# Path to Numbers Writing: A Longitudinal Study with Children from 3.5 to 5.5 Years Old 

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#### Abstract

The issue of the role of number words in preschoolers' arithmetic development is still open. The development of writing Arabic numerals 1 to 9 under dictation was investigated longitudinally, testing a cohort of 20 children at six-month intervals from age 3.5 to 5.5 years. The specific aim was to examine how children arrive at a correct representation of Arabic numerals, and explore whether this development has a unique or multiple paths.


For each subject we were able to observe the types of number notation used at each of the five evaluation periods.

The data obtained indicate that, at each age, most children use different types of number representation but also that the path to correct Arabic representation is not unique. We identified five paths to correct numeral writing, which we defined as 1) linear, 2) forward and backward, 3) no symbols, 4) symbols, and 5) an early number developmental path.

Keywords: preschool, numbers writing, longitudinal study

## 1. Introduction

Many preschoolers are familiar with counting and have some understanding of the meaning of number words (Carey, 2004; Gelman \& Gallistel, 1978; Sarnecka \& Gelman, 2004). However, the debate on the role that number words play in preschoolers' arithmetic development is still open. For example, Carey (2004) claims that knowledge of number words plays a bootstrapping role in the acquisition of mathematical concepts. On the other hand, Gelman and Butterworth (2005) argue that the numerical domain is ontogenetically independent of language, and that conceptual development does not depend on number-word knowledge.
There are many studies on the development of numerical cognition, a body of research that has largely focused on the development of identification or representation of quantity. Over the past two decades, evidence has accumulated to support the hypothesis that children possess innate conceptual principles that guide them in learning how to count during their preschool years (Brannon, 2002; Carey, 2001; Gallistel \& Gellman, 2000; Gallistel, Gellman, \& Cordes, 2006; Muldoon, Lewis, \& Francis, 2007; Wynn, 1995; Xu \& Spelke, 2000). For example, some authors have shown that infants (at six months) are able to represent small quantities of objects, using a visuospatial object-file-based system that allows recognition of the numerosity of 3-4 objects (Feigenson, 2005; Xu, 2003), or to estimate large numerosities (e.g., Xu \& Arriga, 2007). By the same age, infants are able to detect a change in numerosity when elements are added or removed (e.g., Wynn, 1992).

In contrast with the considerable attention given to the acquisition of graphemes as symbolic notation of spoken words (examined intensively since the pioneering studies of Ferreiro \& Teberosky, 1984; Ferreiro \& Gómez Palacio, 1988), few studies have been conducted about the development of number writing in particular.

