VIII. Internationaler Leibniz-Kongress

Einheit in der Vielheit

Leibnitz behauptet, daß nicht zwey Blätter einander völlig ähnlich seyn.

Vorträge 1. Teil

herausgegeben von

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den, kann Leibniz` Konzeption genau das überbrücken, was Locke nicht möglich ist: Die Vermittlung der Perspektive der ersten Person mit den Perspektiven anderer und ihre Relativierung durch diese.

Eine Ursache für das Interesse, das Lockes Ansatz im Unterschied zum leibnizschen immer noch entgegengebracht wird, mag die lange Zeit vorherrschende Fehlinterpretation Lockes als eines säkularen Denkers sein, die ihn in ein gegenwartsnäheres Licht rückte als Leibniz mit seiner berüchtigten Theodizee. Ein weiterer Grund ist vermutlich die explizite Metaphysikkritik Lockes, die er mit wichtigen Gegenwartsströmungen teilt. Freilich kann man dieser Kritik entgegenhalten, dass die reale Alternative zur expliziten Metaphysik nicht Erfahrung, sondern implizite, undiskutierte Metaphysik darstellt. Gleichwohl entspricht Lockes Anspruch, zentrale Fragen der Moralphilosophie möglichst weitgehend auf Erfahrungstatsachen stützen und rational diskutieren können, ohne metaphysische Fragen wie die des Verhältnisses von Leib und Seele oder der Natur geistiger Substanzen beantworten zu müssen, zweifellos mehr dem Gegenwartsverständnis als Leibniz` Versuch, dort, wo es kein sicheres Wissen gibt, mit metaphysischen Hypothesen zu arbeiten.

Ein weiterer Grund liegt vielleicht auch in der impliziten Präsens des Themas der Selbstverantwortung in Lockes Diskussion personaler Identität. Locke war es in seinem *Essay* nicht zuletzt darum gegangen, die Voraussetzungen für einen *verantwortlichen* Umgang mit der eigenen *Meinungsbildung* zu klären. Sein Personverständnis war mit dem Glauben an radikale Selbstverantwortung verbunden, und „selbst" verantwortlich zu sein, bedeutete für ihn, seine Annahmen auf *eigene Erfahrung* (und nicht auf die *Meinungen anderer*) zu stützen.[30] Für Locke und seine Zeitgenossen lag die Ursache der Religionskriege, die Europa verwüsteten, zu einem nicht geringen Teil in der Bereitschaft, unkritisch Auffassungen zu übernehmen, die keiner vernünftigen Prüfung standhalten. Unter diesem Gesichtspunkt hatte seine Behauptung, dass Wahrnehmung immer Reflexion und Selbstrepräsentation einschliesst, auch eine moralische Bedeutung. Denn wenn sie zutrifft, dann sind wir auch jederzeit imstande, unsere Erfahrung kritisch zu reflektieren, und sind entsprechend moralisch verantwortlich für unsere unkritische Haltung: "[Those] taking things upon trust, misemploy their power of Assent, by lazily enslaving their Minds, to the Dictates and Dominion of others, in doctrines, *which it is their duty carefully to examine; and not blindly, with an implicit faith, to swallow.*"[31] Die Tatsache, dass religiös motivierte Verbrechen nicht nur auf Autoritätshörigkeit, sondern auch auf die Irrtümer origineller Köpfe zurückgehen können, oder, mit anderen Worten, dass wir im Selbstdenken nicht weniger irren können als bei der Akzeptanz anderer Meinungen, ändert nichts am Reiz dieser Idee als moralischer Leitidee. Wie Whitehead es einmal ausdrückte: Die Funktion von Ideen liegt nicht in ihrer Wahrheit, sondern darin, dass sie einen „Köder für unsere Phantasie" (*lure for feeling*) anbieten.[32] Und revolutionäre Ansichten, die darauf basieren, dass ein Grundübel ausgemacht wird, das es zu lösen gibt, üben nun einmal einen stärkeren Reiz aus als ausgewogene, die bemüht sind, die Wirklichkeit in all ihren Schattierungen zu verdeutlichen.

[30] Vgl. Locke, S. 101.
[31] Locke, S. 100; Herv. M.-S.L.
[32] Vgl. A. N. Whitehead (1978), *Process and Reality*, New York, S. 25.

