The Meta-didactical transposition: A model for analysing teacher education programs

THE META-DIDACTICAL TRANSPOSITION: A MODEL FOR ANALYSING TEACHERS EDUCATION PROGRAMS

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This RF intends to continue the reflection developed in the RF "Teacher knowledge and teaching: viewing a complex relationship from three perspectives" by D. Ball, C.Y. Charalambous, M. Thames, & J.M. Lewis (PME 33, Thessaloniki, 2009). It presents Meta-Didactical Transposition, as a new model for framing teacher education projects that takes both the research and the institutional dimensions into account. The model is mainly based on the Anthropological Theory of Didactics by Y. Chevallard. In the background of current literature on teacher education, the general aim is to discuss the scope and potentialities of the model, and to use it to study some examples. Two national teacher education programs are analysed using the model: the French Pairform@nce and the Italian M@t.abel.

INTRODUCTION

In the last years, the research on mathematics teacher education has particularly intensified, initially concentrating on pre-service education and gradually enlarging the focus to in-service education, in the perspective of long-life learning and with an emphasis on the role played by specific tools and methods on the professional development of teachers. An overview on this theme (from the knowledge and beliefs of the teacher, to tools and processes, to participation and professional development of teachers), can be found in the 15th ICMI study on teacher education (Even & Ball, 2009) and in the four volumes of the *International Handbook of Mathematics Teacher Education* (Wood, 2008).

Because of the wide number of studies that relate to it, a specific discussion must be developed on the problem of identifying the knowledge that is necessary for the teaching of mathematics. Shulman (1986) identified three categories of teacher knowledge: knowledge of the discipline (Subject Matter Knowledge, SMK), pedagogical content knowledge (PCK) and knowledge of the curriculum (Curriculum Content knowledge, CCK). Shulman was the first to identify PCK as the particular knowledge for teaching, related to content: "the particular form of content knowledge that embodies the aspects of content most germane to its teachability" (Shulman, 1986, p.9). Some researchers tried to adapt PCK to the case of mathematics teaching, observing that it concerns the intertwining of mathematics and pedagogy in relation to the different conditions for and ways of teaching and learning some specific content. Others put into evidence the lack in PCK of specific dimensions of knowledge and knowledge-related skills that are crucial for mathematics teaching. For example Mason

2013. In Lindmeier, A. M. & Heinze, A. (Eds.). *Proceedings of the 37th Conference of the International Group for the Psychology of Mathematics Education*, Vol. 1, pp. 97-124. Kiel, Germany: PME.

(1998) underlines that teacher professional profile should include skills and attitudes that go beyond Shulman's PCK, in particular in terms of content-related awareness.

Taking Shulman's studies as a starting point, Ball & Bass (2003) face the problem of proposing a finer and more effective characterisation of what they refer to as the *mathematical knowledge for teaching* (MKT).

MKT is defined as "the mathematical knowledge, skills, habits of mind, and sensibilities that are entailed by the actual work of teaching", that is "the daily tasks in which teachers engage, and the responsibilities they have to teach mathematics, both inside and outside the classroom" (Bass, 2005).

Ball & Bass (2003) analyse the typical features of mathematics that are involved in teaching and identify the main components of MKT in relation to Shulman's subject matter knowledge (SMK) and pedagogical content knowledge (PCK). Referring to SMK, they identify the *specialized content knowledge* (SCK) as an important sub-domain of mathematical knowledge, strictly connected to the work of teaching. The research forum (Ball et Al., 2009) they proposed together with other researchers from different countries testifies the common aim of bringing together different perspectives in conceptualizing teacher knowledge for teaching.

Another important element arising in the studies on teacher education is the involvement of teachers in the joint analysis and reflection (together with researchers) about the main features of the didactical projects and the incidence of teachers' choices on students' learning (with a constant focus on collective purposes rather than on individual needs). This idea is strictly interrelated with that of a joint collaboration between teachers and researchers, as Krainer (2011) stresses when he suggests to look at researchers as "key stakeholders in practice" and as teachers as "key stakeholders in research". Within the research literature we find therefore typical expressions such as, *community of practice* (Wenger, 1998), *communities of inquiry* (Jaworski, 2006), *adaptive systems*, *collective participation*, *sustained conversation*, and *egalitarian dialogue*. The cornerstone of these studies is the notion of *critical reflection*, conceived not only as a fundamental attitude to be developed by teachers but also as a professional responsibility.

As researchers, we are involved both at the level of teacher education program development and management, and in studying the teaching and learning processes in the classroom, where the actual innovation should take place. This has prompted the emergence of a deeper reflection on the resulting complexity.

We extended our research grounding on the approaches underlined above, which remain basic and preliminary to any further research on the topic. We started analysing the issue of teacher education programs also from another point of view, in a sense more attentive to some components that deeply determine the way such programs are born and evolve. More precisely we noticed the relevance that the institutions play in the school context, to include: national curricula; national assessment tools; the constraints of teachers' time and space; textbooks; etc. We felt the necessity to include also these components in a frame that could account for teacher education programs

development. Our attention was directed towards the theoretical elements that could frame properly these institutional components: we found a suitable frame in the Anthropological Theory of Didactics (ATD) by Chevallard, and specifically in the notion of *didactical transposition* (Chevallard, 1985, 1992, 1999; Bosch & Chevallard, 1999). The complexity arising from the intertwining of the different processes involved during a teacher education program has led us to introduce a descriptive and interpretative model, which considers some main variables in the teacher education processes (community of teachers, of researchers, role of the institutions), and accounts for their mutual relationships and evolution over time. We call the overall resulting process *Meta-Didactical Transposition*. This model provides a tool for studying the transposition from research to teaching practice during teacher education programs.

In the following we present the Meta-Didactical Transposition model focusing on its main components and we contextualise it within two different programs, one from Italy and the other from France. The programs developed in the two countries before and independently from any influence of our theoretical model. We analyse the two programs interpreting them through the lenses of our model, and finally discuss the results of our analysis, pointing out the model's potential with respect to current research in the field.

A NEW PARADIGM: META-DIDACTICAL TRANSPOSITION

The *Meta-Didactical Transposition* model takes into consideration the practices of mathematics educators (researchers) and those of teachers, when both communities are engaged in teachers' education activities. It is an adaptation of the Anthropological Theory of Didactics to teacher education, through the integration with of further elements. It considers:

(i) the complex dynamic interplay, which develops between different communities involved in teacher education activities (e.g. between the teachers and the researchers/math educators);

(ii) the constraints imposed by the institutions that promote such activities (from schools to the Ministry of Education) in view of some specific goals (e.g. promoting teachers' knowledge of new curricula or of new technologies);

(iii) other "institutional" constraints, such as the tradition of the school(s) where the activities develop, the related (intended, implemented, attained) curricula, the textbooks used by the teachers, etc.

Meta-Didactical Transposition is characterized by five intertwined features: the *institutional aspects*, the *meta-didactical praxeologies*, the *double dialectics*, the *brokering* processes, and the dynamics between *internal and external components*. These features, that will be described in the following sections, enable our model to focus on some main aspects of teachers' education programs related to their development: their dynamicity; the dialectic between the communities of teachers and those of the researchers who coach them; the influence of the institutional components and their relationships with the communities.

Institutional aspects

The Anthropological Theory of Didactics by Chevallard focuses on the institutional dimension of mathematical knowledge: ATD puts mathematical activity, hence the activity of studying in mathematics, within the bulk of the human activities and of the social institutions (Chevallard, 1999).

In our view, it is important to consider such an institutional dimension in teacher education activities. As a matter of fact, they are fully situated within and constrained by the context of social institutions (research communities, schools, the Ministry of Education, the policy makers, the teachers associations, etc.). It is contextually relevant to know that in many European countries (e.g. Italy, France), the whole educational system (from kindergarten to university) is public and governed by several institutions at different levels (national, regional, local). Within this context, the importance of the institutional face is also testified by the politics of the European Union. As lifelong education is considered a strategic element for development in Europe, community programs are promoted for prospective or in-service teacher education, which envision a clear cooperation between the research world and the institutional-politic world (see http://ec.europa.eu/education/llp/official-documents-on-the-llp_en.htm).

