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Solution to the range problem for combinatory logic. (English summary)

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In lambda-calculus, the theory \mathcal{H} is obtained from β -conversion by identifying all closed unsolvable terms (or, equivalently, terms without head normal form). The range problem for \mathcal{H} asks whether a closed term has always (up to equality in \mathcal{H}) either an infinite range or a singleton range (that is, it is a constant function).

The paper tackles a modification of the above problem in the combinatory logic setting. A natural notion of head-normal form is defined for combinatory logic, albeit it lacks the equivalence between “terms without head normal form” and “terms being unsolvable”. Let \mathcal{H}_{CL} denote the theory obtained from the weak-reduction by identifying all terms without head normal forms (of combinatory logic). The theory \mathcal{H}_{CL} is weaker than \mathcal{H} . The main result states that for every term M of combinatory logic, the set $\{MP \mid P \text{ closed}\}$ is either a singleton or infinite modulo \mathcal{H}_{CL} -equality.

Although this result adds evidence to the conjecture that the range problem for \mathcal{H} has a positive answer, a negative answer to it was announced in September 2010 by Polonsky.

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