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Business Cycle in a Macromodel with Oligopoly and Agents' Heterogeneity: an Agent-Based Approach

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Abstract. This paper introduces a macromodel with oligopoly and entry/exit in a framework where individuals are heterogeneous in their budget constraints, since they can be workers, new entrant entrepreneurs, incumbent entrepreneurs or unemployed and may change their status due to a stochastic process, associated to entry and exit. Agents' heterogeneity is explicitly modelled in the aggregate demand, that also accounts for the income distribution. Heterogeneity also plays a relevant role in the process of entry/exit in the goods market, which interacts with the labour market and generates macroeconomic fluctuations. As shown in the agent-based numerical simulation, the model provides a new interpretation of the business cycle and an explanation for the empirical phenomenon of countercyclical mark up.

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1 Introduction

The number of existing firms in an economy tends to increase during booms and decrease during recessions, although this empirical fact is not, in general, explicitly modelled in conventional DSGE models. As a partial exception to that, Etro and Colciago (2010) introduce a DSGE model of business cycle with endogenous market structure, differentiated goods and full employment, where two separate benchmark cases of price (Bertrand) competition and quantity (Cournot) competition are separately analyzed. They show that with no product differentiation and with a unique homogeneous good, mark ups can only survive in the case of quantity (Cournot) competition, while they vanish in the case of price (Bertrand) competition, which degenerates into a conventional real business cycle model. They do not explicitly refer to oligopoly (the word "oligopoly" never actually appears in their paper) and use instead a more general notion of "imperfect competition", that may include several sub-cases, where, in any case, the macroeconomic fluctuations are still only driven by technology shocks. They do provide interesting explanations for a number of stylized facts, such as countercyclical mark ups and pro-cyclical business creation, but they do not discuss whether and how does the economic system move between the benchmark cases of Cournot and Bertrand equilibria.

As we know it, in disciplines outside economics such as physics or population genetics, the combinatorial stochastic processes that deal with a large number of interacting individuals or entities amply demonstrate that details of specification of optimizing agents (units) frequently diminish as the number of agents become very large. As discussed in Aoki and Yoshikawa (2007, pp. 28-29), only certain key features of parameters such as correlations among agents matter in determining aggregate behavior. Furthermore, like in most agent-based models, the "optimal" agents' decisions and the aggregate variables resulting out of them are determined as the result of large numbers attempts performed by the individual agents.

In this paper we introduce a macromodel with oligopoly for the specific purpose of explicitly formalizing the interactions among entry/exit, labour market, business cycle and social mobility. The aggregate demand is determined by the sum of individual demand functions and the model also explicitly formalizes the income distribution among workers, incumbent entrepreneurs, new entrant entrepreneurs and unemployed, since the agents are heterogeneous in their budget constraints. The status of worker, entrepreneur or unemployed may stochastically change in each period, potentially generating in this way distributional shocks on the aggregate demand. A modeling feature that characterizes this model as a general equilibrium model is that the wage setting rule, the labor market equilibrium and the entry/exit decisions interact, since the workers are perceived by the incumbent as potential entrants.

2 Consumers and aggregate demand

Let us define the nominal variable "A_t" at time t, which includes liquid "financial assets", assumed to be risk free and not associated to property rights on the firms. It includes Government bonds and deposits, i.e. it corresponds to M3. We also assume that deposits are remunerated and that the interest rate on risk free Government bonds is equal to the interest rate on deposits, since they are both assumed to be risk free financial assets. The monetary policy consists of interest rate setting. It is not explicitly modelled at this stage of our work, but exogenous changes in the money stock can be easily formalized as changes in the the nominal variable "A_t", since M3 is a function of the money base. We assume for simplicity a cashless economy, i.e. the banking sector instantaneously performs all the transactions among individuals through bank transfers. The banking system charges all the incomes a transaction fee ς (excepting for the transactions concerning the lump sum tax to finance the unemployment subsidies), which is a portion (very small in its magnitude) of the transactions. Since, as shown below, all the incomes are spent in consumption goods, these transactions are proportional to the income of the individuals, no matter what is their source of income, and are assumed to be small in magnitude. These transaction fees are the only source of income for the perfectly competitive banking sector, therefore the interest rate on bank lending is equal to the interest rate r on the asset A_t, exogenously set by the policy makers. The banking sector perfectly diversifies its lending risk to the industrial firms and any risk of financial distress in the banking sector is handled by the monetary authorities, acting as a lender of last resort for the banking system.

The entrepreneurs can be incumbent, earning at time $t + i$ the incumbent nominal profits Π_{t+i}^{in} (Π_{t+i}^{inR} in real terms) or new entrants, earning the new entrant nominal profits Π_{t+i}^{e} (Π_{t+i}^{eR} in real terms) which, in general, diverge from Π_{t+i}^{in} because the new entrants have to support the entry costs as shown below. All the agents display the same preferences, represented by the same utility function, while their main source of earnings may be given either by wages, or profits or transferals to the unemployed individuals, assumed to be financed, for the sake of simplicity, by a nominal lump sum τ (τ_R in real terms) .

The entrepreneurs hire the workers, pay them the nominal wages W_t for the period going from t to $t + 1$. They pay themselves the same nominal wage W_t, and receive the residual nominal profits, so that the remuneration for the entrepreneurial activity is given by W_t plus Π_{t+i}^{in} if the entrepreneur is an incumbent or W_t plus Π_{t+i}^{e} if she is a new entrant. When $\Pi_{t+i}^{in} < 0$ and $\Pi_{t+i}^{e} < 0$, (or, equivalently, as shown below $\Pi^{inR}_{t+i} < 0$ and $\Pi^{eR}_{t+i} < 0$) respectively, the incumbent and the new entrant go bankrupt (which happens with a probability to be specified later), the entrepreneur and the workers become unemployed and, until they are hired again by a new firm, they receive a portion of the total unemployment subsidies, which are entirely financed by a lump sum tax τ on the incomes of the employed individuals. Having defined W_{t+i} (w_{t+i} in real terms) as the wage per worker before taxes, we assume that the labour contract is such that each worker receives the wage W_{t+i} for the period going from "$t+i$" to "$t+i+1$", for a fixed amount of hours of work. Let l be the (exogenous and constant) total labour force, n_{t+i} be the number of employed individuals at time $t+i$, h^{in}_{t+i}, (with $0 < h^{in}_{t+i} < h_{t+i}$) the portion of incumbent entrepreneurs at time $t+i$, h^{e}_{t+i} (with $0 < h^{e}_{t+i} < 1$) the portion of new entrants at time $t+i$ (with $h_{t+i} = h^{in}_{t+i} + h^{e}_{t+i}$ and $0 < h_{t+i} < h_{t+i}$), therefore, the portion of workers over the total employed labour force is given by $1 - h^{in}_{t+i} - h^{e}_{t+i} = 1 - h_{t+i}$

Let $(n_{t+i}W_{t+i} + n_{t+i}h^{in}_{t+i}\Pi^{in}_{t+i} + n_{t+i}h^{e}_{t+i}\Pi^{e}_{t+i})\varsigma$ be the transaction fees to the banking system at time $t+i$, since each employed individual pays a nominal lump sum tax τ (τ_R in real terms) to finance the unemployment subsidies, let $n_{t+i}\tau(1-\varsigma)$ be the overall amount of unemployment subsidies, net of the transaction fees to the banking system. We assume that τ_R and ς are constant, very small in their magnitude so that τ_R is assumed to barely cover the survival expenses of the unemployed individuals.

The revenues of the perfectly competitive banking system are given by the transaction fees $(n_{t+i}W_{t+i} + n_{t+i}h^{in}_{t+i}\Pi^{in}_{t+i} + n_{t+i}h^{e}_{t+i}\Pi^{e}_{t+i})\varsigma$.

On the basis of our assumptions, since the effects of tax transferals and bank commissions are simplified out, the aggregate demand can be formalized as follows

$$Y_{t+i} = n_{t+i}(W_{t+i} + h^{e}_{t+i}\Pi^{e}_{t+i} + h^{in}_{t+i}\Pi^{in}_{t+i}) \tag{1}$$

where Y_{t+i} is the aggregate nominal income at time "$t+i$". The right-hand side of the above equation in the first row is income of the workers and entrepreneurs.

Appendix 1 shows how to derive the microfounded aggregate demand (exactly corresponding to the aggregation of the individual demand of each consumer), which is the following:

$$D_t(\cdot) = \frac{\Omega(r_t)}{P_t}\left(A_t + \frac{1}{1+r_t}\sum_{i=0}^{\infty}\left(\frac{1}{(1+E\left(r_{t+i}\right))(1+E(\iota_{t+i}))}\right)^i E(Y_{t+i})\right) \quad (2)$$

Where $\Omega(r_t)$, as discussed in Appendix 1, is an increasing monotonic function of the interest rate r_t at time t, P_t the price level at time t, $E(Y_{t+i})$ the expected nominal output or income at time $t+i$, $E(\iota_{t+i})$ the expected inflation rate at time $t+i$. For what concerns the expected inflation rate, we have to introduce a few more details. Consistently with the modelling features of the industrial sector in our model (where, as explained below, the rivalry among the oligopolistic firms yields an equilibrium in mixed strategies that can be interpreted as an evolutionary equilibrium or as a perturbed game) we assume that the inflation expectations of all the individuals are defined as a "core inflation", contingent on the existing monetary policy regime. The rationale of this assumption goes as follows: the individuals, having observed the past outcomes of the games among the "prime makers" oligopolistic firms, which determine (jointly with the aggregate demand) the price level, formulate an expectation of the long run inflationary trend and this is assumed to be the "core inflation" ι. The current observable inflation may certainly deviate from the "core inflation" due to stochastic shocks (since the actual outcome of the rivalry among the oligopolistic firms may yields a stochastic outcome temporarily deviating from "evolutionary equilibrium"), but the expectations of the "core inflation" are assumed to reflect the inflation target, which is common knowledge (since it is publicly announced by the authorities, assumed to be credible). The "core inflation" is regarded as a long run average inflation and the stochastic deviations from it may be interpreted as temporary deviations (with zero mean) due to the outcome of the rivalry among the oligopolistic firms. The "core inflation" ι is constant under a given monetary policy regime.

The aggregate demand may also be written by showing the income distribution

$$
\begin{aligned}
D_t(\cdot) = {} & [\Omega(r_t)/P_t] \cdot \\
& \cdot \{A_t + [1/(1+r_t)] \sum_{i=0}^{\infty} [(1+E\,(r_{t+i}))(1+E(\iota_{t+i}))]^{-i} \cdot \\
& \cdot n_{t+i}(W_{t+i} + h^{e}_{t+i} E(\Pi^{e}_{t+i}) + h^{in}_{t+i} E(\Pi^{in}_{t+i}))\}
\end{aligned}
\tag{3}
$$

This aggregate demand function may account for potential distributional shocks, entry/exit shocks, expectational shocks and expected changes in the future monetary policy, since it contains the expected future values for the real interest rate and the "core inflation". We assume that the present expected value of each variable is the best predictor for its future income. This formulation also explicitly shows the agents heterogeneity in their income distribution.

3 The firms sector

All the firms use the same production technology to produce the same generic consumption perishable good in regime of oligopoly. The oligopolistic firms imperfectly observe the costs of their rivals and may approximate them on the basis of conjectures: in other words they play a perturbed game. Having defined $\varphi_{i,t}$ as the "purely technological" planned individual real output produced by firm i at time t, the production technology of each individual firm i is described by a Cobb-Douglas production function with labour only:

$$\varphi_{i,t} = \Lambda L^{\alpha}_{i,t} + \psi_{i,t} \tag{4}$$

Where L_t is the (nonnegative and discrete) number of workers employed in the firm at time t for the period from t to $t+1$, Λ is the usual parameter describing the state of technology. We assume that ψ_t is a random variable with uniform distribution. ψ_t is meant to capture an "implementation" shock that works as follows: the oligopolistic firms (as discussed in the next sections) enter a game where they decide the amount of workers to employ (which determines the production capacity) and the equilibrium price and output commitment. The output commitment is not an actual production of commodities, but it is instead a contractual commitment that the firms undertake with their customer, to provide them with an agreed quantity of output. A stochastic shock (that may be interpreted as unpredictable accidents or by unpredictable conflicts between the workers and the entrepreneur of a single specific firm) might occur and prevent the firm from honouring the contractual commitment to provide its customers with the agreed amount of commodity. In that case, the firm, to compensate the customer from breaking the contractual commitment, sell their product (or a portion of it) at a lower price and suffer from a profit loss. As a consequence of that, other firms meet a higher demand than expected (which means that they satisfy a portion of the demand curve with higher price than those hit by the shock). In the simulations we do have a macroeconomic "*ex post*" price, determined by the average price which is determined on the demand curve. The existence of this random implementation shock implies that on average, all the firms have to plan a production level that takes into account the fact that they might either suffer from a negative implementation shock or, on the contrary, meet a higher than expected demand. The aggregate nominal output therefore is

$$Y_t = P_t \left[\sum_{i=1}^{H_t} \left(\Lambda L_{i,t}^{\alpha} + \psi_{i,t} \right) \right]$$

and since $\psi_{i,t}$ has a zero average,

$$Y_t = P_t \Lambda \left(\sum_{i=1}^{H_t} L_{i,t}^{\alpha} \right) \tag{5}$$

We assume an *ex ante* labour contract establishing a fixed number of hours to be worked and hiring a worker implies hiring a fixed number of working hours for the firm. We also assume that starting a new firm involves entrepreneurial and organizational skills that only employed workers have. Being unemployed for one period causes skill loss, so that, with the timing assumptions of our model, only employed individuals can potentially enter the market and are perceived by the incumbent firms as potential entrants.

