre-electoral declarations of political alliance of the candidates; when a citizen is asked to state his/her political orientation in the context of a general survey, this effect is more remote. Therefore the conditional case serves as a sort of theoretical benchmark. There is at least one more reason to use conditional data. The design of an electoral system is not (or, at least, should not be) a contingent measure: it should respond to a long-term goal of (political) efficiency and efficacy. For this reason, studying the properties of an electoral system using only data from real elections is reductive and does not allow a full understanding of its implications. Moreover political parties merge, split, rise and die; people's political orientations are more "stable" in the sense that the concepts of left, centre and right do not depend on the names and on the number of parties on the electoral stage. Therefore the conditional data allow for abstracting the properties (in our case, representativeness) of an electoral system from the contingent political framework. However, since the electoral rules also work under the contingent situation, calculations with data from real elections are also useful to fully

[8] In reality, as I will explain later in the paper, ALEX 4.2 also allows for simulating majority systems, where the coalitions are announced *a priori*. In this case the electors vote strategically, in the sense that they will prefer the leading party of a coalition to another minor party, as the former has more chance than the latter to win seats in a majority system.

understand the different levels of representativeness entailed by different electoral systems.

In this article I use conditional and real electoral data to simulate the outcomes of different electoral systems. Where real data are used, they are either from the World Values Survey (WVS) or from real elections, in both Italy and the Netherlands only one electoral system is at work. Consequently, the outcomes of electoral rules other than those actually applied in these two countries are the result of what would happen if a different system were used. In this sense, in the paper I refer to these results as "simulations". For the simulations presented in this paper, I use data from Italy and the Netherlands. While a study based on a single country has the virtue of homogeneity, the results of a single-country based analysis may also mirror some specific and unobserved characteristics of that country. In this paper, for the cited sake of homogeneity I focus on Italy and Italian data, but I also present some simulations based on Dutch electoral data as a sort of robustness check and for the sake of drawing conclusions that are more general. The main reason to select these two countries is that until 2008 they shared almost the same (pure proportional) electoral system, and while Italy has relatively large chambers (630 and 315 seats), the Netherlands have a relatively small lower chamber (150 seats). This allows for comparing real situations with very different parliament sizes and allows for deepening the analysis of the effects of a (potential) reduction (or increase) in the number of seats in terms of representativeness (which is one of the aims of this paper).

There are also other reasons to use data from Italy and the Netherlands. As Mair (2008) points out, Italy and the Netherlands exhibit very close levels of political instability (i.e. alternation of parties in power); this suggests that a reform of the electoral rules may be beneficial to increase political stability in these two countries. Therefore, a study of the costs of such a reform in terms of representativeness may be useful. Moreover, the

number of major parties running for election is also relatively large (and similar) in both countries so that the electors' behaviour in the two countries is similar as Elff (2007) highlights. The same study reveals that the behaviour of French electors is also similar to the Italians', but France has a different electoral system and a large lower chamber and therefore is less suitable than the Netherlands for the purpose of this paper.

In addition, the political scenario of these two countries features a number of parties sitting in the parliament close to 10 (those present in the artificial benchmark). All these characteristics render Italy and the Netherlands suitable for comparison in studies on political systems and related issues, as confirmed by the number of articles that compare electoral outcomes and issues in these two countries (Calenda and Meijer, 2009; Migheli and Ortona, 2011, de Lange, 2012; Anderson and Kaeding, 2014 and Migheli et al., 2014). The real electoral data are from the Italian elections held in 2006 and 2008 and for the Dutch elections of 2010. The reason for considering two consecutive elections in Italy is that from 2006 to 2008 the number of parties significantly reduced within only two years (during which no radical political change happened); therefore the two close elections in Italy represent an interesting natural experiment to study the sensitivity of the efficiency in terms of representation to the number[9] of parties sitting in the parliament. For studying the sensitivity of the representative efficiency, three different sizes of parliament are assumed (for the simulations with conditional data): 630 seats (corresponding to the Italian *Camera dei Deputati*[10]), 150 (corresponding to the Dutch *Tweede Kamer der Staten-*

[9] These figures refer to the number of parties that got at least one seat in the parliament. This means that all the simulations are not based on all the parties that actually stood candidates for election, but only on those, which had some plausible chance to get seats. In the case of conditional data, the parties considered are only those that would enter the simulated parliament under a one-district pure proportional electoral system

[10] Both corresponding to the lower chambers in the respective constitutional systems.

Generaal[8]) and the intermediate size of 315 (corresponding to the elected members[11] of the Italian *Senato della Repubblica*[12]). Although any size chosen would have been arbitrary, the use of these real examples reduces the arbitrariness of such a choice.

The Italian election of 2006 ended with 14 parties sitting in the lower chamber; in 2008 this figure reduced to 9; while in the Dutch lower chamber 10 parties are currently sitting. While these numbers are large if compared to those of several countries (such as the USA and the UK), they are common for continental Europe and for other countries in the world (such as India and Japan). As mentioned, the number of artificial parties in the *European Value Survey* is originally 10. However, to study the effect of a variation in the number of parties, the simulations are run considering all the 10 parties first, and then reducing (arbitrarily) their number to 7. In this last case the left and the right extreme parties are merged as well as two small left-centrist parties. I would like to highlight here that the distribution of power within a parliament in pure proportional systems tends to mirror the size of each party in 1:1 proportion, as the number of parties increases (Jelnov and Tauman, 2014).

