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Perceiving and creating in the mathematics classroom: A case-study in the early years

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This paper draws on recent research on the theorizing of embodiment in mathematics thinking and learning to adopt a non-dualist perspective that challenges the view that mathematical concepts cannot be perceived and created. This perspective brings out the intensive and immersive aspects of mathematical activity that feed the potential and the mobile in the classroom. Through the analysis of two 8-year old children, who reason on a figural pattern, I show how their ways of talking, moving and feeling allows them to mobilise and invent the mathematics they are learning. In so doing, I propose that perceiving is conceiving and creating is learning.

Keywords*: Creativity, embodiment, mathematics learning, materialism, perception, virtual.*

INTRODUCTION

In the last decade or so, lots of studies of embodied mathematics focused on the role and relevance of bodily activity in mathematics teaching and learning. Far from being an emergent generation of research, this corpus of work has started offering attempts to talk about and understand mathematical activity in *non-dualist* ways. Examples are studies of researchers like Nemirovsky, Radford, Roth and, more recently, de Freitas and Sinclair. No matter what their theoretical stances are, whether phenomenological, semiotic, philosophical, etc., they all embrace visions of a 'multimodal' or 'sensuous' mathematical cognition that recognise a special place to physical bodily aspects in the classroom, without assuming the existence of "two distinctive planes, one internal and one external" (Radford, 2013, p. 144). They all pursue a *participationist* view of teaching and learning that moves away from the constructivist tradition started with Piaget, which considers the mental schemas that students are expected to acquire. And beyond the mechanistic view still present in Lakoff and Nùñez's (2000) seminal embodied cognition theory, which fails to escape the mind/body split by inferring metaphorical mappings in the mind as sites of/for knowledge. Acquisitionist theories, based on structural concept formation, also entail levels of abstract thought confining mathematical concepts to abstractions, to de-personalised intangible and immaterial entities that *cannot* be perceived. Sfard's (2008) communicational theory shares the participationist commitment, by focussing on the ways in which students and teachers change their mathematical discourses, and conies the term 'commognition' to stress how thinking *is* communicating. But, sort of struck by all the 'fuss' about the body, and gesture in particular, it resists discussions about its participation in mathematics classroom discourse.

In this paper, I adopt a non-dualist participationist position that pursues a different vision of perception and creation in mathematics, according to which bodily activities are ways of thinking as well as of communicating (and feeling, I argue). In this way, I hope to contribute to the theorizing about the embodied nature of mathematics and of its learning.

THEORETICAL PERSPECTIVES

Sheets-Johnston challenges our ways of theorizing embodiment in thinking, shifting attention to "our being the animate organisms we are" (2009, p. 397) and "living moving bodies", which "*feel the dynamics*" of their everyday tactile-kinesthetic/affective experiences (2012, p. 393, emphasis in the original). There is no question then but that animate beings are not "*embedded* in the world" or "*embodied* in their actions, their emotions, their cognitions"—as Freedberg and Gallese (2007) would suggest on the basis of research on the mirroring system of the brain, which is in turn rooted in the realities of movement (Sheets-Johnston, 2009, p. 397, emphasis in the original). Animate beings, claims Sheets-Johnston, "are already living, and being already living, are already making sense of themselves and of the world in which they find themselves and of which they are a part." (p. 397). She is here suggesting a new image with respect to the 'I think therefore I am' *à la* Descartes. One that we might refer to as 'I move therefore I am', which entails that moving *is* thinking, so gesturing *is* thinking—as much as communicating *is* thinking (the other way around in the commognitive perspective).

Nemirovsky and colleagues (2013) stress a resonant point of view when they make a parallel between music and mathematics to investigate kinesthetic activity in museum exhibits where learners 'play mathematical instruments'. Fluent use of the instrument involves an interpenetration of perceptual and motor aspects of playing it. Kinesthetic activity is relevant here in two ways: "motor activity is involuntarily enacted as part of perceiving", and "partial motor and perceptual components have the power to elicit the activity as a whole over time" (Nemirovsky et al., 2013, p. 380). Working from a non-dualist approach to tool use, the authors again trouble the dichotomies between thinking and acting, *perceiving and conceiving*. In my own study on multimodality in mathematical activity (Ferrara, 2014), I examine kinesthetic activity in the context of motion detector use and I propose to see mathematical thinking in terms of floating intricate intensive entanglements of ways of perceiving, moving and imagining. Here, I follow Burbules (2006) in claiming that experiences engage our imagination "when we can interpolate or extrapolate new details and add to the experience through our own contributions", so that we may be "making guesses about things that are not immediately present to us" or "anticipating what will happen next in some sequence or development." (p. 41). Imagination, depending on students' active response and engagement in the activities, triggers feelings of *immersion*, senses of "as if", which make the experiences virtual experiences for the students. A key dimension of this quality of immersion that, for Burbules, "makes the virtual seem or feel "real" to us" at that moment, are "our posture, body tension, and startle responses" or "our relaxation, rhythmic movement, and kinesthetic sensations" (p. 42)—and he takes here any truly educational experience as being immersive, or virtual, as much as watching a film, hearing a story and listening to music. As I have stressed, this sense of immersion reconfigures mathematics learning as an alive and *genuinely creative* adventure.

