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Endogenous Economic Growth with Disembodied Knowledge

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Abstract

Mainstream endogenous growth models assume that new knowledge is embodied into either new intermediate or final goods, monopolistically supplied by the patent holder. Recent technological progress, however, often gives rise to pure intellectual contents, such as software codes or business models, directly usable in the production of final goods. Once a content of this type has been produced, it is in fixed supply, that is, the inventor can only rent it out (or sell it) or not; hence the quantity restriction typical of monopoly cannot arise, while competition is viable (Chantrel *et al.* 2012, Marchese and Privileggi 2016). We show that, however, as long as the inventor owning a patent can control through licence activation devices the access to the intellectual content of the workers using her invention in the final goods production, monopolistic exploitation becomes viable and will occur. It turns out that in this framework the income share of labor is smaller than in the Lab-Equipment economy, which represents the setting closest to our model. Moreover, with elastic labor supply also labor employment is negatively impacted. This implies that some standard public policies devised for correcting inefficiencies in development may perform poorly in this framework.

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1 Introduction

In the recent past one way of highlighting the pace of technological progress was to quote the Moore law, describing the expected improvements in a widely used intermediate good, the semiconductor. Nowadays, however, technological progress appears largely disembodied, and characterized by the supply of new immaterial goods such as DNA sequences, software codes, computer programs, internet applications, business models, etc., which

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are patentable[1] and directly usable in the final goods production, without having to be incorporated into capital or intermediate inputs. This feature does not fit well into standard endogenous growth models,[2] which assume that, in a system with Intellectual Property Rights (IPR), the successful inventor has the opportunity of becoming the sole supplier of a new final or intermediate good, embodying the results of her activity. She can thus charge a monopoly price and earn a compensation for her research effort.[3] Boldrin and Levine (2008) identify the rival good in which an invention is embedded with the copies reporting it, which can be used for direct consumption or employed for (time consuming) reproduction. If the inventor is protected by a patent, she gains by restricting the supply of copies. To avoid this monopolistic exploitation Boldrin and Levine suggest to abolish the IPR protection. In this way the inventor, by selling the first copy, would have just the opportunity of recovering the present value of future copies, whose number, however, she cannot restrict. If the compensation for the first copy is large enough to cover the indivisible initial cost of producing it, research is viable and efficiency is reached.

While the desirability of allowing the financing of research through patents is debated, a patent—so goes the mainstream approach—can lead to monopolistic exploitation as long as it enables the inventor to be the sole producer of a rival good whose supply she can restrict. But if knowledge is disembodied only the invention itself can be brought to the market, so that no quantity restriction, and thus no simple monopoly exploitation, can occur.[4] This implication of disembodied knowledge has prompted the elaboration of new endogenous growth models, with competitive markets for knowledge, in which Lindahl prices are paid (Chantrel *et al.* 2012, Marchese and Privileggi 2016). In these markets, even if inventions are protected by patents, the inventor just recovers the present value of the marginal benefits enjoyed by the users of her invention, and does not earn any monopoly profit. Moreover, under the standard assumption that patents refer to marginal addition to the stock of knowledge (which behaves as a continuous homogeneous good when used in the final goods' production), no indivisibility problem arises.

In this paper we study a case in which monopolistic exploitation is possible even if knowledge is disembodied. The basic idea is that disembodied knowledge cannot be used as the sole input in the final goods' production, because we do not live in a world of pure spirits. It must instead be combined with other physical inputs to produce final goods, among which some will be rival; in this paper we focus on labor. In such a framework the patent holder can design a contract by which the invention can be accessed only by workers for whom a personal licence has been activated. That is, as long as the cost of using the necessary software and of enforcement against infringement is not too large, the inventor can set and collect a royalty conditional on the amount of labor "augmented" by the invention.[5] The firms producing final goods can accept such a contract as long as

[1]This trend for the U.S. can be dated back to 1998, when in the so called State Street Bank case a business method was declared patentable. Many other similar rulings followed with respect to software. A 2014 Supreme Court decision stated that the claim for a patent must contain more than abstract ideas of general character; it must be based, *e.g.*, on unconventional steps that confine it to a particular useful application. For patentability in general see Eckert and Langinier (2013).

[2]Like, *e.g.*, in Romer (1990) and the so called Lab-Equipment model in Chapter 13 of Acemoglu (2009).

[3]Alternatively, whoever purchases the patent can reap this monopoly profit and hence recover the cost incurred for compensating the inventor. For the sake of simplicity, and without loss of generality, we will assume in the following that the inventor does not sell the patent and directly exploits it, and we will use the terms inventor and patent holder as synonyms.

[4]Note that also in the Lab-Equipment model the invention per se does not command any profit.

[5]Besides the examples of personal licences for scientific software and databases, which are very familiar

they pay in total for augmented labor (*i.e.*, for the wage and for the licences augmenting it) no more than its marginal product.

