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M. Francaviglia, M. Palese

Department of Mathematics, University of Torino

via C. Alberto 10, I-10123 Torino, Italy

e-mail: marcella.palese@unito.it

Abstract

In the framework of finite order variational sequences generalized Jacobi morphisms for General Relativity are explicitly represented by means of variational vertical derivatives and compared with classical results concerning the variation of the Einstein tensor.

Key words: fibered manifold, jet space, variational sequence, Helmholtz morphism, Jacobi morphism, variation, Einstein tensor.
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1 Introduction

In this note we shall evaluate the Jacobi morphism for the Hilbert–Einstein Lagrangian and verify that the result provides an alternative way to get the variation of the Einstein tensor. We consider the geometrical formulation of calculus of variations on finite order jets of a fibered manifold in terms of *variational sequences* introduced by Krupka [10, 11].

We introduce the notion of iterated variation of a section as an i–parameter 'deformation' of the section by means of vertical flows and thus define the i–th variation of a morphism in the variational sequence, which, in turn, is very simply related to the iterated Lie derivative of the morphism itself. Relying on previous results of us [6] on the representation of the Lie derivative operator in the variational sequence we can then define an operator on the quotient sheaves of the sequence, the *variational vertical derivative*. We relate the second order variation of a generalized Lagrangian with the variational Lie derivative of generalized Euler–Lagrange operators associated with the Lagrangian itself [4, 5]. In particular, we show then that Euler–Lagrange equations as well as Jacobi equations, for a given Lagrangian λ, can be obtained from a unique variational problem for the Lagrangian $\hat{\delta}\lambda$ (the variational vertical derivative of λ), in terms of its generalized symmetries (see also [3]). The generalized Jacobi morphism is then represented as a new geometric object in the variational sequence. It turns out that the generalized Jacobi morphism is closely related with the generalized Helmholtz morphism.

As an application of the above mentioned results, the Jacobi morphism for the Hilbert–Eistein Lagrangian is calculated and a proposition stating its equivalence with the variation of the Eistein tensor is proved.

2 Variational sequences on jets of fibered manifolds

Our framework is a fibered manifold $\pi : Y \to X$, with $\dim X = n$ and $\dim Y = n + m$ (see *e.g.* [12]). For $r \geq 0$ we are concerned with the r–jet space J_rY; in particular, we set $J_0Y \equiv Y$. We recall the natural fiberings $\pi^r_s : J_rY \to J_sY$, $r \geq s$, $\pi^r : J_rY \to X$, and, among these, the *affine* fiberings π^r_{r-1}. We denote multi–indices of dimension n by underlined Greek letters such as $\underline{\alpha} = (\alpha_1, \dots, \alpha_n)$, with $0 \leq \alpha_\mu$, $\mu = 1, \dots, n$; by an abuse of notation, we denote with λ the multi–index such that $\alpha_\mu = 0$, if $\mu \neq \lambda$, $\alpha_\mu = 1$, if $\mu = \lambda$. We also set $|\underline{\alpha}| \doteq \alpha_1 + \ldots + \alpha_n$ and $\underline{\alpha}! \doteq \alpha_1! \ldots \alpha_n!$. The charts induced on J_rY are denoted by $(x^\lambda, y^i_{\underline{\alpha}})$, with $0 \leq |\underline{\alpha}| \leq r$; in particular, we set $y^i_{\mathbf{0}} \equiv y^i$. The local vector fields and forms of J_rY induced by the above coordinates are denoted by $(\partial^{\underline{\alpha}}_i)$ and $(d^i_{\underline{\alpha}})$, respectively.

For $r \geq 1$, we consider the natural complementary fibered morphisms over the affine fibering $J_rY \to J_{r-1}Y$

$$D : J_rY \times_X TX \to TJ_{r-1}Y \,, \qquad \vartheta : J_rY \times_{J_{r-1}Y} TJ_{r-1}Y \to VJ_{r-1}Y \,, \quad (1)$$

which induce the natural fibered splitting

$$J_rY \times_{J_{r-1}Y} T^*J_{r-1}Y = \left(J_rY \times_{J_{r-1}Y} T^*X\right) \oplus \mathcal{C}^*_{r-1}[Y] \,, \qquad (2)$$

where $\mathcal{C}^*_{r-1}[Y] \simeq J_rY \times_{J_{r-1}Y} V^*J_{r-1}Y$.