In addition the new entrant has to bear exogenous sunk costs of entry F (organizational costs and setting up costs), to be supported immediately before the entry decision, at the end of period $t-1$. For this sake, they raise an amount F of financial funds borrowed from the banking and financial system. with a contract set at the end of time $t-1$ and expiring at the end of time t, when the debt has to be repayed back. Therefore, since both the financial sector and the borrowers do not suffer from money illusion, the borrower borrow the nominal amount F (F^R in real terms) at the end of period $t-1$ and will have to repay back (at the end of period t) $(1+E(r_t)(1+E(\iota_t)F$, where $E(\iota_t)$ is the core inflation, defined above (constant, given the current monetary policy regime) and the real interest rate r_t, under the control of the monetary authorities.

The new entrants discount the expected bankruptcy probability at the moment when they decide to enter. Once the new entrants have entered the market, at time t, they enter the oligopolistic game [1].

The labour market displays a particular kind of wage rigidity and unemployment, due to a particular incentive-compatibility constraint, explained in what follows. The wages applying for the next period (from time t to time $t+1$) are set before the potential entrants decide whether or not to enter the market. Since the workers only (and not the unemployed people) may decide to become en-

[1] At the initial time t_0 the existence of entrepreneurs and firms, i.e. different social groups, can be thought of as being determined by a random initial distribution of wealth.

trepreneurs, the oligopolistic entrepreneurs have incentive to keep the wages low, but not so low to trigger entry. Individuals are subject to idiosyncratic informational shocks. Without these shocks, all the individuals would have the same expectations and if the wages were set at a level where the expected profits for new entrants (which take into account the probability of bankruptcy) are lower than the expected future wages, nobody would start a new firm to enter the market. If the wages were irrationally set so low that their expected future value would be lower than the expected future value of the profits of a new entrant, then the workers would prefer to bear the risk of entering the market as entrepreneurs and large scale entry would take place until profits vanish out and the entrepreneurs would only earn the wage they pay themselves and get zero profits.

With unemployment the firms are wage setters and with full employment the workers are wage setters. This means that when the entrepreneurs are wage setters (i.e. when there is unemployment), the wages correspond to a level that discourage entry and entry is only due to idiosyncratic informational shocks in the workers' expectations. We call the wage that does not, ex ante, trigger entry, the "incentive compatible wage" and define it in nominal terms (given the *ex ante* expected price level $E_{t-1}(P_t)$. consistent with the "core inflation") W_t^*. Similarly, we may define the real "incentive compatible wage" as w_t^*.

Let us first consider the case where the firms are wage setters and there is unemployment. Between time $t-1$ and time t the firms coordinate themselves and designate a common representative, who publicly announces the incentive compatible wage W_t^*[2]. With unemployment, all the incumbent firms have exactly

[2] We can imagine that no incumbent firm has incentive to deviate from this "publicly announced" incentive compatible wage by making the following assumption: since there is rivalry among the oligopolistic firms, the publicly announced wage is a form of coordination. If there were no coordination, each firm would have incentive to "steal" the workers from the rivals by offering them a marginally higher wage. We assume that this firm coordination is supported by a retaliation strategy, against a firm that would deviate from the announced wage, performed by all the other firms.

the same incentive not to offer any wage that is greater or equal to the "incentive compatible wage". Every employed worker who is offered (between time $t-1$ and t) the "incentive compatible wage" for the next period (from t to $t+1$), knows that, by rejecting that offer, she would be substituted by an unemployed worker and become unemployed for the next period.

In the case of full employment, the nature of wage setting is radically different. The incumbents do not have any credible way to induce their workers to accept a "no entry wage", because there is no longer any credible threat of offering the same contract to unemployed workers. As a consequence, there is no longer any incentive for the incumbents in coordinating themselves and offering a common wage in the process of wage setting. On the other hand, the rivalry among firms still exists and each rival can push a competitor out of the market by "stealing" its workers and offering them marginally higher wages. This rivalry may be interpreted as a one-shot game among the incumbents. For each firm, the only way to prevent being pushed out of the market by its rivals, is offering wages that eliminate the extra profits, so that the entrepreneurs are only remunerated by the wage they pay themselves. Let us call this wage W^{fu} in nominal terms and w^{fu} in real terms. Any lower wage offered by a firm to its workers would expose the entrepreneur to the risk of being pushed out of the market by her competitors, who could potentially steal her workers by offering them w^{fu}. In this case, w^{fu} is obviously a Nash equilibrium in a one-shot game among the oligopolistic incumbent firms, at the moment where wages are set.

Let us turn now to the case of unemployment. . Once W^* (and w^*) is announced, but still between time $t-1$ and time t, some existing workers may receive an idiosyncratic informational shock that generates the entry decision. Entry/exit, by affecting the number of existing firms, also affects production capacity and aggregate employment.

Since the risk free interest rate r_t is exogenous and under the control of the monetary authorities and the revenue of the perfectly competitive banking and financial system is determined by the transaction fees and not by any interest margin, r_t is also the interest rate that banks charge on their loans. In case of bankruptcy, the entrepreneur would loose her job and not have the right to start a new firm next period, but would keep all of her risk free financial assets A_t. In aggregate terms it is irrelevant who is actually holding the aggregate financial assets A_t. Since any new firm has (different from the entrepreneur) limited liability, any new entrant will borrow from the banking and financial system in order to cover the cost of entry.

The expected remuneration of the entrepreneur is given by the profits (Π_t^e if it is a new entrant, Π_t^{in} if it is an incumbent) plus the wage W_t^* that the entrepreneur pays to herself.

The expected remuneration of the new entrant at time $t-1$ for time t, $(E_{t-1}(\Pi_t^e)+$
$W_t^* - F - \tau)\,(1-\varsigma))$,

is different, in general, from the expected remuneration of the incumbent $(E_{t-1}(\Pi_t^{in}) + W_t^* - \tau)\,(1-\varsigma))$, since, as explained below, it includes entry costs.

Since entry and exit are determined by information shocks, ceteris paribus (with no modifications in the entry costs and for a given level of interest rate r_t and a given "core inflation" rate ι_t), the survival of new entrants depends on their ability to substitute the firms that abandon the market, or by new equilibria configurations in the (oligopolistic) market for goods.

Therefore, the "ex ante" expected nominal profits for the incumbent "i" are:

$$E_t(\Pi_{t+1}^{in}) = [E_t\,(P_{t+1})\,E_t(\varphi_{i,t+1}) - W_{t+1}E_t(L_{i,t+1}^*) - \tau]\,(1-\varsigma) =$$
$$= E_t\,(P_{t+1})\,[E_t\Lambda(L_{i,t+1}^*)^\alpha - w_{t+1}E_t(L_{i,t+1}^*) - \tau_R]\,(1-\varsigma)$$

Where $E_t\Lambda(L_{i,t+1}^*)^\alpha$ is the expected real output, τ_R is the lump sum tax expressed in real terms and $w_{t+1}$the real wage at time $t+1$.

Therefore, the "ex ante" expected real profits for the incumbent "i" are:

$$E_t(\Pi^{inR}_{t+1}) = [E_t\Lambda(L^*_{i,t+1})^\alpha - w_{t+1}E_t(L^*_{i,t+1}) - \tau_R]\,(1-\varsigma)$$

As a consequence, the "ex ante" expected nominal profits for the new entrant "j" are:

$$E_t(\Pi^{e}_{t+1}) = E_t\,(P_{t+1})\,[E_t\Lambda(L^*_{j,t+1})^\alpha - w_{t+1}E_t(L^*_{j,t+1}) - \tau_R -$$
$$-\xi(1+\iota)(1+r_{t-1})F^R]\,(1-\varsigma)$$

And the "ex ante" expected real profits for the new entrant "j" are:

$$E_t(\Pi^{eR}_{t+1}) = [E_t\Lambda(L^*_{j,t+1})^\alpha - w_{t+1}E_t(L^*_{j,t+1}) - \tau_R -$$
$$-\xi(1+\iota)(1+r_{t-1})F^R]\,(1-\varsigma)$$

The above definitions of real and nominal profits show that nominal profits are negative when real profits are negative and nominal profits are null when real profits are null.

The *es ante* real contractual wage w^*_{t+1} is set in advance between time t and $t+1$, but before time $t+1$. With no information shocks, for each incumbent of new entrant, the expected values $E_t(\varphi_{t+1})$, $E_t(L^*_{t+1})$ would be equal to their actual observable values at time. Let us now turn to the definition of the probability of a generic new entrant to stay in the market, which can be interpreted (since the firms are price makers) as the probability distribution function that Π^{eR}_t be greater or equal to zero. *Ex ante*, the probability distribution function $\Pr(\Pi^{eR}_t \geq 0)$ can be defined as follows:

$$\Pr(\Pi^{eR}_t \geq 0) = \Pr\{[\Lambda E_t(L^*_{j,t})^\alpha -$$
$$-w_tE_t(L^*_{j,t}) - \tau^R - \xi(1+\iota)(1+r_{t-1})F^R]\,(1-\varsigma) \geq 0\} \qquad (6)$$

The definition of 6 shows that a reduction in the real interest rate r_{t-1} between time $t-1$ and time t affected by the monetary authorities affects the wage setting and, as a consequence Π^e_t through 6.

For the sake of simplicity, let us assume that $L^*_{j,t}$ is a uniformly distributed random variable, therefore $\Pr(\Pi^e_t \geq 0)$ displays the properties of a distribution function of uniformly distributed random variables and, by exploiting the linearity properties of such a distribution , we get:

$$\Pr(\Pi_t^{eR} \geq 0) = \Pr\{[\Lambda E_t(L_{j,t}^*)^\alpha - w_t E_t(L_{j,t}^*)-$$
$$-\tau^R - \xi(1+\iota)(1+r_{t-1})F^R] \geq 0\} =$$
$$= \Pr\{[\Lambda E_t(L_{j,t}^*)^\alpha - w_t E_t(L_{j,t}^*)-$$
$$-\tau^R] \geq 0\} - \xi[(1+r_{t-1})F]$$

where ξ is a linear parameter and

$\Pr\{[E_t \Lambda(L_{j,t}^*)^\alpha - w_t E_t(L_{j,t}^*) - \tau^R] \geq 0\}$is the probability of the incumbent to survive in the market, i.e., given the linearity properties of the uniform probability distribution function:

$$\Pr(\Pi_t^{eR} \geq 0) = \Pr(\Pi_t^{inR} \geq 0) - \xi(1+\iota)(1+r_{t-1})F^R \tag{7}$$

Therefore, we may introduce here an object that we define "ex ante probability" of survival of the incumbents and we assume it to be a constant. Obviously, in each period of time number of failures of incumbents firms might deviate from the value implied by $\Pr(\Pi_{t+i}^{inR} \geq 0)$, which is an "expected value", therefore an average value, but this does not prevent us from assuming that the average "ex ante" probability of failure is a constant. Therefore, we have $\Pr(\Pi_{t+i}^{inR} \geq 0) = \Pr(\Pi^{inR} \geq 0)$ for every i. $\Pr(\Pi^{inR} \geq 0)$ is then interpreted as an exogenous variable.

3.1 Entry, exit and the labour market

For the sake of simplicity, let us introduce the following definition (interpreted as a "shift parameter" of the aggregate demand):

$$\Psi_t = \sum_{i=0}^{\infty} [(1+E\left(r_{t+i}\right))(1+E(\iota_{t+i}))]^{-i} \cdot$$
$$\cdot E[n_{t+i}(W_{t+i} + h_{t+i}^e E(\Pi_{t+i}^e) + h_{t+i}^{in} E(\Pi_{t+i}^{in}))]\}$$

As we said, we assume that, for a given monetary policy regime, $E(\iota_{t+i}) = \iota$ is a constant. For the other following variables we assume that the expectations of each variable is equal to its last observable value, i.e.

$$
\begin{aligned}
&E\left(r_{t+i}\right) = r_t; E(n_{t+i}) = n_t;\\
&E(w_{t+i}) = w_t; E(h^e_{t+i}) = h^e_t; E(H^e_{t+i}) = H^e_t;\\
&E(h^{in}_{t+i}) = h^{in}_t; E(h^{in}_{t+i}) = H^{in}_t; E(\Pi^e_{t+i}) = \Pi^e_{t-1}; E(\Pi^{in}_{t+i}) = \Pi^{in}_{t-1}
\end{aligned}
$$

Since the decision to enter the market also depends on the market size and since in 3 changes in the term

$$
\begin{aligned}
&\sum_{i=0}^{\infty}[(1+E\left(r_{t+i}\right))(1+E(\iota_{t+i}))]^{-i}\cdot\\
&\cdot n_{t+i}(W_{t+i} + h^e_{t+i}E(\Pi^e_{t+i}) + h^{in}_{t+i}E(\Pi^{in}_{t+i}))\}
\end{aligned}
$$

determines shifts in the aggregate demand, we assume that an increase in the variance of Ψ_t is associated to an increase in the probability of information shocks that induce a worker (potential entrant) to “be more optimistic” and enter the market. The information shock is more likely and more frequent because the complexity of calculus to determine the agents expectations increase, when the variance of the elements composing the term Ψ_t increase. Therefore, at time t, $var(\Psi_t)$, which is a measurable and observable variable, provides a measure of the frequency of prediction mistakes.