### 1.1 Literature Review and Theoretical Framework

Various authors have contributed to the description the development of number writing using a cognitive constructive theoretical approach. The first author was Jean Piaget (Piaget \& Szeminska 1941) who sustained that from the second year of life a child can represent an object throughout another. During preschool years children learn two basilar processes: the production of individual significant (symbols), and the production of
collective significant (signs) related to meaning from a social convention and therefore external to the subject. For the child to reach a full acquisition of the sign, it is essential that the transition from the production of personal meanings to that of the conventional ones occurs. Therefore, the lifetime from 2 to 6-7 years is a key period for development of symbolic processes and the access to conventional mathematical notation. During this period the child acquires both the concept of number and knowledge of conventional signs.
Few studies have been devoted to the development of Arabic number production. One study was carried out by Power and Dal Martello (1990) with seven-year-old children, a second by Seron, Deloche and Noel (1991) with children aged from seven to nine years and a third study was conducted by Seron and Fayol (1994), again with children of primary school. In all the studies, children were asked to write down as digits numbers that were dictated. The task thus required the transcoding of verbal notation into an Arabic one. In all the studies, results indicated that children produced a significant amount of errors that could be classified into two types, lexical errors and syntactical errors. In particular, Seron and Fayol concluded that the children's difficulties in the Arabic written task were localized at the level of the production of the Arabic sequences.
However, these studies have focused on the primary school period, while other authors investigated the ideas these children held about aspects of the written number system, and how this system works, contributing to its description using a cognitive constructive theoretical approach (Agli, \& Martini, 1995; Hughes, 1986; Sastre \& Moreno, 1976; Sinclair, Mello, \& Siegrist, 1988; Sinclair, \& De Zwarth, 1989).
Karmiloff-Smith (1992) emphasized how the comprehension of the notation system, considered as an object of knowledge per se with its own structural principles, precedes the use of notation as a referential-communicative instrument. As early as four years old, children use and discriminate among different symbolic notations. For example, the presence of isolated elements and the repetition of identical ones are considered as appropriated for writing numbers, not words. At around five to six years old, children are able to manipulate the constraints of symbolic systems, for example to write pseudo numbers, using drawings or combinations of letters. At the end of kindergarten age and the beginning of elementary school, the development of these competencies begins to encounter both the cognitive system of quantity and the rules of calculation.
Agli and Martini (1995) confirmed that verbal and conceptual arithmetic precedes written arithmetic. For example, they verified that four-year-olds have no difficulty in counting a few elements but still use iconic and pictographic representation. Moreover, they pointed out that children use different forms of quantitative notation, depending on the type of problem or circumstances in question. In other words, the various forms of quantitative notation do not appear to follow a rigid developmental sequence. These authors concluded that the different written notations are strategies that children discover and use according to the different problems or quantitative representations they have to solve.
In a more recent study of five-year-olds, Brizuela (2001) found that even before any formal schooling, children have many important ideas about the number words (e.g., ideas about the number of digits numbers and about how new numbers can be generated from a set of ten digits and the role of zero).
Arnas, Sigirtmac and Gul (2004) designed a study to investigate the ability of young (60-to 89-months-old) children to write numerals, identifying which they write incorrectly and what sort of mistakes they make in the process. They found that most children wrote numerals correctly and that their skills in writing numerals correctly increased in direct proportion to age. For numerals written upside-down, frequency decreased directly with age. Writing numerals as letters occurred at a low rate in all age groups and decreased with age.
In a very insightful study, Hughes (1986) investigated pre-school children's ideas about written symbolism, asking them to produce their own written representation of number concepts using pencil and paper. He found that children use a variety of ways to represent quantity, divisible into four main categories:

1) idiosyncratic responses or null continuous forms: forms that are meaningless to an external observer but having some numerical meaning for the child concerned;
2) pictographic responses or discrete forms: visual representation of objects with partial reference to their numerosity;
3) iconic responses of biunivocal relationship: a one-to-one correspondence of numerosity made with various forms, bars, objects, letters etc.;
4) symbolic responses: conventional notations of numbers.

Each of these notation categories is characterized by a specific graphical expression. For example, in the pictographic notation, a figurative pictorial format prevails. In the iconic responses, the graphical productions are
roughly abstract signs such as, bars, dots, and letters. Only the conventional notations are represented by Arabic numerals.
Furthermore, Hughes (1986) observed that during development, younger pre-school children favored iconic and idiosyncratic representations, while older children were more likely to produce pictographic and symbolic forms.
Starting from the Hughes' work, in a more recent study, Carruthers and Worthington (2005) analyzed 700 examples of mathematical graphics and identified five common forms of graphics: dynamic, pictographic, iconic, written and symbolic. The most important conclusion that the authors report is that usually teachers fail to recognize children's own mathematical graphics and, consequently, they do not support children in developing their own predisposition.
A limitation of the Hughes' as well as the Carruthers and Worthington's classification is that they do not differentiate all the possible productions of children, especially for the younger ones. Pontecorvo (1985) and, similarly, Sinclair and Sinclair (1984), identified a more analytic classification that permits to classify the evolution of the number writing. So Pontecorvo individuated the following six categories:

1) Continuous forms without any similarity to graphemes or numbers: the child produced continuous forms that have no significance for the observer, but that for the child represent the correct number.
2) Discrete forms without any similarity to graphemes or numbers: the child produced discrete forms that have no significance for the observer, but that for the child represent the correct number.
3) Biunivocal correspondence: there was correct representation of numerosity using any sign or symbol other than Arabic numbers.
4) Use of the number in the second time.
5) Not conventional use of numbers: the child reproduced the sequence of numbers as in counting (e.g. ,1,2, $3,4 \ldots$ ) or repeated the correct number many times as there are things to count e.g., $1,22,333 \ldots$.
6) Correct use of Arabic number.