The above splitting induces also a decomposition of the exterior differential on Y, $(\pi^{r+1}_r)^*\circ d = d_H + d_V$, where d_H and d_V are called the *horizontal* and *vertical differential*, respectively. If $f : J_rY \to I\!\!R$ is a function, then we set $D_{\underline{\alpha}+\lambda} f \doteq D_\lambda D_{\underline{\alpha}} f$, where the operator D_λ is the standard *formal derivative.*

The following sheaves will be needed in the sequel.

i. For $r \geq 0$, the standard sheaves Λ^p_r of p–forms on J_rY.

ii. For $0 \leq s \leq r$, the sheaves $\mathcal{H}^p_{(r,s)}$ and $\mathcal{H}^p_r$ of *horizontal forms*, i.e. of local fibered morphisms over π^r_s and π^r of the type $\alpha : J_rY \to \wedge^p T^* J_sY$ and $\beta : J_rY \to \wedge^p T^* X$, respectively.

iii. For $0 \leq s < r$, the subsheaf $\mathcal{C}^p_{(r,s)} \subset \mathcal{H}^p_{(r,s)}$ of *contact forms*, i.e. of sections $\alpha \in \mathcal{H}^p_{(r,s)}$ with values into $\wedge^p(\mathcal{C}^*_s[Y])$. There is a distinguished subsheaf $\mathcal{C}^p{}_r \subset \mathcal{C}^p_{(r+1,r)}$ of local fibered morphisms $\alpha \in \mathcal{C}^p_{(r+1,r)}$ which project down onto J_rY.

According to [13], the fibered splitting 2 yields naturally the sheaf splitting $\mathcal{H}^p_{(r+1,r)} = \bigoplus^p_{t=0} \mathcal{C}^{p-t}_{(r+1,r)} \wedge \mathcal{H}^t_{r+1}$, which restricts to the inclusion $\Lambda^p_r \subset \bigoplus^p_{t=0} \mathcal{C}^{p-t}{}_r \wedge \mathcal{H}^{th}{}_{r+1}$, where $\mathcal{H}^{ph}{}_{r+1} \doteq h(\Lambda^p_r)$ for $0 < p \leq n$ and h is defined to be the restriction to Λ^p_r of the projection of the above splitting onto the non–trivial summand with the highest value of t.

2.1 Generalized Euler–Lagrange and Helmholtz–Sonin morphisms in variational sequences

We refer now to the theory of variational sequences on finite order jet spaces, as it was developed by Krupka. By an abuse of notation, denote by $d \ker h$ the sheaf generated by the presheaf $d \ker h$. Set $\Theta^*_r \doteq \ker h + d \ker h$.

Definition 1 *The r–th order* variational sequence *associated with the fibered manifold $Y \to X$ is the resolution by exact sheaves of the costant sheaf $I\!\!R_Y$ (see [10]):*

$$0 \to I\!\!R_Y \to \Lambda^0_r \to^{\mathcal{E}_0} \Lambda^1_r/\Theta^1_r \to^{\mathcal{E}_1} \Lambda^2_r/\Theta^2_r \to^{\mathcal{E}_2} \dots \to^{\mathcal{E}_{I-1}} \Lambda^I_r/\Theta^I_r \to^{\mathcal{E}_I} \Lambda^{I+1}_r \to^d \dots \to^d 0 \tag{3}$$

The quotient sheaves in the variational sequence can be conveniently represented. If $k \leq n$, then the sheaf morphism h yields the natural isomorphism $I_k : \Lambda^k_r/\Theta^k_r \to \mathcal{H}^{kh}{}_{r+1} \doteq \mathcal{V}^k_r : [\alpha] \mapsto h(\alpha)$. If $k > n$, then the projection h induces the natural sheaf isomorphism $I_k : (\Lambda^k_r/\Theta^k_r) \to (\mathcal{C}^{k-n}{}_r \wedge \mathcal{H}^{nh}{}_{r+1})/h(d \ker h) \doteq \mathcal{V}^k_r : [\alpha] \mapsto [h(\alpha)]$ [13].

Let $\alpha \in \mathcal{C}^1_r \wedge \mathcal{H}^{nh}{}_{r+1}$. Then there is a unique pair of sheaf morphisms

$$E_\alpha \in \mathcal{C}^1_{(2r,0)} \wedge \mathcal{H}^{nh}{}_{2r+1}\,, \qquad F_\alpha \in \mathcal{C}^1_{(2r,r)} \wedge \mathcal{H}^{nh}{}_{2r+1}\,, \tag{4}$$

such that $(\pi^{2r+1}_{r+1})^*\alpha = E_\alpha - F_\alpha$, and F_α is locally of the form $F_\alpha = d_H p_\alpha$, with $p_\alpha \in \mathcal{C}^1_{(2r-1,r-1)} \wedge \mathcal{H}^{n-1}{}_{2r}$.

