

A Typicality-Based Revision to Handle Exceptions in Description Logics

Roberto Micalizio and Gian Luca Pozzato¹

Abstract. We propose a methodology to revise a Description Logic knowledge base when detecting exceptions. Our approach relies on the methodology for debugging a Description Logic terminology, addressing the problem of diagnosing inconsistent ontologies by identifying a minimal subset of axioms responsible for an inconsistency. In the approach we propose, once the source of the inconsistency has been localized, the identified axioms are revised in order to obtain a consistent knowledge base including the detected exception about an individual x . To this aim, we make use of a nonmonotonic extension of the Description Logic \mathcal{ALC} based on the combination of a typicality operator and the well established nonmonotonic mechanism of rational closure, which allows to deal with prototypical properties and defeasible inheritance.

1 INTRODUCTION

We focus on the problem of revising a Description Logic (DL for short) knowledge base when detecting an exception. We propose a methodology whose aim is to tackle the problem of revising a TBox in order to accommodate a newly received information about an exception represented by an ABox individual x . Our approach is inspired by the weakening-based revision introduced in [6] and relies on the methodology by Schlobach et al. [8, 7] for detecting exceptions by identifying a minimal subset of axioms responsible for an inconsistency. Once the source of the inconsistency has been localized, the identified axioms are revised in order to obtain a consistent knowledge base including the detected exception about the individual x . To this aim, we use a nonmonotonic extension of the DL \mathcal{ALC} recently presented by Giordano and colleagues in [2]. This extension is based on the introduction of a typicality operator \mathbf{T} in order to express typicality inclusions. The intuitive idea is to allow concepts of the form $\mathbf{T}(C)$, whose intuitive meaning is that $\mathbf{T}(C)$ selects the *typical* instances of a concept C . For instance, a knowledge base can consistently express that birds normally fly ($\mathbf{T}(\text{Bird}) \sqsubseteq \text{Fly}$), but penguins are exceptional birds that do not fly ($\text{Penguin} \sqsubseteq \text{Bird}$ and $\text{Penguin} \sqsubseteq \neg \text{Fly}$). The \mathbf{T} operator is intended to enjoy the well-established properties of *rational logic*, introduced by Lehmann and Magidor in [4] for propositional logic. In order to reason about prototypical properties and defeasible inheritance, the semantics of this nonmonotonic DL, called $\mathcal{ALC}_{min}^{\mathbf{R}}\mathbf{T}$, is based on rational models and exploits a minimal models mechanism based on the minimization of the rank of the domain elements. This semantics corresponds to a natural extension to DLs of Lehmann and Magidor's notion of *rational closure* [4].

Given a consistent knowledge base $K = (\mathcal{T}, \mathcal{A})$ and a consistent ABox $\mathcal{A}' = \{D_1(x), D_2(x), \dots, D_n(x)\}$, such that $(\mathcal{T}, \mathcal{A} \cup \mathcal{A}')$ is inconsistent, we define a typicality-based revision of \mathcal{T} in order to replace some inclusions $C \sqsubseteq D$ in \mathcal{T} with $\mathbf{T}(C) \sqsubseteq D$, resulting in a new TBox \mathcal{T}^{new} such that $(\mathcal{T}^{new}, \mathcal{A} \cup \mathcal{A}')$ is consistent in $\mathcal{ALC}_{min}^{\mathbf{R}}\mathbf{T}$ and that \mathcal{T}^{new} captures a notion of *minimal changes*.

2 DESCRIPTION LOGICS AND EXCEPTIONS

The logic $\mathcal{ALC} + \mathbf{T}_R$ is obtained by adding to standard \mathcal{ALC} the typicality operator \mathbf{T} [2]. The intuitive idea is that $\mathbf{T}(C)$ selects the *typical* instances of a concept C . We can therefore distinguish between the properties that hold for all instances of concept C ($C \sqsubseteq D$), and those that only hold for the normal or typical instances of C ($\mathbf{T}(C) \sqsubseteq D$). The semantics of the \mathbf{T} operator can be formulated in terms of *rational models*: a model \mathcal{M} is any structure $\langle \Delta^{\mathcal{I}}, <, \cdot^{\mathcal{I}} \rangle$ where $\Delta^{\mathcal{I}}$ is the domain, $<$ is an irreflexive, transitive, well-founded and modular (for all x, y, z in $\Delta^{\mathcal{I}}$, if $x < y$ then either $x < z$ or $z < y$) relation over $\Delta^{\mathcal{I}}$. In this respect, $x < y$ means that x is “more normal” than y , and that the typical members of a concept C are the minimal elements of C with respect to this relation. An element $x \in \Delta^{\mathcal{I}}$ is a *typical instance* of some concept C if $x \in C^{\mathcal{I}}$ and there is no C -element in $\Delta^{\mathcal{I}}$ more typical than x . In detail, $\cdot^{\mathcal{I}}$ is the extension function that maps each concept C to $C^{\mathcal{I}} \subseteq \Delta^{\mathcal{I}}$, and each role R to $R^{\mathcal{I}} \subseteq \Delta^{\mathcal{I}} \times \Delta^{\mathcal{I}}$. For concepts of \mathcal{ALC} , $C^{\mathcal{I}}$ is defined as usual. For the \mathbf{T} operator, we have $(\mathbf{T}(C))^{\mathcal{I}} = \text{Min}_{<}(C^{\mathcal{I}})$. A model \mathcal{M} can be equivalently defined by postulating the existence of a function $k_{\mathcal{M}} : \Delta^{\mathcal{I}} \mapsto \mathbb{N}$, where $k_{\mathcal{M}}$ assigns a finite rank to each world: the rank function $k_{\mathcal{M}}$ and $<$ can be defined from each other by letting $x < y$ if and only if $k_{\mathcal{M}}(x) < k_{\mathcal{M}}(y)$.