It is important to point out that the stochastic nature of $var(\Psi_t)$ is a direct consequence of the stochastic nature of a mixed-strategy Cournot-Nash equilibrium among the oligopolistic firms (as shown in the sections that follow). In particular, $var(\Psi_t)$ is determined by (and may be interpreted as a function of) the variance of the new entrant profits Π^e and the incumbent profits Π^{in}. If we make the reasonable assumption that the variance of the profits is not constant over time, this also implies that $var(\Psi_t)$ is not constant.

When a firm goes bankrupt both the entrepreneur and the workers lose their job and get unemployed. Therefore an entrepreneur who goes bankrupt at time t, is unemployed at time $t+1$ and can only hope to be hired as a worker at time $t+2$. With this assumption, we do not need to impose any "ad hoc" bankruptcy costs.

As shown in Appendix 2, w_t^* displays a set of rather intuitive properties:

$$\frac{\partial w_t^*}{\partial r_{t-1}} < 0; \frac{\partial w_t^*}{\partial n_{t-1}} > 0; \frac{\partial w_t^*}{\partial \Pi_{t-1}^{in}} > 0; \frac{\partial w_t^*}{\partial \Pi_{t-1}^{e}} > 0$$

Hence we define w_t^* as a generic function

$$w_t^* = w_t^*(\overset{-}{r}_{t-1}, \overset{+}{n}_{t-1}, \overset{+}{\Pi}{}^{in}_{t-1}, \overset{+}{\Pi}{}^{e}_{t-1})$$

In case of unemployment, the oligopolistic firms would set the wage w_t^* on the basis of the incentive compatibility constraint just explained above. When the economy reaches the full employment, the bargaining power is on the side of the workers, and the wages are set at a "zero-profit" level.

As discussed in Appendix 2, For given average market expectations $E_t(\Pi_{t+1}^{eR})$ and $E_t(\Pi_{t+1}^{inR})$, if the variance of the distributions of these two variables increases, there is a higher frequency of prediction mistakes, i.e. there is a larger number of workers who turn "overoptimistic" by observing the profits of the firms where they work: this induces them to decide to enter the market as new entrants. We can summarize and simplify that by the following equation

$$\Pr(entry)_t = \beta(var(\Psi_t))$$

As shown in Appendix 2, the overall probability of exit, for the all existing firms (both new entrants and incumbents) is the following:

$$\begin{aligned}\Pr(exit)_t = {} & 1 - \Pr(\Pi^{inR} \geq 0) + \\ & + \left[\xi(1+\iota)(1+r_{t-1})F^R\right] \cdot \\ & \cdot \left[n_{t-1}h_{t-1}\frac{(1-h_{t-1})}{h_{t-1}}\beta(var(\Psi_t))\right] \cdot \\ \{n_{t-1}h_{t-1}[\Pr(\Pi^{inR} \geq 0) + & \\ & + h_{t-1}^{-1}(1-h_{t-1})\beta(var(\Psi_t))] - \\ & - \xi(1+\iota)(1+r_{t-1})F^R n_{t-2}(1-h_{t-2})\beta(var(\Psi_{t-1}))\}\end{aligned}$$

In the case of full employment, for the reasons explained before, the workers are wage setters, there are no extra-profits and the remuneration of each entrepreneur is given by the wage she pays to herself. For the wage setters workers, it would be irrational to set a wage to the level where some of their firms would go bankrupt, therefore, if new entries takes place by the time where wages are set, the wage is derived by the condition $E_{t-1}(\Pi_t^e) = 0$, which implies

$$W_t^{fu} = \frac{E_{t-1}(P_t) \cdot \Lambda E_{t-1}(L_{j,t}^*)^\alpha - \tau - (1+\iota)(1+r_{t-1})F}{E_{t-1}(L_{j,t}^*)} \tag{8}$$

On the other hand, if full employment is reached after at least one period where there are no new entries, then the condition is $E_{t-1}(\Pi_t^{in}) = 0$, since all the firms are incumbents after one period.

$$W_t^{fu} = \frac{E_{t-1}(P_t) \cdot \Lambda E_{t-1}(L_{j,t}^*)^\alpha - \tau}{E_{t-1}(L_{j,t}^*)}$$

The determination of the wages has a point of discontinuity triggered by the level of full employment. In fact:

$$w_t = \begin{cases} w_t^* & if \quad n_t < l \\ w_t^{fu} & if \quad n_t = l \end{cases} \tag{9}$$

The situation of full employment is subject to a number of shocks and may generate a temporary equilibrium.

Given the production function 4 of the generic firm i, the amount of labour decided *ex ante* by the generic firm i also implies the determination of the optimal output $\varphi_{i,t}$; since the expected value of $\lambda_{i,t}$ is 0 and $\lambda_{i,t}$ is uncorrelated with $\varphi_{j,t}$, we have (for the specific firm "i"):

$$L_{j,t} = \left(\frac{E_{t-1}(\varphi_{j,t})}{\Lambda} \right)^{1/\alpha}$$

Therefore, the amount of labour in real terms employed by all the firms is:

$$L_t^* = \sum_{i=1}^{H_t} L_{j,t} = \sum_{j=1}^{H_t} \left(\frac{E_{t-1}(\varphi_{j,t})}{\Lambda} \right)^{1/\alpha} \tag{10}$$

We introduce now references to Appendix 1 and Appendix 2; they are at ...NOTE FOR THE PUBLISHER: they are at the end of the paper, can be moved in the Supporting information if in use OR placed online in the Git of the simulation package, at `https://github.com/terna/oligopoly/`, reported beginning Section 6.

In Appendix 1 the assumption made on the timing of consumption is discussed: we have assumed that the consumption takes place at the end of each period and at the end of each period firms' bankruptcies and ex post profits are known.

Once the wage for time t is set (between $t-1$ and t), the entry decisions are taken: both the new entrants and the incumbents decide the number of workers to hire for the next period (i.e. from t to $t+1$), on the basis of their profit expectations. Then, at time "t" the firms (both incumbents and new entrants, i.e. the former workers) are bound with contracts to the financial sector, who lent them the money to cover the fixed entry cost $\xi(1+\iota)(1+r_{t-1})F^R$. According to our timing assumptions, the total number of firms is known at time t (and defined in our notation as H_t). The firms determine the Cournot-Nash equilibrium in mixed strategies, which specify both the output and the price for each firm. We further assume that the oligopolistic firms imperfectly observe the cost choices of their rivals, although they can approximate them on the basis of conjectures: in other words they play a perturbed game.

Given the timing assumptions of our model, H_t is determined before n_t, although they both take place at time t.

The dynamics of the model is mainly determined by the birth and death of firms. As shown in Appendix 2, the dynamics of firms is given by the following[3]:

$$
\begin{aligned}
H_t = H_{t-1}\left[\Pr(\Pi^{inR} \geq 0) + \frac{1-h_{t-1}}{h_{t-1}}\beta(var(\Psi_t))\right] - \\
-\xi(1+\iota)(1+r_{t-1})F^R n_{t-2}(1-h_{t-2})\beta(var(\Psi_{t-1}))
\end{aligned}
\tag{11}
$$

Ex ante, if the wage are set by the oligopolistic firms according to the incentive compatibility constraint 49 and if all the individuals had perfectly identical expectations (i.e. if there were no idiosyncratic prediction mistake or informational shocks), $\Pr(entry)_{t+1}$ would be null and there would be no entry. The ex post deviations are those caused by all the possible

stochastic shocks affecting the right-hand side of inequality 49.

4 The nature of the equilibrium among the oligopolistic firms

The existence of an equilibrium in mixed strategies in an oligopolistic market where the firms simultaneously decide prices and quantities is a result due to Gertner (1985), in his unpublished (but very often quoted) PhD thesis, where he proves

[3] Using a different notation, we may write 11 as follows:

$$
\begin{aligned}
n_t h_t = n_{t-1}h_{t-1}\left[\Pr(\pi^{in} \geq 0) + \frac{1-h_{t-1}}{h_{t-1}}\psi(var(\Psi_t))\right] - \\
-(1+r_{t-1})\xi F n_{t-2}(1-h_{t-2})\psi(var(\Psi_{t-1}))
\end{aligned}
$$

where $H_t = n_t h_t$. However, we do not use this notation because it might be misleading, since the number of workers employed by each firm is stochastic, implied by the specific output of the firm, determined by information shocks and this other notation is only true *ex post*.

this result with symmetric firms, in the case of constant and increasing marginal costs (Gertner, 1985, pp. 74-98). Maskin (1986) extends Gertner's results to a case where firms' symmetry is not required and with rather general costs functions. Maskin provides his proofs for a duopolistic market, but he points out that they are valid for a finite number of oligopolistic firms (Maskin, 1986, p. 382). Harsanyi (1973) points out that mixed strategies may be interpreted as pure strategies in a perturbed game, in other words, in a mixed-strategy equilibrium, each player may be using a pure strategy that is contingent on a small random disturbance on his own payoff. For instance, like in our model, a firm might observe its own costs and have slight uncertainty on the costs of its rival. Harsanyi shows that when the uncertainty on rivals' choices vanishes, the mixed-strategy equilibria of the incomplete-information game converge to pure-strategy equilibria. In our model, however, the perspective is different from the one of game theorists: it is mixed strategies equilibria we are interested in and not pure strategy equilibria, since we are modelling a context where a slight uncertainty on costs (and other features) of the rivals may generate a stochastic outcome and this stochastic outcome is a potential source of macroeconomic shocks.

Holt (1994) provides an application of Harsaniy's interpretation of the mixed-strategy equilibrium by considering an oligopolistic market where the firms have incomplete information about a cost parameters of their rivals. The dispersion of privately observed payoff parameters induces the firms to randomize their pricing and as the population distribution of payoff parameters collapses to a point, the distribution of pure-strategy prices converges to the mixed-strategy equilibrium distribution for the limiting complete-information game, even with asymmetric firms.

In another interpretation, discussed by Osborne and Rubinstein (1994, p. 39) the mixed-strategy equilibria are stochastic steady states: each occurrence of the game takes place after n players are randomly chosen from different populations.

This last interpretation is consistent with the assumptions of our model, since the firms interacting in the market, in general, are not the same in each occurrence of the game (since we have entry and exit) and the existing firms are chosen stochastically (since both entry and exit are determined by stochastic shocks).

Therefore, in our model and in our agent-based simulations, the existence of mixed-strategy equilibria in the game among the oligopolistic firms can be justified by Harsanyi's (1973) interpretation (since we assumed that the firms play a "perturbed game") or by Osborne and Rubinstein's (1994) interpretation, although, given the modelling features of our model, they are also consistent with Maskin (1986) results. We assume therefore that the firms simultaneously choose both prices and quantities, that proportional rationing applies and that the profit (payoff) function of the firm is continuous in prices.

We also assume that the amount of work hired by each firm constitutes a capacity constraint, on the basis of the labour contracts set for time t, until time $t+1$.

We further assume that the quantities decided by the firms are not commodities, but "contracts", i.e. "commitments" to sell commodities to the customers and these commitments might be subject to stochastic shocks. In this way, even though a Nash Equilibrium in mixed strategies exists among the oligopolistic firms, these stochastic shocks may cause some firms to go bankrupt and the firms' failure does not implies any lack of rationality in the determination of the Nash Equilibrium in mixed strategies. This point is relevant for the determination of the macroeconomic equilibrium.

Between time $t-1$ and t the wages that apply between time t and $t+1$are set. At the beginning of time t the new entrants operating in the market between time t ant time $t+1$ are known and so are the new entrants and the incumbents of time $t-1$ that have gone bankrupt and have not survived. Once entry has taken place and is known, the game among the oligopolists take place, at time t.

The incumbents and the new entrants hire the workers by setting one-year labour contracts that also specify the amount of hours to be supplied by each worker for the coming period. Both entry costs and labour costs are sunk costs for both the incumbents and the new entrants. Since, as we said, all incomes (profits, labour and unemployment subsidies) for time t are received by all the individuals at the end of time t, labour costs are first accounted as debt of the firms toward the workers, at the beginning of time t, even though they are paid out to workers at the end of time t. This makes them sunk costs.

The amount of labour employed at time t by the generic firm i, $L_{i,t}$, is determined on the basis of *ex ante* expectations. We call it the optimal *ex ante* amount of labour, associated to the optimal *ex ante* individual firm output $E_{t-1}(\varphi_{i,t})$ at time t. All firms (no matter if they are incumbents of new entrants) share the same technology and production function.

