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Stochastic Volatility Models in Gretl

Paolo Chirico
University of Turin

Abstract Gretl allows to perform a wade variety of GARCH models by the gig package, but it doesn't allow to perform directly Stochastic Volatility models yet. This paper suggest how to implement these models by means of the new Gretl's Kalman Filter.

Key words: Stochastic Volatility, Kalman Filter

1. Introduction

According to a very common pattern for the log-returns, r_t, the term "volatility" refers to the coefficient σ_t into the modelling (Taylor, 2008):

$$\begin{aligned} r_t &= \mu + u_t \\ u_t &= \sigma_t\, \varepsilon_t \qquad\qquad \varepsilon_t \sim NID(0;1) \end{aligned} \tag{1}$$

Then, σ_t is a scale coefficient which affects the log-return variability, and, under suitable assumption, σ_t^2 corresponds to the conditional variance of r_t, which is generally non constant (conditional heteroscedasticity).

In Garch modelling σ_t is a deterministic function of the past information:

$$\begin{aligned} \sigma_t &= \sqrt{h_t} \\ h_t &= \omega + \sum_{j=1}^{p} \alpha_j u_{t-j}^2 + \sum_{j=1}^{q} \beta_j h_{t-j} \end{aligned} \tag{2}$$

Actually, model (2) is only the base of a variety of models, which shape conditional heteroscedasticity as a deterministic function of the past information. Gretl allows to perform directly a lot of these Garch variants by the gig package (Lucchetti, Balietti). Below, it is reported the gig version of the E-Garch(1,1) model:

$$\begin{aligned} \sigma_t &= \sqrt{h_t} \\ \ln h_t &= \omega + \alpha_1 |\varepsilon_{t-1}| + \gamma_1 \varepsilon_{t-1} + \beta_1 \ln h_{t-1} \end{aligned} \tag{3}$$

and in the equivalent (less used, but more consistent with the model name) form:

$$\begin{aligned} \sigma_t &= \exp(h_t/2) \\ h_t &= \alpha + \beta_1 h_{t-1} + \alpha_1 \left(|\varepsilon_{t-1}| - \sqrt{2/\pi} \right) + \gamma_1 \varepsilon_{t-1} \end{aligned} \tag{4}$$

where $\alpha = \omega + \alpha_1 \sqrt{2/\pi}$; now h_t is the log-conditional variance of r_t, The E-Garch model is very common because has some suitable features: (i) it does not require parametric constraints to assure positive variance; (ii) it makes the volatility very sensible to big shocks; (iii) it takes into account asymmetric effects of past innovations.

2. Stochastic Volatility models and Kalman Filter

Garch models are widely used in financial analysis although these models are based on a strong assumption: the volatility is a deterministic function of the past. That means the volatility at the next step (day, hour,…) is exactly determinable on the basis of the present information, that is a bit hard to justify with a realistic financial theory. Therefore, some scholars prefer to consider financial volatility as a stochastic process and shape it with models similar to the following:

$$\begin{aligned} \sigma_t &= \exp(h_t/2) \\ h_t &= \alpha + \beta_1 h_{t-1} + \eta_t \qquad \eta_t \sim NID(0; \sigma_\eta^2) \end{aligned} \tag{5}$$

Model (5) is known as *Stochastic Volatility* model[1] (Harvey, Ruiz, & Shepard, 1994) and can be viewed as a stochastic version of the E-Garch model, but we have to note that model (5) is not be able to takes into account asymmetric effects of past innovations. A solution could be the following *Asymmetric Stochastic Volatility* (ASV) model:

$$\begin{aligned} \sigma_t &= \exp(h_t/2) \\ h_t &= (\alpha + \delta u_{t-1}) + \beta_1 h_{t-1} + \eta_t \qquad \eta_t \sim NID(0; \sigma_\eta^2) \end{aligned} \tag{6}$$

Model (6) is quite similar to the model (4), if $\delta u_{t-1} \approx \gamma \varepsilon_{t-1}$; the error η_t, representing the shocks into volatility, stands in for $\alpha_1 \left(|\varepsilon_{t-1}| - \sqrt{2/\pi} \right)$.

