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Thermodynamic instabilities and statistical effects in protoneutron stars

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Abstract.
We investigate the physical properties of the protoneutron stars in the framework of a relativistic mean-field model and we study the finite-temperature equation of state in β-stable matter at fixed entropy per baryon, in the presence of hyperons, Δ-isobar resonances and trapped neutrinos. In this context, we study the possible presence of thermodynamic instabilities and a phase transition from nucleonic matter to resonance-dominated Δ-matter can take place. Such a phase transition is characterized by both mechanical instability (fluctuations on the baryon density) and by chemical-diffusive instability (fluctuations on the isospin concentration) in asymmetric nuclear matter. We show that such statistical effects could play a crucial role in the structure and in the evolution of the protoneutron stars.

1. Introduction

A protoneutron star (PNS) is formed in a stellar remnant after a successful core-collapse supernova explosion of a star with a mass smaller than about 20 solar masses and in the first seconds of its evolution it is a very hot (temperature of up to 50 MeV), lepton rich and β-stable object and a lepton concentration typical of the pre-supernova matter [1].

The knowledge of the nuclear EOS of dense matter at finite temperature plays a crucial role in the determination of the structure and in the macrophysical evolution of the PNS [2]. The processes related to strong interaction should in principle be described by quantum chromodynamics. However, in the energy density range reached in the compact stars, strongly non-perturbative effects in the complex theory of QCD are not negligible [3]. In the absence of a converging method to approach QCD at finite density one often turns to effective and phenomenological model investigations.

In this article, we study a hadronic equation of state (EOS) at finite temperature and density by means of a relativistic mean-field model with the inclusion $\Delta(1232)$-isobars [4, 5, 6] and by requiring the Gibbs conditions on the global conservation of baryon number and net electric charge. Transport model calculations and experimental results indicate that an excited state of baryonic matter is dominated by the Δ resonance at the energies from the BNL Alternating Gradien Synchrotron (AGS) to RHIC [7]. Moreover, in the framework of the nonlinear Walecka model, it has been predicted that a phase transition from nucleonic matter to Δ-excited nuclear matter can take place and the occurrence of this transition sensibly depends on the Δ-meson coupling constants [8, 9, 10, 11, 12].

2. Hadronic equation of state and stability conditions

In this section, we start by introducing the hadronic equation of state (EOS) in the framework of a relativistic mean-field theory. In this investigation we include all the baryon octet in order to reproduce the chemical composition of the PNS at high baryon chemical potential. We also take into account of leptons particle by fixing the lepton fraction $Y_L = Y_e + Y_{\nu_e} = (\rho_e + \rho_{\nu_e})/\rho_B$, where ρ_e, ρ_{ν_e} and ρ_B are the electron, neutrino and baryon number densities, respectively. This is because, in the first stage of PNS evolution, electrons and neutrinos are trapped inside the stellar matter and, therefore, the lepton number must be conserved until neutrinos escape out of the PNS [13].

The Lagrangian density can be written in term of the hadronic [4, 14] plus leptonic component, as follow:

$$\mathcal{L}_{tot} = \mathcal{L}_H + \mathcal{L}_l = \sum_B \bar{\psi}_B [i\gamma_\mu \partial^\mu - (M_B - g_{\sigma B}\sigma)$$

$$-g_{\omega B}\gamma_\mu \omega^\mu - g_{\rho B}\gamma^\mu \vec{\tau} \cdot \vec{\rho}_\mu]\psi_B + \frac{1}{2}(\partial_\mu \sigma \partial^\mu \sigma - m_\sigma^2 \sigma^2)$$

$$-U(\sigma) + \frac{1}{2}m_\omega^2 \omega_\mu \omega^\mu + \frac{1}{2}m_\rho^2 \vec{\rho}_\mu \cdot \vec{\rho}^\mu - \frac{1}{4}F_{\mu\nu}F^{\mu\nu}$$

$$-\frac{1}{4}\vec{G}_{\mu\nu}\vec{G}^{\mu\nu} + \sum_l \bar{\psi}_l [i\gamma_\mu \partial^\mu - m_l]\psi_l \,, \tag{1}$$

where the sums over B and l are over the baryon octet and lepton particles, respectively. The field strength tensors for the vector mesons are given by the usual expressions $F_{\mu\nu} \equiv \partial_\mu \omega_\nu - \partial_\nu \omega_\mu$, $\vec{G}_{\mu\nu} \equiv \partial_\mu \vec{\rho}_\nu - \partial_\nu \vec{\rho}_\mu$, and $U(\sigma)$ is a nonlinear potential of σ meson

$$U(\sigma) = \frac{1}{3}a\sigma^3 + \frac{1}{4}b\sigma^4 \,, \tag{2}$$

usually introduced to achieve a reasonable compression modulus for equilibrium nuclear matter.