These *ex ante* expectations, *ex ante* amount of labour and *ex ante* individual firm's output are all determined on the basis of the *ex ante* demand curve at time t, which is the *ex post* demand for time $t-1$, i.e. the empirically observable demand curve that emerges after the realization of all the stochastic shocks of the model.

The ex post price at time t is determined by the oligopolistic firms, who pick up a specific part of the aggregate demand 2

5 Summarizing the theoretical model for numerical simulations

The next sections contain a description on how the theoretical assumptions of the model are translated into instructions for the agent- based simulation. In particular, the result of existence of a Cournot-Nash equilibrium in mixed strategies among the firms is rendered by the sequence of decisions where the entrepreneurs make decision plans, adapt production plans, decide to hire or fire workers, entrepreneurs produce and the "state of the world" set the price (or, to be more precise, the price distribution).

All the previous sections contain an explanation of the theoretical model, which boils down into the aggregate demand 3, stochastic determination of prices and profits, wage determination (from equation 9) as well as the employment dynamics (such as determined in 10, given 11. As shown in Appendix 2, a more effective algebraic notation for the sake of our numerical simulations is the following:

$$
\begin{aligned}
D(\cdot)_t = {} & (\Omega(r_t)/P_t)\cdot \\
& \cdot[A_t + [1/(1+r_t)]\sum_{i=0}^{\infty}[(1+E\,(r_{t+i}))\cdot \\
& (1+E(\iota_{t+i}))]^{-i}\cdot E\{(n_{t+i}w_{t+i}) + \\
& +[(n_{t+i-1}-H_{t+i-1})\beta(var(\Psi_{t+i}))]\Pi^e_{t+i} + \\
+[H_{t-1}\cdot \Pr(\Pi^{in}\geq 0) & -(1+r_{t-2})\xi F\cdot \\
& \cdot(n_{t-2}-H_{t-2})\beta(var(\Psi_{t-1}))]\Pi^{in}_{t+i}]\}
\end{aligned}
\tag{12}
$$

where β is a constant parameter and $var(\Psi_t)$ is a stochastic shock related to the variance of the profits of the oligopolistic firms, emerging in the Cournot-Nash equilibrium in mixed strategies among the oligopolistic firms.

From the above equation, it is clear that entry and exit generate shocks affecting the aggregate demand.

As we said, P_t, the price level, is interpreted as the average of the price distribution emerging in the Cournot-Nash equilibrium in mixed strategies among the oligopolistic firms in the commodities market. The inflation rate ι_t is defined as $\frac{P_t-P_{t-1}}{P_{t-1}}$

The exogenous and contemporaneous variables appearing in the equation are the liquid assets A_t (positively affecting the level of consumption) and the interest rate r_t (negatively affecting consumption) and the values of the future forward-looking variables $E\,(r_{t+i})$, $E(\iota_{t+i})$, $E(n_{t+i})$, $E(w_{t+i})$, $E(h^e_{t+i})$, $E(\Pi^e_{t+i})$, $E(h^{in}_{t+i})$, $E(\Pi^{in}_{t+i})$, $E(H^{in}_{t+i})$ and $E(H^e_{t+i})$, assumed to be induced by their currently observable value). Wages are predetermined at time t by equation 9:

$w_t = \{^{w_t^* \quad if \quad n<l}_{w_t^{fu} \quad if \quad n=l}$

The *ex ante* amount of workers hired by all the firms is given by

$$L_{j,t} = \left(\frac{E_{t-1}(\varphi_{j,t})}{\Lambda}\right)^{1/\alpha}$$

Therefore, the total amount of employed workers is given by :

$$L_t^* = \sum_{i=1}^{H_t} L_{j,t} = \sum_{j=1}^{H_t} \left(\frac{E_{t-1}(\varphi_{j,t})}{\Lambda}\right)^{1/\alpha}$$

Hence the total amount of employed individuals is given by

$L_t^* + H_t = L_t^* + H_t^e + H_t^{in}$

The dynamics of the model is mainly determined by the birth and death of firms. As shown in Appendix 2, the dynamics of firms is given by the following:

$$H_t = H_{t-1}[\Pr(\Pi^{in} \geq 0)] + [(n_{t-1} - H_{t-1})\beta(var(\Psi_t)] -$$
$$-(1 + r_{t-1})\xi F(n_{t-2} - H_{t-2})\beta(var(\Psi_{t-1}))$$

H_t is pre-determined and so are the terms appearing in the aggregate demand and capturing the effects of income distribution among the different social groups:

$$E[n_{t+i-1}(1 - h_{t+i-1})\beta(var(\Psi_{t+i}))]\Pi_{t+i}^e +$$
$$+E[n_{t-1}h_{t-1} \cdot \Pr(\Pi^{in} \geq 0) -$$
$$-(1 + r_{t-2})\xi F n_{t-2}(1 - h_{t-2})\beta(var(\Psi_{t-1}))]\Pi_{t+i}^{in}]\}$$

or

$$E[(n_{t+i-1} - H_{t+i-1})\beta(var(\Psi_{t+i}))]\Pi_{t+i}^e +$$
$$+E[H_{t-1} \cdot \Pr(\Pi^{in} \geq 0) -$$
$$-(1 + r_{t-2})\xi F(n_{t-2} - H_{t-2})\beta(var(\Psi_{t-1}))]\Pi_{t+i}^{in}]\}$$

The next sections describe how, in terms of individual agents behavior, the equations described in this section are transformed, in sequential steps, into instructions for the agent-based simulation.

6 Introduction to the agent-based simulation

We introduce the use of agent-based simulation (Gilbert and Terna, 2000; Borril and Tesfatsion, 2010; LeBaron and Tesfatsion, 2008; Fagiolo e Roventini, 2012; about oligopolistic behavior look also at Merlone and Szidarovszky, 2015) not only to cope with the heterogeneity of the agent but, must of all, to discover different emerging macro behaviors within the complex structure of the models, as we show in Sections 9 and 10.

The agent-based model uses SLAPP, online at `https://github.com/terna/SLAPP`, as simulation shell. SLAPP has a Reference Handbook at the quoted web address and it is deeply described in Chapters 2–7 in Boero *et al.* (2015). The `oligopoly` project is online at `https://github.com/terna/oligopoly/` and when downloaded will be contained in a stand alone folder, having the same name of the model[4].

The calibration (see Table 1) has been quite lengthy, spanning over the whole construction of the model, built to run in five successive consistent configurations. The process is documented via the file `Oligopoly.pdf`, online at the same address above.

We underline that between (i) the formal presentation of the model in the equation based way, strictly necessary to be consistent with the literature upon which our work is grounded and (ii) the agent-based implementation, the consistency is deeply satisfied, but with a few inevitable differences. The same kind of differences that we run up against when we compare (i) the formalization of a

[4] Please look at the Reference of SLAPP, Section 2.1, to run the simulation shell and to link it to a specific project, as the *oligopoly* one.

phenomenon and (ii) the related observation of the reality (here: an artificial one, simulated).

7 The sequence, repeated at each time step

At time $t = 1$ and uniquely at that time, instead of `adaptProductionPlan` the entrepreneurs will execute the method `makeProductionPlan`.

The sequence, in each step, is:

1. entrepreneurs adaptProductionPlan
2. entrepreneurs hireFireWithProduction
3. entrepreneurs (with probability 0.05) workTroubles
4. entrepreneurs produce
5. entrepreneurs planConsumptionInValue
6. workers planConsumptionInValue
7. WorldState setMarketPrice
8. entrepreneurs evaluateProfit
9. workers toEntrepreneur
10. entrepreneurs toWorker
11. WorldState fullEmploymentEffectOnWages
12. WorldState incumbentActionOnWages

The *methods* or commands are sent the agents in a deterministic way; the agents act in random order; if a probability is set, it is applied to each agent to decide if to activate or not the message.

WorldState is a meta-agent that acts defining or modifying general data of the world, such as the equilibrium price cleaning the market of the wage level.

The following Sections explore the content of each element of the sequence.

A the end of the article we have other Sections with the parameters, two examples of run (via Figures 1 and 2), and the Tables 2 and 3 about the correlation coefficients among the main variables of the model in the two cases.

7.1 makeProductionPlan or adaptProductionPlan

7.1.1 makeProductionPlan

The method (or command) `makeProductionPlan`, acting only at tome $t = 1$, sent to the `entrepreneurs`, orders them to guess their production for the initial period. The production plan $\widehat{\varphi}_t^i$ is determined in a random way, using a Poisson distribution, with mean ν (see below).

The initial production plan is:

$$\widehat{\varphi}_t^i \sim \ Pois(\nu) \tag{13}$$

The `makeProductionPlan` method works uniquely with $t = 1$.

The simulation calculates the initial value ν (used uniquely in the first step) as:

$$\nu = \rho \frac{(N_{workers} + N_{entrepreneurs})}{N_{entrepreneurs}} \tag{14}$$

In this way, about a ρ ratio of the agents is employed in the beginning. The ρ parameter in reported in Table 1 as `expected employment ratio at t=1`.

7.1.2 adaptProductionPlan

The method (or command) `adaptProductionPlan`, working works for time $t > 1$, sent to `entrepreneurs`, orders to the i^{th} firm to set its production plan for the current period to their fraction[5] of the total demand—transformed from its nominal value to the real one (i.e., in quantity)—of the previous period, modified with a random uniform relative correction in the interval $-v$ to $+v$, set in `commonVar.py` as `randomComponentOfPlannedProduction`. The value is shown in Table reported in the Table 1 split in *min* and max effects of the *random rel. component of planned production.*

Being $\widehat{\varphi}_t^i$ the planned production of firm i, we have:

[5] The fraction is equal for all the firms, being i here not relevant

– if $u_t^i \geq 0$

$$\widehat{\varphi}_t^i = \frac{\frac{D_{t-1}}{P_{t-2}}}{N_{entrepreneurs}}(1+u_t^i) \tag{15}$$

– if $u_t^i < 0$

$$\widehat{\varphi}_t^i = \frac{\frac{D_{t-1}}{P_{t-2}}}{N_{entrepreneurs}}/(1+|u_t^i|) \tag{16}$$

with $u_t^i \sim \mathcal{U}(-v, v)$ and P_{t-2} the lagged price.[6]

The double lagged price correction is justified because we are considering the production decisions at time t, which are based on the decisions of consumption at $t-1$, related to the income at time $t-1$; these decisions are made before the determination of the prices at $t-1$ (emerging only when comparing the demand and the predetermined offer). If we want to evaluate the consumption in quantity, without the effect of a too limited or too abundant offer, we have to use $t-2$ prices. This construction will be eliminated with a future version of the model, with the atomic interaction of buyers and sellers in a dispersed way.

7.2 hireFireWithProduction

The method (or command) `hireFireWithProduction`, sent to the `entrepreneurs`, orders them to hire or fire comparing the labor forces required for the production plan $\widehat{\varphi}_t^i$ and the labor productivity π; we have the required labor force (L_t^i is the current one):

$$\widehat{L}_t^i = \widehat{\varphi}_t^i/\pi \tag{17}$$

Now:

1. if $\widehat{L}_t^i = L_t^i$ nothing has to be done;
2. if $\widehat{L}_t^i > L_t^i$, the entrepreneur is hiring with the limit of the number of unemployed workers;

[6] The method is applied with $t > 1$, so the use of the lagged price starts at time 2, at that time P_{t-2} would be the undefined P_0 value; as a simplification we use P_{t-1} in this case.

3. if $\widehat{L}_t^i < L_t^i$, the entrepreneur is firing the workers in excess.

7.3 workTroubles

For each entrepreneur at time t, so for each firm i, we generate a shock $\psi_{i,t} > 0$ due to work troubles, with probability p_ψ (set for all the entrepreneurs via the schedule of the model)[7] and value uniformly distributed between $V_\Psi/2$ and V_Ψ. The shock reduces the production of firm i in a relative way, as in:

$$\varphi c_t^i = \varphi_t^i (1 - \psi_{i,t}) \tag{18}$$

where φ_c means *corrected production*.

The corresponding parameters in Table 1 are:

- p_ψ is set to `from schedule.xls: work trouble probability`;[8]
- $\psi_{i,t}$ is set between `min lost production due to work troubles` and `max lost production due to work troubles`.

If the global logical parameter (see Table 1) `cut also the wages` is *yes*, also wages are cut in the same proportion that the production is suffering. With W indicating the constant basic wage level, cW_t^i is the corrected value at time t and for firm i; the correction is superimposed to the other possible corrections (due to full employment or to artificial barrier creation). For the two runs of Section 9, this option is set to `no`.

[7] If at least one method is linked to a probability, SLAPP displays—in its text output—a dictionary with the method probabilities.

[8] *schedule.xls* is one of the files of the simulation project, as described at `https://github.com/terna/oligopoly/` and particularly into the Oligopoly.pdf reference; it is useful to quote *schedule.xls* to allow the replicability of the simulation.
the sensitivity analysis shows that the model is responsive to this probability, as higher values increase the presence of countercyclical mark ups—see about this effect at the end of the paper— but anyway the key determinant of that effect is the entry/exit phenomenon, as in Section 10.

$$cW_t^i = W(1 - \psi_{i,t}) \quad (19)$$

7.4 produce

The method (or command) `produce`, sent to the `entrepreneurs`, orders them—in a deterministic way, in each unit of time—to produce proportionally to their labour force, obtaining the production φ_t^i, where i identifies the firm and t the time.