A way of re-framing creativity in the mathematics classroom is offered by the new inclusive materialist approach of de Freitas and Sinclair (2013, 2014; see also Sinclair et al., 2013). Creativity is not studied here as

> a property or competence of a learner, as suggested by approaches that seek to measure the flexibility or fluency of a child's thinking. It does not exist independently of its exercise. In other words, it is not that individuals are creative or not creative, but rather that creativity flows across the learning assemblage in a somewhat impersonal way. (de Freitas & Sinclair, 2014, p. 86)

This conception of creativity is not bound to a "personal creativity as a characteristic that can be developed in schoolchildren" (Lev & Leikin, 2013, p. 1204). Indeed, it shifts attention away from the doer, and from the idea of giftedness and high ability in mathematics, to focus on the doing, without lapsing into reading actions as reflections of mental states. It "treats creativity as *an action taken* that emerges in context, without being exhausted by it" (Sinclair et al., 2013, p. 241, emphasis in the original) and *bringing forth the new*.

Thus the inclusive materialism centres on the process of creation of something new, looking at students' actions, with the other material actions in the classroom, as an expression of creativity. Interestingly, it relies upon a re-configuration of the contours of the learners' body, which enables to talk not only about the body *in* but also *of* mathematics. In fact, inspired by the French philosopher Gilles Châtelet and his notion of the virtual, de Freitas and Sinclair explore "how mathematics partakes of the material world" (2014, p. 1) and how this occurs "in operative, agential ways", troubling the tacit belief that "the mathematical concepts (multiplication, cube, zero) are taken for granted, while students collaboratively move towards them." (p. 40). Within a tradition that assumes that abstract thought and materiality are entwined, their philosophical position looks for "how bodies are assembled through activity" (p. 15). For de Freitas and Sinclair, "the body is an assemblage of human and non-human components, always in a process of becoming that belies any centralizing control." (p. 25). Their perspective "moves away from a theory of power as a totalizing, external force and follows power as it flows through sensation and affect, across the surfaces of bodies as they emerge in relation to these flows." (p. 41). In so doing, they open room for post-humanist discourses of subjectivity and agency, for which students are always in a process of *becoming* mathematical subjects through agential relations with the diverse dynamic materialities in the classroom, including the mathematical concepts.

Thus mathematical creativity (or inventiveness) is materially conceived of in terms of the process that "expresses and captures the temporal and dynamic moment when the new or the original comes into (*in-venire*) the world at hand", for example in terms of "the dance between the gesturing and drawing hand" (de Freitas & Sinclair, p. 88, emphasis in the original). Other than bringing forth what was not present before—a feature stressed by Châtelet (1993/2000) in his analysis of inventive moments in the history of mathematics, a creative act has also other characteristics (Sinclair et al., 2013). It is unusual: it does not align with current perceptual habits or practices that are taken as norms and the extent to which it is recognized as creative, depends on the context where it occurs. It is unexpected or unscripted: it is not directly of formally determined by the intentions of the individuals involved. It changes the way language and other signs are used and alters the meanings that circulate in a situation, so that its meaning cannot be exhausted by existing meanings. These qualities "point to the centrality of the body and its movement (actions)—rather than internal mental disposition—in creative acts." (p. 242). De Freitas and Sinclair (2014) discuss how "gesturing and diagramming can together bring about new ways of thinking, moving and imagining, and thereby give rise to inventive processes." (pp. 109–110).