The field equations in a mean field approximation are

$$(i\gamma_\mu \partial^\mu - (M - g_{\sigma B}\sigma) - g_{\omega B}\gamma^0 \omega - g_{\rho B}\gamma^0 \tau_3 \rho)\psi = 0 \,, \tag{3}$$

$$m_\sigma^2 \sigma + a\sigma^2 + b\sigma^3 = g_{\sigma B} <\bar{\psi}\psi> = g_{\sigma B}\rho_S \,, \tag{4}$$

$$m_\omega^2 \omega = g_{\omega B} <\bar{\psi}\gamma^0\psi> = g_{\omega B}\rho_B \,, \tag{5}$$

$$m_\rho^2 \rho = g_{\rho B} <\bar{\psi}\gamma^0\tau_3\psi> = g_{\rho B}\rho_I \,, \tag{6}$$

where $\sigma = \langle\sigma\rangle$, $\omega = \langle\omega^0\rangle$ and $\rho = \langle\rho_3^0\rangle$ are the nonvanishing expectation values of mesons fields, ρ_I is the total isospin density, ρ_B and ρ_S are the baryon density and the baryon scalar density, respectively. They are given by

$$\rho_B = 2\sum_{i=B} \int \frac{\mathrm{d}^3k}{(2\pi)^3}[n_i(k) - \overline{n}_i(k)] \,, \tag{7}$$

$$\rho_S = 2\sum_{i=B} \int \frac{\mathrm{d}^3k}{(2\pi)^3} \frac{M_i^*}{E_i^*}[n_i(k) + \overline{n}_i(k)] \,, \tag{8}$$

where $n_i(k)$ and $\overline{n}_i(k)$ are the fermion particle and antiparticle distributions.

The nucleon effective energy is defined as $E_i^*(k) = \sqrt{k^2 + M_i^{*2}}$, where $M_i^* = M_i - g_{\sigma B}\sigma$. The effective chemical potentials μ_i^* are given in terms of the meson fields as follows

$$\mu_i^* = \mu_i - g_{\omega B}\omega - \tau_{3iB}g_{\rho B}\rho \,, \tag{9}$$

where μ_i are the thermodynamical chemical potentials ($\mu_i = \partial\epsilon/\partial\rho_i$).

The further conditions, required for the β-stable chemical equilibrium and charge neutrality, can be written as

$$\mu_\Lambda = \mu_{\Sigma^0} = \mu_{\Xi^0} = \mu_n\,, \tag{10}$$

$$\mu_{\Sigma^-} = \mu_{\Xi^-} = \mu_n + \mu_e\,, \tag{11}$$

$$\mu_p = \mu_{\Sigma^+} = \mu_n - \mu_e\,; \tag{12}$$

$$\rho_p + \rho_{\Sigma^+} - \rho_{\Sigma^-} - \rho_{\Xi^-} - \rho_e = 0\,. \tag{13}$$

In the case of trapped neutrinos, the new equalities are obtained by the replacement of $\mu_e \to \mu_e - \mu_{\nu_e}$. The total entropy per baryon is calculated using $s = (S_B + S_l)/(T\rho_B)$, where $S_B = P_B + \epsilon_B - \sum_{i=B} \mu_i\rho_i$ and $S_l = P_l + \epsilon_l - \sum_{i=l} \mu_i\rho_i$, and the sums are extended over all the baryons and leptons species.