L_t^i is the number of workers of the firm i at time t. As above, we account also for the entrepreneur as a worker. π is the labor productivity, with its value set to 1, as in Table 1 (parameter `labor productivity`); in this version of the model it does not change with t). P_t^i is the production of firm i at time t:

$$\varphi_t^i = \pi L_t^i \quad (20)$$

The production is reduced for work troubles (as in Section 7.3) calculating the corrected value φc_t^i with:

The production is:

$$\varphi c_t^i = \varphi_t^i(1 - \psi_{i,t}) \quad (21)$$

In absence of work troubles, $\psi_{i,t}$ is 0.

The production (corrected or not) of the firm i is added to the total production: the *common* variable is `totalProductionInA_TimeStep`.

7.5 planConsumptionInValue (sent to entrepreneurs)

The method (or command) `planConsumptionInValue` operates both with the `entrepreneurs` and the `workers`, producing the following evaluations, using the parameters reported into Table 1. The description below is unique for both the cases.

The resulting consumption behavior if the agent i at time t, having income Y_t^i, is:

$$C_t^i = a_j + b_j Y_t^i + u \tag{22}$$

with $u \sim \mathcal{N}(0, common.consumptionRandomComponentSD)$.

Considering W as wage, as above, and Π as profit, the individual i can be :

- case $j = 1$: an entrepreneur, with $Y_t^i = \Pi_{t-1}^i + W_t$;
- case $j = 2$: an employed worker at time t, with $Y_i = W$ and the special [9] case $Y_t^i = Wc_t^i$, with Wc_t^i defined in eq. 19;
- case $j = 3$: an unemployed worker at time t, with $Y_t^i = sW$ (sW = social wage, as a welfare intervention).

The a_j and b_j values are reported in the initial output of each run; here in the Table 1.

The individual C_t^i updates `totalPlannedConsumptionInValueInA_TimeStep` (a *common* value), cleaned at each reset, i.e., at each new time step.

The `totalPlannedConsumptionInValueInA_TimeStep` measure will be then randomly corrected within the `setMarketPrice` method of the *WorldState* meta-agent, see Section 7.7.

7.6 planConsumptionInValue (sent to workers)

Look at Section 7.5.

7.7 setMarketPrice

The method (or command) `setMarketPrice`, sent to the `WorldState`, orders it to evaluate the market clearing price considering each agent behavior and *an external shock, potentially large.*

[9] Activated if the parameter *cut also the wages* is set to *yes*

We introduce a shock Ξ uniformly distributed between $-L$ and $+L$ where L is a rate on base 1, e.g., 0.10. To keep the effect as symmetric, we have the following equations determining the clearing price:

If the shock Ξ is (≥ 0):

$$P_t = \frac{D_t(1+\Xi)}{O_t} \tag{23}$$

if the shock Ξ is (< 0):

$$P_t = \frac{D_t/(1+\Xi)}{O_t} \tag{24}$$

with: P_t, clearing market price at time t; D_t demand in value at time t; O_t, offer in quantity (the total production) at time t.

The boundaries of the Ξ parameter are reported in Table 1 as:
`min of uniform demand relative random shock` and
`max of uniform demand relative random shock`.

The method uses two internal *common* variables:

- `totalProductionInA_TimeStep`, generated by the agents (*entrepreneurs*), via `produce`;
- `totalPlannedConsumptionInValueInA_TimeStep`, generated by the agents (*entrepreneurs* and *workers*) via `planConsumptionInValue` sent to *entrepreneurs* and to *workers*.

7.8 evaluateProfit

The method (or command) `evaluateProfit`, sent to the `entrepreneurs`, orders them to calculate their profit, being φ_t^i the production and L_t^i the labor (accounting also for the entrepreneur, as an employee of the firm).

The use of φ_t^i, the actual production of the entrepreneurs, accounts both for the production plan decided with `adaptProductionPlan`, Section 7.1.2, and for the limits in hiring, if any, in `hireFireWithProduction`, Section 7.2. The sum of all the actual productions of each entrepreneur is used in `setMarketPrice` action.

P_t is the `price`, clearing the market at time t.

W is the `wage` per employee and time unit, set to 1.0 in `common` variable space, not changing with t, but the case of the important events of:

- wage rise due both to full employment (Section 7.11) and
- to the creation of barriers against new entrants (Section 7.12).

γ are the extra costs for new entrant firms. They are calibrated to assure the effectiveness of the action described in Section 7.12, which is working in a non deterministic way, thanks to the movements in prices.

If the *common* variable `wageCutForWorkTroubles` is set to *True* the costs determination takes in account the reduction in the wages (but the wage of the entrepreneur, not changing).

Considering the presence of work troubles (see Section 7.3) the determination of the clearing price, as in Section 7.7, can signal an increase in the equilibrium price, due to the lacking production.

The (relative) shock $\psi_{i,t} > 0$ due to work troubles is defined in Section 7.3.

In presence of work troubles the firm could accept a reduction of its price, to compensate its customers for having undermined the confidence in the implicit commitment of producing a given quantity (the production plan, specified in Section 7.1.1). This option is currently not activated (`price penalty for the firms if work troubles` set to 0 in Table 1).[10]

The profit evaluation, if the parameter `cut also the wages` in Table 1 is set to *yes*, is:

$$\Pi_t^i = P_t(1 - pv_t^1)\varphi_t^i - (W - \psi_{i,t})(L_t^i - 1) - 1W - \gamma \tag{25}$$

being $1W$ the wage of the entrepreneur, unmodified.

[10] The penalty value is *price penalty for the firms if work troubles* defined as a relative measure in Table 1, here shortly pv. Locally, pv_t^i, for the firm i at time t, is set to pv if $\psi_{i,t} > 0$; otherwise ($\psi_{i,t} = 0$) is set to 0 (see Section 7.3).

If the parameter `cut also the wages` in Table 1 is set to *no*, the result is:

$$\Pi_t^i = P_t(1 - pv_t^i)\varphi_t^i - WL_t^i - \gamma \tag{26}$$

The new entrant firms have extra costs γ to be supported, but only for n periods, as stated in `commonVar.py` and activated by method `toEntrepreneur`.

7.9 toEntrepreneur

With the method (or command) `toEntrepreneur`, sent to `workers`, the agent, being a worker, decides to become an entrepreneur at time t, if its employer has a relative profit (reported to the total of the costs) $\geq$ a given *threshold* at time $t-1$. The threshold is retrieved from the parameter `relative threshold to become an entrepreneur` in Table 1.

In actual business world, the decision is a quite rare one, so we have to pass a higher level probabilistic control, that we define with the parameter `max new entrant number in a time step` in Table 1.

This parameter contains a value close to the *potential max number of new entrepreneurs* in each cycle.

Internally, it works in the following way: given an absolute value as number of workers actually becoming entrepreneurs in each cycle, we transform that value in a probability, dividing by the total number of the agent, used as an adaptive scale factor.

The agent starts counting the n periods of extra costs:

- n is set to the value of the parameter `duration (# of cycles) of the extra costs`;
- the amount of extra costs is set in the parameter `new entrant extra costs`.

Both the parameters are in Table 1.

7.10 toWorker

With the method (or command) `toWorker`, an entrepreneur moves to be an unemployed worker if its a relative profit (reported to the total of the costs) at time t is $\leq$ a given *threshold*. The threshold is retrieved from the parameter `relative threshold from entrepreneur to unempl.` in Table 1. Newborn firms are excluded from this control.

The agent changes its internal type.

7.11 fullEmploymentEffectOnWages

The method (or command) `fullEmploymentEffectOnWages`, sent to the `WorldState`, orders it to modify wages accordingly to full employment situation, in a reversible way.

Being U_t the unemployment rate at time t, ζ the unemployment threshold to recognize the *full employment* situation (`full employment threshold` of Table 1), s the proportional increase step (reversible) of the wage level (parameter `wage step up in full employment` in Table 1) and W_t the wage level at time t (being W_b the basic level—parameter `wage base` in Table 1—, remembering that the changes are reversible at the end of a cycle), we have:

$$\begin{cases} W_t = W_b(1+s) \text{ if } U_t \leq \zeta \\ W_t = W_b \text{ if } U_t > \zeta \end{cases} \tag{27}$$

7.12 incumbentActionOnWages

The method (or command) `incumbentActionOnWages`, orders to `WorldState` to modify the wage level for one period, accordingly to the attempt of the incumbent oligopolists to create an entry barrier when new firms are entering into the market.

As a consequence, wage measure contains a floating addendum, set to 0 as regular value and modified temporary in each period.

The current number of entrepreneurs H_t is calculated by the model and the previous one H_{t-1} is extracted from the structural dataframe generated while the model is running.

The wage level has two components, mutually exclusive:

1. the effects of full employment on wages, as in Section 7.11;
2. the effect described in this Section about incumbent oligopolists, which are strategically increasing wages to create an artificial barrier against new entrants; the new entrepreneurs suffer temporary extra costs, so for them the wage increment can generate relevant losses to produce their bankruptcy (about the calibration, considering also the γ parameter, see Section 7.8).

   We have here two levels:

   – K as the (relative) threshold of entrepreneur presence to determine the reaction on wages; the K level is reported via the parameter `trigger level (relative increment of olig. firms)` in Table 1;
   – k as the relative increment of wages; k is reported in parameter `wage relative increment as an entry barrier` in Table 1.

Formally, in case 2 above:

$$\begin{cases} W_t = W_0(1+k) \text{ if } \dfrac{H_t}{H_{t-1}} - 1 > K \\ W_t = W_0 \text{ if } \dfrac{H_t}{H_{t-1}} - 1 \leq K \end{cases} \tag{28}$$

8 Parameters

We use the parameters reported in Table 1 for the two simulation runs detailed in Section 9. The parameters are set via the file `commonVar.py` and, in a few cases, via the `parameters.py` one. Also the file `schedule.xls`, basically dedicated to the definition of the actions in each cycle, can contain parameters triggering the phenomena of the simulation, as explicitly stated in Table 1. This set of specifications is fundamental to allow the replicability of the experiments introduced here, via the program and the project code at the addresses that we display at the beginning of Section 6. The results of Figure 1 and Table 2 are based exactly on the parameters that we introduce here; those of Figure 2 and Table 3 require the modification of the number of `max new entrant number in a time step` from 20 to 0, acting on the variable `absoluteBarrierToBecomeEntrepreneur` in `commonVar.py`.

Table 1: Parameters

Parameter names	**Values**
project version	5b
build	20170403
seed (1 gets it from the clock)	111
wage base	1.0
social welfare compensation	0.3
labor productivity	1
expected employment ratio at t=1	0.8
consumption behavior: a1	0.4
consumption behavior: b1	0.55
consumption behavior: a2	0.3
consumption behavior: b2	0.65
consumption behavior: a3	0
consumption behavior: b3	1
consumption random component (SD)	0.3
relative threshold to become an entrepreneur	0.15
new entrant extra costs	60
duration (# of cycles) of the extra costs	3
min random rel. component of planned production	-0.1
max random rel. component of planned production	0.1
max new entrant number in a time step	20
relative threshold from entrepreneur to unempl.	-0.2
min of uniform demand relative random shock	-0.15
max of uniform demand relative random shock	0.15
full employment threshold	0.05
wage step up in full employment	0.1
wage relative increment as an entry barrier	0.15
trigger level (relative increment of olig. firms)	0.2
min lost production due to work troubles	0.05
max lost production due to work troubles	0.1

Continued on next page

Table 1 – continued from previous page

Parameter names	Values
probability of work troubles, see below	-
cut also the wages	no
price penalty for the firms if work troubles	0
# of cycles	50
from schedule.xls: work trouble probability	0.05

9 Two tales of fifty cycles

Figures 1 and 2 show the results which are related to the parameters of Table 1, with the modification, for Figure 2 to stop the new entrants.

