PLENARIES

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Mathematics teacher education in the institutions: new frontiers and challenges from research

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Abstract: The paper deals with teacher education in the institutional context, using the Meta-Didactical Transposition framework (based on Chevallard theory of transposition) for analysing the processes of researchers and teachers working together in designing tasks of an educational programme, training teachers in these tasks, and following them while experimenting the same tasks with their students. The research aim of the paper is to describe the features of the practices of these communities, to identify them as community of design and community of experimentation, according to the praxeologies they put into action in the processes of designing, training, experimenting. Particularly, this study is referred to a research and educational programme on MERLO (Meaning Equivalence Reusable Learning Objects) items, a didactical tool for involving students in group/individual activities that involve deep understanding in mathematics and assessing them.

Résumé: Cet article concerne la formation des enseignants dans le contexte institutionnel, en utilisant le cadre de la transposition Meta-didactique (fondée sur la théorie de transposition de Chevallard) pour analyser les processus des chercheurs et des enseignants qui travaillent ensemble à la conception des tâches d'un programme de formation, à la formation des enseignants en utilisant ces tâches, et au suivi des expérimentations de ces mêmes tâches avec leurs élèves. Le but de cet article est de décrire les caractéristiques des pratiques de ces communautés, afin de les identifier comme communautés de conception et d'expérimentation, selon les praxéologies qu'elles ont misées en place dans les processus de conception, de formation, d'expérimentation. En particulier, cette étude s'inscrit dans un programme de recherche et de formation sur les tâches MERLO (Meaning Equivalence Reusable Learning Objects), qui est un outil didactique pour engager et évaluer les élèves dans des activités de groupe ou individuelles qui impliquent la compréhension profonde des mathématiques.

This is the era of the teacher

Mathematics teacher education raised to the attention of the community of researchers in the last years in a very deep way and with many facets: educational programmes, use of digital technologies, teaching practices and methodologies, … The variety of articles and books published shows the evidence of a great complexity in the teaching processes, both in terms of teaching practices in the classroom and of teacher education, which involve the use of new technologies, new methodologies of teaching, new kind of students, new claims from the institutions. This complexity can be interpreted using different frameworks, and can be showed in either a more static or more dynamic way, or in a mixt approach, according to the framework chosen and the research aims (see for example respectively Ball & Bass, 2003; Arzarello et al., 2014; Kaiser et al., 2014). With these three kinds of perspective (not to be intended in a rigid way, of course, but as a sort of lens through which teacher processes can be observed and interpreted), we can intend: as static, highlighting features, indicators and elements of the context of teaching that seems important to describe and classify; as dynamic, the complexity of teacher profession in terms of evolution in teaching processes and in professional development; and as mixt approach all the intermediate nuances from the static to dynamic, where different elements of the two approaches are used more or less intensively.

The current research interest in teacher education is a relatively new domain when compared to other research themes relating to mathematics content, curriculum, students, learning, cognitive processes, policy and equity. This era marks an important milestone in the evolution of mathematics education, toward the teacher having an important role to play within the classroom. Sfard (2005) stresses that we are living in the *era of the teacher* and that the advent of this era has brought about a re-conceptualisation of the relationship between the teacher and the researcher, arguing that in most of the international research studies, the question is not *what* is taught in classrooms, but *how* it is taught. This paper is aimed at directing attention to the role of the teacher, and showing some activities, from the point of view of professional development and of daily work in the classroom. The role of the teacher, according to the context in which operates, has many facets. It can be the role of a teacher-researcher, if works with researchers in designing a project, a teaching experiment, an activity of teacher education. Or it can be the role of a teacher-experimenter, if is available to experiment new kind of activities and teaching practices in her/his class, observing the students' processes. It can be the role of teacher-trainer, involved in an education program as teacher of teachers, or broker/tutor of a group of teachers. Or it can be the role of teacher-learner, involved in an education program with other colleagues. These roles are not to be intended rigidly separated: they can be partially or totally overlapped, according to the grade of involvement of the teacher in the life of school, training, and research. From their viewpoint, researchers have choices to make with respect to the 'grain size' of their focus and analysis, not only with respect to the more static or dynamic descriptive approach, but also with the choice of variables to take into consideration, which can range from the individual analyses of teachers/lessons, to studies of the evolution of teaching/learning processes over time, or the observation of communities of teachers, or the introduction of new methodological tools (Clark-Wilson et al., 2014). I would like to stress the necessity of observing mathematics teacher education processes from a systemic point of view, with a focus on the relationships and dynamics between the several "variables" altogether, included in such complex processes as: teachers' knowledge and practices, results from research, institutional constraints (national curricula in particular), traditions, cultural aspects, and so on.