Given standard definitions of satisfiability of a KB in a model, we define a notion of entailment in $\mathcal{ALC} + \mathbf{T}_R$. Given a query F (either an inclusion $C \sqsubseteq D$, or an assertion $C(a)$, or an assertion of the form $R(a, b)$), we say that F is entailed from a KB in $\mathcal{ALC} + \mathbf{T}_R$ if F holds in all models satisfying KB.

Even if the typicality operator \mathbf{T} itself is nonmonotonic (i.e. $\mathbf{T}(C) \sqsubseteq E$ does not imply $\mathbf{T}(C \sqcap D) \sqsubseteq E$), what is inferred from a KB can still be inferred from any KB' with $\text{KB} \subseteq \text{KB}'$, i.e. the logic $\mathcal{ALC} + \mathbf{T}_R$ is monotonic. In order to perform useful nonmonotonic inferences, in [2] the authors have strengthened the above semantics by restricting entailment to a class of minimal models. Intuitively, the idea is to restrict entailment to models that *minimize the untypical instances of a concept*. The resulting logic is called $\mathcal{ALC}_{min}^{\mathbf{R}}\mathbf{T}$, and it corresponds to a notion of *rational closure* on top of $\mathcal{ALC} + \mathbf{T}_R$. Such a notion is a natural extension of the rational closure construction provided in [4] for the propositional logic.

The nonmonotonic semantics of $\mathcal{ALC}_{min}^{\mathbf{R}}\mathbf{T}$ relies on minimal ra-

¹ Dipartimento di Informatica, Università di Torino, Italia, email: {roberto.micalizio,gianluca.pozzato}@unito.it

tional models that minimize the *rank of domain elements*. Informally, given two models of KB, one in which a given domain element x has rank 2 (because for instance $z < y < x$), and another in which it has rank 1 (because only $y < x$), we prefer the latter, as in this model the element x is assumed to be “more typical” than in the former. Query entailment is then restricted to minimal *canonical models*. The intuition is that a canonical model contains all the individuals that enjoy properties that are consistent with the knowledge base. This is needed when reasoning about the rank of the concepts: it is important to have them all represented. A model \mathcal{M} is a minimal canonical model of KB if it satisfies KB, it is minimal and it is canonical². Finally, a query F is minimally entailed from a KB (or, equivalently, F belongs to the rational closure of KB) if it holds in all minimal canonical models of KB. In [2] it is shown that minimal entailment in $\mathcal{ALC}_{min}^R \mathbf{T}$ is in EXPTIME.

3 TYPICALITY-BASED REVISION OF A KNOWLEDGE BASE

Similarly to what done in [6], we define a notion of revised knowledge base, precisely a *typicality-based* revised knowledge base. Given a consistent knowledge base $K = (\mathcal{T}, \mathcal{A})$, we have to tackle the problem of accommodating a further ABox information \mathcal{A}' , describing an individual x that belongs to the extensions of the concepts D_1, D_2, \dots, D_n , and that is an *exception* to the knowledge described by K ; namely, given a consistent ABox $\mathcal{A}' = \{D_1(x), D_2(x), \dots, D_n(x)\}$, we have that the knowledge base $(\mathcal{T}, \mathcal{A} \cup \mathcal{A}')$ is inconsistent.

We revise the TBox \mathcal{T} of K by replacing some standard inclusions $C \sqsubseteq D$ with typicality inclusions $\mathbf{T}(C) \sqsubseteq D$, in a way such that the resulting revised knowledge base is consistent in $\mathcal{ALC}_{min}^R \mathbf{T}$. Given $K = (\mathcal{T}, \mathcal{A})$, we denote with $Rev_{\mathbf{T}, \mathcal{A}'}(K)$ the set of all typicality-based weakenings of \mathcal{T} given a newly received ABox \mathcal{A}' . Among all typicality-based weakenings in $Rev_{\mathbf{T}, \mathcal{A}'}(K)$ we select the one, called \mathcal{T}^{new} , capturing a notion of *minimal changes* needed in order to accommodate the discovered exception. The following example, inspired by the well known problem of the Nixon diamond, shows how we revise a knowledge base after discovering an exception.