Let $\beta \in \mathcal{C}^1_r \wedge \mathcal{C}^1_{(r,0)} \wedge \mathcal{H}^n_r$. Then there is a unique morphism

$$\tilde{H}_\beta \in \mathcal{C}^1_{(2r,r)} \otimes \mathcal{C}^1_{(2r,0)} \wedge \mathcal{H}^n_{2r}$$

such that, for all $\Xi : Y \to VY$, $E_{\hat{\beta}} = C^1_1\left(j_{2r}\Xi \otimes \tilde{H}_\beta\right)$, where $\hat{\beta} \doteq j_r\Xi \rfloor \beta$, C^1_1 stands for tensor contraction and $\rfloor$ denotes inner product. Furthermore there is a unique pair of sheaf morphisms

$$H_\beta \in \mathcal{C}^1_{(2r,r)} \wedge \mathcal{C}^1_{(2r,0)} \wedge \mathcal{H}^n_{2r}\,, \quad G_\beta \in \mathcal{C}^2_{(2r,r)} \wedge \mathcal{H}^n_{2r}\,, \tag{5}$$

such that $\pi^{2r*}_r\beta = H_\beta - G_\beta$ and $H_\beta = \frac{1}{2}\,A(\tilde{H}_\beta)$, where A stands for antisymmetrisation. Moreover, G_β is locally of the type $G_\beta = d_H q_\beta$, where $q_\beta \in \mathcal{C}^2_{2r-1} \wedge \mathcal{H}^{n-1}_{2r-1}$, hence $[\beta] = [H_\beta]$.

2.2 Generalized Jacobi morphisms in variational sequences

The Lie derivative operator with respect to the r-th order prolongation $j_r\Xi$ of a projectable vector field (Ξ, ξ) can be conveniently represented on the quotient sheaves of the variational sequence in terms of an operator, the *variational Lie derivative* $\mathcal{L}_{j_r\Xi}$ [6]. In particular, if $p = n+1$ and $\eta \in \mathcal{V}^{n+1}_r$, then

$$\mathcal{L}_{j_r\Xi}\eta = \mathcal{E}(\Xi\rfloor\eta) + \tilde{H}_{d\eta}(j_{2r+1}\Xi)\,. \tag{6}$$

Definition 2 *Let $\psi^k_{t_k}$, with $1 \leq k \leq i$ and i any integer, be the flows generated by the vertical (variation) vector fields Ξ_k. Let $\sigma : X \to Y$ be a section. An i–th variation of σ generated by $(\Xi_1, \ldots, \Xi_i)$ is a smooth section $\Gamma_i : I \times X \to Y$, $\mathbf{0} \in I \subset I\!\!R^i$, such that $\Gamma_i(\mathbf{0}) = \sigma$ and $\Gamma_i(t_1, \ldots, t_i) = \psi^i_{t_i} \circ \ldots \circ \psi^1_{t_1} \circ \sigma$.* □

Definition 3 *Let $\alpha : J_rY \to \wedge^k T^* J_rY$ and let Γ_i be an i–th variation of the section σ generated by an i–tuple $(\Xi_1, \ldots, \Xi_i)$. We define the i–th variation of the morphism α to be $\delta^i\alpha \doteq \frac{\partial^i}{\partial t_1 \ldots \partial t_i}|_{t_1,\ldots,t_i=0}(\alpha \circ j_r\Gamma_i(t_1, \ldots, t_i))$.* □

The following Lemma states the relation between the i–th variation of a morphism and its iterated Lie derivative (see also [8]).

Lemma 1 *Let* $\alpha : J_r Y \to \wedge^k T^* J_r Y$ *and* $L_{j_r \Xi_k}$ *be the Lie derivative operator with respect to* $j_r \Xi_k$*. Let* Γ_i *be the* i*–th variation of the section* σ *by means of the variation vector fields* $\Xi_1, \dots, \Xi_i$ *on* Y*. Then we have* $\delta^i \alpha = (j_r \sigma)^* L_{j_r \Xi_1} \dots L_{j_r \Xi_i} \alpha$*.*

Definition 4 *We call the operator* $\hat{\delta}^i \doteq [\delta^i \alpha] = j_r \sigma^* \mathcal{L}_{\Xi_i} \dots \mathcal{L}_{\Xi_1}[\alpha]$ *the* i–th variational vertical derivative operator. □

This enables us to represent variations of morphisms in the variational sequence and we can thus state the following important main Theorem [4, 5].