There are communities, and communities…

The term community is actually present in literature, in many declinations and meanings. After a brief review of the use of this term in literature, I will introduce the concept of *community of design* and *community of experimentation*, which are particularly referred to the research reported in this paper.

The introduction of the term *communities of practice* is due to Wenger (1998), who says that they are formed by people who engage in a process of collective learning in a shared domain of human endeavour: for example a tribe learning to survive, a band of artists seeking new forms of expression, a group of engineers working on similar problems, a clique of pupils defining their identity in the school, a network of surgeons exploring novel techniques, a gathering of first-time managers helping each other cope. For Wenger, communities of practice are groups of people who share a concern or a passion for something they do and learn how to do it better as they interact regularly (Wenger, 1998).

Communities of practice are groups of people who share an interest, a scope, a challenge, and so forth: they group spontaneously and they meet and work together. The characteristics of such communities of practice can vary, according to the context and the interests and tasks of participants. Some communities of practice are quite formal in organisation, others informal.

Communities of learners are less spontaneous than communities of practice (Bos-Ciussi et al., 2008). A class of students is a community of learners, in that students take part in the construction of consensual domains, and "participate in the negotiation and institutionalisation of ... meaning" (Roth & Lee, 2006). In a learning community in fact, the educational goal is to let collective knowledge advance, in a way that supports the growth of individual knowledge (Bielaczyc & Collins, 1999). Students are actively engaged in the social process of construction of meanings, and in doing so, they learn as individuals. Teacher is part of the community of course, and takes place in the process of construction of knowledge, designing and guiding it, or coaching from one community to another one (Rasmussen et al., 2009).

In their book, Borba and Villarreal (2006) focus on communities of learners with the tools they use: *humans-with-media* is their paradigm. This point of view overcomes the traditional deep-rooted dichotomy between humans and technology, because it considers learning as a process of interaction amongst humans as a community that includes tools. Media interact with humans in a double sense, namely technologies transform and modify humans' reasoning, as well as humans are continuously transforming technologies according to their purposes.

Yerushalmy and Elikan (2010) describe the unique relationship that must evolve in a class that seeks to function as an *inquiring mathematical community*, in which students' explorations are often based on use of software tools and where students constantly convey their observations, eliciting arguments from peers and from themselves. The assigned tasks required measurements and construction of a model, then the examination of the model itself. The authors claim that the students advanced in knowledge as a community of learners, learned the mathematics of function throughout the elaboration of ideas connected to the modelling actions, and developed new norms of discourse.

The experience of learning together (learning to be with others in mathematics, as written by Radford, 2006) with the use of a technological tool, can be described also in a frame that takes the multimodal production of the students, the teacher and the technology itself into account. In this semiotic-cultural approach learning mathematics is a matter of *being-in-mathematics* (Radford 2006), living in a classroom as a community, working together and sharing activities and results.

Although communities of practice have "insiders" and "outsiders", there are various ways in which communities may be connected across the boundaries that define them. Wenger (1998) discusses several types of connection, including boundary encounters — discrete events that give people a sense of how meaning is negotiated within another practice. The briefest of these encounters is the one-on-one conversation between individuals from two communities to help advance the boundary relationship. For example, after participating in a one-off professional development workshop, a mathematics teacher might have a conversation with the presenter about some aspect of the workshop that could inform the teacher's practice. A more enriching instance of the boundary encounter involves immersion in another practice through a site visit. This may occur when a mathematics educator visits a school over a period of weeks or months to collect classroom data, such as lesson observations or interviews with teachers and students, for a research project. Particularly important in boundary encounters are the brokers (Rassmussen et al., 2009), who mediate the passage of practices and knowledge from one community to another, and possibly are part of both of them. Their role is fundamental in teacher education. For example, Goodchild (2007) extends Wenger's theory of community of practice to conceptualise teacher and didactician learning as taking place reflexively within *communities of inquiry*. Teachers and didacticians together form a *project community*. Through participation in established communities of practice (school or university) teachers and didacticians use inquiry as a critical tool to promote learning within the project. Inquiry results in critical alignment with the norms of established practice, allowing teachers and didacticians to act within their practice while at the same time questioning its dynamics and exploring new ideas (Jaworski, 2006).

Jaworski (2008) proposed the term *community of inquiry* from Wenger's work: "In terms of Wenger's (1998) theory, that *belonging to a community of practice* involves *engagement, imagination* and *alignment*, we might see the normal desirable state as *engaging* students and teachers in forms of practice and ways of being in practice with which they *align* their actions and conform to expectations...