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The influence of Leibnizian ideas on Giuseppe Peano's work[1]

Famous mathematician and brilliant teacher, Giuseppe Peano (1858-1932) graduated from the University of Turin in 1880 and taught there for over fifty years, holding the chairs of Infinitesimal Calculus, Higher Analysis and Mathematics Education. Between 1891 and 1908 he devoted his energies to the edition of the *Formulaire de Mathématiques*, an encyclopaedic treatise in symbolic language that included propositions of classical mathematics, and he moulded a large number of scholars, many of whom collaborated in the numerous activities he began.

In Peano's *Opera Omnia* we find over two hundred quotations of Leibniz, which proves that Peano consciously considered the German mathematician as a distinguished predecessor and inspirer. The influence exerted on him by Leibniz can be documented in the following different fields: logic, geometry, analysis, arithmetic and dyadic, algebra and linguistic. Particularly, I intend to show how and why many studies developed by Peano and by some students of his School are linked to, complete or surpass, start from or stand out from the lines of research established by Leibniz[2]. Some distinctive leitmotifs emerge from

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[2] Some hints about the Leibnizian influence on Peano can be found in E. Carruccio: "Spunti di storia delle matematiche e della logica nell'opera di G. Peano", in A. Terracini (ed.): *In memoria di Giuseppe Peano*, Cuneo 1955, p. 103-114; E. Carruccio: "Tre problemi di Leibniz nelle loro relazioni con la logica peaniana e post peaniana", in: *Memorie dell'Accademia Nazionale di Scienze, Lettere ed Arti di Modena* **6** (1970), p. 197-209; U. Cassina: «L'oeuvre philosophique de G. Peano», in: *Revue de Métaphysique et de Morale* (abbreviated with *RMM*) **40** (1933), p. 481-491; U. Cassina: "Su la logica matematica di G. Peano", in: *Bollettino dell'Unione Matematica Italiana* **12** (1933), p. 57-65; U. Cassina: "Ideografia e logica matematica", in: *Periodico di Matematiche* 4, **80** (1952), p. 65-78; U. Cassina: «L'idéographie de Peano du point de vue de la théorie du langage», in: *Rivista di Matematica dell'Università di Parma* **4** (1953), p. 195-205; U. Cassina: "L'opera scientifica di Giuseppe Peano", in: *Rendiconti del Seminario Matematico di Milano* **7** (1933), p. 323-389; L. Couturat: «La Logique mathématique de M. Peano», in: *RMM* **7** (1899), p. 616-646; P. Freguglia: "Sull'evoluzione algebrica dei principi logici classici", in: *Archimede* **27** (1975), p. 169-175; H.C. Kennedy: *Peano. Life and works of Giuseppe Peano*, Dordrecht 1980, p. 46-47, 65-67; G. Lolli: "«Quasi alphabetum»: logica ed enciclopedia in G. Peano", in: *Le ragioni fisiche e le dimostrazioni matematiche*, Bologna 1985, p. 49-83; M. Malatesta: "On a Particular Meaning of the Principle of Duality: Leibniz, Boole, Peano", in: *Metalogicon* **10** (1997), p. 1-12; M. Mugnai: "Leibniz", in P. Rossi, C.A. Viano (eds.): *Storia della filosofia*, vol. 4 *Il Settecento*, Bari 1996, p. 61-88; E. Pasini: "Segni e algoritmo nell'analisi Leibniziana", in M. Panza, C.S. Roero (eds.): *Geometria, Flussioni e differenziali*, Napoli 1995, p. 385-412; *Peano e i fondamenti della Matematica, Atti del Convegno di Modena del 22-24.10.1991*, Accademia Nazionale di Scienze Lettere e Arti, Modena 1993; C.S. Roero: "Giuseppe Peano, geniale matematico, amorevole maestro", in R. Allio (ed.): *Maestri dell'Ateneo torinese dal Settecento al Novecento*, Torino 2004, p. 138-144; G. Vailati: «La Logique mathématique et sa nouvelle phase de développement dans les écrits de M. J. Peano», in: *RMM* **7** (1899), p. 86-102; G. Vailati: "*Sul* carattere del contributo apportato da Leibniz allo sviluppo della logica formale", in: *Rivista di Filosofia e Scienze affini* **7** (1905), in M. Calderoni, U. Ricci, G. Vacca (eds.): *Scritti di G. Vailati*, Firenze 1911, p. 619-624. Other sources to document the influence of Leibniz can be found in the following correspondences: G. Osimo (ed.): *Lettere di Giuseppe Peano a Giovanni Vacca*, Quaderni PRISTEM, N. 3, Milano 1992, letters n. 43, 90, 107, I; G. Vacca a G. Peano 25.1.1929 in: C.S. Roero (ed.): *Giuseppe Peano. Matematica, cultura e società*, Cuneo 2001, p. 84; G. Arrighi (ed.): *Lettere a M. Pieri (1884-1913)*, Quaderni PRISTEM, N. 6, Milano 1997, letter n. 39; P. Nastasi, A. Scimone (eds.): *Lettere a Giovanni Vacca*, Quaderni PRISTEM, N. 5, Palermo 1995, p. 48-57, 90, 91, 104, 138; G. Lanaro (ed.): *Giovanni Vailati. Epistolario 1891-1909*, Torino 1971, p. 169, 193, 198, 199, 201, 230, 246; A.F. Schmid (ed.): *Bertrand Russell Correspondance sur la philosophie, la logique et la politique avec Louis Couturat (1897-1913)*, Paris 2001; E. Luciano, C.S. Roero (eds.): *Giuseppe Peano-Louis Couturat. Carteggio (1896-1914)*, Firenze 2005, letters n. 6, 8, 9, 22, 30, 31, 34, 36, 37. We can find Leibniz quotations also in some Peano's manuscripts, cf. C.S. Roero, N. Nervo, T. Armano (eds.): *L'Archivio Giuseppe Peano*, cd-rom N. 2,