Theorem 1 *The operator* $\hat{\delta}$ *is a functor defined on the category of variational sequences.*

Proof. Let $\alpha \in (\mathcal{V}^n_r)_Y$. Let d, $\bar{d}$ be the exterior differentials and $\hat{\delta}$, $\bar{\hat{\delta}}$ the vertical variational derivative on $J_r Y$ and $\hat{\delta} J_r Y$, respectively. We have $\bar{d} \hat{\delta} \alpha = \bar{\hat{\delta}} d\alpha$. $\boxed{QED}$

Proposition 1 *Let* $\lambda \in (\mathcal{V}^n_r)_Y$, $\hat{\delta}\lambda \in (\mathcal{V}^n_r)_{VY}$*. We have* $\hat{\delta}^2 \lambda = \bar{\mathcal{E}}(\bar{\Xi}_2 \rfloor \hat{\delta}\lambda) + \tilde{H}_{h(d\hat{\delta}\lambda)}(\bar{\Xi}_2)$*, where* $\tilde{H}_{h(d\hat{\delta}\lambda)}$ *is the unique morphism belonging to* $\mathcal{C}^1_{(2r,r)} \otimes \mathcal{C}^1_{(2r,0)} \wedge \mathcal{H}^n_{2r}$ *such that, for all* $\Xi_1 : Y \to VY$, $E_{j_r \Xi \rfloor d\hat{\delta}\lambda} = C^1_1 (j_{2r} \Xi_1 \otimes \tilde{H}_{h(d\hat{\delta}\lambda)})$*; here* C^1_1 *stands for tensor contraction.*

Proof. In fact we have up to divergences $\hat{\delta}^2 \lambda = \bar{\hat{\delta}} \mathcal{L}_{j_r \Xi_2} \lambda = \mathcal{L}_{j_r \bar{\Xi}_2} \hat{\delta} \lambda$, so that the assertion follows by a straightforward application of the representation provided by Equation 6 and by linearity properties of δ^i.

$\boxed{QED}$

By means of a simple calculation it is very easy to see that the following holds true.

Lemma 2 *Let* $\chi(\lambda) \doteq \tilde{H}_{h(d\hat{\delta}\lambda)}$*. We have* $\chi(\lambda) : J_{2r} Y \to \mathcal{C}^*_r[VY] \wedge (\wedge^n T^* X)$ *and* $\bar{d}_H \chi(\lambda) = 0$*.*

The following is an application of an abstract result due to Kolář [9], concerning a global decomposition formula for vertical morphisms.

Theorem 2 *Let $\chi(\lambda)$ be as in the above Lemma. Then we have $\chi(\lambda) = E_{\chi(\lambda)} + F_{\chi(\lambda)}$, where $E_{\chi(\lambda)} : J_{4r}Y \to \mathcal{C}^*_0[VY] \wedge (\wedge^n T^*X)$, and locally, $F_{\chi(\lambda)} = \bar{d}_H M_{\chi(\lambda)}$, with $M_{\chi(\lambda)} : J_{4r-1}Y \to \mathcal{C}^*_{r-1}[VY] \wedge (\wedge^{n-1} T^*X)$.*

Definition 5 *We call the morphism $\mathcal{J}(\lambda) \doteq E_{\chi(\lambda)}$ the* generalized Jacobi morphism *associated with the Lagrangian λ.* □

3 The Jacobi morphism for the Hilbert-Einstein Lagrangian

In the following we evaluate the Jacobi morphism for the Hilbert–Einstein Lagrangian and verify that the result provides an alternative way to get the variation of the Einstein tensor.

To this aim, let us first of all write explicitely the coordinate expression of $\chi(\lambda) \doteq \tilde{H}_{h(d\hat{\delta}\lambda)}$, for a generic Lagrangian λ.