We show that in this case the inventor can actually earn a monopoly rent. While this case has some features in common with the standard approach about embedded knowledge followed in the literature—*i.e.*, in both cases a rival good is exploited by the patent holder—there are differences due to the fact that, in our case, two prices must be paid for each unit of augmented labor: a wage to the worker and a royalty which goes to the patents' holders. Hence the financing of research involves a kind of "taxation" of wages. As a consequence, unlike in standard Lab-Equipment models, the wage falls short of the labor marginal product. This also implies a reduction of the workers' income share with respect to both the first-best[6] and the Lab-Equipment case. A further source of inefficiency and wages' compression is due to the fact that resources used for activating and maintaining the personal licences represent a pure waste,[7] as their role is to exclude those who do not pay from accessing knowledge, *i.e.*, a pure public good whose marginal cost of use is nil. The negative effects on wages and on the growth rate of the economy turn out to be more pronounced under elastic labor supply.

Our results provide a contribution to the debate on the labor income share's decline in advanced economies. Such a phenomenon is somewhat elusive in empirical terms—as it is difficult to establish a clear boundary between labor and capital income—and is probably too complex to have a unique explanation. Neoclassical explanations (Karabarbounis and Neiman 2014) based on an elasticity of substitution between capital and labor larger than one, together with capital deepening prompted by declining prices of investment goods are not fully supported by empirical analysis (Elsby *et al.* 2013). Also the explanation based on skill-deepening and capital-skill complementarity (Krusell *et al.* 2000) does not seem to accord to empirical observations in terms of timing in which the changes in the relevant variables occurred. Our contribution aims at providing a theoretical model that takes into account some overlooked logical implications of a phenomenon whose relevance is widely recognized and which is roughly speaking coeval with the decline of the labor income share, *i.e.*, dematerialization of technological progress. Our model is kept extremely simple in order to highlight the basic insights. Extensions aimed at improving its descriptive capacity (*e.g.*, by including physical capital and depreciation), calibrations with empirical data and other forms of empirical verification are left for future research.

The paper is organized as follows: in Section 2 we present a simple model characterized by inelastic labor supply and discuss the decentralized static and intertemporal equilibria: we show that the income share of labor is smaller than in a Lab-Equipment economy; moreover, as one should expect, the growth rate in our model is inferior to the

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to researchers, a business model which is not far from that described in this paper is that of Uber, which renders its business model available to partner firms (the car drivers) and asks for a share of the compensation paid by the passengers. This compensation, with some simplification, can be described as a compensation for labor, as long as for the occasional Uber driver the other costs incurred are mainly sunk. Uber is currently applying for patents to protect its model. While some of its applications have been turned down, it might also rely to some extent on patents in the same field belonging to some of its investors.

[6] For an even stronger effect of this type in a competitive economy with disembodied knowledge see Marchese and Privileggi (2016).

[7] Cloud computing might have somewhat improved things as it allows a more flexible access and usage (we are indebted to a referee of this journal for raising this point). Price discrimination that is often observed in this field might, however, still be based on the expected amount of labor augmentation that users are looking for, which might be inferred by their workload demand.

corresponding first-best, social planner one. In Section 3 we consider the case in which labor is elastically supplied and show that in this scenario the decentralized equilibrium performs even more poorly, as also labor supply is being reduced. Finally, Section 4 provides some hints about the implications of the model in terms of policies and concludes.

2 The model with inelastic labor supply

Let us consider an infinite-horizon economy where the representative household aims at maximizing the following lifetime utility function:

$$\int_0^{+\infty} \ln\left[C\left(t\right)\right] e^{-\rho t}\, \mathrm{dt}\,. \tag{1}$$

where $C(t)$ is aggregate consumption at instant t and $\rho > 0$ is the (constant) rate of time preference. There is no demographic growth and labor supply, L, is inelastic.

The instantaneous production function of final goods, echoing those in the Romer/Grossman-Helpman/Aghion-Howitt models (Jones 1995) is

$$Y = X^{\alpha}\left(AL\right)^{1-\alpha}, \tag{2}$$

with $0 < \alpha < 1$, where Y is a composite final good with price 1 (the numeraire), X is an intermediate good composed of final goods also priced at the numeraire, A is knowledge, which, from the final good producers' (F-firms) perspective is a continuous variable representing an aggregate composition of perfect substitutes, implying that whichever new idea is added to the stock, it has the same marginal productivity of all other ideas, while L is labor. Firms cannot dispense with A altogether, since labor is unproductive without it. The relationship between labor and licences is specified in the following assumption.

A. 1 *Knowledge is supplied to F-firms only through personal licences. Each licence is usable and useful only when combined with one worker, and cannot be combined with more than one worker. Each F-firm endows all its workers with the same type and number of licences.*

Assumption A.1 describes a case in which knowledge is excludable; to access it a personal licence is needed. It also rules out the case in which subsets of workers of the same F-firm can be endowed with different numbers and/or types of licences, that is, can access to different amounts or types of knowledge. This is without loss of generality, since one can consider branches using different labor augmentation levels as separate firms. Assumption A.1 implies that each F-firm demand of augmented labor is defined according to the next lemma.

Lemma 1 *Under Assumption A.1 the augmented labor demanded by F-firm i is given by*

$$H_i = A_i L_i \tag{3}$$

where L_i is the total number of workers employed by firm i and $A_i \leq A$ is the amount of knowledge used by firm i.