Chevallard stresses the fact that the very nature of mathematical objects in school is dependent on the person or the institution with which it is related: "An object exists since a person, or an institution acknowledges that it exists (for it itself)" (Chevallard, 1992, p. 9). In the same way we look at teacher education programs focusing on two main communities and taking into account the institutional constraints within which these two communities develop. We therefore analyse the relationships between: (a) the *communities of the researchers*, who design and coach the educational programs, generally as an official task commissioned by the responsible authorities (e.g., school administration, Ministry of Education); (b) the c*ommunities of the teachers* who participate within the projects, either on a voluntary basis or because of an official duty. These two communities sometimes intertwine, and both of them are in relationship with the school: the actual schools where the teachers teach, and the School as an institution with its curricula, the teaching tradition of the country, the textbooks used, etc.

Meta-didactical praxeologies

The ATD proposes a general epistemological model of mathematical knowledge, conceived as a human activity developed for the purpose of addressing specific families of tasks. Its main theoretical tool is the notion of *praxeology* (or mathematical organisation), which is structured as two main levels (García, Gascón, Ruiz Higueras, & Bosch, 2006): (a) The "know how" (*praxis*), which includes a family of similar *problems* to be studied, as well as the *techniques* available to solve them (e.g. 2nd degree equations and their formulae for their solution); (b) The "knowledge" (*logos*), that is the "discourses" that describe, explain and justify the techniques that are used within a more or less sophisticated frame and may even produce new techniques (e.g. the justification of the formula for 2nd degree equations through the completion of

squares or even the theory of algebraic equations and how it encompasses 2nd degree equations)[1].

Hence a praxeology consists in a Task, a Technique, and a more or less structured Argument that justifies or frames the Technique for that Task, encompassing both the know-how and the knowledge, with respect to a family of tasks.

In constructing our model, we consider the *meta-didactical praxeologies*, which consist of the tasks, techniques, and justifying discourses that develop during the process of teacher education. To make clear what we mean, let us consider the example in Sullivan (2008, p.3). There he recalls how during a teacher training course he used the question "which is bigger: 2/3 or 201/301?" in order to prompt teachers for ideas that might be used as the basis of a lesson. The discussion with the teachers promotes at least three points of view, according to which one can answer the question: the mathematics knowledge, the knowledge specific for teaching and the pedagogical knowledge. According to such knowledge, specific interventions could be designed to introduce the students to the task, e.g. to think of baseball champions scores: if a player passes from 200/300 to 201/301 his score increases. All of this can be considered as an example of a *meta-didactical praxeology* in that the task is stimulating the teachers' reflection, and the techniques are those that Sullivan and Clark used in the course to promote the teachers' discussion. During the discussion, it is possible that the two communities of educators and teachers share a common theoretical framework, which justifies the techniques being discussed. For example, based on one's professional experience, they might discuss why the initial question presents difficulties for many students and why the baseball example makes sense in a classroom and can help to overcome the difficulty. Moreover they may scaffold their arguments within specific pedagogical frames: e.g. stressing the necessity to foster the transition from everyday to scientific and formal concepts, according to a Vygotskian approach. The theoretical side of the *meta-didactical praxeology* also includes the reflection made by Sullivan and Clark on the reason why the activity was a good illustration of the way teachers can be made aware of MKT, an aspect that may have been highlighted within Sullivan's exposition.

Within *meta-didactical praxeologies*, what is under scrutiny is not the didactics in the classroom, but the practices and the theoretical reflections developed in teacher education activities. Of course they are the result of the interactions between the concrete practices used by the teachers in their professional activities, the reflections developed by the teachers on these activities and the subsequent reflections developed by the community of researchers about the effects of the educational processes they have previously designed and developed.

We have now the basic ingredients that allow us to introduce the core of our model. Looking at teacher education processes from a dynamic point of view, at the beginning

[1] The "knowledge level" can be further decomposed in two components, i.e. Technologies and Theories. The provided description is sufficient for our purposes.

we find two communities: that of researchers and that of the teachers. Each of them has its own praxeologies: the *researchers' praxeologies*[2] and the *teachers' praxeologies*. Teacher education programs aim for the teachers' praxeologies to evolve towards a new praxeology, which consists of a more or less deeper amalgamation of the components of the two initial praxeologies. This evolution is the result of the interaction with the community of researchers, and for this reason we call it *shared praxeology*. For example, from the discussion of different techniques to address a problem, new ones can be acquired by the teachers, with a suitable theoretical justification, thus replacing or integrating old techniques and resulting in a change in the teacher's MKT frame. Also within this dynamic evolution there are some external components, which may play a crucial role. A typical example is when the activity is developed in response to changes in the official curriculum or because the institution commences a certain type of external assessment for their students.

The community of researchers generally reflects upon the nature of, and reasons for, the changes produced by the teacher education program, expected and unexpected outcomes, and possibly shares such reflections with the community of teachers. In this way, *new researchers' praxeologies* are constituted. Also the teachers' praxeologies may change, and develop into *new teachers' praxeologies*. The overall process can repeat and further refine itself. A global illustration is provided in Fig. 1.

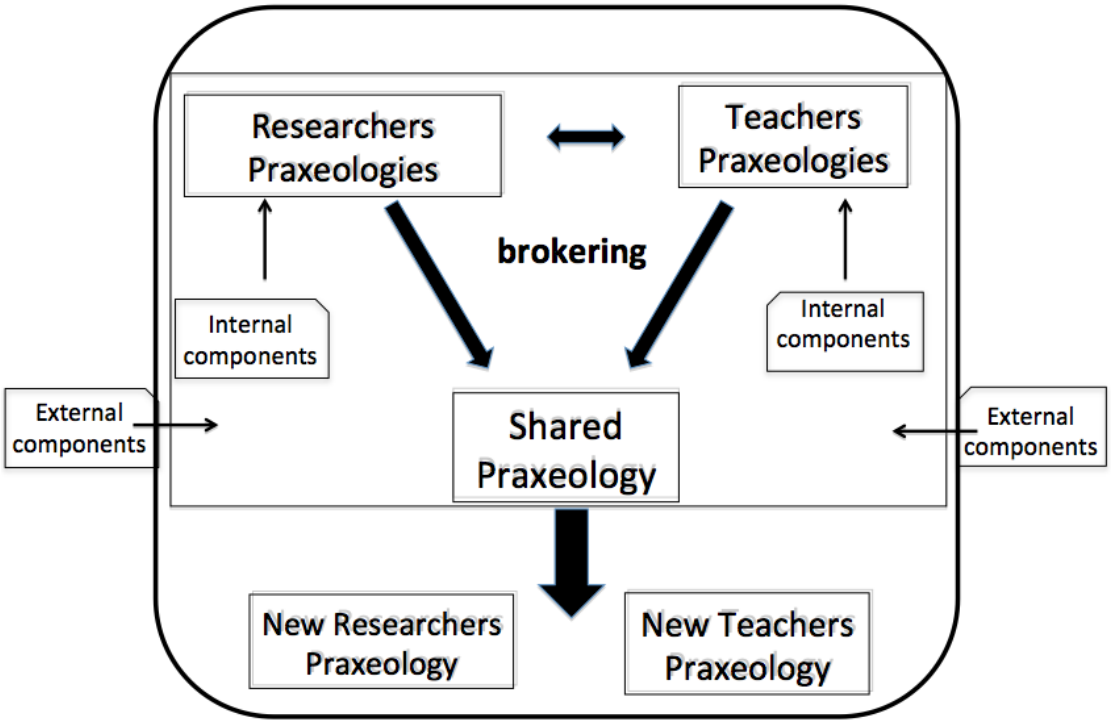

Figure 1: The Meta-Didactical Transposition model.

[2] Of course there may be more than one praxeology referring to researchers, as well as referring to teachers: in the text we will use either singular or plural (researchers praxeologies; teachers praxeologies). In particular the researchers have their own praxeologies as researchers, which concern the praxis and the logos of their research activity; but they have also their praxeologies as teachers' educators, where the praxis and the logos concern the way they coach these activities, and their theories about teachers' educational processes.

