

MR2960042 18B05 16T05 18C50 68Q55

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**A quantum double construction in Rel. (English summary)**

*Math. Structures Comput. Sci.* **22** (2012), no. 4, 618–650.

This paper focuses on the category Rel of sets and binary relations. Rel is a compact closed category, that is, a ribbon category in which the braiding is a symmetry. Bialgebras and Hopf algebras (bialgebras with antipodes) are studied in Rel. In particular, it is proved that various monoidal categories with extra structure, like traces and autonomy, can be derived as the categories of (co)modules of bialgebras in Rel.

An example is considered. For any group  $G$ , by applying the quantum double construction to its associated Hopf algebra in Rel, it is possible to build a noncommutative non-co-commutative Hopf algebra with a universal  $R$ -matrix and a universal twist. The ribbon category of its modules turns out to be a category of crossed  $G$ -sets. Thus a characterization of crossed  $G$ -sets in terms of a quantum double construction is given.

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*Note: This list reflects references listed in the original paper as accurately as possible with no attempt to correct errors.*