this historical reconstruction: Peano adapts Leibniz ideas in order to place the original results in their historical context and he shares Leibniz's humanitarian ideals about the international language and its efficacy in improving collaboration and communication among peoples. Peano absorbs Leibniz's interest in establishing the paternity of mathematical concepts and notations, and he takes Leibniz's works as a starting point to elaborate new theoretic or applicative results in numerical analysis and binary arithmetic. The *corpus* of quotations is effectively articulated around a unique conceptual axis, which can be identified in the semantic link between sign, algorithm and language.

Peano was familiar with Leibniz's works in three different editions present in the library at the Department of Mathematics of the University of Turin and at the Turin National Library. The results in the fields of logic, geometry and arithmetic were taken from *Opera philosophica* edited by J. Erdmann (Berlin, 1840) and from the manuscripts Mss. VIIB and IIIA, cited in L. Couturat *Opuscules* (1903). For the studies on binary arithmetic he referred to the oldest edition by L. Dutens (1768), while, from 1900 on, he used the *Mathematische* and *Philosophischen Schriften* by C.I. Gerhardt. Peano did not really undertake philological or critical analysis of the works of Leibniz, but he read the recent secondary literature on the German mathematician. For example, he followed with interest the historical studies by his collaborator G. Vacca, who in 1899 first analyzed the unpublished manuscripts in Hanover[3], and by Couturat, editor of the volumes *La logique de Leibniz* (1901) and *Opuscules et fragments inédits de Leibniz* (1903). On many occasions he referred to the essays by Vacca and Couturat and he quoted the study of H. Diels, „Ueber Leibniz und das Problem der Universlasprache" (1899) as well. Peano himself edited, in 1901 and 1903, two detailed articles on Leibnizian logic by G. Vacca and G. Vailati in his *Rivista di Matematica* (abbreviated with *RdM*)[4]. I maintain the importance of the study of Leibnizian manuscripts to which Peano's attention was drawn by Vacca and later by Couturat, but I must also point out a plausible connection to E. Schröder and G. Frege's papers of 1880-1897. Peano knew those notes very well and could have been influenced by the quotations of Leibniz *pasigraphia* inserted here[5].

Mathematical logic is the first field of Leibniz influence on Peano[6]. Peano was sensitive to the historical framework of research, and frequently expressed his belief that he had realized the Leibnizian project of the *Characteristica*. In Peano's 'philosophical' thought, Leibniz is considered the mathematician who had conceived as generally as possible the

Torino 2002, mss. N. 103234 and N. 103063.

[3] G. Vacca: "Sui manoscritti inediti di Leibniz", in: *Bollettino di Bibliografia e Storia delle Scienze Matematiche (G. Loria)*, **2** (1899), p. 113-116.