In an inquiry community, we are not satisfied with the normal (desirable) state, but we approach our practice with a questioning attitude, not to change everything overnight, but to start to explore what else is possible; to wonder, to ask questions, and to seek to understand by collaborating with others in the attempt to provide answers to them (Wells, 1999). In this activity, if our questioning is systematic and we set out purposefully to inquire into our practices, we become researchers." Jaworski (2008, pp. 313-314). Moreover, the asking of questions is a developmental tool in drawing students, teachers and didacticians into a deeper awareness of their own actions, motives and goals.

This is an excellent way to teach students to become curious, to questioning, to conjecturing and arguing, just as in a laboratory: the *mathematics laboratory* (Arzarello et al., 2006) is the name Italian research has given to this kind of methodology to be established and used in the classroom. Anyway, it is not automatic that a teacher shifts from a traditional way of teaching (mostly based on lecturing) to an inquiring way, mostly based on laboratory activities. For that reason, it is quite important that teachers learn to be and to work in communities of inquiry of colleagues in order to transfer these practices in the class with their students. So, the term community is particularly useful in considering and treating teachers as learners, in pre-service or in-service educational programmes. The term community in fact has been used also in professional learning of practising teachers (Llinares & Krainer, 2006). Speaking of communities of teachers, we have also to consider the larger social system in which the community is nested. For example, teaching mathematics in a particular secondary school is linked to the larger social system of secondary school education in a region or a country. Communities of practice have a common cultural and historical heritage, and it is through the sharing and re-construction of this repertoire of resources that individuals come to define their relationships with each other in the context of the community. Based on this description, Goos (2014) argue that mathematics teachers and mathematics education researchers are members of distinct, but related, communities of professional practice. Goos (2014) examines ways in which teachers and university-based mathematics educators might work together to develop theoretical and practical knowledge, using an analytical frame (Novotna & Goos, 2007) raised from questions and issues identified by participants to Psychology and Mathematics education (PME) working session in discussing their own experiences in research and development work with teachers, being particularly interested on how the researcher–teacher partnership begins (e.g., a university-based researcher seeks out teachers to participate in a project that has already been planned), and on which are the practices developed by the two communities.

Also Arzarello et al. (2014), Aldon et al. (2013) consider the two distinct communities: the *community of teachers* and the *community of researchers*, interacting in a research and educational programme. With the Meta-didactical transposition, these communities are described and analised in terms of their praxeologies and their possible evolution over time, giving data on the process that takes place and gives changes in both communities. The community of researchers generally reflects upon the nature of, and reasons for, the changes produced by the teachers' education programme, and possibly shares such reflections with the community of teachers. This can result in new researcher praxeologies. Also the teacher praxeologies may change, and develop into new teacher praxeologies, a process that can repeat and further refine itself. As Goos said, there are two distinct communities, but during the process they may become nearer, sharing praxeologies or some components of them, thanks to the interaction. The model of Meta-didactical transposition is useful in describing the evolution to these shared praxeologies.

In this paper, I will introduce the terms *community of design* to indicate the community of researchers and of teacher-researchers working at the task design of activities for classes, and the *community of experimentation* to designate the community of teachers who carry out the teaching experiments involving the tasks designed by the previous community.

Teacher education in the institutions

One of the themes in CIEAEM67 aims exactly at rethinking the complexity of teacher education in terms of resources and obstacles for teaching and learning mathematics.

Taking for granted this complexity, I will use the *Meta-Didactical Transposition* model (MDT), to describe teachers' activities in a dynamic way, namely as processes evolving over time (Arzarello et al., 2014). This framework is properly built to highlight the need to take the complexity of teacher education into account with respect to the institutions in which the teachers operate, alongside the relationships that teachers must have with these institutions. To address this need, the framework is constructed from Chevallard's (1985) *Anthropological Theory of Didactics* (ATD), which is mainly centred on the transposition of mathematics managed by the teacher with the students in the classroom. In particular the model refers to the notions of didactical transposition and praxeology. According to Chevallard, a praxeology is made of a task, a technique, a technology and a theory: the first two are the pragmatic side of it, while the other two are the theoretical counterpart, which justify the first two. Chevallard defines didactical transposition as the transition from knowledge regarded from an epistemological point of view by the community of experts, to knowledge as something to be taught and learnt. Since the aim of the MDT model is to frame and reflect on teacher education programmes, the term "didactical" has been substituted with "meta-didactical" (Aldon et al., 2013; Arzarello et al., 2014), to stress that the processes under scrutiny are, in this case, the practices and the theoretical reflections developed within teacher education activities. In other words, in the case of teacher education programmes, fundamental issues related to the didactical transposition of knowledge are faced at a meta-level, the level of teachers as professional figures and as learners in communities. The complexity of teaching processes can be interpreted particularly well with this framework, because it gives the possibility to describe these processes in a dynamic way, taking into account modification of practices over time, changes in teaching, using materials, and introducing technologies, not only teachers' knowledge at a certain stage.