With real data the real number of seats for the Dutch and the Italian lower chambers are used, in order to prevent some possible bias that may derive from the electors' consciousness of how many seats compose the assembly for which they are voting. In other words, knowing that 150 seats are available rather than 630 may push some electors to vote for larger parties (that have more chance to get at least 0.67% of the votes, the minimum needed to enter a 150-seat parliament under one-district pure

[11] The Italian constitution assigns the faculty of nominating up to 5 senators for life to each President of the Republic. In addition all the past Presidents of the Republic become life members of the Senate. At the time of writing this paper, there are six senators for life in charge.
[12] Corresponding to the higher chamber.

proportionality, while a 630-seat chamber would require only 0.16% under the same rule).

As for the electoral systems considered in the paper, they are: one-district pure proportionality with the D'Hondt method (PP), one-district threshold proportionality with thresholds at 3, 5 and 7 percent[13] (TP), runoff majority (RM), first-past-the-post (FPTP) and FPTP with strategic vote (FPSV)[14]. For the sake of clarity, I would like to give some operational details about how these rules are implemented in the simulations. The proportional systems assume that the country coincides with the (only) electoral district. Then each party receives a share of seats equal to its share of votes (PP). When a threshold applies, the votes (and therefore the seats) of the parties excluded are re-assigned to the parties included in the parliament proportionally to their shares of votes. The residuals (since a seat is not divisible) are distributed using the D'Hondt method. FPTP and FPSV entail a division into several districts, each corresponding to one seat, so that the candidate of the party that gets the majority of votes in a district wins that seat. Therefore, in all the simulations, the number of districts is equal to the number of seats to be assigned. Moreover, the size of all the districts (i.e. the number of the electors) is set equal for all of them, so that, as happens in the real world, each deputy represents the same number of electors. Strategic voting implies that before placing their ballot paper at the poll, voters consider the probability that their preferred party has to win. When this is low (with a probability of 0.6[15]), the elector chooses not to vote for the preferred party. In the RM system, in each district, all the parties minus the two which have the most votes are excluded. A second round is performed with these two only, and the one that has the largest number of votes wins.

[13] This last applied only for the conditional cases with seven parties, as in the other cases no party had less than 5% of the votes.
[14] See footnote 8.
[15] This is a parameter of the simulation, chosen by the author. 0.6 looks in line with the estimations of Cain (1978).

The efficiency of a system is assessed through the computation of some power indices, the differences between their values, and the shares of votes and seats obtained by each party. Finally, the standard deviation and the absolute mean of the distribution of these differences for each electoral system and for each size of parliament are calculated (see Appendix A for more details).

The existing power indices are numerous, and therefore it would have been impossible to use all of them. I have thus chosen some of those most used in the literature, which are also the oldest. On the one hand, being the first to be conceived, they are not particularly sophisticated; on the other hand, the large literature on power indices has already pointed out their merits and their limits, which are relevant for the discussion of the results. Here I use the following indices: Shapley-Shubik (SS), Banzhaf (BA), Holler (HO), Deegan-Packel (DP), Meyerson (ME) and Owen (OW). Each of them features different characteristics: SS is based on all possible voter permutations, from which all the decisive positions for a given party *i* are analysed. The sum of all the decisive positions is divided by all possible orderings (voter permutations) giving party *i*'s share on all pivots (Shapley and Shubik, 1954). BA calculates party *i*'s swings like SS, but while the latter one analyses all possible voter permutations the former index considers each distinct coalition only once thus concentrating on voter combinations (Banzhaf, 1965). DP derives from other indices (among which SS, BA) on the minimal winning coalitions; furthermore it weights each swing for a quantity that is equal to the reciprocal of the number of parties forming the minimal winning coalition (Deegan and Packel, 1979, 1980). HO is based on BA, since the individual voter's swings are counted and then divided by the number of all voters' swings. However, there is a crucial difference with respect to the notion of an acceptable coalition. Where the standardised BA considers all the minimal winning coalitions as acceptable, HO considers only those where each voter must have swing. This

is because free-riders are allowed, and thus no surplus members are allowed in an acceptable coalition (Holler, 1982). Lastly ME and OW allow for establishing majority coalitions; this is a relevant difference with respect to the other indices, as also non-minimal winning coalitions are allowed (Owen, 1981).

More recently, other authors have developed power indices, designed to address several issues that the indices cited above did not assess. I will briefly comment on the most relevant. Aleskerov (2007) proposes and applies (Aleskerov et al., 2014) an index which takes into account the intensity of agents' preferences to coalesce. This is a relevant issue, as the formation and the stability of a coalition depend also on the strength of the members' preferences to coalesce. However, this issue is more relevant after than before elections. On the one hand, these preferences are the private information of the parties, so that the electors vote behind a veil of ignorance; on the other hand, when the electoral system prizes coalitions, their members may convey distorted information about the strength of their preferences for coalescing to maximise the number of votes. To the extent that this behaviour influences the electors' choice, the degree of distortion of the information conveyed has an impact both on the results of the elections and on the governability. There may also be effects on the representativeness, if after the elections the coalition breaks. For this reason, as assessed at the beginning of this section, in this paper I also simulate parliaments with conditional data (taken from the electors' declared political orientation) without pre-electoral coalitions. These electors' preferences should be much less biased than electoral choices in the presence of strategic information conveyed by the parties. Therefore these simulations may serve as a sort of robustness check for the indices calculated using real electoral data.