We can summarise what we have presented in this paragraph defining the "Meta-Didactical Transposition" as the dynamic process through which, thanks to the dialectical interactions between the two communities, both the didactic praxeologies of the community of researchers and the teachers' community change within the institutional environment in which the two communities reside. This dialectical interaction leads to the development of a shared praxeology, which represents the core of our model. One of the main results of the dialectical interaction is the teachers' development of both new awareness (on the cultural level) and new competences (on the methodological-didactical level), which lead them to activate, in their classes, a didactical transposition in tune with the recent educational trends. Therefore, the term "meta-didactical" refers to the fact that important issues related to the didactical transposition of knowledge are faced at a meta-level.

Internal and external components

An important feature of Meta-Didactical Transposition is that some of the components of the two communities' praxeologies change their status in the course of time. Typically, they move from *external* to become *internal* with respect to the community under scrutiny. The internal/external distinction is adapted from Clark & Hollingsworth (2002). Compared with their approach, our model emphasises the process of the teachers' professional evolution, according to which some of the external components become internal as a result of the process of Meta-Didactical Transposition.

To clarify this crucial point we give a brief example. Consider a community of teachers that starts an educational program in which, due to an institutional constraint (e.g. curriculum changes by a competent authority), a community of researchers introduces them to a specific ICT tool (e.g. a dynamic geometry system). Initially, the tool is an external component for the teachers. However, at the end of the educational program it may become an internal component in their praxeologies, featuring their didactic activities within the classrooms, as well as their view on mathematics (geometry).

This example highlights that the Meta-Didactical Transposition process could produce a dynamic change in the teachers' praxeologies, as some components of the researchers' praxeologies may become internal components of the teachers' praxeologies. Discussing the two examples, we will show that this change is only one of the possible transformations that the Meta-Didactical Transposition can produce within the praxeologies of the two interacting communities.

Brokering

The Meta-Didactical Transposition model integrates the ideas of ATD with elements coming from other frameworks. The notion of broker is an example. It is introduced because it describes the role that teachers and researchers often play within the different communities. According to the definition used in Rasmussen et al. (2009), a *broker* belongs to more than one community. Typically a teacher belongs to the community of mathematics experts, to that of his/her school-teachers and to his/her classroom community: "Brokers […] are able to make new connections across

communities of practice, enable coordination, and – if they are good brokers – open new possibilities for meaning" (*ibid*., p.109).

Brokers facilitate the sharing of knowledge and practices from one community to the other (*boundary crossing*), which is accomplished by drawing on *boundary objects*: boundary objects are those objects that both inhabit several communities of practice and satisfy the informational requirements of each of them (Bowker & Star, 1999, p. 297).

Within Meta-Didactical Transposition, brokering is a common habit and frequently the researchers play the role of brokers between the two communities who inhabit the process. A good example of a typical boundary object is the baseball score used by Clark in the example recalled above (Sullivan 2008, p.3). Teachers can use such a boundary object to move students' thinking from the usual meaning of the score to the more mathematical comparison between the two fractions (2/3 and 201/301). At the same time the example, used within an episode of teacher education, is a boundary object used by the researcher to move the teachers from the standard mathematical meaning of fractions to an everyday contextualised meaning that is useful for teaching. In this sense the researcher makes a brokering action with respect to the teachers.

Double dialectic

Another important element of our model is the double dialectic that enters into the *Meta-Didactical Transposition.*

The first dialectic is at a *didactic level* in the classroom in that it is between the personal meanings that students attach to a didactic situation, to which they are exposed in the didactic activity, and its scientific, shared sense (Vygotsky, 1978). The second dialectic is at a *meta-didactic level*. It lies between the interpretation that the teachers give to the first dialectic as a result of their personal meaning, which is a result of their praxeology, and the meaning that the first dialectic has according to the community of researchers, that results from the researcher's praxeology. The second dialectic corresponds to the *scientific shared meaning* of the first dialectic.

Typically the second (meta-didactical) dialectic arises from a contrast/comparison between the researchers' praxeologies and the teachers' praxeologies and the first dialectic engenders the second one as an outcome of a suitable meta-didactical trajectory, which is designed by the researchers. It is through this double dialectic that teachers' and researchers develop a shared praxeology. As a result of this whole process, the teachers and researchers' professional competences could significantly evolve.

THE PAIRFORM@NCE PROGRAM

Pairform@nce is a French national teacher education program from the ministry of education whose objective is to develop in-service teachers' skills for using ICT with students in schools with the general aim of teaching renewal. Pairform@nce addresses all class levels, from primary to secondary and all topics. The implemented principles

of teacher education are the design of resources and peer collaboration, using both face-to-face and on-line work.

Pairform@nce provides web-based platform with a collection of training paths dedicated to particular subjects and a virtual space for their design, use and revision. Training sessions, developed from a path, are set up by teacher educators for a group of trainees at a regional level. These training sessions alternate face-to-face workshops, personal autonomous work, and on-line synchronous or asynchronous exchanges using the web-based platform.

The training paths are available on the platform as sets of pages, that gather texts, links, collaboration tools and other kinds of resources (texts, images, working spaces, files to be downloaded, but no video means for distance communication). Initially, they are all organized in seven steps, but may differ on many aspects. Moreover, a training path is not a training session. It is a tool for the teacher trainer to organize, set up and manage the training session of a group of teachers. Some features of the paths support particularly (i) the appropriation process of the teacher educator that would use the path to implement a training session; and (ii) the face-to-face and at-a-distance intertwining of the blended training (Osguthorpe & Graham, 2003).

The most important Pairform@nce principles are (Guedet et al., 2012):

(i) collaboration among teachers: professional development, especially concerning ICT, results from collective activity and experience with peers;

(ii) implementation in class and reflectivity: a teacher's development program necessarily implies experimentation of resources on the field and afterwards shared reflection;

(iii) continuous work: working efficiently on resources, requires maintaining an ongoing collaboration, intertwining face-to-face and on-line training activities, as well as classroom experiments.

(iv) possibility of appropriation of a path by teacher educators: teacher educators can use (to design their training sessions) a path that they have not designed themselves.

Some of the path features are essential for the distance activities. Some of the paths designed by the IFE research group propose a detailed agenda of the training, describing the activities planned for the thirteen weeks. All the resources (Guedet & Trouche, 2011) for the teachers can also be considered as resources for teacher educators. Moreover, key aspects of the training and designer choices, as well as the detailed activities of each user of the path (teachers and educators), are listed in a series of training guidelines and tools for the educators, within a pedagogical advice section. This section points to organizational notes, resources and comments, which reveal that the designers were concerned with the educator's role and tried to support his/her activity, by providing relevant information for the appropriation and the implementation of the path.

Despite all these resources, the implementation of a path is not straightforward. Educators need to carry out important adaptations. These adaptations and

modifications reveal the documentational genesis. Additionally, if they lead to a new design that incorporates the experience of several educators (as we will discuss below), they are likely to improve the quality of the path.

In this discussion paper we are going to exemplify and analyze the pairform@nce program through a particular training session whose goal is to allow trainers to develop training sessions combining face-to-face instruction with computer-mediated instruction. Eleven teachers trainers' teams volunteered to modify and augment their usual training session by combining on-line and face-to-face instruction. Some projects are disciplinary projects (teaching algorithms and programming, languages teaching, marketing,...) but the most part are related to cross disciplinary projects (using Internet in classrooms, using interactive white board, resources for teachers,...). The main aim of the training session is to allow trainers to change from their usual in-training format to a blended learning system (Graham, 2006).

The next sections are devoted to a first analysis of this in-training session through the lens of the Meta-didactical Transposition. As written in the introduction the Meta-didactical Transposition is characterized by five intertwined features: the *institutional aspects*, the *meta-didactical praxeologies*, the *double dialectics*, the *brokering* processes, and the dynamics between *internal and external components* that will constitute the next sections.