The thermodynamical quantities can be obtained from the thermodynamic potential in the standard way. More explicitly, the baryon pressure P_B and the energy density ϵ_B can be written as

$$
\begin{aligned}
P_B = \quad & \frac{2}{3}\sum_i \int \frac{\mathrm{d}^3k}{(2\pi)^3} \frac{k^2}{E_i^*(k)}[n_i(k) + \overline{n}_i(k)] - \frac{1}{2}m_\sigma^2\sigma^2 \\
& -U(\sigma) + \frac{1}{2}m_\omega^2\omega^2 + \frac{1}{2}m_\rho^2\rho^2\,,
\end{aligned}
\tag{14}
$$

$$
\begin{aligned}
\epsilon_B = \quad & 2\sum_i \int \frac{\mathrm{d}^3k}{(2\pi)^3} E_i^*(k)[n_i(k) + \overline{n}_i(k)] + \frac{1}{2}m_\sigma^2\sigma^2 \\
& +U(\sigma) + \frac{1}{2}m_\omega^2\omega^2 + \frac{1}{2}m_\rho^2\rho^2\,.
\end{aligned}
\tag{15}
$$

Here and in the following, we focus our investigation by considering the so-called GM3 [4] and the SFHo parameter sets [15]. The implementation of hyperon degrees of freedom comes from determination of the corresponding meson-hyperon coupling constants that have been fitted to hypernuclear properties.

Concerning the stability conditions, we are dealing with the study of a multi-component system at finite temperature and density with two conserved charges: baryon number and electric charge. For such a system, the Helmholtz free energy density F can be written as [16]

$$F(T, \rho_B, \rho_C) = -P(T, \mu_B, \mu_C) + \mu_B\rho_B + \mu_C\rho_C\,, \tag{16}$$

with

$$\mu_B = \left(\frac{\partial F}{\partial \rho_B}\right)_{T,\rho_C}\,, \quad \mu_C = \left(\frac{\partial F}{\partial \rho_C}\right)_{T,\rho_B}\,. \tag{17}$$

In a system with N different particles, the particle chemical potentials are expressed as the linear combination of the two independent chemical potentials μ_B and μ_C and, as a consequence, $\sum_{i=1}^N \mu_i\rho_i = \mu_B\rho_B + \mu_C\rho_C$.

Assuming the presence of two phases (denoted as I and II, respectively), the system is stable against the separation in two phases if the free energy of a single phase is lower than the free energy in all two phases configuration. The phase coexistence is given by the Gibbs conditions

$$\mu_B^I = \mu_B^{II}\,, \qquad \mu_C^I = \mu_C^{II}\,, \tag{18}$$

$$P^I(T, \mu_B, \mu_C) = P^{II}(T, \mu_B, \mu_C)\,. \tag{19}$$

Therefore, at a given baryon density ρ_B and at a given net electric charge density $\rho_C = y\,\rho_B$ (with $y = Z/A$), the chemical potentials μ_B are μ_C are univocally determined. An important

feature of this conditions is that, unlike the case of a single conserved charge, the pressure in the mixed phase is not constant and, although the total ρ_B and ρ_C are fixed, baryon and charge densities can be different in the two phases. For such a system in thermal equilibrium, the possible phase transition can be characterized by mechanical (fluctuations in the baryon density) and chemical instabilities (fluctuations in the electric charge density) [9, 16]. As usual the condition of the mechanical stability implies

$$\rho_B \left(\frac{\partial P}{\partial \rho_B} \right)_{T, \rho_C} > 0 \,. \tag{20}$$

By introducing the notation $\mu_{i,j} = (\partial \mu_i / \partial \rho_j)_{T,P}$ (with $i, j = B, C$), the chemical stability for a process at constant P and T can be expressed with the following conditions [9]

$$\rho_B \, \mu_{B,B} + \rho_C \, \mu_{C,B} = 0 \,, \tag{21}$$
$$\rho_B \, \mu_{B,C} + \rho_C \, \mu_{C,C} = 0 \,. \tag{22}$$

Whenever the above stability conditions are not respected, the system becomes unstable and the phase transition takes place. The coexistence line of a system with one conserved charge becomes in this case a two dimensional surface in (T, P, y) space, enclosing the region where mechanical and diffusive instabilities occur.

By increasing the temperature and the baryon density during the high energy heavy ion collisions ($T \approx 50$ MeV and $\rho_B \geq \rho_0$), a multi-particle system with Δ-isobar and pion degrees of freedom may take place.