Proof. Assumption A.1 states that there is perfect complementarity between licences and raw labor, which implies that each F-firm demand of augmented labor is

$$H_i = \min(h_i, l_i)\, A_i, \tag{4}$$

where h_i is the number of workers supplying raw labor and l_i is the number of personal licences per unit of A_i employed by firm i. For each given A_i the solution of (4) is $h_i = l_i = L_i$ (that is, each firm avoids waste), so that the optimal demand necessarily is as in (3). ■

Equation (3) can thus be used to rewrite firm's i production function (2) according to:

$$Y_i = X_i^{\alpha}(H_i)^{1-\alpha}. \tag{5}$$

Moreover, (3), rewritten as $L_i = H_i/A_i$, shows that F-firm i is willing to pay two prices for each unit of augmented labor H_i, namely the salary w/A_i plus a royalty R.

2.1 The instantaneous market equilibrium

In order to study how the equilibrium value of R is determined we introduce the following assumptions.

A. 2 *Personal licences are supplied by many identical firms (R&D-firms) which (already have) produced one unit of A and own the corresponding patent.*

A. 3 *The market for final goods is competitive and F-firms are price takers.*

The instantaneous profit function of F-firm i is:

$$\Pi_i(X_i, H_i) = X_i^{\alpha}(H_i)^{1-\alpha} - X_i - \frac{wH_i}{A_i} - RH_i. \tag{6}$$

Under Assumption A.3 FOCs for (6) completely characterize F-firms profit maximization:

$$\frac{\partial \Pi_i}{\partial X_i} = \alpha\left(\frac{H_i}{X_i}\right)^{1-\alpha} - 1 = 0 \tag{7}$$

$$\frac{\partial \Pi_i}{\partial H_i} = (1-\alpha)\left(\frac{X_i}{H_i}\right)^{\alpha} - \frac{w}{A_i} - R = 0. \tag{8}$$

The inverse demand for licences of firm i as a function of both A_i and L_i, $R_i(A_i, L_i)$, can thus be derived from (8).

In view of Lemma 1 and since, according to Assumption A.2, at each instant the A supply is given and thus inelastic, the royalty will settle at a level such that all the $R\&D$-firms can sell licences pertaining to their patent, and $A_i = A$ must occur in equilibrium for all the F-firms. Thus, with respect to A the market works competitively, as in Chantrel *et al.* (2012) and in Marchese and Privileggi (2016). On the other hand, the royalty also depends on the number of licences: because each $R\&D$-firm, being the sole owner of the patent, can choose how many licences to supply, it enjoys a monopoly. Since $R\&D$-firms are identical, each of them must sell in equilibrium the same number of licences L^*. Because, according to Assumption A.1, all the F-firms endow all their workers with the same number of licences covering the whole A, using (3) we see that

$\sum_i H_i = \sum_i A_i L_i = A \sum_i L_i = AL^*$ must hold. Finally, since labor supply is inelastic, in equilibrium the wage w must settle at a level compatible with full employment, that is, $L^* = L$ must occur. We now proceed in finding the royalty R that arises in equilibrium.

Since (5) is homogeneous of degree 1 in its two variables, intermediate good and augmented labor, and all F-firms use the whole knowledge A and pay the same prices, we can from now on consider a representative F-firm supplying the whole final goods' production and drop suffix i.

The inverse demand of licences faced by each $R\&D$-firm can be obtained by solving for R condition (8):

$$R(L) = (1-\alpha)\left(\frac{X}{AL}\right)^{\alpha} - \frac{w}{A}, \tag{9}$$

where the representative $R\&D$-firm takes the inverse demand as a function of the sole L, while all the other variables are taken as given. The (instantaneous) profit function of the representative $R\&D$-firm is

$$\pi(L) = R(L)L - \beta L = (1-\alpha)\left(\frac{X}{A}\right)^{\alpha} L^{1-\alpha} - \left(\frac{w}{A} + \beta\right)L, \tag{10}$$

where $\beta \geq 0$ is the constant per capita cost of activating the licences for one unit of A. Activation is costly because it entails specific software maintenance and prevention of illegal access. In order the activation cost to be compatible with positive labor supply the following assumption is needed.

A. 4 *The activation cost is not too large; specifically,* $\beta < (1-\alpha)^2 \alpha^{\frac{\alpha}{1-\alpha}}$ *must hold.*

The last expression in (10) is clearly concave in L. The FOC for profit maximization of the representative $R\&D$-firm yields:

$$\frac{\partial \pi}{\partial L} = (1-\alpha)^2 \left(\frac{X}{AL}\right)^{\alpha} - \frac{w}{A} - \beta = 0.$$

By solving it for the optimal quantity of personal licences L^*, one gets:

$$L^* = \frac{(1-\alpha)^{2/\alpha}}{(w/A+\beta)^{1/\alpha}} \frac{X}{A}. \tag{11}$$

By substituting L^* into (9), the royalty, as a function of the equilibrium wage w, turns out to be:

$$R(w) = \frac{1}{1-\alpha}\left(\frac{\alpha w}{A} + \beta\right). \tag{12}$$

The monopoly royalty thus involves a mark-up over the personal licence cost β and is increasing in the wage w. One can also see (12) as a reaction function of the $R\&D$-firm to the wage arising on the market.