Example 1 Let $K = (\mathcal{T}, \emptyset)$ where \mathcal{T} is:

- $$\begin{aligned} Quacker &\sqsubseteq Christian & (1) \\ Christian &\sqsubseteq Pacifist & (2) \\ RepublicanPresident &\sqsubseteq Republican & (3) \\ Republican &\sqsubseteq \neg Pacifist & (4) \end{aligned}$$

and let $\mathcal{A}' = \{Quacker(nixon), RepublicanPresident(nixon)\}$.

It can be shown that there are eleven different typicality-based weakenings in $Rev_{\mathbf{T}, \mathcal{A}'}(K)$, but the one chosen as the typicality-based revision \mathcal{T}^{new} is as follows:

- $$\begin{aligned} Quacker &\sqsubseteq Christian \\ \mathbf{T}(Christian) &\sqsubseteq Pacifist \\ RepublicanPresident &\sqsubseteq Republican \\ \mathbf{T}(Republican) &\sqsubseteq \neg Pacifist \end{aligned}$$

The resulting knowledge base $K^{new} = (\mathcal{T}^{new}, \mathcal{A} \cup \mathcal{A}')$ is consistent.

² In Theorem 10 in [2] the authors have shown that for any KB there exists a finite minimal canonical model of KB.

4 COMPUTING A REVISED TBOX

We have introduced an algorithm that revises a given knowledge base $K = (\mathcal{T}, \mathcal{A})$ according to the typicality-based weakening outlined in the previous section. Our algorithm relies on the computation of a Minimal Unsatisfiability-Preserving Sub-TBoxes (*mups*), introduced by Schlobach et al. in their seminal work [8] about the problem of debugging a DL terminology, that singles out the subset of inclusions strictly involved in the inconsistency.

Definition 1 (MUPS, Definition 3.1 [8]) Let C be a concept which is unsatisfiable in a TBox \mathcal{T} . A set $\mathcal{T}' \subseteq \mathcal{T}$ is a minimal unsatisfiability-preserving sub-TBox (*mups*) of \mathcal{T} if C is unsatisfiable in \mathcal{T}' , and C is satisfiable in every sub-TBox $\mathcal{T}'' \subset \mathcal{T}'$.

Following [8], we have restricted our approach to *unfoldable* TBoxes, only containing unique, acyclic definitions. An axiom is called a definition of A if it is of the form $A \sqsubseteq C$, where A is an atomic concept. An axiom $A \sqsubseteq C$ is unique if the KB contains no other definition of A . An axiom is acyclic if C does not refer either directly or indirectly (via other axioms) to A [1].

5 FUTURE ISSUES

We aim at extending our typicality-based revision also to not *unfoldable* TBoxes: to this aim, in [5, 3] axiom pinpointing is extended to general TBoxes. A set of algorithms for computing axiom pinpointing, in particular to compute the set of *mups* for a given terminology \mathcal{T} and a concept A , is also provided. Furthermore, we intend to develop an implementation of the proposed algorithms, by considering the integration with existing tools for manually modifying ontologies when inconsistencies are detected.

Acknowledgements

The authors are partially supported by the projects “AThOS: Accountable Trustworthy Organizations and Systems” and “ExceptionOWL: Nonmonotonic Extensions of Description Logics and OWL for defeasible inheritance with exceptions”, Progetti di Ateneo Università degli Studi di Torino and Compagnia di San Paolo, call 2014, line “Excellent (young) PI”.

REFERENCES

- [1] F. Baader, D. Calvanese, D.L. McGuinness, D. Nardi, and P.F. Patel-Schneider, *The Description Logic Handbook - Theory, Implementation, and Applications*, 2nd edition, Cambridge, 2007.
- [2] L. Giordano, V. Gliozzi, N. Olivetti, and G.L. Pozzato, ‘Semantic characterization of rational closure: From propositional logic to description logics’, *Artificial Intelligence*, **226**, 1–33, (2015).
- [3] A. Kalyanpur, B. Parsia, B. Cuenca-Grau, and E. Sirin, ‘Axiom pinpointing: Finding (precise) justifications for arbitrary entailments in *SHOIN* (owl-dl)’, *Technical report, UMIACS, 2005-66*, (2006).
- [4] D. Lehmann and M. Magidor, ‘What does a conditional knowledge base entail?’, *Artificial Intelligence*, **55**(1), 1–60, (1992).
- [5] B. Parsia, E. Sirin, and A. Kalyanpur, ‘Debugging owl ontologies’, in *WWW*, pp. 633–640, (2005).
- [6] G. Qi, W. Liu, and D. A. Bell, ‘A revision-based approach to handling inconsistency in description logics’, *Artificial Intelligence Review*, **26**(1–2), 115–128, (2006).
- [7] S. Schlobach, ‘Diagnosing terminologies’, in *Proceedings of AAAI*, pp. 670–675, (2005).
- [8] S. Schlobach and R. Cornet, ‘Non-standard reasoning services for the debugging of description logic terminologies’, in *IJCAI*, pp. 355–362, (2003).