In a different work, de Freitas (2014) claims that Châtelet shows us "how we might study a particular practice for how its lines of flight flourish and act generatively in unfolding new *intensive* dimensions." (p. 290, emphasis in the original). She draws on contemporary theories of perception to focus on the way the student's body, together with its potentiality, can be reconfigured, and the contours of the sensible and the intelligible recoded. She argues that we need to unpack the provisional nature of perception for emphasizing its *virtuality*, or virtual movements: "We never just register visual information from that which is in front of our eyes:", says de Freitas,

> we see potentiality, relationality, mobility, occurrence. [...] In other words, we live on "speculative investments," as though we were surfing "the front edge of a wave-crest" [...]. Perceiving an object entails a prehending of our body's potentiality to walk around the object, to reach out and touch the object, to see the object, to weigh it, to smell it. (p. 298)

In this paper, I want to use these arguments to focus on the *immersive* and *animated* ways in which children talk, move and feel in mathematical activities. These become means to look at how the children *perceive* and *create* in the mathematics classroom, giving rise to inventive moments that mobilise their doing mathematics as much as the mathematics they are doing. My non-representational monist view—aligning with those that challenge the body/concept and matter/thought dichotomy—questions the binary divide between perceiving and conceiving, creating and learning.

METHODOLOGY

In the analysis, I have chosen to focus on a particular pair of 8-year old children, Lara and Filippo, who deal with pattern activities to develop early algebraic thinking. The whole elementary class participates, during regular mathematics lessons, in a 5-year longitudinal study concerning the introduction of the concept of function and the use of variables. Thus the children had already worked on patterns in the previous grades. The selected data refers to the beginning of grade 3, when they are divided into pairs

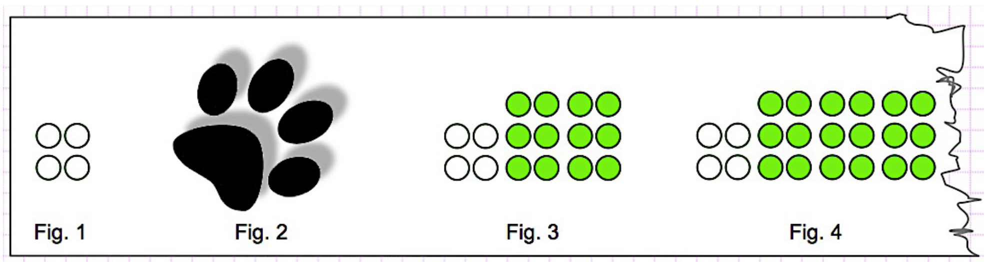

Figure 1: The figural sequence of the activity "Do you remember of Tobia?"

to face an activity called "Do you remember Tobia?", which re-presents a figural sequence to them. Tobia is the name of the imaginary dog that left the track on the ripped paper, having covered up term 2 of the sequence (Figure 1).

In grade 2, the children were asked to extend this sequence up to its sixth term and to find the second term. In grade 3, the purpose of the activity is to shift attention to the relationship between any term of the sequence and the term number—its position in the sequence. To this aim, the pairs are given the task of noticing any regularity in the pattern, and of explaining it.

Lara and Filippo sit on a desk, one in front of the other and turned around to face the other in the discussion. A master degree student, who participates in the lessons as an observer, films them by using a mobile camera. Data of their discourses come from the video-clips, while additional material is given by their written productions. A university researcher (the author) and the teacher are also present in the classroom. The researcher (labelled by R in the data) consistently takes on the role of the guest teacher, teaching the lessons in collaboration with the regular classroom teacher (T below), who has the role of an active observer. In the activities, phases of individual work alternate with pair work and collective discussions.

RESULTS AND ANALYSIS

The results are divided into three sections. The first two focus on the discovery of a remote term within the figural sequence. The last section centres on the way in which Filippo solves the new task about the position of a given total number of circles.

Perceiving numerosity in the particular: What about position twenty-five?

The activity began with Filippo and Lara examining the pattern of Figure 1, with the instruction of looking for regularities. Filippo started by focussing on the bottom row of term 3 and seeing that the number of circles there relates to the term number. In particular, he has found that if one counts the circles on the bottom row and divides this number by 2, one gets the position of the term in the sequence. This introduces in the discourse the new operation of division by 2, which Filippo and Lara share with the researcher as soon as she comes to the pair.