From the main Theorem, by functoriality of $\hat{\delta}$, we have $h(d\hat{\delta}\lambda) = h(\hat{\delta}d\lambda)$. Let now $\lambda = L\omega$, then $d\lambda = \partial^{\underline{\alpha}}_i(L)d^i_{\underline{\alpha}} \wedge \omega$ and $\hat{\delta}d\lambda = \partial^{\underline{\sigma}}_j(\partial^{\underline{\alpha}}_i d^j_{\underline{\sigma}} L) \wedge d^i_{\underline{\alpha}} \wedge \omega$, thus finally $h(\hat{\delta}d\lambda) = h(d\hat{\delta}\lambda) = \partial_j(\partial^{\underline{\alpha}}_i L)d^i_{\underline{\alpha}} \wedge d^j \wedge \omega$. As a consequence we have, with $0 \leq |\mu| \leq 2r+1$:

$$\chi(\lambda) \doteq \tilde{H}_{h(d\hat{\delta}\lambda)} = \tag{7}$$

$$= \left(\partial_j(\partial^{\underline{\mu}}_i L) - \sum_{|\underline{\alpha}|=0}^{s-|\underline{\mu}|} (-1)^{|\underline{\mu}+\underline{\alpha}|} \frac{(\underline{\mu}+\underline{\alpha})!}{\underline{\mu}!\underline{\alpha}!} D_{\underline{\alpha}} \partial^{\underline{\alpha}}_j (\partial^{\underline{\mu}}_i L)\right) \vartheta^i_{\underline{\mu}} \otimes \vartheta^j \wedge \omega \doteq \chi^{\underline{\mu}}_{ij} \vartheta^i_{\underline{\mu}} \otimes \vartheta^j \wedge \omega \tag{8}$$

and by a backwards procedure, which is essentially an integration by parts (see e.g. [9]), we get, up to divergencies:

$$\mathcal{J}(\lambda) = (-1)^{|\underline{\mu}|} D_{\underline{\mu}} \chi^{\underline{\mu}}_{ij} \vartheta^i \otimes \vartheta^j \wedge \omega = \tag{9}$$

$$(-1)^{|\underline{\mu}|} D_{\underline{\mu}} \left(\chi^{\underline{\mu}}_{i\,j} - \sum_{|\underline{\alpha}|=0}^{s-|\underline{\mu}|} (-1)^{|\underline{\mu}+\underline{\alpha}|} \frac{(\underline{\mu}+\underline{\alpha})!}{\underline{\mu}!\underline{\alpha}!} D_{\underline{\alpha}} \chi^{\underline{\mu}+\underline{\alpha}}_{j\;\;i} \right) \vartheta^i \otimes \vartheta^j \wedge \omega \,. \tag{10}$$

Let now $dim X = 4$ and X be orientable. Let $Lor(X)$ be the bundle of Lorentzian metrics on X (provided that it has global sections). Local fibered coordinates on $J_2(Lor(X))$ are $(x^\lambda; g_{\mu\nu}, g_{\mu\nu,\sigma}, g_{\mu\nu,\sigma\rho})$.

The Hilbert–Einstein Lagrangian is the form $\lambda_{HE} \in \mathcal{H}^4_2$ defined by $\lambda_{HE} = L_{HE}\omega$, were $L_{HE} = r\sqrt{\underline{g}}$. Here $r : J_2(Lor(X)) \to I\!\!R$ is the function such that, for any Lorentz metric g, we have $r \circ j_2 g = s$, being s the scalar curvature associated with g, and $\underline{g}$ is the determinant of g.

The function L_{HE} is a *linear* function in the second derivatives of g; thus λ_{HE} is a special Lagrangian. Owing to a well known property of λ_{HE} we have $E_{d\lambda_{HE}} \in \mathcal{C}^1_{(2,0)} \wedge \mathcal{H}^4_2$ and a direct computation shows that $E_{d\lambda_{HE}} = G \doteq R - \frac{1}{2}\, s\, g$, R being the Ricci tensor of the metric g and G the Einstein tensor.