In analogy with the liquid-gas case, we are going to investigate the existence of a possible phase transition in the nuclear medium by studying the presence of instabilities (mechanical and/or chemical) in the system. The chemical stability condition is satisfied if [9]

$$\left(\frac{\partial \mu_C}{\partial y} \right)_{T,P} > 0 \ \text{ or } \ \begin{cases} \left(\dfrac{\partial \mu_B}{\partial y} \right)_{T,P} < 0 \,, & \text{if } y > 0 \,, \\[3mm] \left(\dfrac{\partial \mu_B}{\partial y} \right)_{T,P} > 0 \,, & \text{if } y < 0 \,. \end{cases} \tag{23}$$

In the Fig. 1, we report the baryon and electric charge chemical potential isobars as a function of y, at fixed temperature $T = 50$ MeV and $x_{\sigma\Delta} \equiv g_{\sigma\Delta}/g_{\sigma N} = 1.3$ (the ratio related to the scalar σ meson-Δ coupling constants) in the GM3 parameters set [4].

From the analysis of the above chemical potential isobars, we are able to construct the binodal surface relative to the nucleon-Δ matter phase transition. In Fig. 2, we show the binodal section at $T = 50$ MeV and $x_{\sigma\Delta} = 1.3$.

The right branch (at lower density) corresponds to the initial phase (I), where the dominant component of the system is given by nucleons. The left branch (II) is related to the final phase at higher densities, where the system is composed primarily by Δ-isobar degrees of freedom (Δ-dominant phase). In presence of Δ-isobars the phase coexistence region results very different from what obtained in the liquid-gas case, in particular it extends up to regions of negative electric charge fraction and the mixed phase region ends in a point of maximum asymmetry with $y = -1$ (corresponding to a system with almost all Δ^--particles, being antiparticles and pions contribution almost negligible in this regime). We analyze the phase evolution of the system during the isothermal compression from an arbitrary initial point A, indicated in Fig. 2. In this point the system becomes unstable and starts to be energetically favorable the separation into two phases, therefore an infinitesimal Δ-dominant phase appears in B, at the same temperature and pressure. Let us observe that, although in B the electric charge fraction is substantially
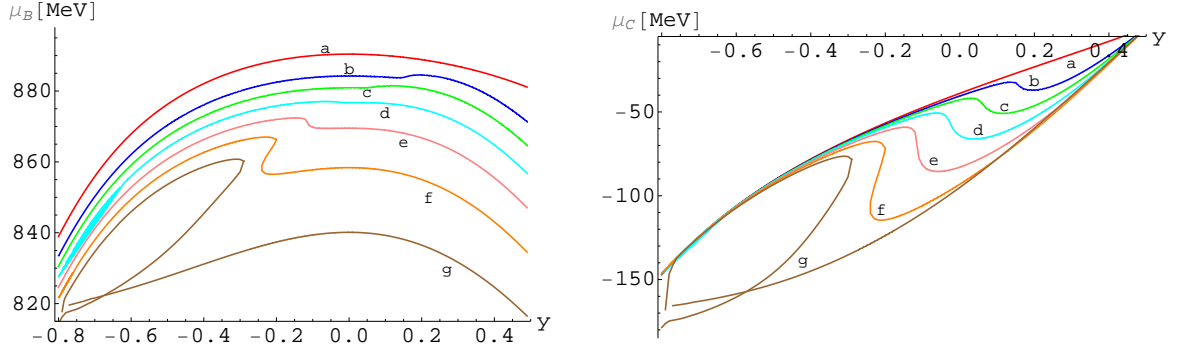
Figure 1. Baryon (right panel) and electric charge (left panel) chemical potential isobars as a function of y at $T = 50$ MeV and $x_{\sigma\Delta} = 1.3$. The curves labeled a through g have pressure P=9,7,6,5,4,3,2 MeV/fm^3, respectively.
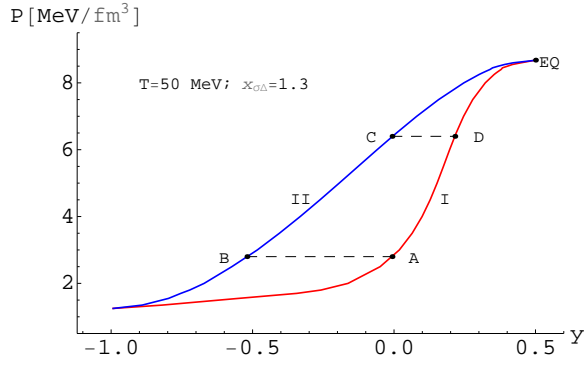