Filippo: For example, you look at this [*Points to term 3 of Figure 1 with the pen in his right hand*], you do, you count the circles below [*Runs the bottom row*], one, two, three, four, five, six [*Counts the circles*], then you do six divided by two [*Looks up at the researcher*] that gives, oh

Lara: Three [*Looks up at the researcher*]

Filippo: And this is, is [*Moves the pen twice around term 3*], oh, the position

Lara: Or you also take this [*Overlaps Filippo's voice. Points to term 4*], you can also take this [*Points to term 4 again*]

R: Oh, and does it also work here? [*Indicates term 4*]

Filippo: This one is equal. One, two, three, four, five, six, seven, eight [*Counts with the pen the circles on the bottom row of term 4. Lara joins him in counting*]. You do eight [*Looks up*] divided by two

Lara: It gives four [*Looks at the researcher*]

Filippo: And this one [*Moves the pen twice around term 4*] is in position four. This one [*Shifts the pen to term 1*], one, two, two [*Looks up at the researcher*] divided by two gives one [*Points with the pen to expression "Figure 1" below term 1. Looks up at the researcher. Smiles*]

R: Very Good! Oh, now I tell you: What about position twenty-five?

Lara: Twenty-five divided by two! [*Laughs. Looks at Filippo*]

Filippo: [*Looks at Lara surprised, looks up at the researcher, looks back at the sequence. Keeps thinking in silence for some seconds, suddenly mimes with his left hand a small rotation towards his torso. Looks up*] you do twenty-five, oh, times two

R: What do you use twenty-five times two for? Explain me.

This short passage shows that Filippo and Lara perceive the first structural relations in the figural sequence, between the numerosity of circles and the number of a given term like 3 or 4 – the children move the discourse beyond the recursive "adding six circles" (that emerged in grade 2), towards looking at the sequence in a functional way, by talking (for example using "position") and gesturing (around the term, to its bottom row). What they perceive is of a very different nature, since it introduces reasoning on the pattern in terms of whole numbers (numbers of circles), even if still per rows, but no longer strictly related just to the spatial structure. Dividing by 2 also comes to the fore as a means to manage the relations between numbers. A certain satisfaction can be grasped in Filippo's explanation about term 1.

Hoping to encourage the children to perceive more than one relation in the sequence, the researcher then introduces a new task for a new (remote) term: the "position twenty-five" task. Lara hurriedly says "Twenty-five divided by two", but Filippo keeps silent, marking his struggle (also expressed by his repeatedly changing gaze). The mathematics of the figural pattern is mobilised. Filippo responds thinking about multiplying by 2, but he gets confused about the kind of numbers in use when invited to explain. Thus he inquires "but do you say position twenty-five or the number twenty-five?". The situation breaks through with the answer "No, the position twenty-five", which prompts Filippo to insist on "you do twenty-five times two". When the researcher then asks "And what do I find?", discourse moves on.

Filippo: You find the number, the number of, of, oh, to put below [*Runs the bottom row of term 4 many times with the pen in his right hand*]. And you put them, at the beginning [*Moves left hand to term 4*] you put two of them and then two [*Indicates the bottom row with the pen, looks up*], then you put two of them and you go by two [*Jumps along the middle row*], and then you don't put any here [*Points to the empty space on the top row with left index finger*] and you always go by two [*Jumps along the top row with the pen*]

Filippo reasons in terms of numbers of circles on the 4th term of the sequence to think about the shape in a remote term like 25. In perceiving the row disposition and composition in term 4, gesturing on its rows with both hands ("always" referring to groups of "two" circles), he conceives of the structure in term 25 in terms of the spatial similarity that is granted by the algebraic structure of the pattern. Through gestures, the circles begin to be mobilised together with the numbers in the sequence.

Creating the new term: You skip the first two and you go

The teacher gets close and Filippo, excited, wants to tell her about term 25.

Filippo: In position twenty-five, you do, to discover that one [*Moves the pen many times around term 4 of the sequence*], this one [*Runs the bottom row of the term*], you do twenty-five times two, twenty-five times two, oh, then you put [*Points to term 4*], oh, wait, twenty-five times two, and, this number, oh, wait, I do no longer remember [*Smiles*]. You do twenty-five times two [*Pauses. Looks around, beats his head*], oh, what did I say? [*Looks at Lara, looks at the sequence*]

Lara: Twenty-five times two [*Pauses, looks at Filippo who points to term 4*], one, two, three, four, five, six, seven, eight [*Counts the circles on the bottom row of term 4*], you do eight divided