Actually, the MDT framework is particularly useful in the description of the evolution of teachers' processes over time, because it gives a model for analysing the different variables involved: *dialectic interactions* between the communities of teachers and researchers; *components of teachers/researchers praxeologies* that change from external to internal or viceversa; and *brokers,* who support teachers, interacting together.

The framework is essential in situations such us national/regional teachers' educational programmes, contextualised in the institutions, where researchers fix a research project in which the educational programme is inserted, then design the programme with its activities and actions, in collaboration with teacher-researchers, and carry it out, involving teachers from schools as learners in communities with colleagues. The involvement of teachers can be done at different geographical levels and institutional modalities: through a national educational programme, or according to professional development needs at local level (region, province, city, net of schools…), or based on a curriculum or assessment change introduced by the Ministry, etc.. In any case, I will use the framework of meta-didactical transposition for describing evolutions in practices of the communities involved in working together, namely:

A. The *community of design*: researchers and teacher-researchers involved in the process of task design (phase 1 below) that guides to a final product in terms of tasks for students;

B. The *community of experimentation*: teachers and trainers involved in the professional development (phase 2 below) and in the teaching experiments in the classrooms (phase 3 below).

The analysis of the mutual interactions between the communities involved (researchers, teacher-researchers – who are also teacher-trainers – and teachers) during the two processes (design and experiments in the classes) can evidence the role of each community, the relationships with the others, and the possible change of the praxeologies or of parts of them.

In this way, we can give ideas on the CIEAEM theme to which I referred at the beginning of this section. One of the main questions of this theme is:

> *How can the social dimension become a resource for teacher education? What are the challenges of programs strongly based on social interaction in communities of practice/enquiry?*

This question is properly fitting with my research interests, on different phases of the project:

1. The process of task design (community A above) in a community composed by researchers and teacher-researchers, working strictly together;
2. The process of teachers' training, where the community of teachers as learners is guided by the community trainers, who make their action of mentoring and brokering and often are the same teacher-researchers who take part of the design phase (community B above);
3. The process of experimentation of the tasks – produced in phase 1 and solved by teachers in phase 2 – by teachers in their classrooms, with the tutoring action of trainers (community B above).

I will examine praxeologies related to the communities involved in the same research and teacher training programme (Master for prospective mathematics teacher trainers) at the Department of Mathematics, University of Turin:

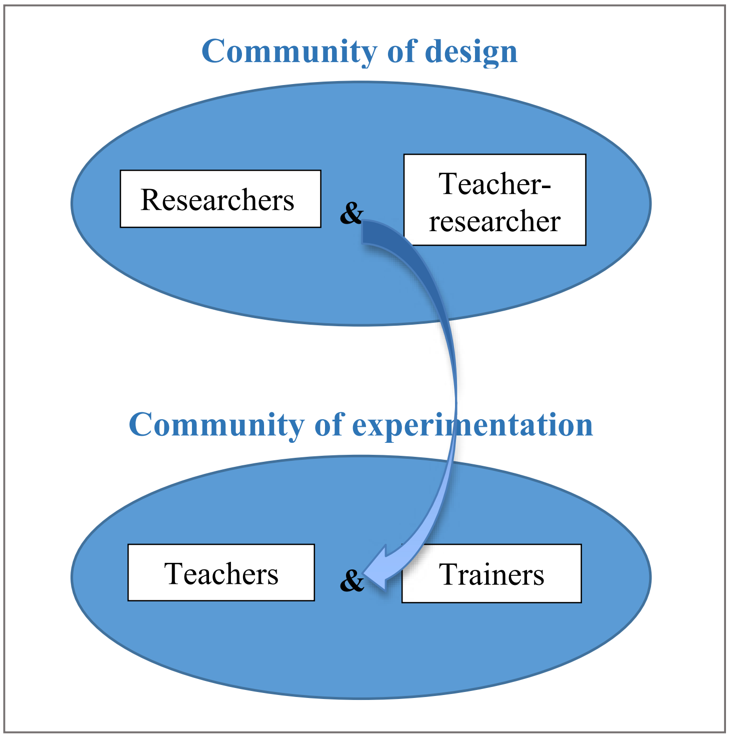


Figure 1: communities of design and experimentation (trainers in the second are often teacher-researcher of the first community)