From (7) one gets:

$$X = \alpha^{\frac{1}{1-\alpha}} AL, \tag{13}$$

which implies that the intermediate good and the augmented labor are used in fixed proportions. By substituting $R(w)$ as in (12) and X as in (13) into (8) for the F-firm the equilibrium wage turns out to be:

$$w = \left[(1-\alpha)^2 \alpha^{\frac{\alpha}{1-\alpha}} - \beta \right] A, \tag{14}$$

which, under Assumption A.4, is strictly positive. Because labor supply is inelastic, the labor market equilibrium wage level w in (14) implies full employment; therefore, $L^* = L^*(w) = L$ is the number of licences that will actually be sold in the market equilibrium. In other words, all the given labor supply will be augmented by A.

The following proposition summarizes the features characterizing the instantaneous market equilibrium.

Proposition 1 *Under Assumptions A.1–A.4 the market equilibrium at any instant $t \geq 0$ is attained at the following conditions:*

i) *R&D-firms maximize their profit by choosing the optimal royalty $R(w)$ in (12) as a function of the equilibrium wage w;*

ii) *the representative F-firm uses the whole A and maximizes its profit taking $R(w)$ and w as given;*

iii) *the labor market clears at the equilibrium wage w and, as labor supply is inelastic, this occurs at full employment.*

According to (8), such characterization implies that the marginal product of labor equals the (augmented) labor marginal cost, which is given by the sum of a) the equilibrium wage w and b) the term $R(w)A$, where $R(w)$ is the optimal royalty in (12); $R(w)A$ is paid for allowing each worker to access the whole available knowledge stock.

The wage in (14) should be compared with the wage $\widehat{w}$, equal to the marginal product of labor, that would arise in a first-best economy in which the government intervenes by collecting a lump-sum tax to finance knowledge production and then releases it for free. Wage $\widehat{w}$ is obtained by setting $R = 0$ and using (13) into (8):

$$\widehat{w} = (1-\alpha) \alpha^{\frac{\alpha}{1-\alpha}} A. \tag{15}$$

Clearly, $\widehat{w} > w$, as workers under monopolistic exploitation of knowledge earn a wage smaller than in a first-best economy at any given A level.

By plugging (14) into (12) we get

$$R = (1-\alpha) \alpha^{\frac{1}{1-\alpha}} + \beta,$$

which, when used in (10), yields the $R\&D$-firm profit:

$$\pi(L) = (1-\alpha) \alpha^{\frac{1}{1-\alpha}} L. \tag{16}$$

Hence, under the assumption of inelastic labor supply, the $R\&D$-firm profit turns out to be constant.

2.2 The intertemporal equilibrium

To keep our analysis simple, we assume that one unit of knowledge is produced at the constant cost δ. In order both to finance the research cost and to satisfy the free entry condition the following equality must hold:

$$V(t) = \int_t^{+\infty} (1-\alpha)\, \alpha^{\frac{1}{1-\alpha}} L e^{-\int_t^v r(s)\,\mathrm{ds}}\,\mathrm{dv} = \delta. \tag{17}$$

It postulates that the present value of future (instantaneous) profits, as defined in (16), must be equal to the (constant) production cost of a unit of new knowledge incurred at instant t. By differentiating with respect to time in (17), the interest rate turns out to be constant:

$$r = \frac{(1-\alpha)\, \alpha^{\frac{1}{1-\alpha}} L}{\delta}. \tag{18}$$

Because the only asset in the economy is the knowledge stock A owned by households, from (17) it follows that the instantaneous household's asset is given by

$$V(t)\, A(t) = \delta A(t), \tag{19}$$

and, taking into account the interest rate as in (18), her problem is that of maximizing (1) under the dynamic budget constraint

$$\dot{A}(t) = \frac{1}{\delta}\left[\pi(L)\, A(t) + wL - C(t)\right], \tag{20}$$

where $\pi(L)$, w are given by (16) and (14) respectively.

Proposition 2 *Suppose that Assumptions A.1–A.4 hold. Then, if*

$$(1-\alpha)\, \alpha^{\frac{1}{1-\alpha}} L > \delta\rho, \tag{21}$$

the economy admits a unique BGP along which knowledge, output, and consumption all grow at the same rate[8]

$$g = \frac{\dot{C}}{C} = \frac{\dot{A}}{A} = \frac{\dot{Y}}{Y} = \frac{(1-\alpha)\, \alpha^{\frac{1}{1-\alpha}} L}{\delta} - \rho. \tag{22}$$

Moreover, there are no transition dynamics: the economy immediately jumps on the BGP starting from $t = 0$.

