ANALYSIS OF ARGUMENTATION PROCESSES IN STRATEGIC INTERACTION PROBLEMS

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In this study we use Habermas construct to analyse argumentation processes related to strategic interaction problems, i.e. problems in which a player tries to figure out what the other player(s) will do, and to choose the best strategy between the possible ones. These problems provide suitable environments to develop and analyse students' planning and control processes, of a paramount importance in mathematical problem-solving. We will integrate Habermas construct with specific theoretical tools to frame students' processes with respect to planning and control, and we will analyse excerpts from a classroom discussion in grade 4, in order to highlight specific features of the argumentative discourses, brought to the fore in strategic games.

Strategic interaction problems

In strategic interaction problems, two or more decision makers can control one or more variables that affect the problem results. Each individual's situation is fully dependent on the move of the opponents, and the players know this fact. The decisions of each player influence the final result of the game: so every player should think, not only about his/her possible moves, but also about what other players should do if they want to construct a successful strategy. Game Theory offers suitable mathematical models for the winning strategies, based on specific assumptions about how ideal, hypercalculating, emotionless players would behave (Von Neumann & Morgestern, 1947). However, analyzing strategic interaction games as problem-solving activities, Simon (1955) argues that the limited capabilities of the human mind (memory system and the development of calculus, attention span, etc.) combined with the complexity of the external environment, make often impossible the elaboration of the strategic choices predicted by Game Theory. The difficulties concerning planning ahead are well documented and studied in Behavioral Game Theory research, which links Game Theory to cognitive science by adding cognitive details. As concerns limits on iterated thinking, the data collected by Camerer (2003) show that, during the resolution of strategic interaction problems faced for the first time, only few subjects are able to develop many thinking ahead steps (*limited strategic thinking*).

Planning and control in mathematical problem solving

Our research is based on the assumption that strategic interaction problems constitute suitable environments to develop and analyse both *planning processes* (e.g. to figure out winning strategies for the game), and *control processes* (e.g. choosing suitable semiotic resources to represent the possible outcomes of a certain move), which are important features of genuine problem-solving. In a problem solving activity, the imagination and planning processes are related to the possible actions to be performed across time. For this reason, the studies about mind times (Guala and Boero, 1999; Tulving, 2002; Atance & O'Neil, 2001) can be useful to interpret specific cognitive processes involved therein. Guala and Boero (1999) identify some examples of mind times (i.e. time of past experience, contemporaneity times, exploration time, synchronous connection time) involved in the imagination of possible actions over time. In particular, they analyse the *exploration time* pointing out the projections that can be developed from the present onward (e.g. "which strategies can I develop to find the solution?", "How can I manipulate the data to solve the problem?") or from the future back to the past (e.g. "I think up a solution and explore it in order to find the operations to be performed, depending on available resources"). The study of the planning processes can be analysed deeper taking into account the cognitive studies about the human ability to remember and imagine fact and situations in the course of time (Martignone, 2007). In particular, considering the mind construction of possible future events, we can distinguish between the knowledge that we possess about an event (*semantic future thinking*), versus thought which involves projecting the self into the future (*episodic future thinking*) to "experience" an event (Atance & O'Neill, 2001). ''Remembering'' and ''projecting'' need the ability to conceive *the self* in the past and future, which goes beyond simple "knowing" about past events and future facts. Knowledge can support and structure the imagination processes by means of the identification of frames or scripts that influence subject's expectations on stereotyped situations.

> Note that in episodic future thinking the imagination is not given free reign, but rather, the projection is constrained. For instance, envisaging my forthcoming vacation might require me to consider such factors as how much spending money I will have, how much work I will have completed before I go, and so on. (*ibid*., p.533)

Besides planning processes, also control processes play a fundamental role in problem-solving activities. As introduced by Schoenfeld (1985), control deals with "global decisions regarding the selection and implementation of resources and strategies" (p. 15). It entails actions such as: planning, monitoring, assessment, decision-making, and conscious meta-cognitive acts. In the context of argumentation and proof activities, Arzarello & Sabena (2011) show how students' processes are managed and guided according to intertwined modalities of control, namely semiotic and theoretic control. *Semiotic control* relates to knowledge and decisions concerning mainly the selection and implementation of semiotic resources. For instance, semiotic control is necessary to choose a suitable semiotic representation for a problem (e.g. an algebraic formula vs a Cartesian graph). *Theoretic control* requires the explicit reference to a theoretical aspects of the mathematical activity: it intervenes when a subject use consciously a certain property or theorem for supporting an argument.