[4] Cf. G. Vailati: «La Logique de Leibniz d'après des documents inédits», in: *RdM* **7** (1901), p. 148-159; G. Vacca: "La Logica di Leibniz", in: *RdM* **8** (1903), p. 64-74. Cf. also G. Vailati: «L. Couturat. La Logique de Leibniz d'après des documents inédits», in: *Bollettino di Bibliografia e Storia delle Scienze Matematiche (G. Loria)* **4** (1901), p. 103-110.

[5] Cf. E. Schröder: „Über Pasigraphie, ihren gegenwärtigen Stand und die pasigraphische Bewegung in Italien", in: *Verhandlungen des Ersten Internationalen Mathematiker-Kongresses, Zürich 9-11.8.1897*, Leipzig 1898, S. 147-148, 161.

[6] Referring to the signatures used in the cd-rom C.S. Roero (ed.): *L'Opera Omnia di Giuseppe Peano*, cd-rom, N. 3, Torino 2002, cf. Peano 1888a, p. VII, X; 1891c, p. 1, 8, 9; 1894c, p.120; 1894g, p. 3; 1894g, p. 52; 1896b, p. 1, 2, 3; 1896i, p. 169; 1896j, p. 566-567; 1897b, p. 63; 1898g, p. 299; 1898h, p. 95; 1900a, p. 3; 1906e, p.154; 1894 g, p. 8, 9, 15; 1895aa, p. 127, 128, p. 129; 1896j, p. 576, 577; 1897b, p. 4, 5, 6, 8, 9, 12, p 13, 18, 32, 34-35, 38, 40, 42; 1898f, p. 13; 1900a, p. 8, 9, 12, 16, 17, 18, 19, 20, 22-23, 24, 26, 27, 28, 30; 1901h, p. 5, 6; 1901j, p. 11, 12, 13; 1902b, p. IV; 1908a, p. 3, 4; 1904d, p. 3; 1910a, p. 33; 1911d, p. 51, 58, 59; 1912h, p. 4; 1915i, p. 1149; 1915k, p. 107, 108, 114, 115; 1916d, p. 40; 1918a, p. 195; 1919e, p. 675, 959; 1921d, p. 189; 1923a, p. 382; 1928a, p. 988.

following problem:

«Leibniz [...] a aussi projeté pendant toute sa vie, depuis son premier travail jusqu'à ses dernières lettres, une *spécieuse générale* ou *une manière de langue ou d'écriture universelle, où toutes les vérités de raison seraient réduites à une façon de calcul*. [...] Donc, selon Leibniz, la question la plus générale qu'on ait posée, est la construction de cette spécieuse»[7].

The *Spécieuse générale* is perceived as a support for the mind and the memory and it provides us with a means for revealing inaccuracies in the deductive procedures, because "sophisms and paralogisms are simply solecisms and barbarisms"[8]. To Peano's mind, the *Characteristica* has a twofold nature because it records the knowledge, and at the same time it produces the knowledge itself. Thus, mathematical logic is a set of procedures that changes the reasoning in calculus; at the same time, it is a tool that does not disconnect the reflection and the written formulation. Peano gave also a suggestive reconstruction of the relevancy of the project about the *Characteristica* within Leibniz's scientific biography:

«Il énonce ce projet dans son premier travail, ou, comme il l'appelle, dans son «essai d'écolier» intitulé «De arte combinatoria a. 1666». [...] Il fixe le temps nécessaire à la former: "aliquot selectos homines rem intra quinquennium absolvere posse puto". Il trouve cette découverte plus importante que l'invention des télescopes et des microscopes; elle est l'étoile polaire du raisonnement. [...] Dans ses dernières lettres il regrette «que si j'avois été moins distrait, ou si j'etois plus jeune, ou assisté par des jeunes gents bien disposés, j'espérerais donner une manière de cette spécieuse (pag. 701)». Il dit aussi (pag. 703) «J'ai parlé de ma spécieuse générale à Mr. Le Marquis de l'Hospital, et à d'autres; mais ils n'y ont point donné plus d'attention que si je leur avois conté un songe».»[9].

Therefore, Peano can conclude that the "Problem of Leibniz", which was forgotten for a long time, is finally accomplished:

«Nous avons donc la solution du problème proposé par Leibniz. Je dis «la solution» et non «une solution», car elle est unique. La Logique mathématique [...] est donc un ensemble de vérités, et non de conventions»[10].