Proof. In order to obtain a Balanced Growth Path (BGP) type of equilibrium note that, because $\pi(L)$ and w in (20) are both constants, the ratio C/A must be constant as well, so that $\dot{A}/A = \dot{C}/C$ must hold; moreover, after replacing (13) into (5), it is clearly seen that $\dot{Y}/Y = \dot{A}/A$ as well. Hence, using (18) in the standard Euler condition [recall that the instantaneous utility in (1) is logarithmic] one easily obtains the growth rate g as in (22), which is positive whenever (21) holds. Note that the transversality condition at infinity holds because, from (18) and (22), it follows that $r > g$. Finally, as (13) requires that $X(0) = \alpha^{\frac{1}{1-\alpha}} LA(0)$, the economy starts immediately on the BGP, that is, there are no transitions. ■

[8] Of course, as postulated by Jones (1995; 1999; 2005) for knowledge-based endogenous growth models, this type of model exhibits the *strong scale effect*; *i.e.*, the growth rate of the economy increases in population size.

2.3 The income share of labor: a comparison with the Lab-Equipment model

As in our model both the final goods' production function and the production function of new knowledge are equal to those used in the standard textbooks' Lab-Equipment model, it is interesting to compare the results with respect to the income share of factors.

We can use (13) to rewrite the total output in (2) as

$$Y = X^{\alpha} (AL)^{1-\alpha} = \left(\alpha^{\frac{1}{1-\alpha}} AL\right)^{\alpha} (AL)^{1-\alpha} = \alpha^{\frac{\alpha}{1-\alpha}} AL, \tag{23}$$

and, accordingly, define income I as the net output:

$$I = Y - X = \alpha^{\frac{\alpha}{1-\alpha}} AL - \alpha^{\frac{1}{1-\alpha}} AL = (1-\alpha)\, \alpha^{\frac{\alpha}{1-\alpha}} AL = (1-\alpha)\, Y, \tag{24}$$

from which we learn that the share of total output distributed to production factors is $1-\alpha$, while the share α is employed in the purchase of the intermediate good X. According to (14) and using (24) the labor share is

$$\sigma_L = \frac{wL}{I} = \frac{\left[(1-\alpha)^2 \alpha^{\frac{\alpha}{1-\alpha}} - \beta\right] AL}{(1-\alpha)\, \alpha^{\frac{\alpha}{1-\alpha}} AL} = (1-\alpha) - \frac{\beta}{(1-\alpha)\, \alpha^{\frac{\alpha}{1-\alpha}}}, \tag{25}$$

while, according to (19) and using (18), the capital share is

$$\sigma_A = \frac{r\delta A}{I} = \frac{(1-\alpha)\, \alpha^{\frac{1}{1-\alpha}} AL}{(1-\alpha)\, \alpha^{\frac{\alpha}{1-\alpha}} AL} = \alpha$$

where we denote as capital our stock variable, *i.e.*, knowledge A, which can be conceived as immaterial capital.

We now turn our attention to the standard Lab-Equipment model, labeling the main variables with a 'LE' superscript. To this purpose, we follow the approach pursued in Chapter 6 of Barro and Sala-i-Martin (2004), BS for brevity from now on, being careful of using our notation in place of that employed there.[9] According to equation (6.12) on p. 292 their optimal amount of intermediate goods turns out to be (recall that their multiplicative constant is set equal to 1) $X^{LE} = \alpha^{\frac{2}{1-\alpha}} AL$, so that, as their aggregate production function in equation (6.13) on p. 292 (again with multiplicative constant set equal to 1) turns out to be

$$Y^{LE} = X^{\alpha} (AL)^{1-\alpha} = \left(\alpha^{\frac{2}{1-\alpha}} AL\right)^{\alpha} (AL)^{1-\alpha} = \alpha^{\frac{2\alpha}{1-\alpha}} AL, \tag{26}$$

and, accordingly, the net output is:

$$I^{LE} = Y^{LE} - X^{LE} = \alpha^{\frac{2\alpha}{1-\alpha}} AL - \alpha^{\frac{2}{1-\alpha}} AL = \left(1-\alpha^2\right) \alpha^{\frac{2\alpha}{1-\alpha}} AL = \left(1-\alpha^2\right) Y^{LE}, \tag{27}$$

[9] Specifically, their aggregate production function is defined as $Y = A\,(LN)^{1-\alpha} X^{\alpha}$, where A is a multiplicative constant, N the number of available inventions and, consistently with our notation, L, X denote labor and the intermediate good respectively; moreover, the constant unit cost to produce new knowledge in terms of final goods is denoted by η. In our model the multiplicative constant is set at $A = 1$, while A, in place of N, denotes the number of available inventions, so that their aggregate production function becomes the same as in (2), $Y = X^{\alpha} (AL)^{1-\alpha}$, while we term the constant unit cost to produce new knowledge by δ in place of η.

so that the share of total output distributed to production factors is $1-\alpha^2$, while the share α^2 is employed in the purchase of the intermediate good X. According to equation (6.5) on p. 289 in BS the equilibrium wage is $w^{LE} = (1-\alpha) Y^{LE}/L = (1-\alpha)\alpha^{\frac{2\alpha}{1-\alpha}} A$; then, using (27) the labor share turns out to be

$$\sigma_L^{LE} = \frac{w^{LE} L}{I^{LE}} = \frac{(1-\alpha)\alpha^{\frac{2\alpha}{1-\alpha}} AL}{(1-\alpha^2)\alpha^{\frac{2\alpha}{1-\alpha}} AL} = \frac{(1-\alpha)}{(1+\alpha)(1-\alpha)} = \frac{1}{1+\alpha}, \tag{28}$$

while, according to equation (6.19) on p. 294 in BS and again using (27), the capital share is

$$\sigma_A^{LE} = \frac{r\delta A}{I^{LE}} = \frac{[(1-\alpha)/\alpha]\alpha^{\frac{2}{1-\alpha}} AL}{(1-\alpha^2)\alpha^{\frac{2\alpha}{1-\alpha}} AL} = \frac{(1-\alpha)\alpha}{(1+\alpha)(1-\alpha)} = \frac{\alpha}{1+\alpha}.$$