In summary, even though these empirical studies have found points of convergence, a unique theory on the development of the competence of the number written does not exist. At present we know that preschool children have many ideas about the number system (Brizuela, 2001), and that writing numbers is a skill that increases directly with age (Arnas, Sigirtmac, \& Gul, 2004), and is linked to verbal and conceptual arithmetic (Agli \& Martini, 1995; Molfese, Beswick, Molnar, \& Jacobi-Vessels, 2006). Moreover, we know that preschoolers favor iconic and idiosyncratic representations, while older children are more likely to produce pictographic and symbolic forms (Hughes, 1986).
However, to know how children progress from pseudo-numerical notations to the correct Arabic numbers, these changes must be considered longitudinally, not simply by observing the performance of children at different ages, as is the case in most current studies. Only this type of investigation design can raise confidence about how children develop their capacity to represent quantities using the conventional rules. For this reason we set up a study to investigate the development of writing Arabic numerals 1 to 9 under dictation, longitudinally testing a cohort of 20 children at six-month intervals from age 3.5 to 5.5 years, with the specific aim to examine how children arrive at a correct representation of Arabic numerals, and explore whether this development has a unique path or multiple paths.

## 2. Method

### 2.1 Participant

Twenty 3.5 years old children ( 10 males, 10 females) were followed longitudinally until they were 5.5 years old.
They were recruited in eight different kindergarten schools after formal agreement with their teachers that no formal instruction on number knowledge would be delivered during the course of the study.
Children were selected to ensure compliance with the following four criteria:

1) Chronological homogeneity: all children have birthdays during the same month.
2) Socio-economic homogeneity: all children were drawn from a socio-economic background of medium level. All parents had gained at least a high school or graduate certificate.
3) IQ homogeneity: the IQ of each child was measured, in order to eliminate any with problems in cognitive development (e.g., mental retardation): this was achieved using the Wechsler Preschool and Primary Scale of Intelligence - III edition (WPPSI-III, Wechsler, 2002). WPPSI is one of the most frequently used IQ assessment
instruments available in Italy, and has good psychometric properties, including in the Italian version (e.g., reliability: range $0.53-0.97$ ). All children obtained standardized IQ scores within a normal level range ( $M=105$; $\mathrm{DS}=4.3$ ).
4) All children were not exposed to formal numerical literacy.

### 2.2 Tools of Analyses

In a precedent exploratory pilot study that examined the main hypotheses on the development of numerical knowledge, including writing numbers, Lucangeli and Tressoldi (2002) collected "typical" children's responses to the writing numbers task and used both Hughues' and Pontecorvo's models to analyze the data. Such pilot study revealed that the children's responses found a better fit within the categories described by Pontecorvo (Lucangeli \& Tressoldi, 2002). In the corrent study, with respect to the Pontecorvo's classification and according to our pilot study results (Lucangeli \& Tressoldi, 2002), we added a "graphemes" category, not considered as an option to write numerals. This category consists in the fact that the child wrote a grapheme, and/or "other pseudo-symbolic forms" instead of a number. Moreover, we distinguished two steps in the writing of Arabic numbers: correct and wrong; in other words, we considered as a separate condition when the child tried to write a number but $\mathrm{s} / \mathrm{he}$ made mistakes as, for example, when $\mathrm{s} / \mathrm{he}$ wrote the a specular number. (Figure 1 shows a prototypical response for each of the categories of the theory).
Consequently, we classified all children's productions, according to the following categories:

1) Continuous forms without any similarity to graphemes or numbers ( CF ): the child produced continuous forms that have no significance for the observer, but that for the child represent the correct number (fig. 1-a).
2) Discrete forms without any similarity to graphemes or numbers (DF): the child produced discrete forms that have no significance for the observer, but that for the child represent the correct number (fig. 1 -b).
3) Graphemes (G): the child wrote a letter instead of the dictated number (fig. $1-\mathrm{c}$ ).
4) Other pseudo-symbolic forms (OS): the child produced peculiar symbols that indicate the number (fig. 1-d).
5) Biunivocal correspondence ( BC ): there was correct representation of numerosity using any sign or symbol other than Arabic numbers) (fig. 1 -e).
6) Different Arabic number (wrong number, WN): the child wrote a number, but it was incorrect with respect to the number dictated or it was written in wrong spatial form i.e., 6 as 9 (fig. $1-\mathrm{f}$ ).
7) Correct Arabic number $(\mathrm{CN})$ : the number was written in the correct conventional form.


Figure 1. Examples of quantitative notations

### 2.3 Procedure

Testing was carried out by a research assistant in individual sessions in a room of the child's school. Each participant was asked to write the number dictated by the examiner on a blank sheet of paper. Numbers 1 to 9 were dictated in pseudo-random order, avoiding dictation of numbers in succession (i.e., 3, 4). Each number was dictated three times (in pseudo-random order) in order to obtain a more reliable result for a total of 27 items.
The experimenter, with the supervision of one of the authors, classified every "number" produced by the child in one of the seven categories to achieve a consensus of $100 \%$.
For each age the percentage of each category was calculated with respect the total of items produced.
This task was repeated at six-month intervals a total of five times until each participant reached 5.5 years old.

## 3. Results

Table 1 gives the percentages of the different categories observed at each age. At age 3.5 only 12 subjects ( $60 \%$ ) were able to produce one sign or more. From age four onwards, all subjects produced at least one sign for every number proposed. (Note again that the numbers from 1 to 9 were presented three times.)

Table 1. Percentages of the different numbers notations categories

| Age (years) | Cf | Df | G | Os | BC | WN | CN |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 3.5 | 9.6 | 19.3 | 6.4 | 25.8 | 12.9 | 19.3 | 6.4 |
| 4 | 0 | 11.4 | 22.8 | 17.1 | 11.4 | 14.2 | 22.8 |
| 4.5 | 0 | 0 | 14.7 | 20.5 | 20.5 | 22.8 | 20.5 |
| 5 | 0 | 0 | 0 | 9.5 | 0 | 23.8 | 66.6 |
| 5.5 | 0 | 0 | 0 | 0 | 0 | 16.6 | 83.3 |

Table 1 clearly shows the development from an undifferentiated representation of numbers to their correct Arabic representation, passing through the use of graphemes or signs representing the numerosity of the dictated number (biunivocal correspondence), confirming the data obtained with transversal studies (Pontecorvo, 1985).
Another interesting finding is the presence of multiple forms of number representation even at the earlier ages, 3.5 and 4.0 with a presence of more advanced representation (WN and CN categories) of $25.7 \%$ at age 3.5 and $37 \%$ at age 4.
However, this way of summarizing the data masks the individual development of the number representation, giving a false impression of a linear development from CF (continuous forms) to CN (correct Arabic number) categories. Instead, when analyzing the single pattern of development, we found multiple paths of number representation.
For each subject we were able to observe the type of number notations used at each of the five evaluation periods, reconstructing each individual developmental pattern. At each age, we considered not just the prevalent use of the various notations, but all notations used independently from their frequency.
Analyzing all notations observed for each participant at each age point, we could recognize five developmental trajectories qualitatively distinguishable. This finding constitutes a new contribution to the literature.

### 3.1 Linear Developmental Pattern

This developmental pattern, (see Fig. 2, 3 and 4), is characterized by the correct representation of numbers at age five, after the use of almost all other representation forms at earlier ages.


