Detecting global bridges in networks

Supplementary Informations

S1. Modified Brandes algorithm

Bridgeness algorithm, inspired by Brandes' "faster algorithm" [5]

```
SP[s,t] ←precompute all shortest distances matrix/dictionary
CB[v] ← 0, v ∈ V ;
for s ∈ V do
 S ← empty stack;
 P[w] ← empty list, w ∈ V ;
 σ[t] ← 0, t ∈ V ; σ[s] ← 1;
 d[t] ← −1, t ∈ V ; d[s] ← 0;
 Q ← empty queue;
 enqueue s → Q;
 while Q not empty do
  dequeue v ← Q;
  push v → S;
  foreach neighbor w of v do
   // w found for the first time?
   if d[w] < 0 then
    enqueue w → Q;
    d[w] ← d[v] + 1;
   end
   // shortest path to w via v?
   if d[w] = d[v] + 1 then
    σ[w] ← σ[w] + σ[v];
    append v → P[w];
   end
  end
 end
 δ[v] ← 0, v ∈ V ;
 // S returns vertices in order of non-increasing distance from s
 while S not empty do
  pop w ← S;
  for v ∈ P[w] do δ[v] ← δ[v] + σ[v]/σ[w] · (1 + δ[w]);
  if SP[w,s]>1 then CB[w] ← CB[w] + δ[w];
 end
end
```

S2. Case study on a synthetic network community

The specificity of bridgeness and the influence of the degree, which prevents BC from identifying correctly the most important bridges, can be exemplified by examining the scores of nodes in cluster 5 of the synthetic network. This cluster is linked to cluster 13 by 5 connections (through nodes 248, 861, 471, 576 and 758) and to cluster 1 by a single connection (through node 232). BC gives roughly the

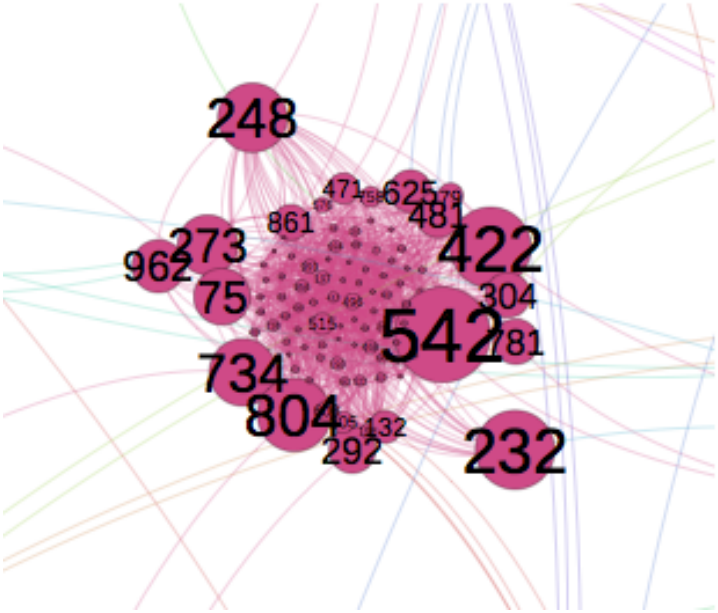

FIG. S1. Zoom on cluster 5 of the synthetic network. The numbers show node's labels, while the size of the nodes is proportional to their BC score.

same score to nodes 232 and 248, while bridgeness attributes a score almost 4 times higher to node 232, correctly pointing out the importance of this single bridge between clusters 5 and 1. This is because BC is confused by the high degree of node 248 (41) as compared to node 232 low degree (20). Therefore, by counting all the shortest paths, BC attributes too high a bridging score to node 248. Second problem with BC, it gives a high score to nodes that are not connected to other communities, merely because they are local centers, *i.e.* they have a high degree. For example, node 515 obtains a higher BC score than node 758 (Table S1), even if node 515 has no connection to other communities (but degree 49), contrary to node 758 (connected to cluster 5, but degree 23). Bridgeness never ranks higher local centers than global bridges: here, it correctly assigns a 5 times higher score to node 758 than to node 515.

Table S1. Nodes in community 5 of the synthetic network, ranked by decreasing BC (see text)

Id	Stirling	Modularity Class	Betweenness	Bridgeness	Degree
542	0.0222	5	9173.71	2644.62	44
422	0.0278	5	7714.27	3855.62	35
232	0.0950	5	7551.22	5846.86	20
804	0.0285	5	6995.63	2824.64	34
248	0.0082	5	6588.65	1624.30	48
734	0.0907	5	6410.31	4373.72	21
273	0.0322	5	5698.28	2631.59	30
75	0.0868	5	5349.47	3558.31	22
962	0.0399	5	4989.66	2951.45	24
292	0.0399	5	4377.77	1939.06	24
481	0.0256	5	4305.68	1796.92	25
781	0.0475	5	4257.93	2200.21	20
304	0.0434	5	4221.64	2467.65	22
625	0.0202	5	3964.21	1314.62	32
861	0.0108	5	3295.01	714.44	36
132	0.0200	5	2985.45	1157.49	24
471	0.0154	5	2865.07	1296.38	25
79	0.0302	5	2256.02	1004.28	21
205	0.0208	5	1921.65	788.51	23
515	0.0000	5	1884.07	86.45	49
758	0.0166	5	1791.80	435.66	23
608	0.0200	5	1777.54	522.75	24