If $\beta < 1$, h_t is a Gaussian stationary process, and then:

$$(h_{t+1}/2)|I_t \sim N\left(\frac{1}{2}\hat{h}_{t+1|t}; \frac{1}{4}P_{t+1|t}\right) \tag{7}$$

where:

$$\hat{h}_{t+1|t} = E[h_{t+1}|I_t] \tag{8a}$$

$$P_{t+1|t} = Var[h_{t+1}|I_t] \tag{8b}$$

As a result, the volatility σ_t is a log-normal stochastic process:

$$\sigma_{t+1}|I_t \sim logN\left(\frac{1}{2}\hat{h}_{t+1|t}; \frac{1}{4}P_{t+1|t}\right) \tag{9a}$$

That means:

$$\hat{\sigma}_{t+1|t} = E[\sigma_{t+1}|I_t] = \exp\left(\frac{1}{2}\hat{h}_{t+1|t} + \frac{1}{8}P_{t+1|t}\right) \tag{9b}$$

Unfortunately, h_t is not easy to forecast, because it is a latent process, not observable (determinable). Therefore h_t has to be filtered in some way; the main method to filter and forecast h_t consists in using

[1] More specifically, model (5) is a Stochastic Volatility model of order 1, SV(1), because h_t is an autoregressive process of order 1.

the Kalman Filter (KF). Therefore, if we set $y_t = \log u_t^2$, we have the following structural time series model:

$$\begin{cases} y_t = -1.27 + h_t + w_t & w_t \sim ID(0; 4.93) \\ h_t = (\alpha + \delta u_{t-1}) + \beta_1 h_{t-1} + \eta_t & \eta_t \sim NID\left(0; \sigma_\eta^2\right) \end{cases} \tag{10}$$

where $w_t = \log \varepsilon_t^2 + 1.27$, being $E(\log \varepsilon_t^2) = -1.27$ and $Var(\log \varepsilon_t^2) = 4.93$; $\delta, \alpha, \beta_1, \sigma_\eta^2$ are the model parameters.

In case of heavy tailed standardized errors, ε_t, i.e. standardized Student's t-distributions with v degrees of freedom, the values -1.27 and 4.93 in (10) should be replaced by the values:

$$\theta_0 = -1.27 - \psi^0\left(\frac{v}{2}\right) + \ln\left(\frac{v-2}{2}\right) \tag{11}$$

$$\theta_1 = 4.93 + \psi^1\left(\frac{v}{2}\right) \tag{12}$$

where ψ^0 and ψ^1 are the *di-gamma* and *tri-gamma* functions respectively (Chirico, 2017).

As known, the KF allows to filter and forecast optimally the latent components of structural time series models, but, in this case, the application of the KF is sub-optimal since the noise w_t is not Gaussian. Nevertheless, the KF remains optimal among the linear estimators/predictors (MMSLEs) and it works well for the sample sizes typically encountered in financial analysis (Ruiz, 1994).

2. Implementation in Gretl's Kalman Filter

In 2016, Gretl's Kalman Filter was restyled with a new interface and commands: now the KF is implemented as a bundle generated by the function "ksetup". At the moment, Gretl's Kalman Filter doesn't allow the use of exogenous variables (u_t) in the state equation directly. In this case we can consider u_t as a false latent variable. That means transforming the model (10) in the following bi-dimensional structural time series model:

$$\begin{cases} y_t = -1.27 + h_t + w_t & w_t \sim ID(0; 4.93) \\ r_t = \mu + u_t & \\ & \\ h_t = \alpha + \beta_1 h_{t-1} + \delta u_{t-1} + \eta_t & \eta_t \sim NID\left(0; \sigma_\eta^2\right) \\ u_t = v_t & v_t \sim NID\left(0; \sigma^2\right) \end{cases} \tag{11}$$