Institutional aspects in Pairform@nce

The Pairform@nce program as said in the introduction is a national program with different local levels. The institutional aspect is, for our example, very important because the initial demand came from the local education authority in Lyon which wanted to enhance in-service teacher education without increasing costs. In this pairform@nce program three levels had to be taken into account:

- the trainer's level: persons in charge of the training course both researchers and institutional trainers,
- the trainers-trainees' level: trainees in this training course but working as trainers,
- the teachers-trainees' level: at the end point of the string, the target of the in-training course.

Depending on the context, the same person can have different positions as illustrated in Figure 2.

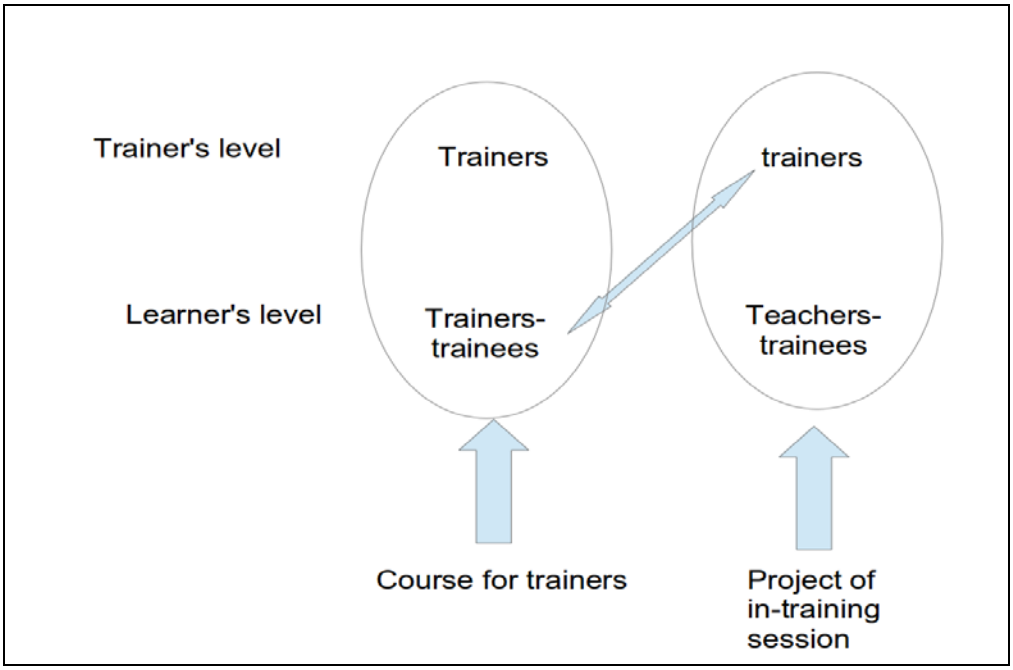

Figure 2: Different positions relatively to the context in Pairform@nce.

The end point of this string, which must be added to these three levels, is the level of the students and of the enhancement of learning through actualized teaching methods. These three communities intertwine with one another. The community of researchers, who, on demand of the local authority, both participates to the design and coaching of the course and supervises the general program has a specific position in this project.

The dialogs between these three communities have to take into account the reality of each of the institutions:

- the local education authority with its own policies and rules; for example, the choice of trainers-trainees had been done through the rules and under the responsibility of the institution, which had effect, for the candidates, to have a particular presentation of their project leaned towards the institutional acknowledgement,
- the institutional context of the teachers' status which, in France is particularly sensitive; for example, the work done in distant phases (synchronous and even asynchronous) has to be recognized officially by an institutional acknowledgment,
- finally the national curricula and the actual school play an important role relatively to the content of the in-training course; for example, what is possible to do in a primary school is not necessarily allowed in a secondary school due to local organization of the school.

Meta-didactical praxeologies in Pairform@nce

A meta-didactical praxeology consists in a Task, a Technique and a Theory giving sense and justifying the technique when accomplishing the task. In the particular example of the in-training course that we presented above, it is possible to highlight a

large number of tasks that need to be accomplished and justified through techniques and theories. Let's take one example.

In the case of on-line and asynchronous sessions, trainers have to present a particular work, to encourage and to guide trainees in the accomplishing of a particular task without leaving anybody behind them. One possible technique allowing the realization of the task is tutoring. It is a technique that, in the good conditions, enables each learner to become aware of the work he/she has to do, the useful tools available, and the way of using them, the success of his/her work, and so on. The justification of the tutoring technique is founded both on empirical experiments; in the trainers-trainees praxeology, the justification of tutoring comes from the analysis of trainees feedbacks (Ehlers, 2004) and from the trainers' experience. From the perspective of the community of researchers, the justification of the tutoring comes from a theoretical frameworks allowing to define precisely the tutor's role in the new organization of learning through distant courses (Lebrun, 1999; Oztok et al., 2013). The tutors' roles appear to be linked with the concept of community of practices (Wenger, 1998) and learning theories in a Vygotkian approach. These frameworks give the theoretical support to justify the use of the tutoring technique during the in-training course. The trainers praxeologies enrich themselves from the theoretical frameworks when the researchers' praxeologies feed on the empirical data (Figure 3).

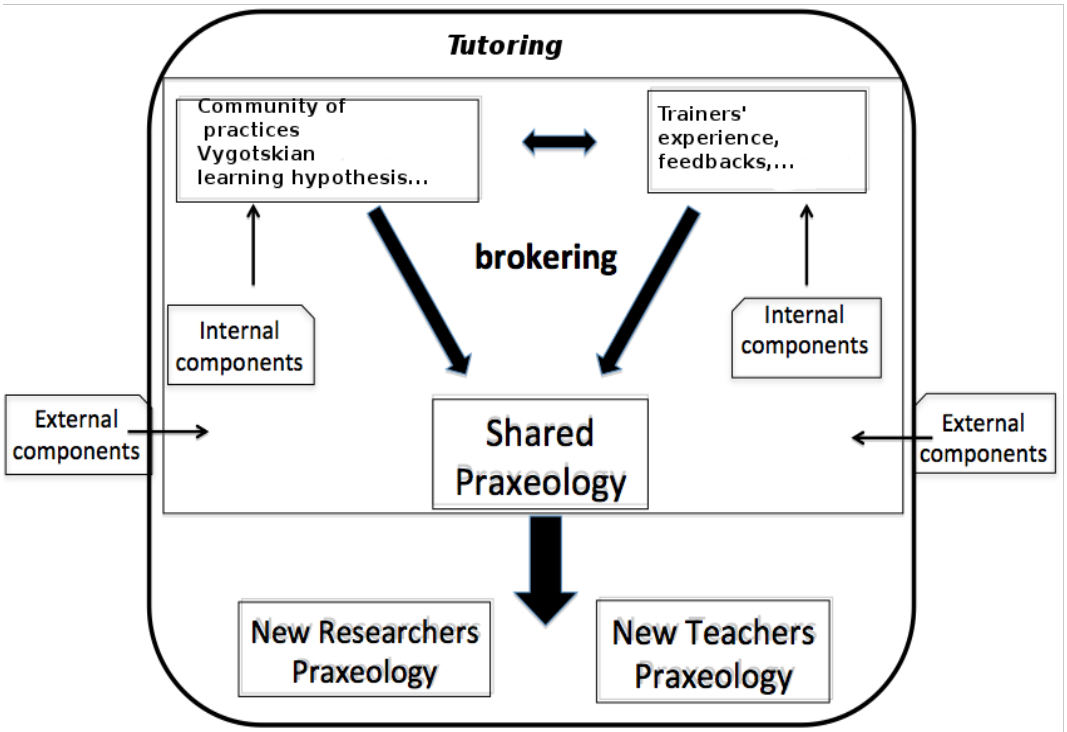

Figure 3: Meta-didactical praxeologies in tutoring process in Pairform@nce.

In that case, the trainers-trainees could share during the course with the researchers some elements of the praxeology that could become a shared praxeology. Trainers-trainees, too, could share this praxeology with teachers-trainees in a dynamic evolution.