The solution of the Leibnizian problem consists in the construction of a symbolic language, the *ideografia* - as Peano preferred to call it - which is modelled on the features of the *Characteristica*: it has to be as expressive and economic as possible, and it must provide "Ariadne's thread" for the thought. Further, the symbols - as Leibniz established - are not abbreviations of words, but rather represent ideas, so ideography is not tachygraphy[11]. Consequently Peano recovered Leibniz reflections on the excellence of Chinese ideography, which permits the maximum reciprocity between characters and ideas. From these quotations emerges Peano's strong conviction of having solved the problem of the *Characteristica*, a conviction largely shared in his School by C. Burali-Forti, U. Cassina, A. Padoa, M. Pieri, G. Vacca and G. Vailati[12]. Actually, notwithstanding the perplexity on the part of some authors[13] that Peano should seek inspiration in Leibnizian works of logic, the results of the German mathematician cited are so numerous that it seems difficult to ascribe them to chance. The tumultuous development of the mathematics in the years 1880-1930

[7] Peano 1896i, p. 169.

[8] A I, II, 240.

[9] Peano 1896b, p. 1.

[10] Peano 1896b, p. 2.

[11] Peano 1896j, p. 566.

[12] Cf. for example C. Burali-Forti: *Introduction a la Géométrie différentielle suivant la méthode de H. Grassmann*, Paris 1897, p. VI; C. Burali-Forti: *Logica matematica*, Milano 1919, p. XVII-XVIII; U. Cassina: «L'oeuvre philosophique de G. Peano», in: *RMM* **40** (1933), p. 489-491; A. Padoa: "Il contributo di G. Peano all'ideografia logica", in: *Periodico di Matematiche* 4, **13** (1933), p. 15-22; M. Pieri: "Uno sguardo al nuovo indirizzo logico-matematico delle scienze deduttive. Discorso per l'inaugurazione dell'a.a. 1906-1907 nella Università di Catania", in M. Pieri: *Opere sui Fondamenti della Matematica*, Roma 1980, p. 389-448; G. Vacca: *Origini della Scienza*, Roma 1946, p. 31.

[13] H.C. Kennedy: *Peano. Life and works of Giuseppe Peano*, Dordrecht 1980, p. 46.

gave rise to the need to reorganize and place in historical perspective a large body of results and "suddenly, what Leibniz had said about the *Characteristica Universalis* assumed the value of a prophecy"[14]. Peano's attempt to consider the new symbolical tendencies in mathematics as a repercussion of Leibnizian intuitions can be re-evaluated today, but it gained significance in the context of the so-called *Leibniz Renaissance*. This opinion founds evidence, for example, in this statement by Padoa, Peano's collaborator:

"Ma, si badi, nel 1889, cioè quando il Peano negli *Arithmetices principia, nova methodo exposita* [...] riuscì primo ad esporre con linguaggio simbolico una teoria deduttiva (postulati, definizioni, teoremi e dimostrazioni), il contributo di Leibniz alla sua *Ars characteristica* era ancora in gran parte ignoto: e fu non lieve merito di G. Vacca, già discepolo ed allora assistente del Peano, d'essersi poi recato espressamente in Hannover, a riordinare e decifrare in quella biblioteca numerosi fogli manoscritti inediti del grande matematico, incitando così L. Couturat a più vasta opera di analisi e di volgarizzazione del pensiero di Leibniz"[15].

Peano also inserted a substantial set of sentences by Leibniz, which were quoted *verbatim* from the notes "Specimen calculi universalis", "Difficultates quaedam logicae", "Non inelegans specimen demonstrandi in abstractis" and "Addenda". Thanks to these, Peano attributed to the German mathematician the introduction of the relation of equality as "identity of indistinguishable", the definition of the logic sum and product, the law of simplification, the proofs of the commutative and distributive properties of logic sum and product and some theorems of the 'algebra of the logic' concerning the negation.