Tangian (2013a) uses an index based on the comparison between public opinion polls and the position of German parties on 30 social issues. This approach has the merit of

using the real position of parties, instead of using a simple left-right position that is generally assumed to represent the average position of a party about the generality of social issues. This is a very interesting and powerful approach[16] to study the representativeness within one country, but becomes problematic when, as in the case of this paper, more than one country is considered, as electors may formulate priorities in different ways and these may differ between countries, rendering international comparisons weak. Moreover, as Tangian (2015) shows, the traditional approach of using the left-right location of parties represents well their positions about different social issues (in this case the author covers 38 of these and compares the resulting relative positions of parties with the traditional positioning on a left-right axis).

Migheli et al. (2014) propose an index which explicitly excludes the non-contiguous post-electoral coalitions (i.e. those formed by parties that are not seated contiguously in the parliament). This constraint to the formation of coalitions is very useful for mirroring the real world, where non-contiguous coalitions are very rare and allowing for them increases the power of parties artificially. However, this index is still not well-established in the literature and therefore, for the sake of caution it is not employed in the present study.

All the indices listed in the first part of this section are applied; in this way I consider both the *a priori* measures of power not based on coalitions, and also those which are based on them. Then, I compute the differences between each of them and the shares of votes and seats of each party under the different electoral systems. In this way for each index (and under each system) I obtain a distribution of differences of power. For each distribution I then compute the standard deviation and the mean absolute deviation. Then, for each system I take the mean values of these two indicators. These figures are a

[16] See also Tangian (2013b)

measure of the inefficiency of a system in being perfectly representative. In particular, the larger the system, the more a system is distant from perfect representativeness. Since all the seats and the power are distributed between parties, and since the values of the indices are normalised to 1, then the mean of each distribution of differences between power and shares of votes or of seats is equal to 0. This reduces the formula for the computation of the standard deviation from $\sigma_{j,\lambda} = \sqrt{\frac{1}{n}\sum_{i=1}^{n}\left(\Delta_{i,j,\lambda} - \mu_{j,\lambda}\right)^2}$ to $\sigma_{j,\lambda} = \sqrt{\frac{1}{n}\sum_{i=1}^{n}\Delta_{i,j,\lambda}{}^2}$, where $\Delta_{i,j,\lambda} = p_{i,j,\lambda} - s_{i,j,\lambda}$ and $\mu_{j,\lambda} = \frac{1}{n}\sum_{i=1}^{n}\Delta_{i,j,\lambda}$, $p_{i,j,\lambda}$ is the power of party *i* under system *j*, according to power index λ and $s_{i,j,\lambda}$ is the share of votes (or of seats: it depends on whether we are concerned with weak or strong representative efficiency) of party *i* under system *j*, according to power index λ. As it can be noticed, $\sigma_{i,j,\lambda}$ depends on $\Delta_{i,j,\lambda}$ positively, meaning that $\sigma_{i,j,\lambda}$ increases as the distance between the actual power of a party and its share of seats/votes increases in absolute terms.

We may notice that if, in a given parliament, Δ_i increases by 1 percentage point, while Δ_j decreases by the same amount (as it has to[17]), then the standard deviation increases or decreases, depending on whether $|\Delta_i|$ is larger or smaller than $\frac{1}{n}\sum_{i=1}^{n}|\Delta_i|$. This is not surprising, as the standard error is a non linear statistic. For this reason, I also compute and present the mean absolute deviation of the distribution, namely $\mathrm{M}_{j,\lambda} = \frac{1}{n}\sum_{i=1}^{n}\left|\Delta_{i,j,\lambda} - \mu_{j,\lambda}\right|$, which simplifies to $\mathrm{M}_{j,\lambda} = \frac{1}{n}\sum_{i=1}^{n}\left|\Delta_{i,j,\lambda}\right|$, since, as before, $\mu_{j,\lambda} = 0$. The

[17] Actually if Δ_i varies by 1 percentage point, then $\sum_{k=1;\, j\neq i}^{n-1}\Delta_k$ must vary by -1 percentage point. What is written in the text serves just as exemplification.

use of $M_{j,\lambda}$ allows for understanding whether changes in the value of $\sigma_{j,\lambda}$ are simply due to intra-distribution adjustments.

The indices of dispersion summarised above generate two figures for each power index and for each electoral system. I then compute the average of the values over the power indices, obtaining a single value for each indicator and for each electoral system. Hence, the figures contained in the tables of the paper are the following: $\sigma_j = \frac{1}{6}\sum_{\lambda=1}^{6}\sigma_{j,\lambda}$ and $M_j = \frac{1}{6}\sum_{\lambda=1}^{6}M_{j,\lambda}$. To clarify how these indices work, Appendix A proposes a numerical example.

Another way to measure to what extent the distribution of seats between the parties mirrors the share of votes received is to calculate how many electors are not represented in terms of seats[18]. To this goal, I calculate two measures: the first is obtained as follows: $R = \sum_{i=1}^{n} v_{i,j} - s_{i,j}\ if\ v_{i,j} - s_{i,j} > 0$, where $v_{i,j}$ is the share of votes obtained by party i under the electoral system j; these are summed only if the share of seats of party i is smaller than its share of votes (i.e. if the share of seats underrepresents the voters). When the share of seats attributed to party i overrepresents the voters, nothing is added to the index. This is a "weak" measure of the degree of non-representativeness, as the index captures only the cases in which voters are underrepresented. I also propose a "strong" version of this index: $\Psi = \sum_{i=1}^{n}\left(v_{i,j} - s_{i,j}\right)^2$, which measures the squared share of "misallocated" seats; here "misallocation" means any non-zero difference between the share of seats of party i and its share of votes. Thus, this second index captures both

[18] I would like to thank an anonymous referee for suggesting this extension.

under- and overrepresentation. Differences are squared for two reasons: first, to render all of them positive, and second, for their weighting to increase with their magnitude (i.e. the more the share of seats is distant from the share of votes, the higher this difference is weighted).