Figure 2. Subject D
Legend: $\mathrm{CF}=$ continues forms without any similarity to graphemes or numbers; $\mathrm{DF}=$ discrete forms without any similarity to graphemes; $\mathrm{G}=$ graphemes; $\mathrm{OS}=$ other pseudo symbolic forms; $\mathrm{BC}=$ biunivocal correspondence; $\mathrm{WN}=$ Wrong Arabic number; $\mathrm{CN}=$ Correct Arabic number.


Figure 3. Subject G
Legend: $\mathrm{CF}=$ continues forms without any similarity to graphemes or numbers; $\mathrm{DF}=$ discrete forms without any similarity to graphemes; $\mathrm{G}=$ graphemes; $\mathrm{OS}=$ other pseudo symbolic forms; $\mathrm{BC}=$ biunivocal correspondence; $\mathrm{WN}=$ Wrong Arabic number; CN = Correct Arabic number.


Figure 4. Subject I
Legend: $\mathrm{CF}=$ continues forms without any similarity to graphemes or numbers; $\mathrm{DF}=$ discrete forms without any similarity to graphemes; $\mathrm{G}=$ graphemes; $\mathrm{OS}=$ other pseudo symbolic forms; $\mathrm{BC}=$ biunivocal correspondence; $\mathrm{WN}=$ Wrong Arabic number; $\mathrm{CN}=$ Correct Arabic number.

As it can be seen from the figures, children use discrete forms without any similarity to graph (DF), graphemes (G), other pseudo symbolic forms (OS), biunivocal correspondence (BC) and finally arrive to the Arabic number (CN), even if in a first moment making several errors (WN). In other words, they present a linear developmental pattern using all the possible representation forms.

### 3.2 Forward and Backward Developmental Pattern

This developmental pattern, observed in three subjects (see Fig. 5, 6, 7), is characterized by the use of more "primitive" forms of number representation after the use of a more "advanced" form at an earlier age.


Figure 5. Forward and backward developmental pattern (Subject A)
Legend: $\mathrm{CF}=$ continues forms without any similarity to graphemes or numbers; $\mathrm{DF}=$ discrete forms without any similarity to graphemes; $\mathrm{G}=$ graphemes; $\mathrm{OS}=$ other pseudo symbolic forms; $\mathrm{BC}=$ biunivocal correspondence; $\mathrm{WN}=$ Wrong Arabic number; CN = Correct Arabic number.


Figure 6. Forward and backward developmental pattern (Subject B)
Legend: $\mathrm{CF}=$ continues forms without any similarity to graphemes or numbers; $\mathrm{DF}=$ discrete forms without any similarity to graphemes; $\mathrm{G}=$ graphemes; $\mathrm{OS}=$ other pseudo symbolic forms; $\mathrm{BC}=$ biunivocal correspondence; $\mathrm{WN}=$ Wrong Arabic number; $\mathrm{CN}=$ Correct Arabic number.


Figure 7. Forward and backward developmental pattern (Subject N)
Legend: $\mathrm{CF}=$ continues forms without any similarity to graphemes or numbers; $\mathrm{DF}=$ discrete forms without any similarity to graphemes; $\mathrm{G}=$ graphemes; $\mathrm{OS}=$ other pseudo symbolic forms; $\mathrm{BC}=$ biunivocal correspondence; $\mathrm{WN}=$ Wrong Arabic number; $\mathrm{CN}=$ Correct Arabic number.

This pattern is characterized by the fact that children use more advanced and simple form at the same time. For example subject N (fig. 7) at 3.5 years use the biunivocal correspondence (BN), at age 4 use both biunivocal correspondence ( BN ) and graphemes (G), at age 4.5 the child use graphemes (G) and other pseudo symbolic forms (OS) that are more "primitive" than biunivocal correspondence (BN) that was used at age 3.5.