Model (11) is implemented in Gretl by the following commands:

```
# Preliminary setup
series r = log-returns          # input log-returns
scalar mu = mean(r)
scalar sigma = sd(r)
scalar alpha = alpha0           # starting value for alpha
```

```
scalar beta = beta0               # starting value for beta
scalar gamma = gamma0       # starting value for gamma
scalar sigmeta = sigmeta0     # starting value for sigmeta

matrix H = {1, 0; 0, 1}
matrix F = {beta, gamma; 0, 0}
matrix Q = {sigmeta^2, 0; 0, sigma^2}
series y = log( (r-mu)^2 )
matrix yr = {y, r}

# Model definition
bundle SVM = ksetup(yr, H, F, Q)
SVM.obsxmat = {-1.27, mu}
SVM.obsvar = {4.93, 0; 0, 0}
SVM.stconst = {alpha; 0}
```

Generally the values of model parameters, $\alpha, \beta_1,\ \delta, \sigma_\eta$, are not know; therefore these parameters have to fix initially with suitable starting values[2], then the optimal values are found by maximum likelihood estimation:

```
mle ll = ERR ? NA : SVM.llt
   SVM.statevar[1,1] = sigmeta^2
   SVM.stconst[1,1] = alpha
   SVM.statemat[1,1] = beta
   SVM.statemat[1,2] = gamma
   ERR = kfilter(&SVM)
params alpha beta gamma sigmeta
end mle --hessian
```

Finally, the volatility is drawn got by the commands:

```
ksmooth(&SVmod)
series s_sv=exp( SVM.state[,1]/2 + SVM.stvar[,1]/8 )
series eps_sv = (r-mu)/s_sv              # standardized errors
```

3. A case study: Generali stock

We considered the Generali stock price from 2009-04-16 to 2011-04-08 (517 observation); we calculated the price log-returns, r_t, then, we modeled the volatility of r_t by an E-Garch(1,1) and an ASV(1) (model 6); the E-Garch model was estimated by the Gretl's gig package; the ASV model by the script above. Table 1 and Table2 report the estimation results; Figure 1 reports the volatility series

[2] For example, *inialpha* and *inibeta* could be the values of the corresponding parameters in the E-Garch model; *inisigmeta* is fixed equal zero.

by both methods; Figure 2 reports the Quantile-Quantile plot of the standardized errors by both methods.

Table 1: EGARCH(1,1) [Nelson] (Normal)

Dependent variable: ld_generali
Sample: 2009-04-17 -- 2011-04-08 (T = 516), VCV method: Robust

	coefficient	std. error	z	p-value	
omega	-0.606608	0.176318	-3.440	0.0006	***
alpha	0.164792	0.0581530	2.834	0.0046	***
gamma	-0.0885252	0.0329905	-2.683	0.0073	***
beta	0.941855	0.0218515	43.10	0.0000	***

Log-liklihood: 1380.67 (1311.84, sample 2009-05-30, 2011-04-08)

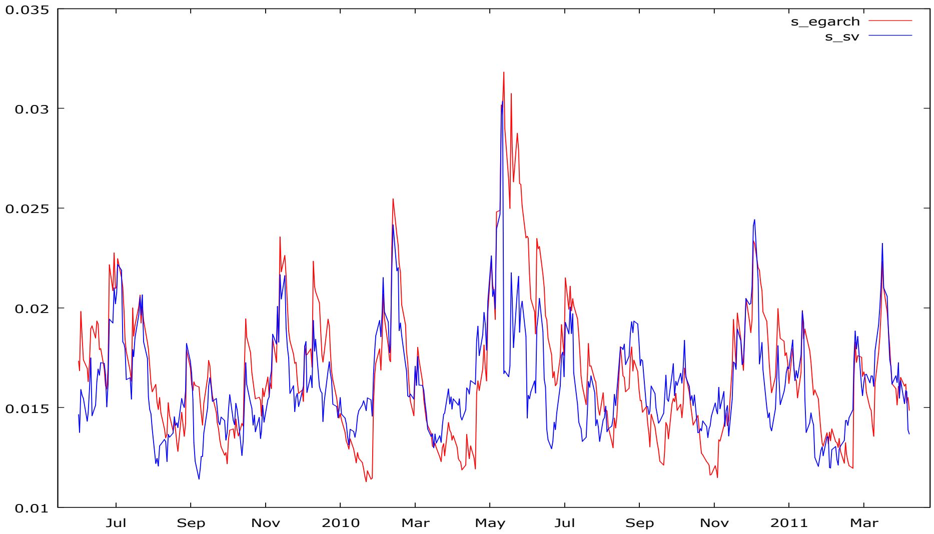
Table 2: ASV(1) [Kalman Filter] (Normal)