3. Results

The results are presented first for the conditional cases (since they serve as a theoretical benchmark), then for Italy and finally – for the reasons explained above – for the Netherlands. Table 1 shows the results for weak representative efficiency (WRE) using the conditional Italian data.

[Table 1 about here]

WRE often decreases with the number of seats, while the system that maximises it depends on the size of the parliament: with 150 seats the FPTP system is the worst (in terms of representativeness), while RM is the best. This latter is actually always the best to minimise WRE (as the majority party wins a share of seats that is very close to its power). With 315 seats representativeness is maximised by PP and TP (3%), while with 630 seats the worst systems are FPTP and FPSV. It is noteworthy that the first five systems presented in the table do not generate results that are very different from each other. The very best system is the RM, while the others do not entail significant differences, especially in terms of standard deviations. Indeed, when the mean absolute deviations are considered, TP and the majority systems appear to be more representative-efficient than PP. Both the index of misrepresentation and the share of non-represented voters suggest that representativeness decreases as the system deviates from pure proportionality. The

share of non-represented voters in FPTP, FPSV and RM is much larger than 50%. RM performs slightly better than FPTP and FPSV, and this last one performs better than FPTP, but this is due to the strategic behaviour of the voters. In fact, strategic voting moves some votes from the most desired party to the closest with the highest probability of winning seats. In such a case, then, the reduction in representativeness is induced rather than the consequence of a really better performance of FPSV with respect to FPTP. The value of the index of misrepresentation and that of the share of non-represented voters almost do not vary with the number of available seats. There is an apparent contradiction between the indices of representative efficiency and the measures of representativeness. However, the first evaluate representative efficiency in terms of power, while the others evaluate in terms of how the share of voters is not represented in the parliament. The different stories told by the two groups of measures suggest that a system such as RM can lead to large under-representation in terms of electors without a corresponding representative in the parliament. However, the same system is very efficient in representing the power of the electors. This apparent contradiction arises from the fact that the share of power that a party has in a chamber does not necessarily coincide with the share of votes obtained by that party at the elections. As Migheli et al. (2014) show, the power of a party in the parliament depends not only on its share of votes, but also on its position (on a left-right basis) in the chamber. This dichotomy between the share of power and that of votes produces the figures reported in Table 1 (and in other tables) and explains the apparent contradiction in the results of the paper.

Let us then consider strong representative efficiency (SRE, Table 2). We notice a clearly different situation: the most efficient systems in terms of representativeness are PP and TP, whilst the three majority systems are all equally inefficient. The explanation is that the latter allow only a few parties in the parliament, and so a large number of the voters

are not represented in the assembly. This fact increases the differences between the (share of) power that a party has in the chamber and the share of votes obtained in the election.

[Table 2 about here]

In the case of majority systems we should remark that the two types of representative efficiency appear to be substitutes rather than complements.

The situation is not very different when the number of parties sitting in the parliament under PP reduces by 30% (in our case from 10 to 7). Tables 3 and 4 are analogous to Tables 1 and 2 respectively. Notice that to render the application of a threshold meaningful in the TP system, this had to be fixed at 7%, as no party had a share of votes lower than that.

[Table 3 about here]

[Table 4 about here]

We can notice that representativeness in terms of power increases as the number of parties decreases. However, while RM remains the best system in terms of WRE, PP is (weakly[19]) better than FPTP in large parliaments, but this is not the case for a small 150-seat chamber. The situation with seven parties is also worse than that with ten in terms of SRE. Apparently, the fact that votes mass on a smaller number of subjects increases the representativeness of a system. Moreover this effect is very strong: considering weak representative inefficiency, the case with seven parties is absolutely worse than that with

[19] While in the 315 and 630 cases the standard deviation of PP is smaller than that of FPTP, the average absolute deviation is still smaller for FPTP than for PP in the medium size parliament (with 315 seats).

ten: the minimum of representativeness inefficiency in the former case (and in terms of s.d.) is 4.91 under the TP system (7%)[20], a figure higher than the worst in the ten-party simulation (4.18 under FPTP[21]). Something very similar holds when we compare strong efficiency for "homogeneous" blocks of electoral systems (either proportional or majority): the seven-parties case is absolutely worse than the ten-parties scenario.

The misallocation and the share of non-represented voters slightly reduce when the number of parties decreases from 10 to 7. The reason is that the number of parties non-represented in the parliament when a non-proportional system or a threshold is applied decreases with the number of parties that run for the election. This suggests that electoral systems that deviate from pure proportionality are more efficient in assuring representativeness in elections with few rather than with many parties.

So far the simulations run using conditional data suggest that: 1) the proportional systems display very similar levels of WRE and SRE, while proportional systems lead to much less SRE than majority systems. 2) An increase in the total number of seats generally worsens the representativeness of any system. 3) A reduction in the number of parties (that enter the parliament under one-district PP) decreases both WRE and SRE. 4) The magnitude of the representative bias of an electoral system varies with the measure adopted to assess it; in particular, RM is much more efficient in terms of representativeness when it is evaluated in terms of power rather than in terms of misallocation between votes and seats. We then expect to observe similar patterns in the analyses with real data.