Internal and external components in Pairform@nce

In the case of our in-training example, the distant aspect of the in-training course is an external components of trainers-trainees, whereas they all are experimented and institutionally recognized trainers. Our today's hypothesis is that, through the Meta-Didactical Transposition process, the concept of distant courses should become an internal component for trainers-trainees and modify deeply their way of thinking and proposing in-training courses for teachers. Different tools and activities are external components, like, for example forums, wikis, or chats, which become internal in the perspective of distant communications needed by a scheduled monitoring. From the point of view of researchers, different theories could highlight from an external point the tutoring process, as for example the documentational genesi which allow to take into account, in the tutoring process of the instrumentation and instrumentalisation of resources.

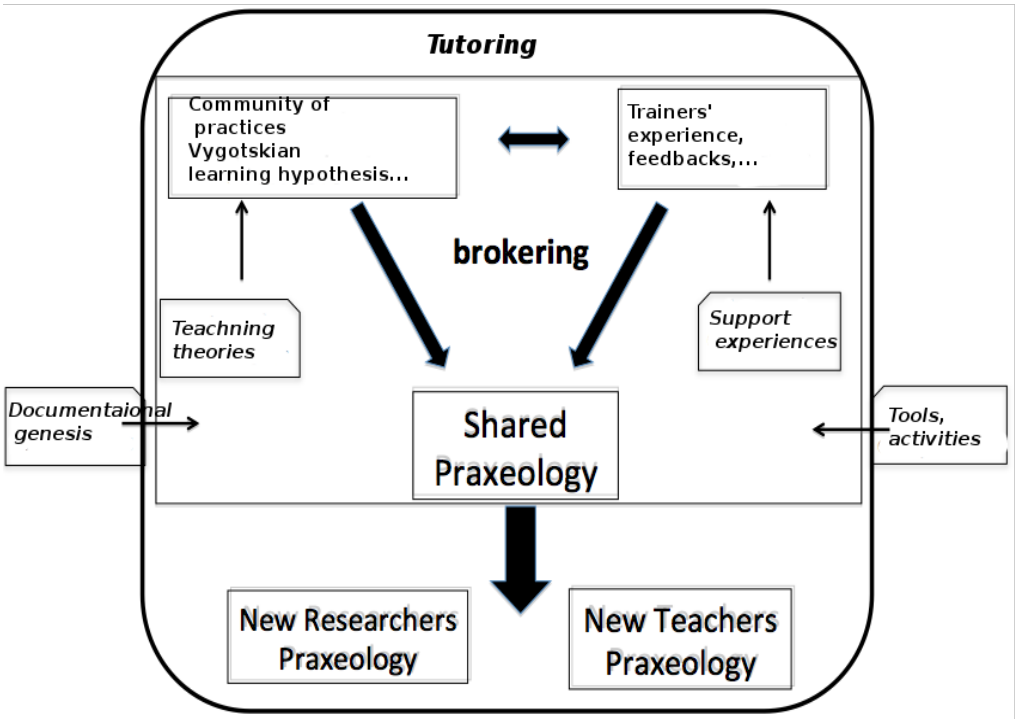

Figure 4: Internal and external components in Pairform@nce.

It is an issue of our present work to see how deeply the internalization could be done and to follow the trainers-trainees, particularly, in the fourth level, to observe whether there is an impact on teachers and students and more generally on teaching and learning.

Brokering in Pairform@nce

Brokers, in this case study, may be different persons due to the context and the actual role of each actor. However, in the case of tutoring, the go-between role is mostly played by trainers-trainees who are trainees preparing an in-service training courses. Where does this brokering take place?

The concept of brokering is directly linked to the boundaries objects that can be considered as a field where brokers can establish a link between two communities. The

concept of tutoring remains a good example of boundary object that can bring trainers, teachers and researchers to move from one point of view to another. In all teaching situations, teachers or trainers support the students or trainees' work, associating resources and activities, clues and helps in a planned order. Even if it is often implicit in the organization of a face to face lesson, it becomes fundamental and compulsory in a distant course. Trainers can use the scenario of their own in-training course to make trainers-trainees become aware of the different roles that tutors could play and of the implications of these roles; which questions can be asked to define the activities and organize the path between these activities? Which kind of activities will be used? Which resources, which tools will be available?... In another way, trainers-trainees in their position of trainers can present, share and discuss examples of tutoring that can be used in a distant course. Trainers-trainees play the role of brokers between the two communities and finally participate to the creation of a shared praxeology through the double dialectic.

Double dialectic in Pairform@nce

In one hand, the didactical dialectic takes into account the effective interaction between teachers and trainers in a particular situation of teaching-learning. In the case of tutoring, the in-training situation is built on an isomorphism model, that is to say it fosters a focus and a reflection on tutoring in the same time tutoring is implemented and enables the involved subjects to think about the tools they may use as trainers when using them as ~~a~~ trainees. The meta-didactical dialectic arises from the comparison between praxeologies, and, more precisely, between the theories allowing a justification of the implemented technique in the realization of the task. Reflections about tools, resources and their implementation in the course may foster, thanks to the brokering process, the creation of a shared praxeology, that, in our case, has to be finally shared with teachers in a perspective of enhancement of teaching methods.
This analysis of this in-training session through the lens of the Meta-didactical Transposition has to be completed and augmented, both with a deeper look on different tasks and with a confrontation with the contingency.

THE M@T.ABEL PROJECT

The *M@t.abel* Project is an Italian national teacher education program for in-service mathematics teachers supported by the Ministry of Education. It started in 2006 and, to date, has involved more than ten thousand secondary school teachers distributed across the whole of Italy. Teachers are involved in professional development through face-to-face participation and distance-learning sessions coordinated by tutors alongside the trialling of activities in their classes.

M@t.abel has its roots in the Italian research for innovation paradigm (Arzarello & Bartolini Bussi, 1998), and in particular, a previous project called "*Matematica per il cittadino*" (Mathematics for the citizen, 2001-2005 - http://umi.dm.unibo.it/old/italiano/Matematica2001/matematica2001.html). From this project it took the main issues and most of the activities.

Matematica per il cittadino was carried out by the Italian Mathematical Union (thanks to an agreement with the Ministry of Education) and consists in an innovative curriculum in mathematics, from primary to secondary school (Anichini, Arzarello, Ciarrapico & Robutti, 2004), which influenced the Italian school reforms of the new millennium.

The main issue of *Matematica per il cittadino* is the idea of the *mathematics laboratory*, intended as teaching practices (coordination of collaborative working groups, manage of collective discussion, orchestration of materials and technologies, interactive lessons) in which the students can learn by doing, seeing, imitating and communicating, under the guidance of the teacher, in a word: "practicing", as in a Renaissance workshop. These teaching practices foster close interaction between novices (students) and expert (teacher), in the frame of *cognitive apprenticeship*. This phrase "refers to the fact that the focus of the learning-through-guided-experiences is on cognitive and meta-cognitive, rather than on physical, skills and processes" (Collin, Brown & Newman, 1989, p. 458). Another important issue of the project is the particular attention to processes (and not only to contents), such as: the use of different communication strategies, conjectures, argumentation and proof, problem posing and problem solving, measuring and modelling.

Most of the 200 activities presented in *Matematica per il cittadino* curriculum have been improved in order to be used in the *M@t.abel* Project. For this use, they have been implemented in a platform and in a website in a double form: text and multimedia.

Institutional aspects in M@t.abel

The reformed Italian National Curriculum has been influenced by *Matematica per il cittadino* curriculum, but the school reality is quite far from being broadly influenced by this new perspectives: innovation is bounded to isolated cases (teachers, schools, or networks of schools) and to primary or middle schools. In order to improve school mathematics education at secondary level, the Ministry of Education (MIUR) alongside the Agency of School (Indire) promoted the M@t.abel project from 2006 to today. To reach this aim, M@t.abel requires the teachers to follow a professional development made of two phases deeply intertwined together:

1. Teacher education phase: face-to-face and at distance meetings in a group with other teachers and trained by a tutor;
2. Experimentation phase: in their own classrooms some M@t.abel activities with the related praxeologies (different from traditional lecture-exercises), based on: problem solving, discovering-conjecturing-arguing and proving, through group-work, discussions, and using ICT technologies.