Methodology

Methodology is based on teaching-experiments, planned and analyzed with the collaboration of classroom teachers. Activities develop around classical strategy games, such as NIM, Chomp, Prisoner Dilemma, etc., and alternate game phases with reflection phases (collective discussions, written reports). Video-recordings of the discussions and students' written reports are collected and analysed on a qualitative and interpretative base.

In the following we consider a case-study in grade 4, based on a strategic interaction game called "Race to 20", used by Brousseau to illustrate the Theory of Didactical Situation (Brousseau, 1997). The rules of the game are the following. There are two players: they know the possible alternative choices and the relative outcomes, they don not cooperate and they do not know in advance the adversary strategies. The first player must say a number between 1 and 2. The second player must add 1 or 2 to the previous number, and says the result. Now the first player adds 1 or 2, and so on… The player who says 20 wins the game.

Analysis of the collective discussion

We analyse some excerpts of the collective discussion organized by the teacher after the children have played the game several times, at first individually, and then in teams. In the discussion, students are asked i) to describe possible winning strategies and ii) to justify them. In particular, the attention (both on didactical and research planes) is on the development of argumentation processes: we analyse them as rational discourses in Habermas model. Numbers 14 and 17 are soon identified as "winning numbers". Justifications are based on the possible moves of the two players. We report Elena's contribution:

> Elena: it's necessary to arrive before at 14 and then at 17. Because if you do from 14 you do plus 1 and arrive at 15 and then you do plus 2 and arrive at 17, which then…you do plus 1 and arrive at 18 and the other does plus 2 and arrives at 20. Rather, if you do plus 2 from 14, you arrive at 16 and the other does 1 and arrives at 17, the other if he does plus 2 arrives at 19, you do plus 1 and arrive at 20. Hence anyway from 14 to 17 you arrive anyway at 20.

Elena carries out her argumentation by describing the possible moves of the players (*episodic future thinking*) who start from two particular positions (14 and 17). The *teleological aspect* is clear: she wants describe the winning choices. Because the steps of thinking are limited (*limited strategic thinking*), she manages to plan ahead only close to the winning end. When teacher asks students if there are other number like 14 and 17, different scenarios are explored by students. The number line helps them to control the winning positions and strategies, relying on the semiotic representation at the blackboard (Fig. Y).

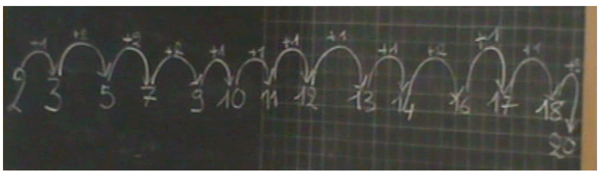

Figure Y. The written representation used to play the game.

The students' attention is focused both on backwards movements, in search of previous winning position, and on the forward movements, to check the efficacy of the strategies.

Diego: 11 maybe is an important number, because maybe my team adds 2 and it is 13, the other team adds 1 and arrives at 14, I add 1, 15, they add 2 and it is 17

Elisa: but if they are stupid they make plus 1 and arrive at 18, we make plus 2 and arrive at 20; but they are not so stupid to do plus 1, eh!