Tables 5 and 6 present the results for real Italian data (WRE and SRE respectively).

[20] See Table 3.
[21] See Table 1.

[Table 5 about here]

[Table 6 about here]

Between the 2006 and 2008 elections the number of parties sitting in the Italian lower chamber decreased, and the representative inefficiency in terms of power increased (with a notable exception), as expected. In particular, the levels of WRE and SRE under PP and TP are very similar. Consider first Table 5: the most representative system using the 2006 data would be TP (5%), while it would be RM with the 2008 data. This result may depend on the number of parties considered for the simulation. In 2008, nine parties (a figure close to the ten used in the conditional case) entered the parliament. In this case, the most representative system is RM, as in the simulations with ten parties. In 2006 fourteen parties were sitting in the Italian lower chamber. It is plausible that with such a wide distribution of the votes (i.e. number of parties), the number of parties that do not enter the parliament under RM is large enough to generate the observed loss of representativeness. In terms of SRE, PP and TP are definitely better than FPTP, FPSV and RM. Again, this result was expected, as proportional rules entail a larger number of parties to win seats than majority systems.

What was observed in Tables 1 to 4 on the misallocation and the share of non-represented voters is confirmed by the results of the simulations with the real Italian data. The value of the indices is lower in 2006 than in 2008, mirroring the smaller number of parties that ran in the 2006 elections compared to 2008. We can also notice that, in the Italian case, the introduction of a threshold in the proportional systems generates levels of misallocation and shares of non-represented voters, which are rather high. The relatively large number of parties that got a share of votes between 2% and 5% explains this result.

In the simulations with conditional data, the number of parties falling into that interval was smaller.

Lastly Table 7 reports the results obtained using the Dutch data. As already highlighted, this further analysis is justified both in terms of a robustness check and by the aim of applying the proposed indices of representative inefficiency to a second case other than Italy to allow for conclusions that are more general.

[Table 7 about here]

The Dutch lower chamber shows lower levels of representative inefficiency than the Italian one. In the light of the results obtained with conditional data, a major reason seems to be the small size (only 150 seats) of this assembly. Moreover eleven parties sit in the *Tweede Kamer der Staten-Generaal*: this is a relatively large number, and it is likely to enhance representativeness. Among the majority systems, RM engenders the highest level of representativeness, although PP and TP work better. Comparing the Dutch and the Italian figures it also emerges that the introduction of a FPTP scheme in the Netherlands would lead to a loss of representativeness larger than that generated by the same reform in Italy.

The figures for the Netherlands confirm what has already been observed in the case of Italy and with the conditional data in terms of misallocation and shares of non-represented voters. However, I would like to underline that in the real cases these two measures of representative efficiency depict a somewhat better scenario than that obtained using conditional data. Indeed, in this last case the share of non-represented voters always exceeds 50%, while in the real cases this figure is hardly attained.

As already mentioned in the introduction, the power indices used in the paper are among the best known indices of this sort. However all of them evaluate the power of a party before elections (or immediately after, assuming that the coalitions announced during the electoral campaign are maintained and persist unaltered during the legislature). This entails that the real power of a party may differ from that calculated, as a consequence of defections, modified alliances, or other factors, which were unobservable *a priori*. Hence the simulations and the figures presented here have a qualitative rather than a precise quantitative meaning, and should serve as a starting point for conducting additional evaluations of the electoral rules.

4. Conclusions

The representation of people's needs and preferences in a parliament is one of the most important advances that democracy has brought to modern societies. However, parliaments are elected under different rules, which may result in more or less effective mirroring of the desires of the electors. Defining as perfectly efficient in terms of representativeness a system that assigns to each political party in a parliament the same power as its share of electoral ballots (SRE) or as its share of seats in the assembly (WRE), this paper evaluates different electoral rules under this aspect, as well as the sensitivity of efficiency in representativeness to the number of potential parties sitting in the assembly, and to the number of available seats.

The main conclusions of the paper rely on simulations based on both conditional and real electoral data. These show that: 1) large parliaments are generally less likely than small ones to be highly representative (contrary to what one might expect); 2) representative efficiency tends to be larger in proportional than in FPTP and FPSV

systems; however 3) RM works better than PP and TP (at least with conditional data); 4) as the number of parties that obtain at least one seat under one-district PP increases, so does representativeness. 5) Finally, the results vary considerably according to the measure chosen to assess representative inefficiency. In sum this evidence seems to suggest that electoral systems that seek representative efficiency (as it is defined in this paper) should allow several parties to compete for a restricted number of seats, and adopt either RM or a proportional method.

The distinction between WRE and SRE also deserves some more discussion. Although one may find the definition of representative efficiency somewhat arbitrary, it should be taken more as an ideal benchmark than as a normative dogma to pursue as a primary objective of an electoral system. Of course, perfect representativeness is not a must. To pursue it exclusively may hinder governability. Hence, the paper does not imply that perfect representativeness is an absolute good. Indeed, the aforementioned governability (i.e. its ability to sustain an effective and stable government – also over time) is one of the other fundamental aspects of a political system. As Benoit (2004) points out: "Governability implies a general rather than a partisan interest because governability is concerned with maximising the seat share of the largest party, rather than the seat share of any particular party."[22] And majority systems sacrifice representativeness to this goal.

I would like to add a further remark to these conclusions. I am writing this paper at the end of an international economic crisis, whose consequences are severely affecting the public finances of several European countries. The governments of some of these are committed to reduce public expenditure, also through a reform of the political system,

[22] Benoit (2004), p. 369.

which also entails a possible reduction in the number of members of the parliaments[23]. The results presented in this paper may be of some help to support these future reforms.