By comparing (25) and (28) it is easily seen that the former is always smaller than the latter, as

$$\sigma_L = (1-\alpha) - \frac{\beta}{(1-\alpha)\alpha^{\frac{\alpha}{1-\alpha}}} \leq (1-\alpha) < \frac{1}{1+\alpha} = \sigma_L^{LE}, \tag{29}$$

where the last inequality holds as $\alpha \in (0,1)$. We have just proven the following result.

Proposition 3 *Under Assumptions A.1–A.4 the income share of labor in (25) is strictly smaller than that arising in the Lab-Equipment model.*

For example, setting the exclusion cost at its lower limit of $\beta = 0$ and with a value of $\alpha = 0.35$, the income share of labor in our model is 0.65, while it is 0.74 in the Lab-Equipment model. Actually, according to (29), the share of income obtained by labor in our model is even smaller as long as $\beta > 0$, because in this case some income is wasted to ensure the exclusion of those who do not pay for knowledge.

2.4 The first-best, social planner equilibrium

To compare g as in (22) to the rate of growth that would arise in a first-best economy, let us consider the corresponding social planner problem. The resource constraint at instant t is

$$C(t) + J(t) = Y(t) - X(t) = I(t), \tag{30}$$

where $J(t)$ represents the amount of output invested in R&D activity, and on the RHS the total output net of the intermediate goods, *i.e.*, what we have labeled as 'income' in the last subsection, is considered. Dropping time dependency for simplicity, in order to obtain a dynamic constraint in the only variables A (state) and C (control) a social planner first considers maximization of the net output $I = Y - X = X^\alpha (AL)^{1-\alpha} - X$ with respect to X for a given stock A at instant t: the solution turns out to be the same as in (13), $X^S = \alpha^{\frac{1}{1-\alpha}} AL$, where the superscript 'S' denotes the level of intermediate good chosen by the social planner. Hence, net output is again given by (24): $I^S = Y^S - X^S = (1-\alpha)\alpha^{\frac{\alpha}{1-\alpha}} LA$. Under our assumption of a constant cost equal to δ required to produce one unit of new knowledge, $J = \delta\dot{A}$, so that (30) can be rewritten as

$$\dot{A} = \frac{1}{\delta}\left[(1-\alpha)\alpha^{\frac{\alpha}{1-\alpha}} LA - C\right]. \tag{31}$$

Denoting by $\lambda(t)$ the costate variable associated to the unique dynamic constraint (31) and dropping the time argument for simplicity, the current-value Hamiltonian of the social planner problem is

$$H(A, C, \lambda) = \ln(C) + \lambda \frac{(1-\alpha)\alpha^{\frac{\alpha}{1-\alpha}} LA - C}{\delta}.$$

Necessary conditions are

$$\frac{1}{C} = \frac{\lambda}{\delta} \tag{32}$$

$$\dot{\lambda} = \rho\lambda - \lambda \frac{(1-\alpha)\alpha^{\frac{\alpha}{1-\alpha}} L}{\delta} \tag{33}$$

$$\lim_{t\to+\infty} \lambda(t) A(t) e^{-\rho t} = 0, \tag{34}$$

where (34) is the transversality condition. Differentiating (32) with respect to time and coupling the result with (33), using (31) and rearranging terms we obtain the following consumption growth rate:[10]

$$g^S = \frac{\dot{C}}{C} = \frac{\dot{A}}{A} = \frac{\dot{Y}}{Y} = \frac{(1-\alpha)\alpha^{\frac{\alpha}{1-\alpha}} L}{\delta} - \rho. \tag{35}$$

As $0 < \alpha < 1$ implies $\alpha^{\frac{\alpha}{1-\alpha}} > \alpha^{\frac{1}{1-\alpha}}$, a comparison among g^S in (35) and the growth rates g in (22) immediately allow us to establish the following result.

Proposition 4 *Under Assumptions A.1–A.4 the growth rate in (22) is strictly smaller than the social planner growth rate obtained in (35).*

This result, establishing that our decentralized economy grows at a lower rate with respect to the social planner model, is in a sense somewhat unexpected, because a system with personal licences, although characterized by monopolistic exploitation, involves features that should sustain efficiency, namely: i) the whole labor and knowledge are used, like in the first-best; ii) the burden of monopolistic exploitation is borne in full by workers, through a reduction of their wage which settles strictly below their marginal product; iii) instantaneous efficiency in production is preserved, because the composite factor $H = AL$ is paid its marginal product; iv) research is financed in a non distortionary way, as the difference between the marginal product of labor and the wage—which is cashed as a profit by the *R&D*-firms—represents the equivalent of a non distortionary tax on labor, *i.e.*, on a factor which is in inelastic supply. The intuition for the result relies on a dynamic inefficiency arising because the profit obtained by patent holders falls short of the marginal product of knowledge. This implies a too small interest rate and thus a too small growth.