Also Diego and Elisa rely on the *episodic future thinking* to imagine the possible moves of the players and justify their hypothesis about number 11. The steps of thinking ahead are always "close" to the new winning numbers (*limited strategic thinking*). In the rational discourse of Elisa, the *teleological component* is linked to the goal of the game, getting to number 20, and it is guided by her knowledge (stressed in the discourse) that the other players have the same information and capacity. After that many students have expressed similar arguments, a general rule, which can drive all strategies, emerges:

Giulio: I think that for the winning numbers you always remove 3: from 20 you remove 3 and you arrive at 17; from 17 you remove 3 and you arrive at 14, I think that another winning number could be 11, could be…8, could be…5, could be…2

Teacher: Explain well this idea

Giulio: Because…that is I don't know, if I arrive at 2…I don't know, I begin, I make 1, no I make 2, he arrives and makes 1 (gesture in Fig. Xa), I put 2 and I arrived at 5 (Fig. Xb), which I think is a winning number… yes, arrived at 5…it is a winning number, I think. Then…he adds 2, say (Fig. Xc), I add 1 and I arrived at 8, which is another winning number. He adds 1, I add 2 and I arrive at…12, which is another winning number. He adds 2, I add 1, and I arrived at 14, which is another winning number, he adds 1 I add 2, we arrive at 17 which is a winning number, he adds 1 or 2, I add 1 o 2 and I win

Figure Xa,b,c. Giulio's gestures in his argument.

Giulio at first states a general rule to find out all the winning numbers, beyond those already found. His sentence expresses the rule in a general way, as an a-temporalized relationship between numbers ("you always remove 3" from 20). Giulio's argument is based on the *backward induction* (similar to what described in Game Theory): he starts from the winning result and moving backward he identifies the best strategy to win the game. In the argumentative discourse on the winning strategy, *the teleological and epistemic components* of rationality are on the foreground. The epistemic plane relies on the relationships between numbers, and in particular on the control over the number line model, recalled in the written schema introduced by the teacher to play the game (Fig. Y). This representation has now become a thinking tool for Giulio, who shows to have a semiotic control over it. Asked to better explain his ideas (*communicative component*), the boy imagines a match, and describes the moves in a temporalized way (*episodic future thinking*). The subtraction turns into an onward movement starting from the very first move (the number 2). This movement is produced by means of a rhythmical repetition of the same linguistic structure: “he adds…I add… and I arrive at…, which is a winning number”. Linguistic repetition is co-timed with a gesture repetition during the entire argument: gestures are synchronous with the added and numbers in the imagined game. Gestures and words together constitute a schema through which the generality of the argument is conveyed. Gesture in Fig. Xa (open hand as holding something) is co-timed with the words “makes 1”: its metaphorical nature indicates that while saying “1”, Giulio is indeed meaning “let’s say 1” or “any move of the player”, something similar to what Balacheff in proving processes called “generic example” (Balacheff, 1987). This interpretation is confirmed, besides by the voice intonation, by the gesture-speech combination of Fig. Xc: a similar gesture is performed within a similar speech schema, but now the generic nature of the example is made explicit by the word “say”.

Concluding remarks

In this paper we analysed some excerpts of a discussion about the winning strategies in a particular strategic interaction problem: the Race to 20. It provided a suitable context to study the students’ argumentations, intended as rational discourses in the Habermas construct. The analysis integrated Habermas construct with cognitive studies about mind times, and semiotic aspects of control processes. As a result of the use of the different interpretative tools, an important distinction in the teleological component of the argumentations emerged. In fact, we can identify two teleological planes: a *pragmatic* plane, related to the goal of the game (“Which strategy can I choose or develop in order to win the game?”), and a *theoretical* plane, related to justifying the chosen strategy (“How can I justify that my strategy is the best one?”). On the theoretical plane, the teleological dimension strongly intertwines with the *communicative* one, when students are asked to explain and justify their strategies to their mates. Furthermore, as we could see in the reported excerpts, the two planes are deeply intertwined: theoretical considerations can fruitful ground on pragmatic ones, and—more important—can also be justified on a pragmatic base (see Giulio’s argument). This feature is inherent to strategic interaction problems, what makes them suitable activities to develop complex argumentation activities from early school grades. We plan to validate the scope of these results on a more extended empirical base.

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