These two phases of professional development (education and experiment in the classes) are finalised to obtain a deep dissemination of the issues and teaching practices of M@t.abel project in order to obtain as a result a *change* in the current didactic praxeologies used by teachers and enhance mathematics learning.

Meta-didactical praxeologies in M@t.abel

The project is planned and carried out under the supervision of a mathematics education research group: the Scientific Commettee. An important feature of the project are the tutors, a small number of teacher-researchers or expert teachers, who are usually part of a research team of University researchers. Teacher-researchers, according to the Italian paradigm of research for innovation, are teachers who take part of research projects in mathematics education side-by-side with University researchers (Zan & Malara, 2008). The tutors first follow a training program for teacher educators and then are involved as tutors in the teacher education program.

The two steps:

- *First step.* Tutors education (made by University researchers): the praxeologies of the researchers make contact with those of tutors, giving rise to shared praxeologies (Fig. 5a).
- *Second step.* Teachers education (made by tutors): the shared praxeologies of the first step become the base of the second step as Researchers/Tutors praxeologies and make contact with teachers praxeologies (Fig. 5b).

Each step can be considered as a Meta-Didactical Transposition process.

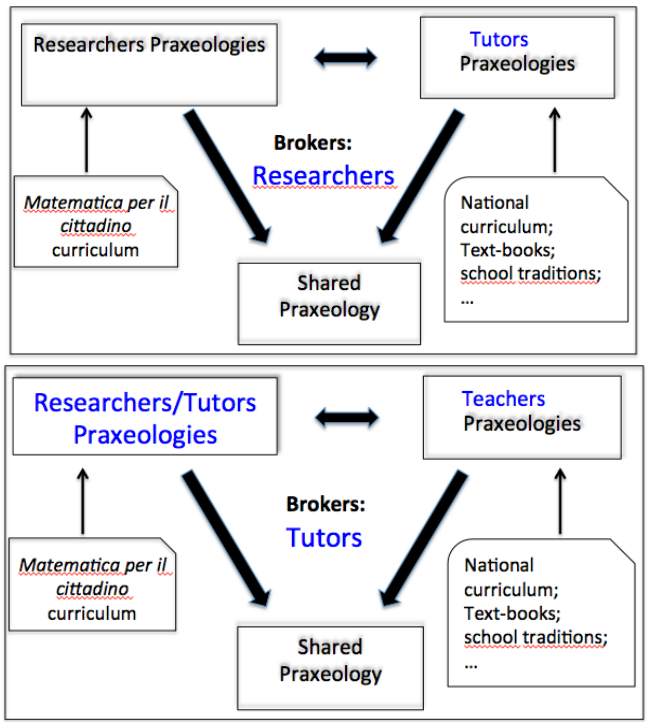

Figure 5 a, b: The two-step process of Meta-Didactical Transpositions in M@t.abel.

The role of the researchers is fundamental in both the steps of Meta-didactical transposition: they are called upon to plan all the components of the tutor and the teacher education program, to implement the educational meetings for the tutors, and to prepare materials for the teachers. In the developing of materials they are helped by teacher-authors, who are also involved in the tutor education program along with researchers. These teacher-authors of the materials (activities and suggestions on

teaching practices) act as brokers during the tutor education (Fig. 5a). The *researchers' praxeologies* are the product of the past and current research activity of the mathematics educators involved in the project: for example, they include the methodology of mathematical laboratory and its theoretical background (i.e. the cognitive apprenticeship approach, in many cases coordinated with other approaches), the research for innovation paradigm, and the focus on argumentation, proving, and communicative processes, as mentioned above.

The role of the tutors is first to participate to the first step as trainees and then to be trainers in the second step. Depending on the individual case, the initial tutor's praxeologies may be more or less similar with the researchers' ones: for instance they are very close in the case of "teacher-researchers" who have taken part of research groups for many years. Whatever the initial praxeologies are, the participation to the first step allow the researchers and the tutors to constitute a core of *shared praxeologies*, which will be the base of the second step of the Meta-didactical transposition process.

In the second step teachers are the protagonists: they come into contact with the researchers/tutors shared praxeologies and, according to their experience, they compare their individual praxeologies (*teachers' praxeologies*, Fig. 5b, made up of tasks, techniques and theoretical discourses) to the researcher/tutors praxeologies that can be far apart form theirs. For instance, some of the teachers involved in the program often used quite traditional tasks and techniques, that is traditional lessons (lectures-exercises-applications), which are quite far from mathematics laboratory approach and are based on traditional textbooks, old curriculum, and tradition.

During the educational program the tutors' and teachers' praxeologies can evolve and change, through meta-didactical trajectories, towards convergence of *shared praxeologies* as a result of the Meta-didactical transposition (Fig 5b). Such shared praxeologies – present in M@t.abel activities – may include for instance at task-technical level the use of exploration-conjecture-argumentation tasks, the mathematical laboratory methodology and the introduction of new tools (like DGS, spreadsheets, etc.), but also the attention to students' processes (see below the discussion about the logbook). Furthermore they can include a theoretical dimension, producing a new vision of mathematics learning and teaching that is shared by the communities of teachers and researchers/tutors.

In the following, we will focus on the second step of the Meta-didactical transposition, highlighting the role of tutors and the development of shared praxeologies involving teachers.

Brokering in M@t.abel

In order to develop the shared praxeologies, the teachers are organised into *communities of inquiry* (Jaworski, 2008), composed of 15-20 teachers and supervised by a tutor. Tutors act as *brokers* between the two communities of teachers and of researchers. The figure of tutors can play this role because, on the one hand, they are

confident with the innovative paradigms that come from research[3], and on the other hand they share with teachers the same status and experience in the classroom.

Teachers work remotely through an e-learning platform and during some face-to-face meetings with tutors. Initially, the tutor outlines the spirit of the project, ~~and~~ presents the activities during some meetings and asks the teachers to analyse them from a didactical point of view. Then, the tutor coordinates the groups of teachers remotely through synchronous meetings (i.e. with the possibility of sharing the screen and interacting on it) and a-synchronous discussions (emails, forums). Having shared activities and methods, the teachers choose four activities and experiment them in their own classes. These trials are a fundamental part of the teaching education program and, during the experimentations, the tutor asks the teachers to carefully observe their students' processes, and to record their notes in a logbook, to be uploaded on the platform. The logbook is required also on an institutional base, since the teachers have to compile it in order to obtain the certification of attendance to the project.

In the researchers' praxeologies, the logbook is meant to be a tool that helps the teachers to plan, monitor, and control their own work, and in particular to organise the observation of what happens in the classroom, focusing their attention on processes rather than on products. In this sense, it can enable the teachers to orient or re-orient their didactic practice, and contribute to improve the teaching practice by means of teachers' self-reflection. Furthermore, logbooks can be a valuable means of exchange between teachers working around the same mathematics topic, or the same school level, and may provide information tools for external observers.

Table 1 reports a summary of the main information that can be widespread through the logbook, presenting the different aspects that teachers are asked to highlight.

General aspects/Context of the activity	• title of the activity • name of the teacher • name of the school • involved class • period of time within which the activity was developed • number of hours devoted to the activity (in the class and before or after the lessons)
Aspects related to the organization and the development of the activity	• methodology of work with students (working group activities and composition of groups, class discussion, interconnections with other subjects); • reactions of the students to the proposed activity; • collaboration between students and students and between teacher and students;

[3] According to the Italian paradigm of "research for innovation", in this second step the tutors praxeologies may be assimilated to the researchers ones: as said above, in many cases the tutors are teachers-researchers, i.e. are experienced with research studies and methodologies, having been part of research teams in mathematics education for many years. Of course this is not always the case. For the purpose of the paper, we privilege clarity, taking the risk of over simplification.