The heritage of Leibniz's reflections also stands out clearly in the field of geometry[16]. Peano and some of his collaborators were leaders in studies on the foundations of mathematic and they developed research on geometric calculus, referring again to an intuition of Leibniz. In fact, the German mathematician had hinted at the *calculus situs*, with the aim of linking it to the formulation of the *Characteristica Geometrica*. Leibniz had often treated it but Peano referred chiefly to those *échantillons* inserted in a letter to C. Huygens and sketched in the writings "Characteristica Geometrica" and "De analysis situ". In the opinions of both Leibniz and Peano, the *analysis situs* must directly express the properties of the geometric objects, and provide the foundation of the analytic method. In order to develop the *Characteristica Geometrica* it is sufficient to formalize this analysis systematically and, after having selected the primitive concepts, to translate it into symbolic language. Consequently, research in the foundations of geometry is propaedeutic to the creation of the geometric calculus, and the *calculus situs* provides the natural forerunner of the modern establishment of vector theories:

"La Geometria analitica cartesiana fu certo un grande progresso. Ma [...] essa tratta le questioni geometriche per via indiretta, e spesso con lunghi ragionamenti e formule complicate arriva a risultati facilissimi ad ottenersi colla geometria sintetica elementare. Questa osservazione trovasi a più riprese nelle opere di Leibniz, il quale anzi tentò colla sua *characteristica geometrica* di porvi rimedio: «Je ne suis pas encor content de l'Algèbre, en ce qu'elle ne donne ny les plus courtes voyes, ny les plus belles constructions de Géométrie. C'est pourquoy lorsqu'il s'agit de cela, je croy qu'il nous faut encor une autre analyse proprement geométrique ou lineaire, qui nous exprime directement *situm*, comme l'Algèbre exprime *magnitudinem* » (Leibniz a Huygens, 8 sett. 1679). Ma questa, come altre idee di Leibniz, impiegò molto tempo a maturare; e il lavoro più profondo che abbiasi su questo soggetto è senza dubbio l'*Ausdehnungslehre* pubblicato dal Grassmann or sono appunto cinquant'anni"[17].

With regard to the analysis[18], the purpose is that of showing Peano's need to go back to

[14] M. Mugnai: *Leibniz e la logica simbolica*, Firenze 1973, p. 3.

[15] A. Padoa: "Il contributo di G. Peano all'ideografia logica", in: *Periodico di Matematiche* 4, **13** (1933), p. 15-22.

[16] Cf. Peano 1888a, p. VI, VII, VIII, X; 1891b, p. 3; 1894f, p. 167-168; 1896d, p. 952; 1896d, p. 953; 1903a, p. 8 and 1908a, p. 265.

[17] Peano 1894f, p. 167.

[18] Cf. Peano 1884c, p. VIII, XX; 1888f, p. 190; 1908a, p. XXXI, 72, 94, 223, 224, 258, 263-264, 277, 336, 342, 343, 395, 431; 1913a, p. 47, 48, 49, 56, 58, 61, 66, 67, 68-69; 1915j, p. 167; 1919b, p. 799. 1916 a, p. 283; 1916a, p. 284;

the original sources and situate each concept historically in the encyclopaedic *Formulario Mathematico* (1908a). He was also interested in the history of mathematical notations and carefully studied the symbolism adopted by Leibniz in the note "Sulla forma dei segni di algebra" (1920b), which is considered an ideal sequel of the Leibnizian "Monitum de characteribus algebraicis". For example, Peano attributed to Leibniz the introduction of the *vinculum*, the use of dots to distinguish different propositions, the symbol '*molesa*' to express the 'modulus' and the notations for differential and integral calculus. Research into the evolution of the symbols was developed in the note *Historia* of the *Formulario* and in the valuable essay "Derivata e differenziale" (1913a). In this last, the history of mathematical stenography was inserted in a critical analysis of the concepts themselves. Peano put forth a "very simple logical argument", according to which Leibniz and the pioneers of the infinitesimal calculus thought in terms of derivatives and not in terms of differentials. In order to support this thesis he analyzed the differential in Leibniz, wondered about the possibility of thinking of dx, dy as infinitely small increments and surmised the identity between Newtonian fluxions and Leibnizian differentials, concluding with this sentence:

"Però spetta a Leibniz il merito d'aver introdotti dei simboli per le operazioni, di aver mostrato come si operi su questi. E questo merito è ritenuto grande da tutti coloro che pensano che l'analisi è essenzialmente un linguaggio ben fatto"[19].

The interpretation of history as "research on the paternity of the single results" that is evident in the *Formulario* is a clear heritage of Leibnizian thought:

«[...] il n'y a rien de si important que de voir l'origine des inventions, qui valent mieux à mon avis que les inventions mêmes, à cause de leur fécondité et par ce qu'elles contiennent en elles la source d'une infinité d'autres»[20].