New frontiers from research to teacher education: MERLO items

MERLO is the acronym for Meaning Equivalence Reusable Learning Objects; it is a didactical and methodological tool developed and tested since the 1990s by Uri Shafrir and Masha Etkind at Ontario Institute for Studies in Education (OISE) of University of Toronto, and Ryerson University in Toronto, Canada (Shafrir & Etkind, 2010). MERLO is a very adaptable tool, suitable for several subjects and based on equivalence of meaning across different kinds of representation. It is particularly recommended in mathematics: as it is well known, mathematical objects are sophisticated cultural products that are accessible only by means of representations (Duval, 2006), that is through suitable semiotic representations. As Duval points out "there is no noesis without semiosis" (Duval, 1995, p. 5, 22: Noesis is the intentional act of intellect, and can be defined as the action and the effect of understanding). This is the main reason why semiotic systems are central to mathematical activities and understanding, as pointed out by many scholars, such as Duval himself, Johnson-Laird (1983), Peirce (1931-1958, 1977), Sfard (2000) and Leung, Graf and Lopez-Real (2006). A first consequence of such a situation is what Duval calls the "cognitive paradox" of access to mathematical objects: "How can [students] distinguish the represented object from the semiotic representation used if they cannot get access to the mathematical object apart from the semiotic representations?" (Duval, 2006, italics in the original). A second consequence is that for grasping the meaning of mathematical objects one must cope with the multiple semiotic representations of the same mathematical object in more than one semiotic register, and with their mutual relationships. These capabilities are fundamental for understanding mathematics and consequently crucial for its effective learning (Duval, 2006). The ability to shift from one representation of an object to another representation of the same object is a competence that students should acquire in order to access the underlying meaning (Duval, 2006). The ability to shift between representations is evaluated in international (PISA, TIMSS) and national assessment tests (INVALSI, in Italy). MERLO approach is in line with these directives. Moreover, it is also a didactical tool for avoiding or overcoming the so called "duplication obstacle" (Duval, 1983), a real source of difficulty in mathematical learning as reported in literature (Fishbein, 1987), sometimes ignored or underestimated by teachers and currently difficult to be solved in the practice of the didactics of mathematics. This kind of obstacle leads students to the consideration of two representations of the same mathematical object as two different mathematical objects, but also, conversely, it may represent students' inability in grasping two different meanings of a mathematical object in only one representation.

MERLO (Arzarello et al., in press; Etkind, Kenett, Shafrir, 2010) is a collection of items, which allows the sorting and mapping of relevant concepts within a discipline, through multi-semiotic representations in multiple sign systems (numeric, graphical, symbolic, verbal, …). Specifically, each element in the MERLO database is a structured item, anchored to a target statement that describes a conceptual situation and encodes different features of an important concept; each element also includes other statements that may or may not share the meaning with the target. In a mathematical context, for example, an element of MERLO database could be about "parabola": then this element could include a target statement with the definition of parabola, and other statements in different kinds of representations (symbolic notation, Cartesian graph, table) that share or not share the meaning with the definition of parabola. The figure below is a template for constructing an item anchored to a single target statement.

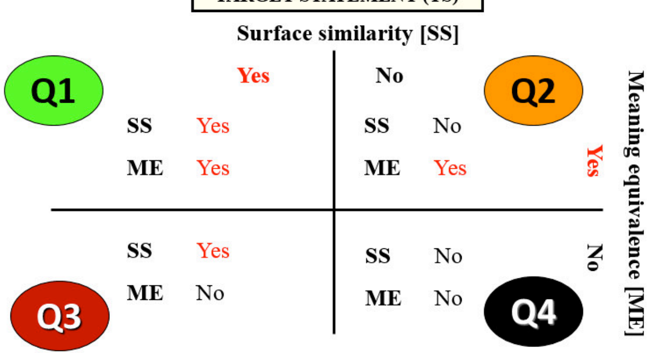


Figure 2: template for constructing a MERLO item

Statements in the four quadrants of the template - Q1, Q2, Q3 and Q4 - are thematically sorted by their relations to the target statement that anchors the particular item. They are classified by two sorting criteria: *Surface Similarity* and *Meaning Equivalence* with respect to the target. The term *Meaning Equivalence* designates a commonality of meaning across several representations. The term *Surface Similarity* means that representations "look similar": they are similar only in appearance, sharing the same sign system, but not the meaning. Hence, for example, at an intuitive level the statements "two plus one" and "two plus three" share Surface Similarity but not Meaning Equivalence, while the statements "two pair" and "2+2" share Meaning Equivalence but not Surface Similarity.

