Daily Prediction of Demand and Supply Curves

Previsione giornaliera di curve di domanda e offerta

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Abstract We propose a model for the analysis of functional time series subjected to monotonicity and bound-constraints on the codomain. In detail, we provide the space of constrained functions with a suitable pre-Hilbert structure and then model them by mean of a functional autoregressive model. The autoregressive lagged operators and the non-centrality function of the model are estimated by minimizing the squared L^2 distance between functional data and functional predictions with a penalty term based on the Hilbert-Schmidt squared norm of the autoregressive lagged operators. Moreover, we prove the existence and uniqueness of the corresponding estimators. Finally, the method is successfully applied to the analysis of daily demand and supply curves of the Italian Natural Gas Balancing Platform.

Abstract *Proponiamo un modello per l'analisi di serie storiche di dati funzionali soggetti a vincoli di monotonia e limitatezza nel codominio. In dettaglio, dotiamo lo spazio delle funzioni vincolate di una struttura pre-Hilbertiana per poi modellarle tramite un modello autoregressivo funzionale . Gli operatori di ritardo del modello autoregressivo e la funzione di non centralità sono stimati minimizzando il quadrato della distanza quadratica L^2 tra i dati funzionali e le loro predizioni funzionali penalizzata tramite un termine basato sul quadrato della norma Hilbert-Schmidt degli operatori di ritardo del modello autoregressivo. Inoltre dimostriamo l'esistenza e l'unicità degli stimatori corrispondenti. In fine il metodo applicato con esito positivo all'analisi delle curve giornaliere di domanda e offerta del Mercato Nazionale del Bilanciamento del Gas Naturale.*

Key words: Functional Time Series, Constraints, Supply and Demand Curves.

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1 Motivation

Motivated by price prediction in Italian natural gas balancing market, we propose a model to forecast supply and demand curves evolving day by day. The approach is of general interest and can be generalized in any situations in which one has to deal with constrained functions which evolve through time. Functional Data Analysis provides an extremely useful set of tools to deal with data that can be modeled as functions (e.g., demand and supply curves). Differently from the most common framework in Functional Data Analysis, we hereby focus on functions that are constrained (i.e., monotonic, lower and upper bounded, and with an equality constraint on one edge of the domain and an inequality constraint on the other edge) and temporally dependent.

2 Model

The approach we are going to present is in line with the so called transform/back-transform method. The idea is to find a bijective map from the space of the generic functions to the subspace of the functions satisfying some given constraints. The idea is indeed to transform data such to perform an unconstrained estimation on the transformed data and then back-transform the estimated function to the constrained subspace.

In detail, during the talk we will introduce a suitable pre-Hilbert structure on the set of monotonic, lower and upper bounded functions satisfying an equality constraint on one edge of the domain and an inequality constraint on the other one. In the rest of this document we will refer to this pre-Hilbert space as the $\mathscr{M}^2$ space. In particular, we will provide $\mathscr{M}^2$ with a vector space structure defining the operations of addition and scalar multiplication and provide it with a suitable inner product and the corresponding norm and distance. Then, we will introduce a suitable bijective map from $\mathscr{M}^2$ to L^2 and build an entire geometry on $\mathscr{M}^2$ which makes the former map isometric with respect to the geometry induced by the usual inner product in L^2 and thus allowing us to work in an unconstrained framework.

To deal with temporal dependence we will introduce a non-concurrent functional autoregressive model (FAR) in the space $\mathscr{M}^2$. We will basically assume to deal with a collection of random functions in $\mathscr{M}^2$ generated sequentially through discrete time t. In detail, we will assume that today function linearly depends (in an $\mathscr{M}^2$ sense) on the values assumed by the random functions earlier appearing in the sequence potentially at each domain location.

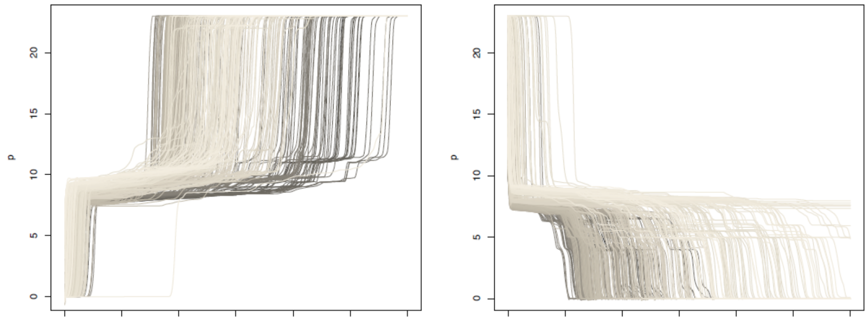

Fig. 1 Smoothed functional time series of demand (left) and supply (right) curves. Color denotes time, with the oldest curves in dark and the most recent ones in bright.

3 Estimation

Before moving to the application, we will present the estimation method used in the application which is based on the direct minimization of the prediction error. In detail, we estimate the lagged autoregressive operators and the non-centrality function by minimizing of an objective function being a weighted sum of a goodness-of-fit term and a penalty term. The first term of the objective function is the sum of the squared residuals (between the observed values and their predictions) according to the L^2 distance. The lower this term, the better the fit of predictions to data. The second term is instead the sum of the squared Hilbert-Schmidt norms of the lagged autoregressive operators. The lower this term, the lower is the autocorrelation associated to the estimated model. For any weighting of the two terms we prove the existence and uniqueness of the estimators of both the lagged autoregressive operators and the non-centrality function and provide also their explicit expressions. Moreover we prove that the plug-in predictions belong to $\mathscr{M}^2$.

4 Data Analysis

The data used in our analysis refer to the first thirteen months of the PB-GAS, namely from December 1st, 2011 to December 31st, 2012. The data are available at [2]. Before applying the model described in Section 2 row data has been converted to functional data in $\mathscr{M}^2$. The obtained smoothed functional time series are plotted in Figure 1.

To asses model goodness of fit, for increasing value of the autoregressive order p, we will compare the root mean squared error (in an $\mathscr{M}^2$ sense) between the predicted curves and the original ones. We will show the results of the analysis and pointing out the fact that: (i) the autoregressive order p has similar impacts in the estimation of both the demand and supply curves (i.e., for fixed p, the errors for the demand and supply curves are roughly of the same order of magnitude); (*ii*) as expected, increasing p the estimates improve, with the larger improvements observed moving from $p = 1$ to $p = 2$ and moderate ones moving from $p = 0$ to $p = 1$ and from $p = 2$ to $p = 3$ suggesting $p = 2$ as a good candidate value; (*iii*) our procedure is outperforming the simpler functional exponential smoother.

We will conclude our talk by giving an example of the tremendous additional insights that the whole curve forecast can give to traders with respect to the simple trading price prediction.

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References

1. Canale, A., Vantini, S. (2014): Constrained Functional Time Series: an Application to Demand and Supply Curves in the Italian Natural Gas Balancing Platform, MOX Report 42/2014, Dept. of Mathematics, Politecnico di Milano.
   Available at https://www.mate.polimi.it/biblioteca/add/qmox/42-2014.pdf
2. Gestore Mercati Energetici: https://www.mercatoelettrico.org/En/Mercati/Gas/PGas.aspx