3 The case of elastic labor supply

In this final section we briefly discuss a slight generalization of our model that includes elastic labor supply. Let us normalize the labor potential supply of the households to one; that is, the actual instantaneous labor supply is now $0 \le L(t) \le 1$. We focus on a case in which labor supply, while being variable, tends to a constant in the steady state,[11] in

[10] The transversality condition (34) holds because, by differentiating (32) with respect to time, we get $\dot{\lambda}/\lambda = -\dot{C}/C$, so that $\dot{\lambda}/\lambda + \dot{A}/A = -\dot{C}/C + \dot{A}/A = 0 < \rho$. Again the economy starts immediately on the BGP.

[11] For this approach see Barro and Sala-i-Martin (2004), pp. 422–431, and Chu *et al.* (2012).

which the economy evolves along an Asymptotic Balanced Growth Path (ABGP). Let us adopt the following specification of the household utility function:

$$\int_0^{+\infty} u(t)\, e^{-\rho t}\, \mathrm{d}t,$$

with

$$u(t) = \ln C(t) + \gamma \ln [1 - L(t)], \tag{36}$$

where $\gamma \geq 0$ indicates the preference for leisure and $C(t)$ refers to individual consumption (equal to total consumption). As from now on we focus on the steady state in which labor supply is constant, all the results pertaining to profits and prices presented in the previous sections hold, with L being now some value between 0 and 1. We need to further restrict the range of values for the activation cost in order to allow for positive labor supply in the equilibrium.

A. 5 *The activation cost* β *satisfies* $0 \leq \beta < (1-\alpha)^2 \alpha^{\frac{\alpha}{1-\alpha}} - \gamma\rho\delta$.

The constrained maximization of utility in (36) implies the following consumption-leisure optimality condition:

$$C = \frac{w(1-L)}{\gamma}, \tag{37}$$

while, when monopolistic exploitation occurs, the household budget constraint is the same as in (20) and can be rewritten in the more general form

$$\dot{A} = rA + \frac{1}{\delta}(wL - C). \tag{38}$$

In the steady state the economy evolves along the ABGP at the constant rate given by the Euler equation, $g = \dot{C}/C = \dot{A}/A = \dot{Y}/Y = r - \rho$, so that (38) can be rewritten as:

$$\begin{aligned} C &= r\delta A + wL - \delta\dot{A} = r\delta A + wL - \delta g A = wL + (r-g)\,\delta A \\ &= wL + \rho\delta A, \end{aligned} \tag{39}$$

where in the last equality we used the Euler equation. Coupling (39) with (37) and using (14) one gets:

$$L = \frac{1}{1+\gamma}\left[1 - \frac{\gamma\rho\delta}{(1-\alpha)^2 \alpha^{\frac{\alpha}{1-\alpha}} - \beta}\right], \tag{40}$$

which, under Assumption A.5, yields a positive amount of labor L along the ABGP equilibrium. It turns out that the larger the licence activation cost β, the smaller is labor supply. That is, β, by reducing the wage, reduces labor supply via the substitution effect. Under variable labor supply the economy is thus more deeply impacted by the personal licence system than with inelastic labor supply.

As the interest rate is the same as in (18), along the ABGP the growth rate is again given by the Euler equation and is equal to g in (22), but with L given by (40) instead of the inelastically supplied value used there. To compare this growth rate to that of a first-best economy, again we consider the corresponding social planner problem. The resource constraint is the same as in (30), and after following the same steps as in Section 2.4, one easily gets the same dynamic constraint as in (31). Hence, denoting by λ the costate variable associated to it, now the current-value Hamiltonian of the social planner problem is

$$H(A,C,L,\lambda)=\ln(C)+\gamma\ln(1-L)+\lambda\frac{(1-\alpha)\,\alpha^{\frac{\alpha}{1-\alpha}}LA-C}{\delta}.$$

Necessary conditions are

$$\frac{1}{C}=\frac{\lambda}{\delta} \tag{41}$$

$$\frac{\gamma}{1-L}=\lambda\frac{(1-\alpha)\,\alpha^{\frac{\alpha}{1-\alpha}}A}{\delta} \tag{42}$$

$$\dot{\lambda}=\rho\lambda-\lambda\frac{(1-\alpha)\,\alpha^{\frac{\alpha}{1-\alpha}}L}{\delta}, \tag{43}$$

plus the same transversality condition as in (34). Differentiating (41) with respect to time and coupling the result with (43), using (31) and rearranging terms once more we easily obtain the same expression for the growth rate[12] as in (35), only that now g^S is determined by the optimal amount of labor L^S chosen by the social planner to be employed in the producing sector, instead of the amount obtained in (40):

$$g^S=\frac{\dot{C}}{C}=\frac{\dot{A}}{A}=\frac{\dot{Y}}{Y}=\frac{(1-\alpha)\,\alpha^{\frac{\alpha}{1-\alpha}}L^S}{\delta}-\rho. \tag{44}$$