The Jacobi morphism for λ_{HE} is

$$
\begin{aligned}
& \mathcal{J}(\lambda_{HE}) \doteq E_{\chi(\lambda_{HE})} = \\
= \; & \frac{1}{2}[-\nabla_\lambda \nabla^\lambda w^\beta_\alpha + r^{\beta\lambda} w_{\lambda\alpha} - r_{\alpha\lambda} w^{\lambda\beta} - 2R^\beta_{\rho\alpha\lambda} w^{\rho\lambda} + \delta^\beta_\alpha r_{\rho\lambda} w^{\rho\lambda} + \\
+ \; & (G^\beta_\alpha + \frac{1}{2} s \delta^\beta_\alpha) w + \nabla^\beta \nabla \lambda w^\lambda_\alpha + \nabla_\alpha (g^{\beta\gamma} \nabla_\lambda w^\lambda_\gamma) - \delta^\beta_\alpha \nabla_\lambda (g^{\lambda\gamma} \nabla_\rho w^\rho_\gamma)]\,,
\end{aligned}
$$

where ∇ is the covariant derivative with respect to the metric connection, $w_{\alpha\beta} = \gamma_{\alpha\beta} - \frac{1}{2} g_{\alpha\beta}\gamma$, $\gamma = \gamma^\lambda_\lambda$ and $\gamma_{\alpha\beta} = \delta g_{\alpha\beta}$ is the variation of the metric tensor.

Comparing with results in the literature [1, 2], we can conclude that $\mathcal{J}(\lambda_{HE}) \doteq E_{\chi(\lambda_{HE})} = \hat{\delta} G$. In fact, as a consequence of Proposition 1, we have the following (see also [3, 8]).

Proposition 2 *Let λ_{HE} be the Hilbert–Einstein Lagrangian, G the corresponding Einstein tensor (i.e. the corresponding generalized Euler–Lagrange morphism in the variational sequence) and $\hat{\delta}$ the variational vertical derivative operator. We have (up to divergencies):*

$$
\hat{\delta}^2 \lambda_{HE} = E_{\lambda_{HE}} + E_{\chi(\lambda_{HE})} \equiv G + \hat{\delta} G\,. \tag{11}
$$

This has relevant consequences. Among them, we recall that the variation of the Einstein tensor has proved to be an important tool for the study of the positivity of the energy in General Relativity. The Jacobi morphism, being the adjoint operator of the Hessian [7], can be used to provide global results in this direction. This topic will be developed elsewhere.

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References

[1] É. Blancheton: Équation aux variations de la relativité générale, *Comptes Rendus Acadèmie des Sciences* **245** (1957) 284–286.

[2] É. Blancheton: Les équation aux variations de la relativité générale, *Publ. Sci. Univ. Alger Sér. A* **9** (1962) 31–116.

[3] B. Casciaro, M. Francaviglia, V. Tapia: On the Variational Characterization of Generalized Jacobi Equations, *Proc. Diff. Geom. and its Appl.* (Brno, 1995); J. Janyška, I. Kolář, J. Slovák eds., Masaryk University (Brno, 1996) 353–372.

[4] M. Francaviglia, M. Palese: Second Order Variations in Variational Sequences, *Steps in differential geometry* (Debrecen, 2000), 119–130, Inst. Math. Inform., Debrecen, 2001.

[5] M. Francaviglia, M. Palese: Generalized Jacobi morphisms in variational sequences, in Proc. XXI Winter School Geometry and Physics, Srni 2001 *Rend. Circ. Matem. di Palermo. Serie II, Suppl.* **69** (2002) 195–208.

[6] M. Francaviglia, M. Palese, R. Vitolo: Symmetries in Finite Order Variational Sequences, *Czech. Math. J.* **52(127)** (2002) 197–213.

[7] M. Francaviglia, M. Palese, R. Vitolo: A geometric formulation of Hessian and Jacobi tensors for higher order Lagrangians, submitted to *Diff. Geom. Appl.*

[8] H. Goldschmidt, S. Sternberg: The Hamilton–Cartan Formalism in the Calculus of Variations, *Ann. Inst. Fourier, Grenoble* **23** (1) (1973) 203–267.

[9] I. Kolář: A Geometrical Version of the Higher Order Hamilton Formalism in Fibred Manifolds, *J. Geom. Phys.* **1** (2) (1984) 127–137.

[10] D. Krupka: Variational Sequences on Finite Order Jet Spaces, *Proc. Diff. Geom. and its Appl.* (Brno, 1989); J. Janyška, D. Krupka eds., World Scientific (Singapore, 1990) 236–254.

[11] D. Krupka: Topics in the Calculus of Variations: Finite Order Variational Sequences, *Proc. Diff. Geom. and its Appl.* (Opava, 1993) 473–495.

[12] D.J. Saunders: *The Geometry of Jet Bundles*, Cambridge Univ. Press (Cambridge, 1989).

[13] R. Vitolo: Finite order Lagrangian bicomplexes, *Math. Proc. Cambridge Phil. Soc.* **125** (1) (1998) 321–333.