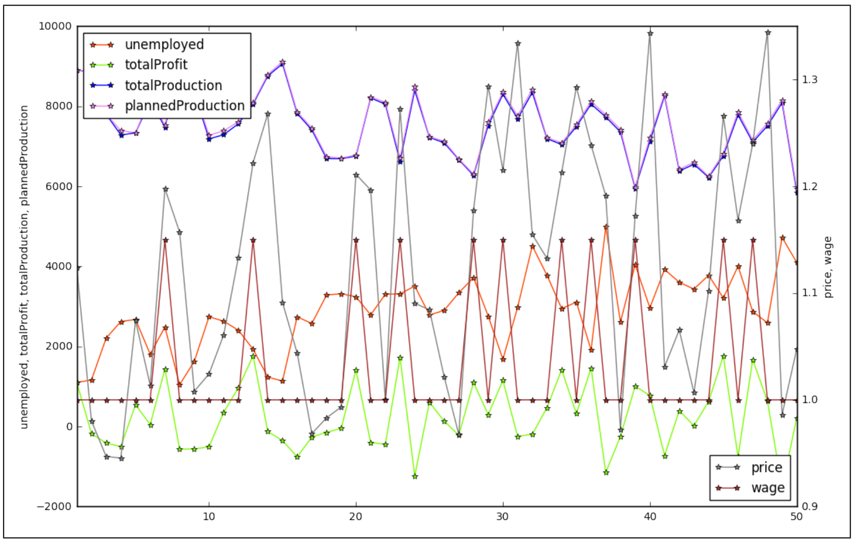

Fig. 1: A 50 cycle run with new entrant firms

From a general point of view, the main result of the simulation within the *oligopoly project* is that of showing an emergent business cycle from the proposed system of rules. If we compare the two Figures, it is easy to notice that the second one (Figure 2) displays series a lot more regular in the cycles. The most of

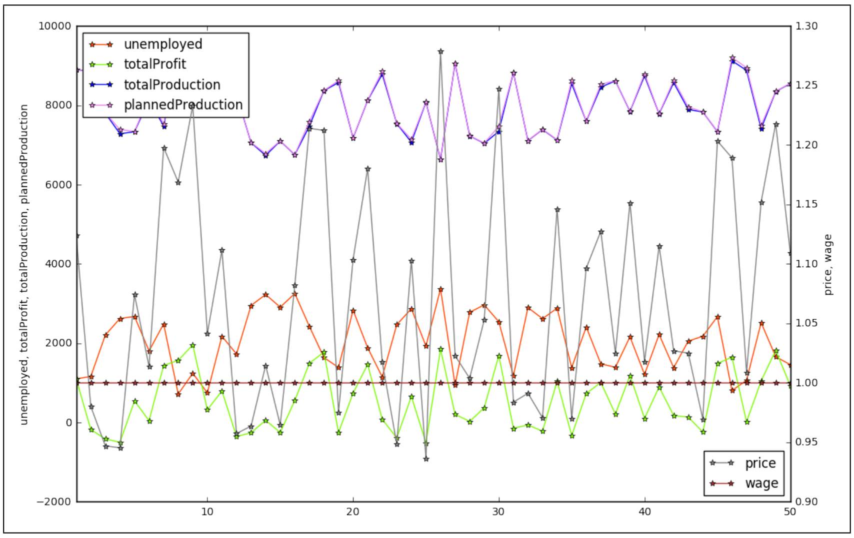

Fig. 2: A 50 cycle run without new entrant firms

the complexity of this artificial world is coming from the presence of the new entrant oligopolists, generating and entry/exit mechanism, while without the entry possibility we also have no exit (no firms in bankruptcy, in other words).

10 Looking for correlations

The correlations in Tables 2 and 3 are linked to the experiments of Figures 1 and 2. They are quite reasonable, with an interesting discovery.

We observe in Tables 2 that *profits* are positively related to *prices*, negatively to *total production* (high production reduces prices), negatively to *unemployment* (low unemployment increases demand and prices), positively to *wages*: here we have a complicated loop, as high profits are attracting more firms and so incumbent oligopolists increase wages to create a barrier.

That described above is a clear countercyclical mark up phenomenon (negative correlation between profits and production).

The phenomenon completely disappears in Tables 3, where the simulation does not allow the entry of new firms. The hint is that the key element of our simulation engine is properly the entry/exit mechanism about firms.

	unempl.	totalProfit	totalProd.	plannedP.	price	wage
unemployed	1.00	-0.18	-0.57	-0.56	-0.02	-0.02
totalProfit	-0.18	1.00	-0.36	-0.37	0.53	0.77
totalProduction	-0.57	-0.36	1.00	1.00	0.02	-0.25
plannedProduction	-0.56	-0.37	1.00	1.00	0.02	-0.25
price	-0.02	0.53	0.02	0.02	1.00	0.46
wage	-0.02	0.77	-0.25	-0.25	0.46	1.00

Table 2: Correlations among the time series of the model, with new entrant firms

	unempl.	totalProfit	totalProd.	plannedP.	price	wage
unemployed	1.00	-0.02	-1.00	-1.00	0.05	NaN
totalProfit	-0.02	1.00	0.02	0.02	0.99	NaN
totalProduction	-1.00	0.02	1.00	1.00	-0.05	NaN
plannedProduction	-1.00	0.02	1.00	1.00	-0.05	NaN
price	0.05	0.99	-0.05	-0.05	1.00	NaN
wage	NaN	NaN	NaN	NaN	NaN	NaN

Table 3: Correlations among the time series of the model, without new entrant firms

In Table 3, the NaN (not available number) results are due to the wage level, never moving in this case. This is also a interesting side effect resulting from the no-entry situation.

11 Concluding remarks

We have introduced here a theoretical macroeconomic framework for an agent-based model of an oligopolistic economy with heterogeneous agents and wage rigidity where the macroeconomic fluctuations are determined by the interaction among the oligopolistic firms. The process of entry/exit of the oligopolistic firms potentially interact with distributional shocks among workers and entrepreneurs. In this framework the agents have the same preferences, modeled with a conventional CRRA utility function, are heterogeneous in their budget constraint and may change their social status in each period according to a stochastic process which interacts with labour market and with the process of entry/exit.

The agent-based simulations show that the model can generate cyclical fluctuations in the economy, with the entry/exit mechanism associated to the social mobility and informational shocks.

The simulations most of all show that the model provides an explanation for countercyclical mark ups.

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[Note for the editors and the publisher: Appendix 1 and Appendix 2 are are supposed to be published on a website containing supporting information.]

Appendix 1 - Microfoundation of consumption and aggregate demand

The aggregate demand is microfounded and explicitly formalized as the aggregation of the individual demand functions of each consumer. Let us assume that the preferences of the individuals be represented by a CRRA utility function and the consumer problem be formalized as a standard intertemporal optimization problem, similar to the one presented in Bagliano and Bertola (2004, ch.1):

$$\max U_t = E_t \left[\sum_{i=0}^{\infty} \left(\frac{1}{1+\rho} \right)^i u(c_{t+i}) \right] \tag{29}$$

$$c_{t+i}, i = 0, ...\infty \tag{30}$$

for each $i = 0, 1, ...\infty$

subject to the following constraint:

$$E(b_{t+i+1}) = (1 + r_{t+i})E(b_{t+i}) + E(y_{t+i}) - c_{t+i} \tag{31}$$

and

$$c_{t+i} \geq 0$$

at every time $t + i$ from $i = 0, ..., \infty$

Where $\left(\frac{1}{1+\rho}\right)$ is the subjective discount factor, r is the real interest rate on the financial asset and controlled by the central bank, c_t the real consumption at time t for the consumers, $y_{j,t}$ consumer j's real income, $b_{j,t}$ the real wealth held by the individual consumer j at time t, which may be defined as a random portion δ_j of the aggregate real wealth a_t, i.e. $b_{j,t} = \delta_j a_t$ with $0 < \delta_j < 1$, so that summing up to all the l individuals we have $\sum \delta_j a_t = a_t$ and $\sum \delta_j = 1$

The financial assets are risk free and do not include shares: in this simplified model, investing in shares is a time consuming activity and implies being an entrepreneur. The budget constraint 31 also holds for any time $i = 0, ..., \infty$.

The transversality condition is the following

$$\lim_{j \longrightarrow \infty} b_{t+j} \left(\frac{1}{1 + r_{t+j}} \right)^j \geq 0$$

Since the marginal utility of consumption is always positive the transversality condition is always satisfied in terms of equality. The financial wealth $b_{j,t}$ and the individual human capital (let us define it $g_{j,t}$) are assumed to be valued at the beginning of period t, while $X_{j,t} = (1 + r_t)(b_{j,t} + g_{j,t})$ represents the individual total wealth, which is valued at the end of period t, but before consumption $c_{j,t}$, that absorbs part of the available resources. We also assume that both profits and wages are paid at the end of the period, when consumption takes place. The human capital valued at the beginning of time t is the following:

$$g_{j,t} = \frac{1}{(1 + r_t)} \sum_{i=0}^{\infty} \left(\frac{1}{1 + E_t(r_{t+i})} \right)^i E_t(y_{j,t+i}) \tag{32}$$

and, as above

$$X_{j,t} = (1 + r_t)(b_{j,t} + g_{j,t}) \tag{33}$$

hence

$$E_t(X_{j,t+1}) = (1 + r_t) \left[E_t(b_{j,t+1}) + \frac{1}{(1 + r_t)} \sum_{i=0}^{\infty} \left(\frac{1}{1 + E_t(r_{t+i})} \right)^i E_t(y_{j,t+1+i}) \right] \tag{34}$$

Substituting in the budget constraint the definition of $b_{j,t+1}$ we get:

$$E(X_{j,t+1}) = (1+r_t)\left[(1+r_t)b_{j,t} + y_{j,t} - c_{j,t+i} + \right.$$
$$\left. + \frac{1}{(1+r_t)}\sum_{i=0}^{\infty}\left(\frac{1}{1+E_t(r_{t+i})}\right)^i E_t(y_{j,t+1+i})\right]$$

where is the income in real terms. Hence

$$E(X_{j,t+1}) = [(1+r_t)(b_{j,t} + g_{j,t}) - c_{j,t+i}]$$
$$= (1+r_t)(X_{j,t} - c_{j,t+i})$$

and, generalizing

$$E(X_{j,t+i+1}) = (1+r_{t+i})(X_{j,t+i} - c_{j,t+i})$$

Where $X_{j,t+i+1}$ is the state variable.

$$g_{j,t} = \frac{1}{(1+r_t)}\sum_{i=0}^{\infty}\left(\frac{1}{1+E_t(r_{t+i})}\right)^i E_t(y_{j,t+i}) \tag{35}$$

where, again, summing up over the all population we get the aggregate human wealth H_t i.e. $\sum g_{j,t} = H_t$ and, as above

$$X_{j,t} = (1+r_t)(b_{j,t} + g_{j,t}) \tag{36}$$

where $X_{j,t}$ is the overall (human and financial) wealth of the individual j at time t. Like in Bagliano and Bertola (2004, ch. 1) it is assumed that consumption takes place at the end of each period.

Let us assume now that the instantaneous utility be represented by the following function:

$$u_{j,t} = \frac{c_{j,t}^{1-\gamma}}{1-\gamma} \tag{37}$$

with $0 < \gamma < 1$

Therefore the consumer problem boils down into the following Bellman and Euler equations respectively:

$$V(X_{j,t}) = \max_{c_{j,t}} \left[\frac{c_{j,t}^{1-\gamma}}{1-\gamma} + \left(\frac{1}{1+\rho} \right) E(V(\omega_{j,t+1})) \right] \tag{38}$$

$$u'\left(c_{j,t}\right) = \frac{1+r_t}{1+\rho} E_t u'_{j,t}\left(c_{j,t+1}\right) \tag{39}$$

Subject to

$$E(X_{t+1}) = (1+r_t)(X_t - c_{j,t}) \tag{40}$$

Where X_{t+i+1} is the state variable.

Now we assume (and later verify) that the value function has the same analytical form of the utility function, i.e.

$$V(X_{j,t}) = \Theta \frac{X_{j,t}^{1-\gamma}}{1-\gamma} \tag{41}$$

Where Θ is a positive constant whose exact value will be shown later. By using the definition of $V(\omega_t)$ 41, the Bellman equation can be rewritten as follows:

$$\Theta \frac{X_{j,t}^{1-\gamma}}{1-\gamma} = \max_{c_t} \left[\frac{X_{j,t}^{1-\gamma}}{1-\gamma} + \frac{1}{1+\rho} E\left(\Theta \frac{X_{j,t}^{1-\gamma}}{1-\gamma} \right) \right] \tag{42}$$

hence, using the constraint 40 and deriving with respect to $c_{j,t}$, we get the F.O.C:

$$c_{j,t}^{-\gamma} = \frac{1+r_t}{1+\rho} \Theta \left[(1+r_t)(X_{j,t} - c_{j,t})\right]^{-\gamma}$$

and solving for $c_{j,t}$ we get the individual consumption (demand) function:

$$c_{j,t} = \frac{1}{1 + (1+r_t)^{\frac{1-\gamma}{\gamma}} (1+\rho)^{-\frac{1}{\gamma}} \Theta^{\frac{1}{\gamma}}} X_{j,t}$$

where Θ is the constant to be determined.