Peano was also convinced that teaching could and should be enriched by historical and bibliographical information. In this context it is noteworthy that in an academic lecture he proposed a procedure in numerical analysis in order to evaluate the rest in the series of $\frac{\pi}{4}$, which Leibniz found in 1674 and published in 1682. Peano's method was expounded in 1930 by F. Audisio, one of his students in the course of Mathematics Education at the University of Turin, who under Peano's direction discussed a graduate thesis in Mathematics concerning the history of π[21].

The year 1900 was significant for Peano, as he attended in Paris the International Congresses of Philosophy and of Mathematicians. From this date on, probably under the influence of his friend Couturat, he began to devote himself with increasing enthusiasm to philology and international language. In 1903 Peano proposed his project of international language in the note "De latino sine-flexione" (1903d), which he developed taking inspiration from the Leibnizian studies[22]. Further, in 1908 he became President of the

1920b, p. 44, 45-46, 48.

[19] Peano 1913a, p. 69.

[20] G.W. Leibniz to J.P. de La Roque, 1675, in: A III, I, 346. Cf. also GM V, 392: "Utilissimum est cognosci veras inventionum memorabilium origines, praesertim earum, quae non casu, sed vi meditandi innotuere. Id enim non eo tantum prodest, ut Historia literaria suum cuique tribuat et alii pares laudes invitentur, sed etiam ut augeatur ars inveniendi, cognita methodo illustribus exemplis".

[21] F. Audisio: "Calcolo di π colla serie di Leibniz", in: *Atti della R. Accademia dei Lincei* 4, **11** (1930), p. 1077-1080. This exposition was completed by an introduction, with quotations from Leibniz and Newton. In this circumstance Peano did not really manage an historical search about Leibniz results and he simply took them as hint for original mathematical developments.

[22] Cf. Peano 1903d, p. 74, 75, 76, 80, 81, 82; 1904a, p. 273, 277-278, 279-280, 281; 1904c, p. 3, 6; 1905a, p. 358;

Akademi internasionale del Volapük, later Academia pro Interlingua; he used the *latino sine-flexione* both in the last edition of his *Formulario Mathematico* and in the two periodicals of the Academia, *Discussiones* and *Circulares*. The *latino sine-flexione* is characterized by a rational and linear grammar and its lexicon depends on the principal European languages. This interlingua is built according to a logical procedure, that of simplifying classical Latin step by step. Thus Peano starts by abolishing the declensions and the endings for gender and number, while the verbal conjugations are reduced to the imperative. Here Peano showed an accurate knowledge of Leibnizian studies and at the beginning of each paragraph he quoted a sentence:

"Nominum casus semper eliminari possunt substitutis in eorum locum particulis quibusdam. Leibniz. Ed. Couturat a. 1901, p. 67"[23].

With the aim of 'justifying' the link between his studies in logic and philology, Peano also inserted an interesting account of the rediscovery of Leibnizian manuscripts by Vacca and Couturat, in the years 1899-1903:

"Dopo due edizioni delle sue *Opera omnia*, del Dutens e del Gerhardt, si credeva generalmente che i manoscritti di questo gran pensatore, giacenti nella Biblioteca di Hannover, non contenessero più cosa importante. Ma il dott. Vacca, allora assistente di Calcolo nell'Università di Torino, accortosi di lacune nei lavori stampati, andò a consultare i manoscritti, e inviò copia di quelli relativi alla Logica matematica, da cui risultò che parecchi teoremi di questa scienza, attribuiti a Boole, Schröeder e ad altri, già erano stati enunciati dal Leibniz. Il Prof. Couturat, uno degli autori del libro che ho presentato, e che si occupa pure con successo della Logica matematica, per incarico del Governo francese si recò alla sua volta ad Hannover, e raccolse larga messe di documenti, che pubblicò in un grosso volume. Fra questi manoscritti, ora venuti alla luce, c'è pure il progetto di lingua razionale, ideato dal Leibniz; risultava prima, dalle varie lettere, che Leibniz si era profondamente occupato della questione; ma solo ora si sa (almeno parzialmente) quanto egli disse. Ora Leibniz, applicando metodi analoghi a quelli del Calcolo logico, già aveva ridotto le regole grammaticali"[24].