### 3.3 No Symbols Developmental Pattern

This developmental pattern, observed in three subjects (see Fig. 8), is characterized by the complete absence of the use of graphemes or other symbols and by the use of the biunivocal correspondence representation before the use of numbers.


Figure 8. No symbols developmental pattern (Subject E, F, H)
Legend: $\mathrm{CF}=$ continues forms without any similarity to graphemes or numbers; $\mathrm{DF}=$ discrete forms without any similarity to graphemes; $\mathrm{G}=$ graphemes; $\mathrm{OS}=$ other pseudo symbolic forms; $\mathrm{BC}=$ biunivocal correspondence; $\mathrm{WN}=$ Wrong Arabic number; $\mathrm{CN}=$ Correct Arabic number.

As can be seen in Figure 8, these children do not use graphemes and other pseudo-symbolic forms.

### 3.4 Symbols Developmental Pattern

This pattern, present in five children, is characterized by the use of symbols before the use of correct Arabic numbers. This pattern is the opposite of the previous one (No symbols developmental patter): all these children are characterized by the fact that they do not use the biunivocal correspondence, but symbols (see Fig. 9). In
other words, these children use symbols before use numbers.


Figure 9. Symbols developmental pattern (Subject L, M, O, P, Q)
Legend: $\mathrm{CF}=$ continues forms without any similarity to graphemes or numbers; $\mathrm{DF}=$ discrete forms without any similarity to graphemes; $\mathrm{G}=$ graphemes: $\mathrm{OS}=$ other pseudo symbolic forms; $\mathrm{BC}=$ biunivocal correspondence; $\mathrm{WN}=$ Wrong Arabic number; $\mathrm{CN}=$ Correct Arabic number.

### 3.5 Early Number Developmental Pattern

The final pattern observed in six subjects, the "early number developmental pattern" is characterized by an early use of numbers, exclusively or associated with other less advanced representation forms at earlier ages (see Fig. 10).


Figure 10. Early number developmental pattern (Subject R, S, T, U, V, C)
Legend: $\mathrm{CF}=$ continues forms without any similarity to graphemes or numbers; $\mathrm{DF}=$ discrete forms without any similarity to graphemes; $\mathrm{G}=$ graphemes: $\mathrm{OS}=$ other pseudo symbolic forms; $\mathrm{BC}=$ biunivocal correspondence; $\mathrm{WN}=$ Wrong Arabic number; $\mathrm{CN}=$ Correct Arabic number.

Most of these children show a less advanced representation at age 3.5 and then a very advanced representation as biunivocal correspondence (BC) or wrong Arabic number (WN) or correct Arabic number (CN). For example subject U and T showed a BC at age 3.5 and subject V showed WN at age 3.5 ; subjects $\mathrm{S}, \mathrm{T}$ and U reached the (CN) representation at age 4.

## 4. Discussion

An overview of the literature reveals that preschool children have many ideas about the number system (Brizuela, 2001), even before any formal schooling. Most research in this area has focused on the child's understanding of the relationship between the notations they use and the quantities these represent in terms of collections of
objects, and have considered this relationship by comparing children of different ages or levels of arithmetic abilities. However, a more accurate approach is to observe these changes longitudinally, to gain a clearer understanding of how children develop their capacity to represent quantities using conventional rules.
Data obtained to date suggest that all children pass through the same developmental phases. However, there is no support in our longitudinal data for the hypothesis of a unique linear progression from continuous forms (CF) without any similarity to graphemes or numbers; discrete forms without any similarity to graphemes or numbers (DF); graphemes (G); other pseudo-symbolic forms (OS); biunivocal correspondence (BC); wrong number (WN); correct number (CN), as hypothesized by other authors (Pontecorvo, 1985; Hughes, 1986; Carruthers \& Worthington, 2005;). Only 3 out 20 subjects ( $15 \%$ ) showed a linear progression from symbols different from graphemes to biunivocal correspondence to WN and CN writing, while the other children showed several paths of development. The data obtained clearly indicate that at each age, most children use a variety of types of number representation, but that the path to correct Arabic representation is not unique. We identified five paths to correct number writing, which we define as follows:

- linear, characterized by the correct representation of numbers at age five, following the use of almost all other representation forms at earlier ages and that represent the hypothesized linear progression present in literature (Pontecorvo, 1985; Hughes, 1986; Carruthers \& Worthington, 2005) ;
- forward and backward, characterized by the use of more "primitive" forms of number representation after the use of a more "advanced" form at a previous age;
- no symbols, characterized by the complete absence of the use of graphemes or other symbols, and by the use of the biunivocal correspondence representation before the use of numbers;
- symbols, characterized by the use of symbols before the use of correct Arabic numbers;
- early number developmental path, characterized by an early use of numbers, exclusively or associated with other less advanced representation forms at earlier ages.
What emerges is that at each age, most children use different notations to represent quantities, an indicator that multiple concepts of its representation may coexist. As early as 3.5 years old, children represent numbers using forms other than graphemes, suggesting an early conceptual differentiation between writing words and numbers.
A second important finding is that the paths to number writings rely very little on capacity to write graphemes. In fact, only two subjects ("O" and "P") used graphemes without any other symbols or numbers at one age level, whereas six ("G", "B", "L", M" and "C") used graphemes with other symbols or numbers; the remaining twelve subjects made no use of graphemes at all.
Such findings suggest that from 3.5 years old, children begin to acquire a conceptual representation of how quantities can be graphically represented; furthermore, in most cases it is already clear that these graphic forms are - or should be - different from those needed to represent words.
Once again numerical knowledge appears to be a distinct conceptual domain that follows specific and independent developmental paths.
It should be pointed out, however, that despite its advantage of observing children longitudinally, this study has some limitations. For example, it does not address why there are so many different patterns to numbers writing.
Although our procedure controlled that no formal instruction on number education was delivered to our subjects by their teachers, we cannot exclude children having possible exposure to numbers and number use through observing their older peers and family members. This "number exposure" is very difficult to have under control: any interpretation of why subjects present a particular developmental pattern must therefore be purely speculative. Moreover, our sample is little, so we could not control for the potential influence of fine motor control, or other cognitive characteristics (apart IQ) that can limit the child's writing abilities.
However, despite these limitations, our findings have important implications in education, in view of their confirmation of the existence of individual pathways of development. Our data underline the importance of educators/teachers being acquainted with the developmental models of number writing that explain trends, but also their need to take individual profiles into account. It is the differences in individual profiles that identify which processes should be enhanced through education in order to ensure that all components of an individual's pattern develop. In line with the findings of Carruthers and Worthington (2005), we underline that it is crucial for teachers to recognize and develop the individual's pattern of writing number, because in doing so they will help children translate between the production of their personal significant (informal marks) to the conventional one (abstract symbolism).

How should teachers handle the different pathways? If teachers fail to recognize each child's individual characteristics, their teaching will not be personalized, and the benefits of an individual teaching plan - starting out from each child's own level of development and targeting their particular weak areas - will be lost. As already pointed out in a study by Van Loosbroek (2009), children with difficulties in mathematics continue to present difficulties at school age, being both slower and making more errors. It can be postulated that a careful approach to number writing already in preschoolers can lower the risk of difficulties arising in primary school mathematics.
In summary, we can conclude that our findings provide confirmation that common patterns exist (e.g., Hughes, 1986; Pontecorvo, 1985); however, longitudinal data indicate the existence of very marked individual qualitative differences, working as different paths to reach a same goal. It is these individual differences that should be targeted by educational strategies.
It is important to recall that the writing representation of numbers follows an innate capacity, albeit limited, to master numerosity, counting and number calculation (Butterworth, 2005). Consequently, counting experiences (i.e., with objects, symbols, etc.) may favor the use of different representations of quantities. Further investigations specifically designed to explore this relationship would clarify their mutual influence.
Our data provide a useful point of departure for observational and experimental investigations exploring the cognitive and educational variables that favor one number representation path or another.

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