To find L^S, by coupling (41) and (42) we first obtain the optimal consumption/knowledge ratio, which, again, must be constant along the ABGP:

$$\frac{C}{A}=\frac{(1-\alpha)\,\alpha^{\frac{\alpha}{1-\alpha}}}{\gamma}\left(1-L^S\right),$$

and then, by replacing it into the resource constraint (31) and using (44),

$$\frac{\dot{A}}{A}=\frac{(1-\alpha)\,\alpha^{\frac{\alpha}{1-\alpha}}L^S}{\delta}-\frac{(1-\alpha)\,\alpha^{\frac{\alpha}{1-\alpha}}}{\gamma\delta}\left(1-L^S\right)=g^S=\frac{(1-\alpha)\,\alpha^{\frac{\alpha}{1-\alpha}}L^S}{\delta}-\rho,$$

we easily get

$$L^S=1-\frac{\gamma\rho\delta}{(1-\alpha)\,\alpha^{\frac{\alpha}{1-\alpha}}}. \tag{45}$$

Through a quick comparison between (45) and (40) it is immediately seen that $L^S>L$ for any $\gamma\geq 0$, which, in turn, as $\alpha^{\frac{\alpha}{1-\alpha}}>\alpha^{\frac{1}{1-\alpha}}$, when used in (44), implies that the social planner growth rate is again strictly larger than the decentralized one in (22):

$$g^S=\frac{(1-\alpha)\,\alpha^{\frac{\alpha}{1-\alpha}}L^S}{\delta}-\rho>\frac{(1-\alpha)\,\alpha^{\frac{1}{1-\alpha}}L}{\delta}-\rho=g. \tag{46}$$

Note that when labor supply is elastic the gap between g^S and g is even larger than that with inelastic labor supply: in (46), besides the terms $\alpha^{\frac{\alpha}{1-\alpha}}>\alpha^{\frac{1}{1-\alpha}}$ already making the difference in Section 2, according to (45) and (40) the equilibrium labor amounts $L^S>L$ further add to such a difference. In fact when labor supply is elastic not only the intertemporal allocation of resources is distorted, but also the instantaneous efficiency of the economy is negatively affected, since workers react to the drop of wages by restricting their labor supply.

[12]The same argument as in footnote 10 establishes that the transversality condition (34) holds.

4 Conclusions

In this paper we considered an economy in which knowledge is dematerialized, patented, and directly usable in the final goods' production. The patent holders are able to earn monopoly profits because they provide on an exclusive basis personal licences to the workers involved in final goods' production, as a condition for letting them access knowledge contents. Contracts providing the activation of personal licences for agents operating within firms are actually used in the fields of software, data banks, access to on-line commercial and financial platforms, etc.. Economic growth and welfare are negatively affected in an economy in which knowledge is financed in this way. Wages fall short of the labor marginal product. At the same time, also the compensation received by research is lower than its marginal product, thus implying insufficient incentives for knowledge accumulation and, in turn, slower growth. Moreover, costs borne to secure the exclusion of those not holding a licence from accessing knowledge, which is a public good, imply that resources are wasted.

A notable implication of the model is the compression of the labor income share, *e.g.*, with respect to that predicted by the Lab-Equipment model, at the benefit of the income share accruing to the patent holders. The decline of the income share of labor at the advantage of the share going to intangibles and particularly to holders of patents that occurred in many countries since the 1980s, is a stylized fact that has attracted much attention (Corrado *et al.* 2009, Karabarbounis and Neiman 2014, Koh *et al.* 2015) and whose motivations are widely debated. The model presented in this paper provides a possible rationale, based on the shift that occurred in the last decades from an economy in which technological progress was mainly embedded in physical capital to an economy characterized by the diffusion of information technology and by knowledge dematerialization. Hence, nowadays on the one hand in many instances knowledge directly augments the labor productivity but, on the other hand, it also commands a larger share of the ensuing income.

As far as policies aimed at correcting the inefficiencies are concerned, a standard suggestion arising in Lab-Equipment models is to provide subsidies, financed by non-distortionary taxes, to support the demand of the capital inputs which embody new ideas. In the case here considered the relevant demand, however, is that of labor. But the benefits of subsidies to support labor demand would flow only partially to finance research, while, according to equation (12), for the remaining part they would boost the wage. The latter effect does not contribute at all to economic growth if labor supply is inelastic. Even if it is not, a single tool cannot be tuned to pursue two objectives, *i.e.*, bringing both the labor and the knowledge supply at the efficient level. More appropriate policy intervention may instead involve the public funding of research, in order to release the results for free and get rid of the exclusion costs. If this approach is deemed undesirable as it would endanger the market incentives for research, the government could, *e.g.*, sponsor tournaments for eliciting inventive activity (Taylor 1995), or buy patents and put the results into the public domain (Marchese *et al.* 2014). The sole abolition of the patent system, instead, does not seems to be a suitable remedy against monopolistic exploitation when knowledge is dematerialized, because in this case the research results are very close to a pure public good, and thus highly exposed to the danger of large free riding.

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