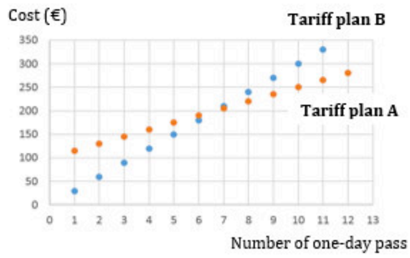
A typical MERLO activity contains five statements: a target statement plus four additional statements of type Q2, Q3 and Q4; they can be in a variable number, provided that at least one Q2 statement is present, in addition to the TS. The inclusion of Q1 statements is avoided, because creators' experience shows that inclusion of this kind of statement makes the activity too easy (Etkind M., Kenett R.S., Shafrir U., 2010), for their equivalence both in appearance and in meaning with TS.

Here is an example of MERLO activity designed by a group of teacher-researchers enrolled to a Master programme for Italian in-service teachers (future teacher educators), held in the University of Turin, Department of Mathematics. This MERLO activity is inspired by a question asked in a test of INVALSI, the National Evaluation Institute for the School System (INVALSI, 2012) and requires recognition of relations and functions in different semiotic systems.

Mario is going on holiday at a ski area. To take advantage of ski lifts (chair lifts, cable cars, ...), he can choose between two offers, A and B, both valid for the entire winter season.

<table>
<tr><td></td><td>TS</td><td>Q2</td></tr>
<tr><td>1. Mark the statements that share the same meaning (two or more).
2. Write the reasons that guided you in the choice</td><td>A[ ]

Tariff plan A: initial fixed cost 100 € plus 15 € per day (for every day you use ski lifts).

Tariff plan B: 30 € per day, with no initial cost.</td><td>B[ ]
</td></tr>
<tr><td>Q2</td><td>Q2</td><td>Q4</td></tr>
<tr><td>C[ ]
<table><tr><td>Number of days in which you use ski lifts</td><td>**Tariff plan A** (cost in €)</td><td>**Tariff plan B** (cost in €)</td></tr><tr><td>1</td><td>115</td><td>30</td></tr><tr><td>2</td><td>130</td><td>60</td></tr><tr><td>3</td><td>145</td><td>90</td></tr><tr><td>4</td><td>160</td><td>120</td></tr><tr><td>5</td><td>175</td><td>150</td></tr><tr><td>6</td><td>190</td><td>180</td></tr></table></td><td>D[ ]

Tariff plan A: $c = 100 + 15\,g$

Tariff plan B: $c = 30\,g$

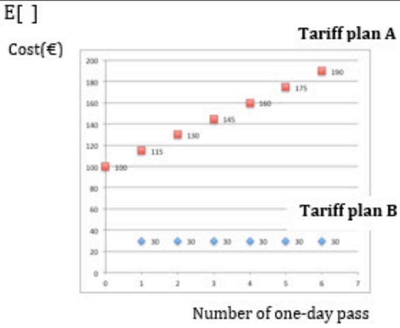
g number of one-day pass
c cost (€)</td><td>E[ ]
</td></tr>
</table>

Figure 3: an example of MERLO activity (teacher's papersheet)

As we can see from the figure the activity is linked with a real life context and shows:

- A natural language description of two tariff plans, chosen as target statement TS;
- The same tariff plans represented in a different way (Cartesian graph, table and formal language) as Q2 statements, that share meaning, but do not share surface similarity with TS;
- Another Cartesian graph, chosen as Q4 statement, that does not share neither meaning, nor

surface similarity with TS.

In the version of MERLO activity for students, obviously, the type of each statement is not revealed and the position of statements can be changed by the teacher. The task for students (in a student's paper-sheet, without labels Q, TS) is to recognize the statements in multiple representations that share the same meaning and to write the reasons for the choice. In this way MERLO activity combines multiple-choice (recognition) and short argumentation answer (production). The correct solution of a MERLO activity gives a feedback on students with two main scores: recognition score and production score. The first score comes from the recognition of statements with shared meaning among the given 5 statements, while the second score comes from the writing production of reasons for the decisions. This feedback is useful also to the teacher, for getting information about the level of understanding (the so called "deep understanding") of their students on a particular conceptual knowledge, and for that reason MERLO can be used in formative assessment.