Although what is presented in the paper is a simulation and, as such, a theoretical exercise, some policies may be suggested. First of all, an electoral law that is intended to enhance representativeness should adopt a pure proportional rule, or, at most, it should fix low thresholds of votes for a party to benefit from the allocation of parliamentary seats. However, concerns of efficiency may play a role. Where several parties present candidates at elections, the Buchanan-Tullock theorem suggests that transaction costs inside the parliament are likely to be high (at least higher than when fewer parties enter the assembly). For this reason, legislators may want to limit the number of parties that obtain seats in the parliament, although this may entail large losses in representativeness. However, these losses in representativeness are likely to decrease as the number of parties involved in an electoral contest decreases.

The results of this paper suggest that the use of electoral systems that deviate from pure proportionality is less costly in terms of representativeness, where few parties run for elections.

Perhaps it is not the case that FPTP and RM characterise countries such as the UK and the USA, where the number of parties is small. The small number of parties may also be the consequence of the electoral rules applied: where contestants with small shares of voters cannot access any seat, there is an incentive to merge into large parties. Small parties merged within a larger body can persist in the form of factions, which still represent the different positions originally expressed in as many parties. In such a case, a FPTP or a RM system would also be efficient in terms of representativeness. The price to pay during the

[23] For example, in Italy such a proposal is currently being discussed in the Parliament.

transition depends on the speed of adjustment (i.e. on how fast the small parties merge into larger bodies). In the absence of such a process, countries with a large number of parties and interested in maximising the representative efficiency of their electoral systems, should hesitate to adopt rules that significantly deviate from pure proportionality.

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Appendix A. Numerical examples.

Let suppose that ten parties run for election under a pure proportional system and that the chamber to be elected has 150 seats. The distribution of the votes is as shown in Table A1:

Table A1. Distribution of votes and of seats

	Share of votes	Number of seats
Party 1	7.18%	11
Party 2	6.79%	10
Party 3	12.37%	19
Party 4	12.92%	19
Party 5	24.20%	37
Party 6	15.66%	24
Party 7	8.17%	12
Party 8	5.99%	9
Party 9	3.08%	4
Party 10	3.64%	5

Since seats are indivisible, the share of seats of each party is slightly different from the share of votes obtained in the election (Table A2):

Table A2. Share of votes and share of seats compared

	Share of votes	Share of seats
Party 1	7.18%	7.33%
Party 2	6.79%	6.67%
Party 3	12.37%	12.67%
Party 4	12.92%	12.67%
Party 5	24.20%	24.67%
Party 6	15.66%	16.00%
Party 7	8.17%	8.00%
Party 8	5.99%	6.00%
Party 9	3.08%	2.67%
Party 10	3.64%	3.33%

The share of votes is the basis for computing the index of weak representative inefficiency, while the share of seats is used to compute the index of strong representative inefficiency.

At this point, the power indices are computed, using ALEX 4.2 software (the manual of the software can be freely downloaded at https://alex.unipmn.it/software.php?L=IT). Table A3 reports the values of the indices used in the paper for the example used in this appendix

Table A3. Power indices under a pure proportional electoral system.

	Banzhaf	Shapley-Shubik	Holler	Deegan-Packel	Meyerson	Owen
Party 1	7.28%	7.10%	10.05%	9.94%	7.82%	12.50%
Party 2	6.34%	6.19%	9.78%	9.47%	7.30%	12.50%
Party 3	12.13%	12.30%	11.14%	11.26%	12.02%	16.67%
Party 4	12.13%	12.30%	11.14%	11.26%	12.02%	16.67%
Party 5	28.73%	28.77%	11.68%	13.44%	25.63%	41.67%
Party 6	15.30%	15.75%	11.14%	11.49%	14.44%	0.00%
Party 7	7.28%	7.10%	10.05%	9.94%	7.10%	0.00%
Party 8	5.41%	5.32%	10.05%	9.46%	6.11%	0.00%
Party 9	2.43%	2.30%	7.07%	6.46%	3.49%	0.00%
Party 10	2.99%	2.86%	7.88%	7.28%	4.05%	0.00%

At this point, we can compute the difference between the power of each party according to the indices reported in Table A3 and the share of seats that they have in the parliament. This is the first step for evaluating what the paper defines as "strong representative inefficiency". Table A4 reports these differences based on the figures reported in Tables A3 and A2:

Table A4. Differences between the power as computed by the power indices and the power as share of seats.

	Banzhaf	Shapley-Shubik	Holler	Deegan-Packel	Meyerson	Owen
Party 1	-0.05%	-0.23%	2.72%	2.61%	0.49%	5.17%
Party 2	-0.33%	-0.48%	3.11%	2.80%	0.63%	5.83%
Party 3	-0.54%	-0.37%	-1.53%	-1.41%	-0.65%	4.00%
Party 4	-0.54%	-0.37%	-1.53%	-1.41%	-0.65%	4.00%
Party 5	4.06%	4.10%	-12.99%	-11.23%	0.96%	17.00%
Party 6	-0.70%	-0.25%	-4.86%	-4.51%	-1.56%	-16.00%
Party 7	-0.72%	-0.90%	2.05%	1.94%	-0.90%	-8.00%
Party 8	-0.59%	-0.68%	4.05%	3.46%	0.11%	-6.00%
Party 9	-0.24%	-0.37%	4.40%	3.79%	0.82%	-2.67%
Party 10	-0.34%	-0.47%	4.55%	3.95%	0.72%	-3.33%

These figures represent the first part of the index of strong representative inefficiency used in the paper. For each of the indices reported in Table A4, the standard deviation and the mean absolute deviation are then computed over the group of all the parties that enter the parliament (ten in the case of the example proposed in this appendix). This leads to the figures reported in Table A5.