Filippo: Ah! You do twenty-five times two [*Looks at the teacher*], and you put the result here below [*Mimes with the pen the arranging of the first circles on the bottom row of term 4. Looks up at the teacher. Figure 2a*], you put the circles, all, of the result [*Continues the gesture outside of the paper. Figure 2b*]. When you arrive at the result with the circles, there [*Shifts the pen to a position towards the desk side. Figure 2c*], you stop and you go above [*Shifts the pen to indicating the middle row of term 4, keeps reference to it with left index finger*] and you always put twenty-five, no, always the result [*Mimes the arranging of circles on the middle row, moving to the desk side. Figure 2d*]. Then here [*Points with index finger and pen to the initial empty space on the top row of term 4*], you skip this, you skip the first two and you go [*Keeps the finger as a reference, mimes the arranging of the circles on the row with the pen. Figure 2e*], oh, and you put, you do [*Looks at the teacher*], oh, you take two away from the result and put those ones! [*Repeats the miming of the top row. Looks at the teacher, smiles*]

Figure 2: (a-c) miming the bottom row of term 25; (d, e) miming the other two rows

Filippo thinks of the pattern in terms of numbers as results of operations (no longer just numbers of circles). He uses his hands and fingers to enact the exact shape of the figure in the 25th position, with the reference of the 4th term. But he also gestures outside of the paper to talk about/imagine a term that would appear after the sequence––and, in any case, would be made of longer rows. Filippo mobilises the static diagram on paper through his gestures as a form of diagramming. He actualizes the virtual movement of the sequence, rather than only realising its logical possibility through numbers (that exists in the given only). Thus the circles are mobilised and the mathematics of the pattern is invented in the moving assemblage of the child, the pattern and the mathematics. This allows the creation of new mathematics as the new term is figuratively brought forth through Filippo's gestures.

Without distinguishing between perceiving and conceiving, as I am encouraged to do in materialist terms, I might say that, for Filippo, term 4 *is* term 25 here. In a similar way, without distinguishing between creating and learning, we might say that Filippo is starting to reason in algebraic terms and the children's discourse is moving to a more functional one compared to the previously discussed.

Perceiving and creating the unknown: What position is?

Filippo explains to the teacher the term in position Pippo that was introduced by the researcher as a challenge after term 25. The children faced the task with some tension with respect to using expressions like "the result of Pippo times two". During this interaction, the teacher poses the new task of having the total number of circles in a term: "I have a position, which I don't know, which has twenty-two circles, how can I discover what position is?", specifying that she means "as a whole".

Filippo: Twenty-two, oh, you take away four from twenty-two [*Mimes the operation moving his hands together in front of his torso from right to left. Figure 3a*] and you get eighteen, and it's the first group [*Mimes a grouping. Figure 3b*] of four [circles], eighteen. Then, you take away six from eighteen [*Mimes a block, with a vertical movement of his right hand. Looks at the teacher. Figure 3c*] and you get twelve [*Shifts right hand on the right, moves closer*] that, so, are four and a row of six [*Mimes the grouping again and a new block. Looks at the teacher. Figure 3d*]. Then [*Moves on the right again with both hands open to mime the remaining circles, bends his head*], take away six from twelve and it gives six, so they are four [*Marks a grouping with left hand*] and two rows of six [*Mimes a block with right hand*]. Then [*Pauses*], take away six from six, so it gives three rows of six [*Keeps still left hand, mimes the three blocks with right hand. Looks at the teacher. Figure 3e*], plus four [*Marks the grouping again with left hand. Look at the teacher. Smiles. Figure 3e*], and then, oh, there are no more [*Turns more towards the teacher*], you do, they are four plus three rows of six [*Mimes the grouping with left hand, the blocks with right hand*]

R: Ok, so what position is?

Figure 3: (a-c) miming the first operation; (d, e) miming the resulting blocks

Filippo: So, it's, one, two... three, four... five, six... four! [*Mimes the counting of the circles on the bottom row of the term, moving from left closer to the camera up to disappear almost entirely from its view. Smiles*]

This short episode shows how Filippo's ways of talking and moving become ways of diagramming the specific term in the sequence, and of imagining its position. It is as if Filippo had the term (big) in front of his eyes, could touch it and see it from various points of view, moving around it. His gestures, gazes, postures, smiles, all creatively tend to the term, with his body clearly marking the position and making space for the teacher. Like in the case of term 25, the invention of new mathematics is allowed as the given circles are mobilised, the imaginary blocks are animated and their position is created in the evolving body-material assemblage, through gestures of repeatedly subtracting six and the body shift closer and closer to the camera.