	• difficulties met by the teachers in developing the activity and strategies adopted to overcome them; • planning of the final test.
Aspects related to students' motivation and to students' learning	• difficulties met by students during the proposed activity (from both the metacognitive and cognitive point of view) and choices made by the teacher in order to make them overcome these difficulties; • positive metacognitive results (changes in students' attitude toward the discipline, growth in their interest…); • positive cognitive results; • students' results in the final test;
General evaluation of the activity	• effectiveness of the activity in terms of recovery of students' difficulties; • role of the activity as a stimulus for gifted students; • effects of the activity on the teacher in relation to his/her didactical planning and to this/her attitude toward the discipline; • suggestions for the improvement of the activity.

Table 1: A summary of the main information documented in M@t.abel logbook.

Internal and external components in M@t.abel

As mentioned above, the Meta-didactical transposition in the case of M@t.abel has its strength in the use of a *platform* for synchronous and a-synchronous activities among teachers. The teachers, coached by the tutor, use the platform's functionalities during the year of their professional education that is, as a forum, for chat, for the download and upload of materials, and as synchronous laboratories of discussion with the possibility of working on a shared desktop, as if it were a whiteboard. The platform is the environment that gives new techniques to teachers, influencing and supporting them in changing their praxeologies. Particularly, if a teacher has worked for many years in a traditional way, mainly alone or with sporadic face-to-face meeting with school colleagues, only for sharing old curricular plans, now she is forced to discuss new methodological issues through ICT (Arzarello et al, in press)

The platform in particular, together with the brokering function of the tutors, has accomplished the aim of building a community of teachers with *shared praxeologies*. Besides being a *communication infrastructure*, allowing synchronous and a-synchronous interactions, for sharing ideas, materials and methods, the platform worked also as a *representational infrastructure* (Hegedus & Moreno-Armella, 2009), fostering the use of a shared desktop where the teachers can work together on-line on the same topic or mathematical object. In the Meta-didactical transposition model, the platform constitutes an example of an *external component* (for both researchers/tutors' and teachers' initial Praxeologies) that becomes an *internal* one in the course of the Project. Also the activities, methodologies, and theoretical ideas presented in the

Matematica per il cittadino project are initially external components to the teachers' initial praxeologies, and become internal to the shared praxeologies.

Another outcome of the project is that the elaborated activities (which initially came from *Matematica per il cittadino* but then are further elaborated by the teachers) are published in an open website and made available for all teachers and interested people, included those not involved in the project (http://risorsedocentipon.indire.it/ offerta_formativa/f/): this is an example of *external* component which becomes *internal* to the whole process.

The evolution of the praxeologies over time in M@t.abel

Finally, the MDT model includes two important effects of the *Meta-Didactical Transposition*, that are the changes brought about by the project in the researchers' praxeologies, and in those of the teachers.

A first analysis at the national level, concerning year 2009-10, has confirmed the validity of the followed method involving the mathematics laboratory methodology. The final analysis of the logbooks showed that the new methodology carried out in the classrooms had some impact on teachers' teaching praxis. In about of the 25.29% of the logbooks[4], the teachers said that they changed their teaching approach and attitude towards the mathematical contents involved in the teaching units, thanks to the activities carried out in their classrooms. This does not happen for 26.56% of the teaching units tested that did not seem to have a significant impact on the normal teaching practice. In about half of the logbooks (48,15%) the answers provided by the teachers were missing (9.45%) or in the written answers the teachers:

- emphasized the strengthening of the teaching methodology usually adopted and then there were not a change prompt by the activities tested (13.28%);
- underlined the utility that the unit of work had for students without saying anything about their teaching approach (25.42%).

The provision of structured materials according to this methodology has helped the teachers to experiment whilst instilling the feelings of security and confidence. Gradually, the teachers gained expertise in the new methodology and became able to transfer it to new didactic activities. The group-work methodology, in particular, was most incisive in changing the teachers' practices. These changes in teachers' didactic activity may give birth to new teachers' praxeologies in their everyday professional life.

The researchers' praxeologies also change during the project. In the case of M@t.abel, their ongoing reflection is prompted by considering the evolution of the system over time and the analysis of the internal/external components. Figure 6 highlights this in order to give an overall picture of the Meta-Didactical Transposition within the M@t.abel project (the second step):

[4] Information come from a document internal to the project, unpublished.

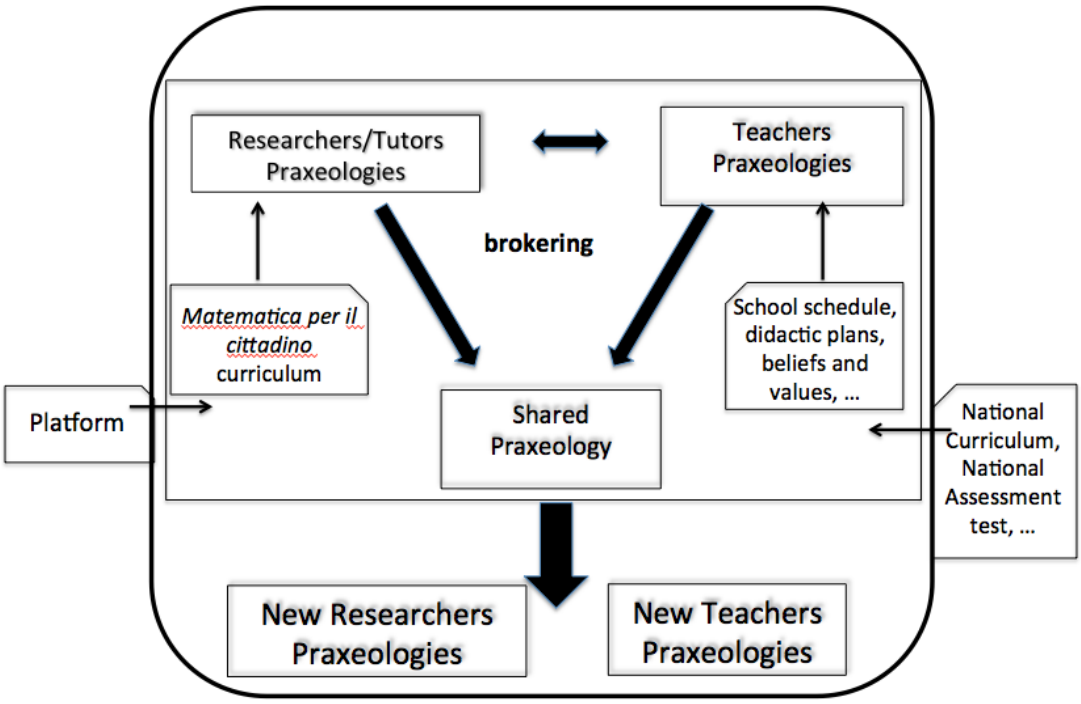

Figure 6: An overall picture of the Meta-Didactical Transposition within M@t.abel (second step).

As a picture of a dynamic process, Figure 6 cannot capture this temporal evolution. If we imagine the model evolving in time, as a set of pictures forming a film, we can focus our attention on the *occurred* and *not-yet-occurred evolutions*. Concerning the *occurred evolutions* in M@t.abel, we find two external components that become internal ones: the platform (as described above), and the *Matematica per il Cittadino* curriculum (which was internal for researchers/tutors' praxeologies but not for those of the teachers). The former is an example of evolution concerning the "know how" part (*praxis*) of the meta-didactical praxeology, whereas the latter concerns the evolution of the "knowledge" part (*logos*) of the praxeology.

The logbook constitutes a suitable lens for observing the occurred and not-occurred evolutions within the whole process over the time. The logbook is in fact at the beginning an external component for the teachers' praxeology. Once introduced to it, many teachers appreciated the tool of the logbook, and used it as part of their praxeology, in order to carry out reflections on the classroom activity, using some theoretical lens shared by the community of researchers (*shared praxeology)*, and to annotate possible changes in their future praxeologies (*new teachers' praxeologies*).