Peano's celebrity is usually due to his research in arithmetic, with the list of the famous axioms for natural numbers. However, the historical research in algebra and arithmetic managed in Peano's school is still today largely unknown. It is used to enrich the textbooks for secondary schools edited by Peano and his followers, but it is mainly documented in the *Formulario*[25]. In this treatise we find many results of Leibniz in these fields, supplied with historical notes by Vacca. These include an elegant proof of the 'small Fermat theorem', some properties on the multiples and greatest common divisor, the studies of the determinants and prime numbers, the combinatorial techniques, the theory of divisibility, some new algorithms for linear systems and the extension of the binomial theorem to polynomial expressions. Quotations from Leibniz were mostly taken from a set of 1679-1680 manuscripts, including "De primitivis et divisoribus ex tabula combinatoria", "Conspectus calculi" and "Invenire triangulum rectangulum in numeris, cujus area sit quadratus".

Finally we touch on the less well-known studies by Peano on binary arithmetic[26]. Peano was actually concerned with problems related to typography and, at the 13 November 1898 session of the Turin Academy of Sciences, he submitted the note "La numerazione binaria applicata alla stenografia" (1898m). Here he advanced the drafting of a new form of stenography and a printing-press based on the properties of the binary code, which had been studied by Leibniz. The German mathematician is considered the first to have introduced the algorithm in dyadic and singled out the periodicities in binary sequences. Peano recalled as well the Leibnizian studies on the *Libro delle variazioni*, with the mathematical interpretation of the sixty-four Fohy hexagrams:

"Già Leibniz fece vedere che le proprietà d'ogni sistema di numerazione sono, in questa base, ridotte a forma semplicissima. [...] Leibniz riscontrò in un libro cinese, detto "Libro delle variazioni", delle figure, in cui riconobbe i numeri scritti nel sistema binario. [...] Leibniz si fece tradurre da missionarii questo libro; ma esso riuscì poco intelligibile, poiché già i cinesi da lungo tempo (egli dice), ne hanno perduto il significato. E si limita a conchiudere (t. III, p. 394): «Je ne sçai s'il y a jamais eu dans l'écriture Chinoise un avantage approchant de celui qui doit être nécessairement dans une Caractéristique que je projette. C'est que tout raisonnement qu'on peut tirer des notions, pourrait être tirée de leurs Caractères par une manière de calcul qui seroit un des plus importants moyens d'aider l'esprit humain»"[27].

In conclusion, I maintain the relevance of the influence of Leibniz's reflections on Peano's writings in order to show in greater depth how Peano's intellectual path travelled between logic and international language, in accordance with his studies on Leibniz's *Characteristica* and *Lingua Universalis*. Also noteworthy is Peano's contribution to the diffusion among mathematicians of some original works of Leibniz and the encouragement he gave to his friends and collaborators Couturat, Russell, Vacca and Vailati in their pursuit of historical and critical research into Leibniz.

1908a, p. 17; 1909b, p. 79, 86; 1909d, p. 5; 1909e, p. 13; 1911e, p. 7; 1912a, p. 4; 1913i, p. 51; 1913k, p. 50; 1913l, p. 3, 4; 1914f, p. 78; 1915b, p. VII, VIII; 1917e, p. III; 1921b, p. V, IX; 1922h, p. I; 1923b, p. 13; 1927e, p. 1. Cf. also L. Couturat, L. Leau: *Histoire de la langue universelle*, Paris 1903, p. 23-28; L. Couturat, L. Leau: *Les nouvelles langues internationales*, Paris 1907, p. 70-76 and C.S. Roero: "I matematici e la lingua internazionale", in: *Bollettino dell'Unione Matematica Italiana*, **8**, **2-A** (1999), p. 159-182.

[23] Peano 1903d, p. 74.

[24] Peano 1904a, p. 277-278.

[25] Cf. Peano 1898d, p. 69, 70, 71, 78; 1898f, p. 41, 48; 1899b, p. 71, 76, 81, 85; 1902b, p. 61, 132; 1908a, p. 56, 61, 62, 92, 129, 146.

[26] Cf. Peano 1898f, p. 29; 1898m, p. 47, 48-49; 1908a, p. 256. Cf. also G. Vacca: "*Sulla* storia della numerazione binaria", in: *Atti del II Congresso Internazionale di Scienze Storiche, Roma 9-12.4.1903*, **12**, Roma 1904, p. 63-67.

[27] Peano 1898m, p. 47-49.