Figure 2. Binodal section at $T = 50$ MeV and $x_{\sigma\Delta} = 1.3$.

negative, the relative Δ^- abundance must be weighed on the low volume fraction occupied by the phase II near the point B. During the phase transition, each phase evolves towards a configuration with increasing y, in contrast to the liquid-gas case, where each phase evolves through a configuration with a decreasing value of y (with the exception of the gas phase after the maximum asymmetry point). We will see in the next section how the presence of such features are relevant in the structure and in the evolution of the PNS.

3. Results and discussion

We investigate the relevance Δ-isobar degrees of freedom and the stability conditions in the bulk properties of compact star and PNS.

Let us start by considering β-stable and electric-charge neutral nuclear matter at $T = 0$. In Fig. 3, we report the mass-radius relations in absence (no Δ) and in presence of Δ-isobars with different scalar coupling ratios ($x_{\sigma\Delta} = 1.0$ and $x_{\sigma\Delta} = 1.2$) in the GM3 model [4]. The presence of Δ-isobar degrees of freedom smooths the equation of state enlarging the effect of thermodynamic instabilities and reduces the maximum gravitational mass. On the other hand very compact object with smaller radii can be formed.

In Fig. 4, we show the temperature as a function of the baryon density (in units of the saturation nuclear density), in absence (np) and in presence (npH) of hyperons in the SFHo model [15, 17]. We limit our analysis in the first two phases: in the left panel, the first leptonic rich state ($s = 1, Y_L = 0.4$) and, in the right panel, the maximum heating phase ($s = 2, Y_{\nu_e} = 0$). Indeed in the cold-catalyzed phase ($s = 0, Y_{\nu_e} = 0$), the temperature is very low (fews MeV), and the above statistical effects due to thermodynamical instabilities
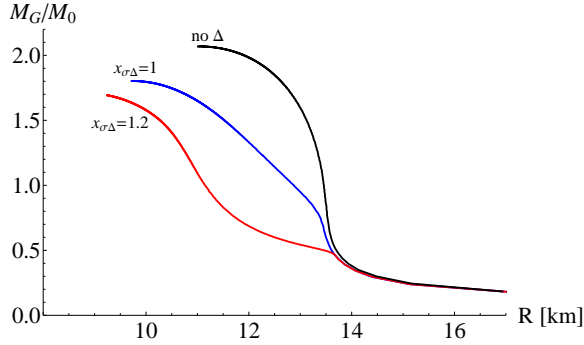
Figure 3. Mass-radius relations in absence (no Δ) and in presence of Δ-isobars with different scalar coupling ratios ($x_{\sigma\Delta} = 1.0$ and $x_{\sigma\Delta} = 1.2$).

and Delta-isobar formation may be neglected. In both previous cases, we observe a reduction in temperature in presence of hyperons and Delta-isobar degrees of freedom. Note also that, when hyperons are present, for $s = 1$ and $Y_L = 0.4$, the system evolves in a quasi isothermal configuration above $\rho_B = (2.5 \div 6)\rho_0$. The different behavior in the stellar temperature have important consequences in the PNS evolution and in its particles concentration. Finite temperature properties of matter at high density influence the diffusion of neutrinos, being the neutrino mean free paths strongly temperature dependent [13]. In particular, neutrino opacity is very sensitive to the inner temperature (in general proportional to T^2) and, therefore, this would affect sensibly the cooling of the PNS.
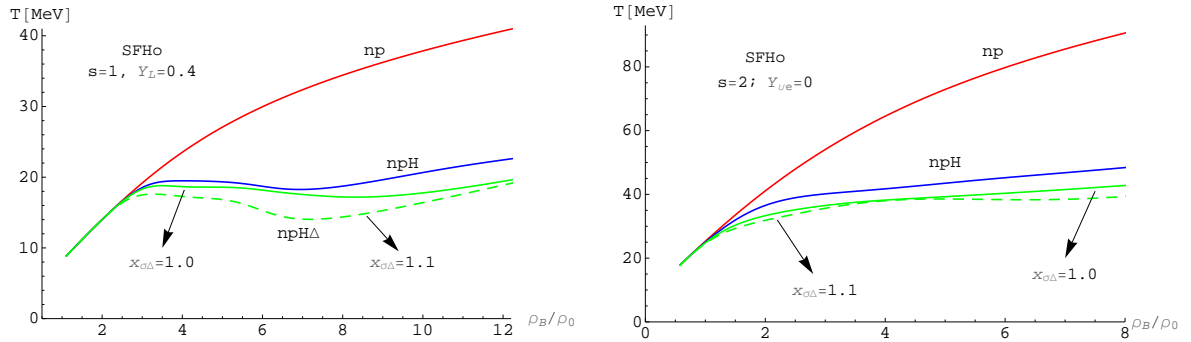