Table A5. Standard deviations and mean absolute deviations of power as computed by the indices proposed in the paper.

	Banzhaf	Shapley-Shubik	Holler	Deegan-Packel	Meyerson	Owen
Standard deviation	1.44%	1.46%	5.52%	4.84%	0.87%	9.17%
Mean absolute deviation	0.81%	0.82%	4.18%	3.71%	0.75%	7.20%

Now, the last step to get the result reported in the tables of the paper is to compute the arithmetic means of the figures reported in each row of Table A5. This leads to the following figures: mean standard deviation 3.88% and average of the mean absolute deviations 2.91%. These two last figures express the strong representative inefficiency of a pure proportional system where ten parties run for 150 seats and obtain the share of votes reported in Table A1.

Appendix B

The following Tables report the input data for the simulations of "conditional Italy" and real Italy and Netherlands.

Table B1. Input data for conditional Italy

Ten-parties case		Seven-parties case	
Party 1	7.18	Party 1	13.97
Party 2	6.79	Party 2	12.37
Party 3	12.37	Party 3	12.92
Party 4	12.92	Party 4	24.20
Party 5	24.20	Party 5	15.66
Party 6	15.66	Party 6	14.16
Party 7	8.17	Party 7	6.72
Party 8	5.99		
Party 9	3.08		
Party 10	3.64		

Figures represent the percentage of the interviewees who positioned themselves at a given point on a ten-digit scale. Party 1 is extreme left; party 10 is extreme right.

Table B2. Electoral data for Italy	
	2008
Popolo delle Libertà	37.38
Partito Democratico	33.18
Lega Nord	8.30
Unione di Centro	5.62
Italia dei Valori	4.37
Sinistra Arcobaleno	3.12
Destra-Fiamma Tricolore	2.43
Movimento per le Autonomie	1.31
Sudtiroler Volkspartei	0.41
	2006
L'Ulivo	31.27
Forza Italia	23.72
Alleanza Nazionale	12.34
Unione dei Democratici	6.76
Rifondazione Comunista	5.84
Lega Nord	4.58
Rosa nel Pugno	2.60
Comunisti Italiani	2.32
Italia dei Valori	2.30
Federazione dei Verdi	2.06
UDEUR	1.40
Nuovo Partito Socialista Italiano	0.75
Sudtiroler Volkspartei	0.48

Table B3. Electoral data for the Netherlands	
	2010
Christen-Democratisch Appel	26,50
Partij van de Arbeid	21,20
Socialistische Partij	16,60
Volkspartij voor Vrijheid en Democratie	14,70
Partij voor de Vrijheid	5,90
Groen Links	4,60
Christen Unie	4,00
Democraten 66	2,00
Partij voor de Dieren	1,80
Staatkundig Gereformeerde Partij	1,20

Table 1. Weak inefficiency of power allocation : conditional Italy with 10 parties (figures are in percentage points)

	150 seats				315 seats				630 seats			
	Standard deviation	Mean absolute deviation	Index of misrepresentation	Percentage of non-represented voters	Standard deviation	Mean absolute deviation	Index of misrepresentation	Percentage of non-represented voters	Standard deviation	Mean absolute deviation	Index of misrepresentation	Percentage of non-represented voters
Pure proportionality	3,88	2,91	2,53	1,27	3,92	2,94	1,38	0,69	3,91	2,96	0,88	0,44
Threshold proportionality (3%)	3,88	2,91	2,53	1,27	3,92	2,94	1,38	0,69	3,91	2,96	0,88	0,44
Threshold proportionality (5%)	3,83	2,65	13,44	6,72	3,71	2,53	13,44	6,72	3,84	2,6	13,44	6,72
First past the post	4,18	2.00	146.27	73.13	3,77	1,71	147,16	73,58	4,04	2,03	143,35	71,67
First past the post with strategic vote	3,78	1,87	132,93	66,47	3,89	1,84	134,46	67,23	4,11	2,06	131,28	65,64
Runoff majority	1,07	0,53	131.60	65.8	0,93	0,44	133,19	66.7	1,76	0,83	130.97	65.48

Table 2. Strong inefficiency of power allocation: conditional Italy with 10 parties (figures are in percentage points)

	150 seats				315 seats				630 seats			
	Standard deviation	Mean absolute deviation	Index of misrepresentation	Percentage of non-represented voters	Standard deviation	Mean absolute deviation	Index of misrepresentation	Percentage of non-represented voters	Standard deviation	Mean absolute deviation	Index of misrepresentation	Percentage of non-represented voters
Pure proportionality	3,88	2.90	2,53	1,27	3,88	2.90	1,38	0,69	3,92	2,97	0,88	0,44
Threshold proportionality (3%)	3,88	2.90	2,53	1,27	3,88	2.90	1,38	0,69	3,92	2,97	0,88	0,44
Threshold proportionality (5%)	4,33	3,28	13,44	6,72	4,26	3,25	13,44	6,72	4,34	3.30	13,44	6,72
First past the post	26,94	15,16	146.27	73.13	26,94	15,16	147,16	73,58	26,94	15,16	143,35	71,67
First past the post with strategic vote	26,94	15,16	132,93	66,47	26,94	15,16	134,46	67,23	26,94	15,16	131,28	65,64
Runoff majority	26,94	15,16	131.60	65.80	26,94	15,16	133,19	66.70	26,94	15,16	130.97	65.48