These items are completely new in Italy, both in the community of researchers and that of teachers at national level. For that reason, their introduction with teachers is particularly delicate and has to be discussed not only in the value of the mathematics involved, but particularly for what deals with their use with students. They can be used as well as a tool for an activity aimed at discussing and arguing, and as a tool for formative assessment (at this moment, we do not have data about the use of MERLO for summative assessment).

If interested in deepening the research, the teacher education, and the kind of collaboration between researchers and teachers in practical activities (of design-training-experimentation of MERLO items), one can read in these Proceedings the paper (Arzarello et al., 2016).

Community of design and community of experimentation in the context of MERLO project

The work of the communities and their dialectic interactions are specifically described introducing their tasks, and the theoretical reasons of them. The communities involved have been described above (A and B) and are respectively a community of design and a community of experimentation. The components of praxeologies involved in the work and interaction of communities are referred to the tool studied, designed, and applied in the teaching experiment: What the design community do (phase 1), is to produce MERLO items to be used at school by teachers, who are involved in a teaching education programme (Piano Lauree Scientifiche), aimed at professional development and introduction of teaching experiments in the classes of the teachers participating at it. What the experimentation community do (phase 2 and 3) is to use the MERLO items in the educational programme, then to experiment them in their classes, observing students' processes.

Stating from the experience of working on design of MERLO items, we can have a general description of the steps to go through in designing an activity of this type. First, the concept and the clusters are part of the choice of the conceptual node on which you plan to concentrate the didactical activity. It is a decision to be taken before the definition of TS, according to the curricular reference given by the institutions. For example, the design community chose to create an item MERLO starting from an INVALSI question (namely, a question taken from the national school assessment test). This experience of the community allowed to face with several theoretical aspects, which are critical and important for the design phase, and related to the experimentation phase: to touch a recent mathematical aspect from the point of view of its definition, representations, or properties, or to direct questions of consolidated work made in the past, soliciting a deep understanding of studied concepts. The design community worked at preparing a wide set of MERLO items, following the most important concepts of the National Curriculum (Indicazioni Nazionali). The community prepared the items according to an organisation of concepts in grapes:

in such a way, the teacher may have more than one item linked to the same concept (or concepts strictly related to it), to be used in individual/group activities, in class or at home, and in assessment (formative/summative). This phase of design is particularly delicate, for the choice of the different statements of MERLO items within a boundary of meaning, according to each grape. The boundary of meaning represents the set of representations that share the same meaning with a representation chosen as TS: all of them are meaning equivalent.

The design of an item passes through some steps, in order to obtain different forms of meaning equivalence: different semiotic representations of a concept, or situations requiring the application of the concept, or logical consequence of a statement. So, preparing the items, the community of design encountered some situations where to make choices, obtaining then a methodological procedure in the design, and - accordingly - developing and applying some praxeologies typical of the design. Some of them are described in the following:

a. In preparing the statements TS and Q2, the designers choose a concept or a grape of concepts, and write a statement as TS, then find one or more Q2 (statements that share the same meaning with TS, in various ways as written above – different representations, or logical consequence, and so on). In this phase, the designer made the choice to consider a Q2 statement more or less close to the TS, according to the kind of meaning sharing (very close if only a translation of representation in another register, or far if necessary to make some passages to obtain the Q2 from the TS, with other nuances in the middle). The three distances from TS - very close, in the middle or far – are what the designers called distance from the target.

b. Willing to prepare a Q3, the designers have to use surface similarity to represent a concept in the same semiotic register as TS, but with different meaning: these Q3 do not belong to the same boundary of meaning as TS and Q2.

c. For Q4, the work is different, because it does not have meaning equivalence, nor surface similarity with TS and Q2, so, it is out of the boundary of meaning, but also out of the same register of TS. A choice that the community did, is to have Q4 that do not share meaning with Q3, to avoid the risk that the students have in front two sets (one set: TS and the Q2, the other set: Q3 and Q4) with separate boundaries of meaning, and to avoid the risk of choosing Q3 as TS and Q4 as Q2. Since the students does not know which is the Target Statement TS (they have to choose only all the statements in the item that share a meaning), the designer have to put attention in preparing some statements that share a meaning, and the others completely not related among them.

d. Having prepared a set of five statements, they group together to obtain a MERLO item (as in Figure 3), and have in this way the teacher's paper-sheet (with labels TS, Qi), and the student's paper-sheet (without labels).

e. Generally, the MERLO item is composed of two or more TS and Q2 altogether, and at least one Q3, because it allows investigating the capability of the students to go beyond the outward appearance of a mathematical object. In addition or alternatively there may be a Q4 (or more than one), which plays a similar role. The quantity of Q2 is not determined a priori, but depends on the production of equivalent representations that you can do.