Figure 4. Temperature as a function of the baryon density (in units of the saturation nuclear density ρ_0) in the SFHo model and for different value of the Delta coupling constant $x_{\sigma\Delta}$. Left panel: entropy per baryon and neutrino fraction $s = 1, Y_L = 0.4$, right panel: $s = 2, Y_{\nu_e} = 0$. The labels np and npH stand for nucleons and nucleons plus hyperons.

In Fig. 5, we show the variation of the maximum baryonic mass in units of solar mass $M_\odot$ as a function of the central baryon density ρ_c, for pure nucleonic (np) and hyperonic plus Delta-isobars (npH) stars in the first leptonic rich state (left panel, $s = 1, Y_L = 0.4$) and in the maximum heating phase (right panel, $s = 2, Y_{\nu_e} = 0$). For a comparison, in the figure we have considered the two models GM3 and SFHo model in presence of Δ-isobars with different scalar coupling ratios ($x_{\sigma\Delta} = 1.0$ and $x_{\sigma\Delta} = 1.1$).

Let us note the strong reduction of the maximum baryonic mass with the introduction of hyperons and Delta-isobar degrees of freedom. This effect is remarkable stronger in the maximum heating phase ($s = 2, Y_{\nu_e} = 0$) and for a greater value of the $x_{\sigma\Delta}$ coupling due also to the presence of thermodynamical instabilities conditions.

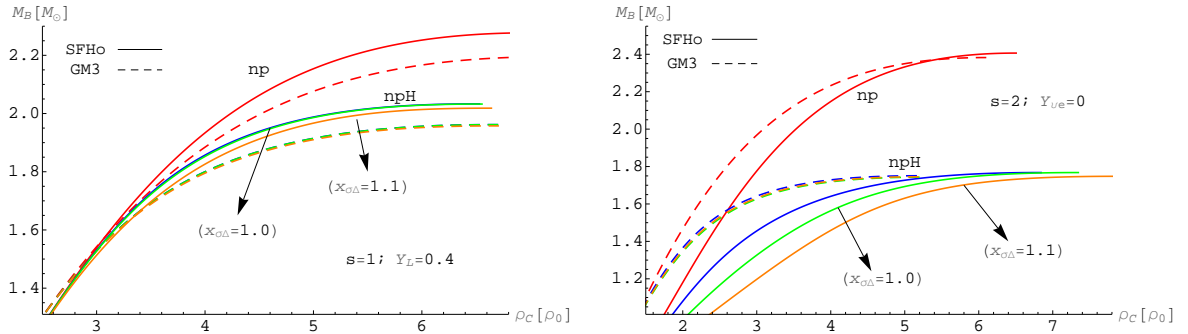In the presence of hyperons, when the stellar core contains non-leptonic negative charges,

Figure 5. Maximum baryonic mass M_B in units of solar mass $M_\odot$ as a function of the central baryon density ρ_c (in units of the nuclear saturation density ρ_0) for nucleons (np) and hyperons stars (npH) stars in the case $s = 1$ and $Y_L = 0.4$ (left panel) and during the maximum heating phase $s = 2$ and $Y_\nu = 0$ (right panel).

the maximum masses of neutrino-trapped stars result to be significantly larger than for low temperatures and for lepton poor matter. Hence, there exists a window of initial masses for which the star becomes unstable to gravitational collapse during deleptonization and a black hole can take place [13]. We can see that the formation of such a metastable phase strongly depends on the presence of statistical effects and the window of metastability grows with the value $x_{\sigma\Delta}$.

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