To complete the solution, we still use the Bellman equation 42, substitute the consumption function in it and we set:

$$M \equiv (1+r_t)^{\frac{1-\gamma}{\gamma}} (1+\rho)^{-\frac{1}{\gamma}}$$

just to simplify the notation. Then we get:

$$\begin{aligned} \Theta \frac{X_{j,t}^{1-\gamma}}{1-\gamma} = & \frac{1}{1-\gamma} \overbrace{\left(\frac{X_{j,t}}{1+M\Theta^{\frac{1}{\gamma}}} \right)}^{c_{j,t}}{}^{1-\gamma} + \\ & + \frac{1}{1+\rho} \frac{\Theta_t}{1-\gamma} \underbrace{\left[(1+r_t) \frac{M\Theta^{\frac{1}{\gamma}}}{1+M\Theta^{\frac{1}{\gamma}}} X_{j,t} \right]^{1-\gamma}}_{X_{j,t+1}} \end{aligned} \tag{43}$$

The value of K satisfying equation 43 can be obtained by equating the coefficients of $X_{j,t}^{1-\gamma}$ in the two sides of the equation and solving for K:

$$\Theta = \left(\frac{1}{1-M} \right)^{\gamma}$$

Under the condition $M < 1$ the consumption (expenditure) function is fully specified:

$$V(X_{j,t}) = \left(\frac{1}{1-(1+r_t)^{\frac{1-\gamma}{\gamma}} (1+\rho)^{-\frac{1}{\gamma}}} \right)^{\gamma} \frac{X_{j,t}^{1-\gamma}}{1-\gamma}$$

and

$$c(r_t, X_{j,t}) = \left[1-(1+r_t)^{\frac{1-\gamma}{\gamma}} (1+\rho)^{-\frac{1}{\gamma}} \right] X_{j,t}$$

i.e.

$$c(X_{j,t}) = \left[1-(1+r_t)^{\frac{1-\gamma}{\gamma}} (1+\rho)^{-\frac{1}{\gamma}} \right] (b_{j,t} + g_{j,t})$$

Where $c(w_t)$ can be interpreted as the individual demand function, i.e..

$$c(r_t, X_{j,t}) = D_t(r_t, X_{j,t}) = \left[1 - (1+r_t)^{\frac{1-\gamma}{\gamma}} (1+\rho)^{-\frac{1}{\gamma}}\right] (b_{j,t} + g_{j,t}) \tag{44}$$

Recalling that $b_{j,t} = \delta_{j,t} a_t$ and summing up for all the individuals, we have:

$$\begin{aligned}
C_t &= \sum_{j=1}^{l} c_i(r_t, X_{i,t}) \\
&= \sum_{i=1}^{l} \left[1 - (1+r_t)^{\frac{1-\gamma}{\gamma}} (1+\rho)^{-\frac{1}{\gamma}}\right] (\delta_{i,t} a_t + g_{j,t}) \\
&= \left[1 - (1+r_t)^{\frac{1-\gamma}{\gamma}} (1+\rho)^{-\frac{1}{\gamma}}\right] \sum_{i=1}^{l} (\delta_{i,t} a_t + g_{j,t}) \\
&= \left[1 - (1+r_t)^{\frac{1-\gamma}{\gamma}} (1+\rho)^{-\frac{1}{\gamma}}\right] (a_t + H_t) \\
&= \left[1 - (1+r_t)^{\frac{1-\gamma}{\gamma}} (1+\rho)^{-\frac{1}{\gamma}}\right] W_t
\end{aligned}$$

where W_t is the aggregate overall wealth. This is the aggregate demand

We can rearrange the aggregate demand $D_t(\cdot)$ in order to account for income distribution and expected future variables, under rather general assumptions. Let us start by defining

$$\Omega = \left[1 - (1+r_t)^{\frac{1-\gamma}{\gamma}} (1+\rho)^{-\frac{1}{\gamma}}\right]$$

Since ρ is constant and $0 < \gamma < 1$, then $\partial C(\cdot)/\partial r_t < 0$

Looking at 2 and defining the aggregate real income as y_{t+i}, then we have:

$$D(\cdot)_t = \Omega(r_t) \left(a_t + \frac{1}{1+r_t} \sum_{i=0}^{\infty} \left(\frac{1}{(1+E\,(r_{t+i}))(1+E(\iota_{t+i}))} \right)^i E(y_{t+i}) \right)$$

Defining the nominal aggregate financial wealth as $A_t = a_t \cdot P_t$, and the nominal aggregate $Y_t = P_t \ \cdot y_t$, we may re-write the aggregate demand function as follows:

$$D(\cdot)_t = \frac{\Omega(r_t)}{P_t} \left(a_t + \frac{1}{1+r_t} \sum_{i=0}^{\infty} \left(\frac{1}{(1+E\,(r_{t+i}))(1+E(\iota_{t+i}))} \right)^i E(Y_{t+i}) \right)$$

in addition, if we want to explicitly formalize the income distribution between workers and entrepreneurs, we get:

$$D(\cdot)_t = \frac{\Omega(r_t)}{P_t}\{A_t + (1+r_t)^{-1}\sum_{i=0}^{\infty}[(1+E\,(r_{t+i}))(1+E(\iota_{t+i}))]^{-i}\cdot$$
$$\cdot E(n_{t+i}(w_{t+i} + h^{e}_{t+i}\Pi^{e}_{t+i} + h^{in}_{t+i}\Pi^{in}_{t+i}))\} \qquad (45)$$

which is a unit elastic aggregate demand function.

Appendix 2 - Algebraic explanation of the interactions among labour market, "incentive compatible" wages, entry/exit and aggregate demand

For what concerns the behavior of the profits, we have:

$$\Pr(\Pi^{eR}_t < 0) = 1 - \Pr(\Pi^{eR}_t \geq 0)$$

and

$$\Pr(\Pi^{inR}_t < 0) = 1 - \Pr(\Pi^{inR}_t \geq 0)$$

At time t the new entrant survive with probability $\Pr(\Pi^{eR}_t \geq 0)$, goes bankrupt with probability $\left[1 - \Pr(\Pi^{eR}_t \geq 0)\right]$ and, in that case, both the entrepreneur and the workers will get the unemployment subsidy $\tau \cdot n_t\,(l - n_t)^{-1}$. At time t, the new entrant will have 2 possible outcomes, or "future paths". At time *t+1,* if successful, she will be an incumbent and survive with probability $\Pr(\Pi^{inR}_{t+1} \geq 0)$ or fail with probability $[1 - \Pr(\Pi^{inR}_{t+1} \geq 0)]$; still at time *t+1*, the unsuccessful new entrant will be unemployed and have a certain probability of still being unemployed and another probability of being hired as a worker, and so on. In other words, at time *t=1* there will be 2 possible outcomes (or "future paths") for the new entrant, at time *t=2* there will be 4 possible "future paths", at time *t=3*, there will be 8 possible "future paths", at time *t=n* there will be 2^n possible "future paths". Similarly, the worker who decides not to enter the market as an entrepreneur, with probability $\Pr(\Pi_t^{inR} \geq 0)$ will earn the real wage w_t and (in the event that her firm goes bankrupt) loose the job with probability $\left[1 - \Pr(\Pi_t^{inR} \geq 0)\right]$ and earn the unemployment subsidy $\tau^R \cdot n_t \left(l - n_t\right)^{-1}$. However, if we move on in time, for instance, at time $t+2$, the surviving entrant will get with probability $\Pr(\Pi_{t+1}^{inR} \geq 0)$ the wage and the profit of the incumbent Π_{t+1}^{inR} and with probability $\left[1 - \Pr(\Pi_{t+1}^{inR} \geq 0)\right]$ the unemployment subsidy. Valuating the expectation of future profits for the new entrant means valuating a tree of outcomes where from *t+1* onwards where in each period the firm can survive (with a certain probability) or going bankrupt (with the complementary probability). In other word, the rational forward-looking decision maker that makes plans at time $t = 1, 2, 3...n$ (i.e. for all the future periods from t onwards), faces 2^k different "future paths" for every $k-$periods interval in its future. In other words, we have a degree of on-going uncertainty which is increasing in the length of future time expectations. The variance of such expectations is higher the further away is the forecast.

Let us define as $J_{t+1}$the expected future stream of real income from time $t+1$ onwards for the successful entrant at time t and let us define as Γ_{t+1} the expected future stream of real income from time $t+1$ onwards for the worker who decides not to enter the market. We introduce then an approximation and define as Υ_{t+1} the expected stream of real income from time $t+1$ onwards of an individual unemployed at time $t+1$.

Υ_{t+1} Positively depends on the probability of being hired as a worker by a firm the next period, and negatively on the number of unemployed individuals. Therefore the term $\{1 - \Pr(\Pi_t^{eR} \geq 0)]\}(1+\rho)^{-1} \cdot E_{t-1}[(\tau^R \cdot n_t \left(l - n_t\right)^{-1} + \Upsilon_{t+1}]$ is the expected future stream of income for the unsuccessful entrant from time t onwards, weighted with the probability of going bankrupt in the first period.

If the new entrant at time t is successful, her expected stream of future income, from time $t+1$ onwards is

$$J_{t+1} = \frac{1}{1+\rho}\{[\Pr(\Pi_{t+1}^{inR} \geq 0)][E_t(\Pi_{t+1}^{inR}) + E_t(w_{t+1}) - \tau^R + J_{t+2}] +$$
$$+[1 - \Pr(\Pi_{t+1}^{inR} \geq 0)][\tau^R \cdot n_{t+1}\,(l - n_{t+1})^{-1} + \Upsilon_{t+2}]\}$$

We assume that *ex ante*, with no stochastic shocks $E[\ \Pr(\Pi_{t+1}^{inR} \geq 0)] = \Pr(\Pi_t^{inR} \geq 0) = \Pr(\Pi^{inR} \geq 0)$, $E_t(\Pi_{t+1}^{inR}) = \Pi_t^{inR}$ and $E(n_{t+1}) = n_t$. Hence:

$$J_{t+1} = \frac{1}{1+\rho}\{[\Pr(\Pi^{inR} \geq 0)][E_t(\Pi_{t+1}^{inR}) + E_t(w_{t+1}) - \tau^R + J_{t+2}] +$$
$$+[1 - \Pr(\Pi^{inR} \geq 0)][\tau^R \cdot n_{t+1}\,(l - n_{t+1})^{-1} + \Upsilon_{t+2}]\}$$

On the other hand, if the potential new entrant at time t decides not to enter the market (i.e. to remain a worker), we define her expected stream of future income, from time $t+1$ onwards as Γ_{t+1}, i.e. the expected stream of future income of a worker who keeps on being a worker:

$$\begin{aligned}\Gamma_{t+1} &= \frac{1}{1+\rho}\{[\Pr(\Pi^{inR} \geq 0)][E_t(w_{t+1}) - \tau^R + \Gamma_{t+2}] +\\ &+[1 - \Pr(\Pi^{inR} \geq 0)][\tau^R \cdot n_{t+1}\,(l - n_{t+1})^{-1} + \Upsilon_{t+2}] =\\ &= w_t \cdot \left[\sum_{i=1}^{\infty}(1+\rho)^{-i} \cdot \Pr(\Pi^{inR} \geq 0)^i\right] -\\ &-\tau^R \cdot \left[\sum_{i=1}^{\infty}(1+\rho)^{-i} \cdot \Pr(\Pi^{inR} \geq 0)^i\right] +\\ &+\sum_{i=1}^{\infty}(1+\rho)^{-i} \cdot [1 - \Pr(\Pi^{inR} \geq 0)]^i \left[\tau^R \cdot n_{t+i}\,(l - n_{t+i})^{-1} + \Upsilon_{t+1+i}\right]\end{aligned} \qquad (46)$$

Therefore J_{t+1} may be also defined as follows:

$$J_{t+1} = \Gamma_{t+1} + \sum_{i=1}^{\infty}\left[\left(\frac{1}{1+\rho}\right)^i E_t(\Pi_{t+i}^{inR})[\Pr(\Pi^{inT} \geq 0)]^i\right] \qquad (47)$$

We are now enabled to write the incentive compatibility constraint for wage setting with unemployment. In this case the wage is set by the oligopolistic firms in such a way to discourage entry, i.e. it has to satisfy the incentive compatibility constraint saying that the expected future discounted stream of income from time $t+1$ onwards for the worker employed by an incumbent surviving at the beginning of time $t+1$ has to be greater than or equal to the expected future discounted stream of income from time $t+1$ onwards for the new entrant.

$$\Pr(\Pi_t^{eR} \geq 0)\,(1-\varsigma)\,(1+\rho)^{-1}[E_{t-1}\left(\Pi_t^{eR}\right) + w_t - \tau^R +$$
$$+J_{t+1}(\cdot)] + \Pr(\Pi_t^{eR} < 0)\,(1-\varsigma)\,(1+\rho)^{-1}\cdot$$
$$\cdot\left\{E_{t-1}[\tau^R \cdot n_t\,(l-n_t)^{-1}] + \Upsilon_{t+1}\right\} \leq (1+\rho)^{-1}\,(1-\varsigma)\,\{[\Pr(\Pi^{inR} \geq 0)]\cdot$$
$$\cdot[w_t - \tau^R + \Gamma_{t+1}(\cdot)] + (1-\varsigma)\,(1+\rho)^{-1}\cdot$$
$$\cdot[\Pr(\Pi^{inR} < 0)]\cdot E_{t-1}[\tau^R \cdot n_t\,(l-n_t)^{-1}] + \Upsilon_{t+1}\} \tag{48}$$

The term $(1+\rho)^{-1}\cdot\Pr(\Pi^{inR} \geq 0)\cdot[w_t\left(1-\tau^R-\varsigma\right) + \Gamma_t(\cdot)]$ is the expected stream of future income for the worker who decides to remain worker and whose firm survives. The term $\{\Pr(\Pi^{inR} < 0)\cdot E_{t-1}[n_t(w_t + h_t^e \Pi_t^{eR} + h_t^{in}\Pi_t^{inR})\tau^R(l-n_t)^{-1} + \Upsilon_{t+1}]\}$ is the expected stream of future income for the worker who decides to remain worker and whose firm goes bankrupt. As we said, for τ very small and negligible, the term $E_{t-1}[n_t(w_t+h_t^e\Pi_t^{eR}+h_t^{in}\Pi_t^{inR}))\tau^R(l-n_t)^{-1}]$ will be very small and negligible. The term Υ_{t+1} will also be very small. The incentive compatible constraint 48 yields the following wage setting rule with unemployment:

Simplifying out inequality 48, and reminding that $\Pr(\Pi_t^{eR} \geq 0) = \Pr(\Pi^{inR} \geq 0) - \xi(1+\iota)(1+r_{t-1})F^R$, we get:

$$[\Pr(\Pi^{inR} \geq 0) - \xi(1+\iota)(1+r_{t-1})F^R]\cdot[E_{t-1}\left(\Pi_t^{eR}\right) + w_t -$$
$$-\tau^R + J_{t+1}(\cdot)] + [1 - \Pr(\Pi^{inR} \geq 0) + \xi(1+\iota)(1+r_{t-1})F^R]\cdot$$
$$\cdot\left\{E_{t-1}[\tau^R \cdot n_t\,(l-n_t)^{-1}] + \Upsilon_{t+1}\right\} \leq [\Pr(\Pi^{inR} \geq 0)]\cdot[w_t - \tau^R + \Gamma_{t+1}(\cdot)] +$$
$$+[1 - \Pr(\Pi^{inR} \geq 0)]\cdot E_{t-1}[\tau^R \cdot n_t\,(l-n_t)^{-1}] + \Upsilon_{t+1}]\}$$

Then, substituting 46 and 47 in the above inequality and simplifying out again, we get the following:

$$w_t\xi(1+\iota)(1+r_{t-1})F^R\left\{1+\sum_{i=1}^{\infty}[\Pr(\Pi^{inR}\geq 0)]^i(1+\rho)^{-i}\right\}\geq$$
$$\geq [\Pr(\Pi^{inR}\geq 0)-$$
$$-\xi(1+\iota)(1+r_{t-1})F^R]\cdot$$
$$\cdot\{E_{t-1}(\Pi_t^{eR})+$$
$$+\sum_{i=1}^{\infty}(1+\rho)^{-(i+1)}\cdot$$
$$\cdot E_{t-1}(\Pi_{t+i}^{inR})[\Pr(\Pi^{inR}\geq 0)]^{i+1}\}+$$
$$+\xi(1+\iota)(1+r_{t-1})F^R\tau^R\cdot$$
$$\cdot\{1+\sum_{i=1}^{\infty}[\Pr(\Pi^{inR}\geq 0)]^i\cdot$$
$$\cdot(1+\rho)^{-i}\}+$$
$$+\xi(1+\iota)(1+r_{t-1})F^R\cdot$$
$$\cdot\{E_{t-1}[\tau^R n_t(l-n_t)^{-1}+$$
$$+\Upsilon_{t+1}]-$$
$$-\sum_{i=1}^{\infty}[(1+\rho)^{-i}[1-\Pr(\Pi^{inR}\geq 0)]^i\cdot$$
$$\cdot E_{t-1}[\tau^R n_{t+i}(l-n_{t+i})^{-1}+$$
$$+\Upsilon_{t+i+1}]$$

Then, using the properties of the geometric series for the terms:
$\xi(1+\iota)(1+r_{t-1})F^R\left\{1+\sum_{i=1}^{\infty}[\Pr(\Pi^{inR}\geq 0)]^i(1+\rho)^{-i}\right\}$,
$\sum_{i=1}^{\infty}(1+\rho)^{-(i+1)}\cdot E_{t-1}(\Pi_{t+i}^{inR})[\Pr(\Pi^{inR}\geq 0)]^{i+1}\}$,
$\xi(1+\iota)(1+r_{t-1})F^R\tau^R\left\{1+\sum_{i=1}^{\infty}[\Pr(\Pi^{inR}\geq 0)]^i(1+\rho)^{-i}\right\}$,
$\sum_{i=1}^{\infty}[(1+\rho)^{-i}[1-\Pr(\Pi^{inR}\geq 0)]^i\cdot E_{t-1}[\tau^R n_{t+i}(l-n_{t+i})^{-1}+\Upsilon_{t+i+1}]$
and reminding that, *ex ante*, with no unexpected shocks, $E_{t-1}(n_t)=E_{t-1}(n_{t+1})=E_{t-1}(n_{t+1+i})$ and $E_{t-1}\left(\lim_{t\to\infty}\Upsilon_{t+1}\right)=E_{t-1}\left(\lim_{t\to\infty}\Upsilon_{t+i+1}\right)$, since, with no unex-

pected shocks, $E_{t-1}(\Upsilon_{t+1})$ and $E_{t-1}(\Upsilon_{t+i+1})$ can also be represented as a geometric series, then taking the limit for $t \to \infty$, then we can further simplify and get:

$$\begin{aligned} w_t^* \geq & \frac{E_{t-1}(\Pi_t^{eR})[\Pr(\Pi^{inR} \geq 0) - \xi(1+\iota)(1+r_{t-1})F^R]}{\xi(1+\iota)(1+r_{t-1})F^R \frac{1+\rho}{1+\rho-\Pr(\Pi^{in} \geq 0)}} + \\ & + \frac{E_{t-1}(\Pi_t^{inR}) \cdot \Pr(\Pi^{inR} \geq 0)}{\xi(1+\iota)(1+r_{t-1})F^R(1+\rho)} [\Pr(\Pi^{inR} \geq 0) - \xi(1+\iota)(1+r_{t-1})F^R] + \tau + \\ & + \frac{1+\rho-\Pr(\Pi^{inR} \geq 0)}{1+\rho} \cdot \\ & \cdot E_{t-1}[\tau^R n_{t+1}(l-n_{t+1})^{-1} + \Upsilon_{t+1}] \cdot \\ & \cdot \left[1 + \frac{1-\Pr(\Pi^{inR} \geq 0)}{\rho + \Pr(\Pi^{inR} \geq 0)}\right] \end{aligned} \tag{49}$$

which is the wage determination equation with unemployment 49. Considering 49 as a binding equality, for a given exogenous value of $\Pr(\Pi^{inR} \geq 0)$, reminding that, on the basis of our assumptions, $\Pr(\Pi^{inR} \geq 0) - \xi(1+\iota)(1+r_{t-1})F^R = \Pr(\Pi_t^{eR} \geq 0) \geq 0$, and reminding that, being w_t set between time $t-1$ and time t, with no stochastic shocks, the expected values for Π_t^{inR}, Π_t^{eR}, n_{t+i} are equal to their observable values at time $t-1$, then w_t^* displays a set of rather intuitive properties:

$$\frac{\partial w_t^*}{\partial r_{t-1}} < 0; \frac{\partial w_t^*}{\partial \tau^R} > 0; \frac{\partial w_t^*}{\partial n_{t-1}} > 0; \frac{\partial w_t^*}{\partial \Pi_{t-1}^{inR}} > 0; \frac{\partial w_t^*}{\partial \Pi_{t-1}^{eR}} > 0$$

As explained above, τ is assumed to be constant and to have a very small magnitude, which also makes small the magnitude of Υ_{t+1}.

For the sake of our numerical simulations, we may approximate the behaviour of w_t^* with a generic function $w_t^*(r_{t-1}, n_{t-1}, \Pi_{t-1}^{inR}, \Pi_{t-1}^{eR})$.

Defining the right-hand side of inequality 49 as Φ_t, we can introduce **the probability of entry,** $\Pr(entry)_t$, which may be interpreted as **the sum of the stochastic idiosyncratic information shocks generating entry decisions, i.e. the integral (over the whole population of workers $n_t(1-h_t)$ at time t) of the**

perceived probability that the wage is set at a lower level than the expected present discounted value of the future profits as an entrepreneur in case of entry.

$$\Pr(entry)_t = \int_0^{n_t(1-h_t)} (\Pr(w_t < E_{t-1,i}(\Phi_t))_i di \qquad (50)$$

Given the equilibrium equality $w_t = E_{t-1,i}(\Phi_t)$, the expression

$$\int_0^{n_t(1-h_t)} (\Pr(w_t < E_{t-1,i}(\Phi_t))_i di$$

is the integral of all the idiosyncratic information shocks on $E_{t-1,i}(\Phi_t)$for any individual worker i at time t, which depends on the variance of the expectation $E_{t-1,i}(\Phi_t)$ which is induced, as we said, by $var(\Psi_t)$, a stochastic variable related to the variance of the profits of the oligopolistic firms, emerging in the Cournot-Nash equilibrium in mixed strategies among the oligopolistic firms.

For the sake of our numerical simulation, we simply assume that the workers who decide to turn entrepreneurs and enter the market are those who observe "large than expected" profits in the firm where they work. The frequency of this event increases with the variance of the firms' profits (i.e. with the volatility of firms profits). When an increasing number of workers notice that their firms are enjoying "larger than average" profits, (i.e. when the variance of the profits increases), they make the "optimistic mistake" and enter the market. The "optimistic mistake" amounts to an informational shock.

Another rationale for this assumption is that an increase in the volatility of Ψ_t forces the individuals (including the potential entrants) to update their expectations, by collecting and processing data. Although the expectations are, on average, correct, the frequency of mistakes increase (i.e. the variance of the expectations increase) because data collecting and data processing are costly and time consuming. To put it another way, the integral

$$\int_0^{n_t(1-h_t)} (\Pr(w_t < E_{t-1,i}(\Phi_t))_i di$$

represents the portion of "optimistic" workers counting on a successful entry.

On the basis of the previous definitions, the total number of existing firms at time t is determined as follows:

$$
\begin{aligned}
n_t h_t = n_{t-1}h_{t-1} + &\overbrace{n_{t-1}h_{t-1}\frac{1-h_{t-1}}{h_{t-1}}\beta(var(\Psi_t))}^{\text{Number of new entrants for time } t} - \\
- &\underbrace{[1-\Pr(\Pi^{inR}\geq 0)]n_{t-1}h^{in}_{t-1}}_{\substack{\text{Non surviving incumbents from}\\ \text{time } t-1}} - \\
- &\underbrace{[1-\Pr(\Pi^{eR}\geq 0)]n_{t-1}h^{e}_{t-1}}_{\substack{\text{Non surviving new entrants from}\\ \text{time } t-1}}
\end{aligned}
$$

Hence, given the definition of $\Pr(\Pi^{eR}\geq 0)$ and since $h_t = h_t^{in} + h_t^e$ and

$$h_t^e = \frac{n_{t-1}(1-h_{t-1})\beta(var(\Psi_t))}{n_t}$$

and

$$
\begin{aligned}
h_t = n_t^{-1}\cdot\{n_{t-1}h_{t-1}&\left[\Pr(\Pi^{inR}\geq 0) + (1-h_t)h_t^{-1}\beta(var(\Psi_t))\right] - \\
&-\xi(1+\iota)(1+r_{t-1})F^R(1-h_{t-2})\beta(var(\Psi_{t-1}))\}
\end{aligned}
$$

we get:

$$
\begin{aligned}
n_t h_t = n_{t-1}h_{t-1}&\left[\Pr(\Pi^{inR}\geq 0) + \frac{1-h_{t-1}}{h_{t-1}}\beta(var(\Psi_t))\right] - \\
&-\xi(1+\iota)(1+r_{t-1})F^R n_{t-2}(1-h_{t-2})\beta(var(\Psi_{t-1}))
\end{aligned}
$$

or, using a different notation (where $H_t = n_t h_t$, $H_t^e = n_t h_t^e, H_t^{in} = n_t h_t^{in}$)

$$
\begin{aligned}
H_t = H_{t-1}&[\Pr(\Pi^{inR}\geq 0)] + [(n_{t-1}-H_{t-1})\beta(var(\Psi_t)] - \\
&-\xi(1+\iota)(1+r_{t-1})F^R(n_{t-2}-H_{t-2})\beta(var(\Psi_{t-1}))
\end{aligned}
$$

By substituting the definition of H_t into the aggregate demand, multiplying all the real variables for the price level P_t and rearranging, we get the following definition of the aggregate demand:

$$
\begin{aligned}
D(\cdot)_t = {} & (\Omega(r_t)/P_t) \cdot \\
& \cdot [A_t + [1/(1+r_t)] \sum_{i=0}^{\infty} [(1 + E\left(r_{t+i}\right)) \cdot \\
& (1 + E(\iota_{t+i}))]^{-i} \cdot E\{(n_{t+i} w_{t+i}) + \\
& + [(n_{t+i-1} - H_{t+i-1})\beta(var(\Psi_{t+i}))] \Pi^{e}_{t+i} + \\
+[H_{t-1} \cdot \Pr(\Pi^{in} \geq 0) - (1 + r_{t-2}) \xi F \cdot \\
& \cdot (n_{t-2} - H_{t-2})\beta(var(\Psi_{t-1}))] \Pi^{in}_{t+i}]\}
\end{aligned}
$$

This last notation shows the effects of the predetermined variables of the model, given the assumed valued of the (exogenous) variable $\Psi(\cdot)$.