Table 3. Weak inefficiency of power allocation: conditional Italy with 7 parties (figures are in percentage points)

	150 seats				315 seats				630 seats			
	Standard deviation	Mean absolute deviation	Index of misrepresentation	Percentage of non-represented voters	Standard deviation	Mean absolute deviation	Index of misrepresentation	Percentage of non-represented voters	Standard deviation	Mean absolute deviation	Index of misrepresentation	Percentage of non-represented voters
Pure proportionality	5.30	4.00	1,67	0,84	5,29	4.00	0,93	0,47	5,59	4,18	0.60	0.30
Threshold proportionality (7%)	4,91	3,31	13,44	6,72	4,96	3,35	13,44	6,72	4,97	3,37	13,44	6,72
First past the post	7,13	4,38	193.60	69.80	5,63	3,36	140.81	70.40	6,63	4,17	135.41	67.70
First past the post with strategic vote	6,38	4.00	123.60	61.80	5,57	3,36	128.11	64.05	6,84	4,31	122.39	61.20
Runoff majority	2,78	1,71	120,93	60,47	2,66	1,54	128,11	64,05	3,95	2,31	121,44	60,72

Table 4. Strong inefficiency of power allocation: conditional Italy with 7 parties (figures are in percentage points)

	150 seats				315 seats				630 seats			
	Standard deviation	Mean absolute deviation	Index of misrepresentation	Percentage of non-represented voters	Standard deviation	Mean absolute deviation	Index of misrepresentation	Percentage of non-represented voters	Standard deviation	Mean absolute deviation	Index of misrepresentation	Percentage of non-represented voters
Pure proportionality	5,31	4,04	1,67	0,84	5,31	4,04	0,93	0,47	5,57	4,18	0.60	0.30
Threshold proportionality (7%)	5,64	4,28	13,44	6,72	5,64	4,28	13,44	6,72	5,64	4,28	13,44	6,72
First past the post	33,55	21,66	193.60	69.80	33,55	21,66	140.81	70.40	33,55	21,66	135.41	67.70
First past the post with strategic vote	33,55	21,66	123.60	61.80	33,55	21,66	128.11	64.05	33,55	21,66	122.39	61.20
Runoff majority	33,55	21,66	120,93	60,47	33,55	21,66	128,11	64,05	33,55	21,66	121,44	60,72

Table 5. Weak inefficiency of power allocation: real Italy (figures are in percentage points)

	2006 Elections				2008 Elections			
	Standard deviation	Mean absolute deviation	Index of misrepresentation	Percentage of non-represented voters	Standard deviation	Mean absolute deviation	Index of misrepresentation	Percentage of non-represented voters
Pure proportionality	6.04	3.89	4.01	0.22	10.30	7.73	8.73	0.42
Threshold proportionality (3%)	5.8	3.16	27.40	11.91	11.29	7.98	39.58	4.15
Threshold proportionality (5%)	5.62	2.71	36.56	16.49	11.46	6.56	63.72	11.64
First past the post	14.74	5.53	81.99	39.20	14.52	6.46	177.95	46.04
First past the post with strategic vote	15.08	6.06	84.53	40.47	15.61	6.49	147.79	29.69
Runoff majority	16.09	6.10	84.53	40.47	6.35	2.82	197.15	54.37

Table 6. Strong inefficiency of power allocation: real Italy (figures are in percentage points)

	2006 Elections				2008 Elections			
	Standard deviation	Mean absolute deviation	Index of misrepresentation	Percentage of non-represented voters	Standard deviation	Mean absolute deviation	Index of misrepresentation	Percentage of non-represented voters
Pure proportionality	5.97	3.87	4.01	0.22	10.19	7.63	8.73	0.42
Threshold proportionality (3%)	5.95	4.06	27.40	11.91	10.65	8.01	39.58	4.15
Threshold proportionality (5%)	5.95	3.9	36.56	16.49	11.24	7.87	63.72	11.64
First past the post	20.51	9.66	81.99	39.20	25.29	13.72	177.95	46.04
First past the post with strategic vote	20.51	10.75	84.53	40.47	25.29	13.72	147.79	29.69
Runoff majority	23.3	10.79	84.53	40.47	25.29	13.72	197.15	54.37

Table 7. Inefficiency of power allocation: real Netherlands (figures are in percentage points)

	Weak inefficiency				Strong inefficiency			
	Standard deviation	Mean absolute deviation	Index of misrepresentation	Percentage of non-represented voters	Standard deviation	Mean absolute deviation	Index of misrepresentation	Percentage of non-represented voters
Pure proportionality	4.47	3.47	3.63	1.26	4.32	3.34	3.63	1.26
Threshold proportionality (3%)	4.58	3.52	7.18	3.04	4.74	3.83	7.18	3.04
Threshold proportionality (5%)	4.72	3.4	13.66	6.28	5.02	3.97	13.66	6.28
First past the post	20.61	9.6	91.44	45.17	26.7	14.89	91.44	45.17
First past the post with strategic vote	21.1	9.73	102.66	50.78	28.55	15.87	102.66	50.78
Runoff majority	11.1	5.16	101.32	50.11	13.66	10.4	101.32	50.11