Unfortunately, for many other teachers, the logbook has not functioned as a helpful day-to day observation tool, that is to say it did not become an internal component in his/her praxeology. They completed the logbook at the end of their experience in the project, as a sort of compulsory homework that was required by the system (see T1 in Table 2). It is possible that many of them had viewed this tool as a formal action that was needed in order to obtain the certification, i.e. just another bureaucratic requirement, which pervaded their daily professional life. As a matter of fact, we have

to say that the project enables the participating teachers to gain certification that is useful within their career progression, and that the completion of the logbook is a required element of this accreditation.

T1: *Everything could be better if we weren't obliged to write, specify and document all the things we have done… this is the opinion of many of us.*
T2: *I do not agree. Without the logbook we wouldn't be able to become aware of what we have done with our students. […]We wouldn't have the possibility to reflect on our mistakes, achievements, motivations…*
T3: *Through the logbook, written little by little, as a draft, as a writer's first draft, we make the portrait of a journey. A journey planned at the departure and then described step by step: what we desired and what happened unexpectedly. Only at the end the draft can be handled, ordered, and made clear and understandable; and it is at this point that it is like looking back and noticing the route taken and noting that, while head down and breathless we were running after classes, programs, meetings, evaluations, we were building something that before was not there and that is not in the schoolbooks.*

Table 2: Different comments on the Logbook by teachers in the platform.

In general, by looking at the evolutions in time of internal and external components, researchers can identify those features of *the Meta-didactical Transposition* that are in need of further reflection and work. This kind of consideration is part of a new praxeology for researchers, which in the model is called *new researchers' praxeology*.

The discussions in the Research Forum will help us to further develop our researchers praxeologies.

DISCUSSION

In this paper we introduced the *Meta-Didactical Transposition* model to consider how a complex dialectic between research and institutional dimensions may be productive in teacher education programs. This model is based on Chevallard's anthropological theory, and it is complemented with suitable elements that point out some dynamic features occurring when teachers and researchers are engaged in teacher education activities. We outlined and analysed two examples, one from the French context, the other from the Italian context, in order to show how this model can provide a tool for studying the complexity of diverse types of teacher education programs.

The first example discussed is the Pairform@nce program, a French national teacher education program whose objective is to develop in-service teachers' skills for using ICT with students with strong innovative didactical aims (from primary to secondary level). The implemented principles of teacher education are the design of resources and peer collaboration, using both face-to-face and on-line work. The persons involved in the project can have three different functions depending on the context (Fig. 2): the trainer's level, the trainers-trainees' level, the teachers-trainees' level. Each of these functions has been analysed according to the main components of the model of the meta-didactical praxeologies (distinguishing their own tasks, techniques and theories): internal Vs/ external components, the brokering activities, and the double dialectic.

The second example is the M@t.abel Program. It is an Italian national-scale project in which teachers are involved in professional development through the participation to face-to-face and distance sessions coordinated by tutors, and the experimentation of activities in their classes. These activities, according to the methodology of the mathematics laboratory, are based on working groups around problem solving, discovering-conjecturing-arguing and proving tasks, using digital technologies.

The Meta-Didactical Transposition model allowed us to identify and describe the dynamics between external and internal components (Figg. 4 and 6), and to identify strong and weak points of the projects.

For example, we found a relevant *occurred evolution* from external to internal components in the Meta-Didactical transposition of both projects, regarding the use of the platform. The platform is a technological device, which at the beginning is external to both teachers' and researchers' praxeologies. The passage from external to internal has been fostered by the brokering actions of the trainers/tutors respectively in the two projects. This platform not only enabled the communication of ideas, feelings, and didactic plans between teachers and tutors/trainers, but it also opened up a concrete space for the development of didactic activities that involved the use of software. Another example of the evolution from external to internal praxeologies is the *Matematica per il cittadino* curriculum for the M@t.abel Program: it was at first internal to the researchers praxeologies, and external to the teachers' ones.

Sometimes the model lens allows pointing out some feebleness in the developing of a program. For example, in the case of M@t.abel, the logbook results as delicate element. It was initially an external component to the teachers' praxeologies. Throughout the project, for many teachers the logbook became an internal component of the shared praxeology as they used it as a technique to organise their classroom observation and to plan their work better. This dynamic transformation from an external to an internal component did not occur for all teachers. Many of them, in fact, wrote their logbook at the end of the whole project, despite the constant prompts of their tutors. For these teachers, the logbook remained an external component and did not enter into their praxeologies.

More generally, the Meta-Didactical Transposition model offers also a powerful framework to support the analysis of the evolution of praxeologies over the time. For example, it can be used to focus on the researchers' praxeologies related to the changing of logos and praxis. The Meta-Didactical Transposition model is useful to analyse not only how the praxeologies changed but why these are modified in relation to the teacher education program development: it allows us to examine with a fine grain the evolutions of praxeologies over the time as well maintaining a systemic view.

At the end of the Meta-Didactical Transposition process both researchers and teachers have acquired new praxeologies, changing some of their techniques and/or knowledge that explain and justifies these techniques. This could become the new starting point of a new Meta-Didactical Transposition process.

The model can also be used to point out role played by an appropriate "meta-didactical trajectory" in (i) helping teachers become aware of the first-level dialectic related to the contrast/interaction between the personal sense their students attribute to the activities and the institutional meaning of the same activities; and in (ii) enabling researchers and teachers highlight the second-level dialectic which is related to the contrast/interaction between the different interpretations of the dynamics realised in the classes, given by teachers and researchers also referring to specific theoretical lenses. The tension developed thanks to this double-level dialectic may possibly foster the evolution of both researchers and teachers praxeologies. In particular, it highlights the strict interrelation between this evolution and the chosen methodology of work with teachers.

Our two examples illustrate how the Meta-Didactical Transposition model is "useful to study whether and how different approaches to teacher development have different effects on particular aspects of teachers' pedagogical content knowledge" (Ball *et al*., 2008, p. 405). We have already noticed that a similar construct was used by Clark & Hollingsworth (2002) to underline that teachers' education programs can produce changes in teachers' teaching strategies, "that represented in themselves new pedagogical knowledge" (*ibid.*, p. 953) for those teachers and that "were subsequently put into practice" (*ibid.*, p. 954). In other words, teacher education programs can produce changes in teachers' praxeologies. In fact, our model is similar but not identical to that of Clark & Hollingsworth, since it underlines more the interdependence of such changes from the institutions (according to the ATD approach), and focuses on the Meta-didactical components of the processes, which on the contrary remain more implicit in Clark & Hollingsworth's approach.

The Meta-Didactical Transposition model is deeply related also with the MKT construct of Ball and others (e.g. Ball *et al*., 2008). Both models represent powerful theoretical tools for focusing on the intertwining of the theoretical knowledge and the common practices needed by teachers in their work. Each one stresses different foci of the matter. The MKT focuses on the structure of the mathematical knowledge for teaching; the Meta-Didactical Transposition stresses more the dynamic evolution of its components. In particular, as illustrated in the examples above, it shows the relevance of the double-level dialectic and of the evolution from external to internal components in promoting and supporting the processes of teachers' education.

As the MKT model refines Shulman's PCK model (Pedagogical Content Knowledge, 1986), so the Meta-Didactical Transposition model may be seen as enriching the MKT model, introducing a dynamic point of view and a systemic perspective.

In fact, our model introduces the temporal dimension, the double level dialectic, and the internal-external dynamics, which are all elements that allow us to focus on the dynamic evolution of teachers' educational programs, which eventually produce the specificity of the different "domains of mathematical knowledge for teaching" (according to Ball *et al.*'s terminology). The lens of the meta-didactical praxeology allows the dynamicity of the process to be made evident.

Furthermore, the components of MKT are so embedded in their proper institutional context, where they evolve, and their evolution can be monitored using the model. The difference in focus between the two models may be explained by cultural reasons, which relate to the historical school traditions in the different countries.

However, this also introduces a fresh and promising strand of future investigation, which could produce further results concerning the nature of the domains of mathematical knowledge for teaching and the underlying processes of teachers' education.

A last (but not least) observation is that the preparation of this paper and the consequent discussions that will take place during the Research Forum can help us to further develop our researchers praxeologies.

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