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The search for a reduction in combinatory logic equivalent to $\lambda\beta$ -reduction.

(English summary)

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Finding reductions and conversions of combinatory logic (CL) modelling those of lambda-calculus (or vice versa) is a tricky issue. The nub of the matter, addressed by the paper under review, is the search for a satisfying notion of reduction for CL modelling the beta-reduction of lambda-calculus. Motivations and classical results are surveyed, with special care for historical remarks and references.

Many alternative representations of lambda-abstraction in CL are provided to settle many results. Three main proposals for modelling beta-reduction in CL are considered and refined on the basis of both practical and theoretical analysis. On the historical side, quoting the paper: “A full account of Curry’s interest in CL, why originally he was interested in equality rather than reduction, and why he eventually became interested in CL reduction, will be found in Appendix”.

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Note: This list reflects references listed in the original paper as accurately as possible with no attempt to correct errors.