Dependent variable: ld_generali
Sample: 2009-04-16 -- 2011-04-08 (T = 517), ML, SVM.llt

	coefficient	std. error	z	p-value	
sigmeta	0.0747468	0.165669	0.4512	0.6519	
alpha	-1.02031	1.01330	-1.007	0.3140	
beta	0.877496	0.122228	7.179	7.01e-013	***
gamma	-8.44457	4.56377	-1.850	0.0643	*

Log-liklihood: 1376.5 (1315.4, sample 2009-05-30, 2011-04-08)

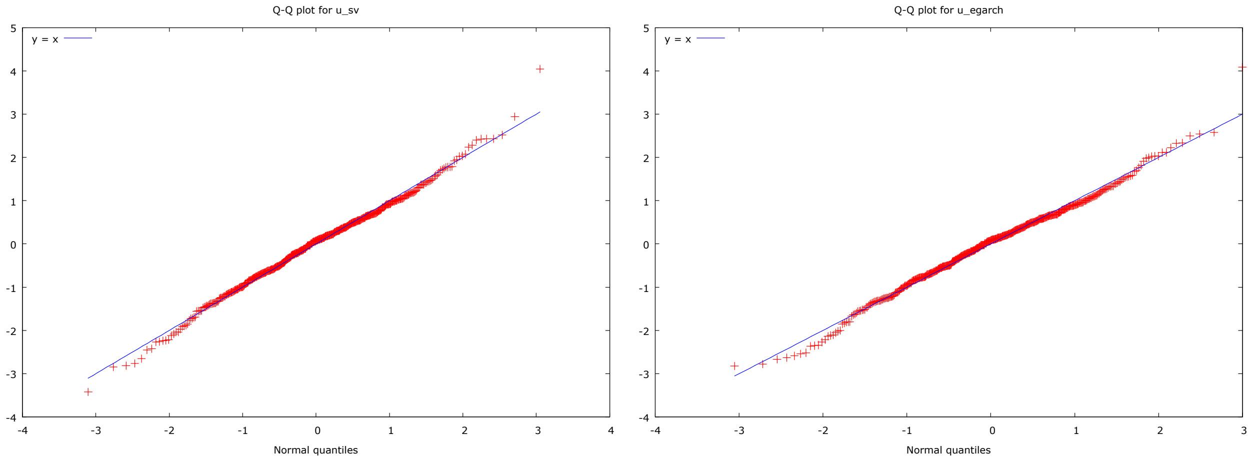
Figure 1: Volatility with E-Garch and ASV models



On the basis of these results, the ASV model doesn't seem inferior to the E-Garch. Obviously, the volatility are different between the models, but present the same dynamics.

On the full sample, the E-Garch log-likelihood is higher than ASV likelihood, but the ASV likelihood is higher if we ignore the first two weeks. As known, the Kalman Filter generally requires some steps to fit well series.

Figure 2: Quantile-Quantile plot of the standardized errors



Conclusions

The paper illustrates a way to implement stochastic volatility models in Gretl using the Gretl's Kalman Filter on suitable state space models. The implementation of the model is not very difficult, but the estimation results depend extremely on the starting values of the parameters and on the starting values of the state components. At the moment, I haven't find a standard solution for this problem, but I am confident that this drawback can be limited!

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