CONCLUSIONS

My goal in this paper has been to examine the children's ways of talking, moving and feeling as immersive and animated ways of perceiving and creating in mathematics, *the* mathematics. In a monist materialist perspective, the episodes have shown that the children's gestures play relevant roles as for the claim that their perceiving *is* conceiving and their creating *is* learning. Filippo and Lara begin to learn to think algebraically in the first and second episodes, when they create the shape of term 25 using the reference of term 4 of the sequence. Filippo sometimes gestures on the 4^{th} term as if the 4^{th} term was the 25^{th}, in other times he gestures beyond it for reaching the new imaginary term. Without these gestures, the diagram would stay static and the children would only use numbers to realise the possible given in the figure. In the third episode, Filippo is learning that a given total number of circles (not only the number of circles on the bottom row) can have a position in the figural pattern, when he creates the position for twenty-two circles without any reference to specific terms on paper. In the episodes, the child's hand gestures are never iconic representations of one term in the pattern. Rather, they are conceptions and creations that allow reducing distance between the physical and the mathematical. They transform what is static/possible towards the mobile/virtual––algebraic thinking here. At the heart of this virtuality, the figural pattern is a part of Filippo's body––and feelings––and *is* the mathematics in that mathematics is implicated by the pattern. As a consequence, the duality of the child-pattern and pattern-mathematics can be rethought of as *one*, the child-pattern-mathematics, which is a learning assemblage in the classroom.

REFERENCES

Burbules, N. C. (2006). Rethinking the virtual. In J. Weiss, J. Nolan, J. Hunsinger, & P. Trifonas (Eds.), *The international handbook of virtual learning environments* (pp. 37–58). Dordrecht, The Netherlands: Springer.

Châtelet, G. (1993/2000). *Figuring space: Philosophy, mathematics and physics*. (R. Shore & M. Zagha, Trans.). Dordrecht, The Netherlands: Kluwer Academy Press. (Original work published as *Les enjeux du mobile*, 1993).

de Freitas, E. (2014). How theories of perception deploy the line: Reconfiguring students' bodies through topo-philosophy. *Educational Theory, 64*, 285–301. doi: 10.1111/edth.12062

de Freitas, E., & Sinclair, N. (2013). New materialist ontologies in mathematics education: The body in/of mathematics. *Educational Studies in Mathematics, 83*, 453–470. doi: 10.1007/s10649-012-9465-z

de Freitas, E., & Sinclair, N. (2014). *Mathematics and the body: Material entanglements in the classroom*. Cambridge, UK: Cambridge University Press.

Ferrara, F. (2014). How multimodality works in mathematical activity: Young children graphing motion. *International Journal of Science and Mathematics Education, 12*, 917–939. doi: 10.1007/s10763-013-9438-4

Freedberg, D., & Gallese, V. (2007). Motion, emotion and empathy in aesthetic experience. *Trends in Cognitive Sciences, 11*, 197–203. doi: 10.1016/j.tics.2007.02.003

Lakoff, G., & Núñez, R. (2000). *Where mathematics comes from: How the embodied mind brings mathematics into being*. New York, NY: Basic Books.

Lev, M., & Leikin, R. (2013). The connection between mathematical creativity and high ability in mathematics. In B. Ubuz, Ç. Haser, & M. A. Mariotti (Eds.), *Proceedings of the Eighteenth Congress of the European Society for Research in Mathematics Education* (pp. 1204–1213). Ankara, Turkey: Middle East Technical University and ERME.

Nemirovsky, R., Kelton, M. L., & Rhodehamel, B. (2013). Playing mathematical instruments: Emerging perceptuomotor integration with an interactive mathematics exhibit. *Journal for Research in Mathematics Education, 44*, 372–415. doi: 10.5951/jresematheduc.44.2.0372

Radford, L. (2013). Sensuous cognition. In D. Martinovic, V. Freiman, & Z. Karadag (Eds.), *Visual mathematics and cyberlearning* (pp. 141–162). New York, NY: Springer.

Sfard, A. (2008). *Thinking as Communicating: Human development, the growth of discourses, and mathematizing*. Cambridge, UK: Cambridge University Press.

Sheets-Johnston, M. (2009). Animation: The fundamental, essential, and properly descriptive concept. *Continental Philosophy Review, 42*, 375–400. doi: 10.1007/s11007-009-9109-x

Sheets-Johnston, M. (2012). Movement and mirror neurons: A challenging and choice conversation. *Phenomenology and the Cognitive Sciences, 11*, 385–401. doi: 10.1007/s11097-011-9243-x

Sinclair, N., de Freitas, E., & Ferrara, F. (2013). Virtual encounters: The murky and furtive world of mathematical inventiveness. *ZDM – The International Journal on Mathematics Education, 45*, 239–252. doi: 10.1007/s11858-012-0465-3