The design of MERLO items and of their modalities of implementation at school has been described in details with the previous points, which can be seen as praxeologies of the design communities of researchers and teacher-researchers: in term of task, the construction of MERLO items; in terms of technique, the specific issues described in the previous points, in relation to the TS and Qi compositions and connections (see the points above, particularly b and c); in terms of technology, the mathematical (epistemological) and didactical/institutional (related to the curriculum) reasons to have a coherent MERLO item, and in terms of theory the mathematical meaning of a concept, its semiotic representations and all the references (Duval, and so on) described before.

The training phase took place after the design phase, and involved those teacher-researchers who participated to the design, now with the role of teachers’ educators in the Piano Lauree Scientifiche programme. The praxeologies of the trainers, who have an important involvement as brokers - because members of different communities – can be described briefly in the following. They have the task of educate teachers in using MERLO items in their classes, and their technique is the presentation of the MERLO activities to teachers, giving them examples to solve and to discuss on (Arzarello et al., 2016); and the theoretical reasons (technology and theory) lie on the corresponding of the praxeologies of design community, on the didactical side as dissemination of the research side (the use of MERLO in mathematics laboratory, with discussions, group work). For example, the trainers make the teachers aware of the design phase, with the connections and implications and choice of TS and Qi, and on the sharing of meaning in different representations. Particularly useful for teachers in the training phase is to be available in experimenting new didactical methodologies, with the use of new praxeologies or components of them, and to discuss with brokers about the pedagogical implications of the use of MERLO items in class.

As described in another paper of these Proceedings (Arzarello et al., 2016), MERLO pedagogy as carried out in our teaching experiments, is made of:

- An *individual phase*, where students, have to identify in a MERLO item those representations that share the same meaning and to write the reasons for their choice.
- A *group phase*, where students in in groups have to compare their personal choices with those of their group-mates, discussing for arriving at the ultimate goal of a shared answer.
- A *class discussion*, coordinated by the teacher with the aim to clarify and reflect on the deep understanding of the concept in relation to its representations that share the same meanings.

In this experimentation phase of MERLO items, the teachers do what they have learnt in the training phase in terms of MERLO pedagogy, and those teachers who feel not sure to do it by themselves, are supported by a trainer in the class. Their praxeologies have as task and technique the application of MERLO items with their students, using MERLO pedagogy (individual-group-class phase described above), while as technology and theory the elements that justify task and technique: the equivalence of meaning, the boundary of meaning, the use of mathematics laboratory, instead of lecturing and assessing in a traditional way.

In the related paper in these Proceedings (Arzarello et al., 2016) there are several examples referring to the design of MERLO items, their implementations in teaching experiments and data on the results.

Discussion

From the research point of view, what is important is to study these praxeologies or even only one or more components of them, which at the beginning of the training phase are absolutely external to teachers’ community, and at the end of this phase and the experimentation phase become internal. This means that in the teacher’s professional life something has changed, for example passing from a traditional lecturing to the use of MERLO items in individual and group work among students, or in the formative assessment. Not all the teachers can do it in the same time and in the same ways, but everyone follows her professional history, motivation, and aims. And this is the main result observed through the lens of meta-didactical transposition, in this experience but also in other experience of teacher education. When a teacher says (Arzarello et al., 2016):

> Using MERLO in oral questions in class, it is easier for me to know students mental processes. Because some of them make a choice but do not write anything about arguing, for several reasons…

we have reached a twofold aim: one didactical, as an improvement of her pedagogy according to theoretical constructs from mathematics education, and one of research, because the teacher has changed something in her approach to teaching, with respect to the past.

Moreover, we have the possibility to observe the birth and growth of new professional figures:

- The teacher trained in MERLO pedagogy and involved in her professional change, who becomes a leader in her school, organising meeting with colleagues for comparing results, discussing about MERLO pedagogy, and connecting this novelties with the institutional constraints (curriculum, assessment, books, …);
- The teacher trainer who not only applies in educational programmes for teachers, but also wants to jump in the research context, and participate to national/international seminar, congresses not only for learning, but also for presenting her experience of designer of MERLO items, of broker in the teachers' community, of researcher herself.

These new professional profiles are a product of the process of meta-didactical transposition not only for their roles, but also for all the competencies and praxeologies they are carriers and witnesses in other communities. Therefore, a challenge for future research in teachers' professional development could be the study of these professional profiles in details.

And also for teacher education and teaching to students there can be new challenges: the introduction and application of MERLO items in the institutions, namely for all the curricular themes and concepts, according to the institutional guidelines on curriculum and assessment.

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