# **SUPPLEMENTARY INFORMATION**

Magnetic states of nanostructures containing Ni<sup>2+</sup> ions at the surface of SiO<sub>2</sub> nanospheres

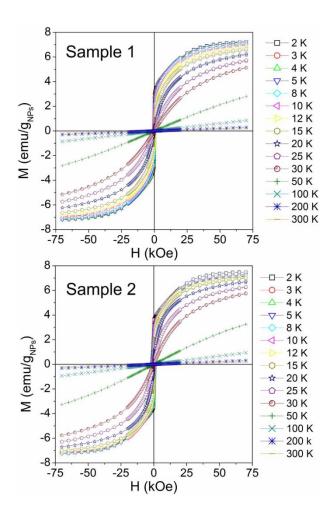
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## 1. Additional Results

The magnetization of the two samples is reported in Fig. S1 as a function of magnetic field at different temperatures between 2 and 300 K. The quantity is defined as the measured magnetic moment per unit mass of the sample, whose weight is dominated by the diamagnetic silica NPs. In both cases, a linear, paramagnetic behavior is observed at high temperatures (above 50 K); magnetic hysteresis appears below 8 K (sample 1) and 12 K (sample 2).



**Figure S1.** Isothermal magnetic hysteresis loops for samples 1 and 2.

The imaginary part of the AC susceptibility  $\chi$ ''(T) is shown in Figure S2 (top panel) for sample 2 (a closely similar behavior is found in sample 1). The temperature of the maximum of  $\chi$ ''(T<sub>max</sub>) increases with increasing frequency. When the logarithm of measurement frequency is plotted as a function of  $1/T_{max}$  the data are well aligned on a straight line (Figure S2, bottom panel).

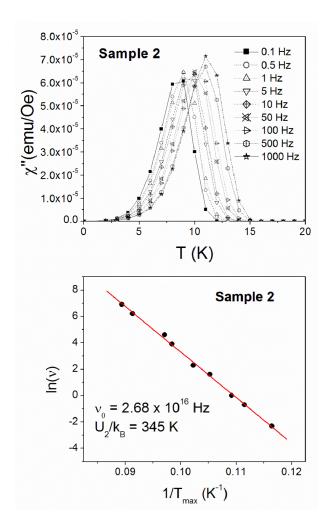


Figure S2. Top panel: Imaginary part of the AC susceptibility ( $\chi$ ") as a function of temperature at different frequencies in sample 2; bottom panel: symbols: logarithm of measurement frequency as a function of the reciprocal of the temperature of the maximum of  $\chi$ "; line: best fit to the Arrhenius law.

# 2. Magnetic viscosity of an assembly of independent quantum nanoparticles with random anisotropy axes

A model is introduced to describe the magnetic viscosity displayed by a three-dimensional system of quantum nanoparticles evolving towards equilibrium through crossing of an anisotropy energy barrier via coherent quantum tunneling (both resonant and phonon-assisted). The anisotropy axes are assumed to point along random directions in space.

When the magnetic field  $H_A$  points along a direction defined by the angle  $\phi$  with respect to the easy axis of a NP (the z-axis in a local reference frame) the magnetic field  $H_A$  can be considered as split in one contribution parallel to the easy axis ( $H_{A//} = H_A \cos \phi$ ) and one perpendicular to it ( $H_{A\perp} = H_A \sin \phi$ ); in the first case, the energy is given by  $E = U \sin^2 \theta - \mu H_{A//} \cos \theta$ , where U is the anisotropy barrier,  $\theta$  is the angle between M and the z-axis; in the second case,  $E = U \sin^2 \theta - \mu H_{A\perp} \sin \theta$ ; this implies that  $H_{A\perp}$  does not make the double well asymmetric; it just decreases the height of the barrier and displaces both minima towards the perpendicular direction ( $\theta = \pi/2$ ); as a result, no imbalance of  $N_1$  and  $N_2$  is produced by  $H_{A\perp}$ . The behavior of  $S_V$  with field H at fixed temperature for a system of independent particles with randomly distributed easy-axis directions is obtained considering for each angle  $\phi$  the evolution of the fractions  $N_{1\phi}$ ,  $N_{2\phi}$  of particles in the double wells starting from the off-equilibrium state induced by the applied field  $H_A$ .

We consider nanoparticles of total spin S and magnetic moment  $\mu=2\mu_B S$  described as doublewell systems with anisotropy barrier U.

#### a) Magnetic field aligned with the easy axis direction ( $\phi$ =0)

The double well is symmetric when H=0. When a magnetic field  $H_A$  is applied at the temperature  $T_A$  the well becomes asymmetric with  $E_1 < E_2$  (see Figure 6(a) of main text) The energy difference is  $E_2 - E_1 = 2\mu H_A$ . Defining  $\beta = \frac{2\mu H_A}{k_B T_A} \gg 1$ , the equilibrium population in each well is

$$N_{01} = N_0 \frac{1}{1 + e^{-\beta}} \approx N_0$$

$$N_{02} = N_0 \frac{e^{-\beta}}{1 + e^{-\beta}} \ll N_0$$

 $N_0$  being the total number of double-well systems with easy axis aligned with the field direction.

At the temperature  $T \ll T_A$ , immediately after setting the field to  $H \ll H_A$ , the magnetization takes the off-equilibrium value:

$$M_{0i} = (N_{01} - N_{02})_i \mu = \frac{1 - e^{-\beta}}{1 + e^{-\beta}} \mu$$

the final (equilibrium) value being instead:

$$M_{0f} = (N_{01} - N_{02})_f \mu = \frac{1 - e^{-\alpha H}}{1 + e^{-\alpha H}} \mu$$

where  $\alpha=\frac{2\mu}{k_BT}$ . Note that when H=0 the final value of  $M_0$  is expected to be zero, corresponding to  $N_{01}=N_{02}=N_0/2$ . If the logarithmic relaxation of M(t) is related to the presence of a flat distribution of energy barriers p(U) in the interval  $U_1 \leq U \leq U_2$  with the

associated distribution of relaxation times  $p(\tau)$  in the interval  $\tau_1 \le \tau \le \tau_2$  at the temperature T, the magnetic viscosity  $S_V(T,H)$  is:

$$S_{V}(T,H) = \frac{1}{\ln \frac{\tau_{2}}{\tau_{1}}} \left( 1 - \frac{M_{0f}}{M_{0i}} \right) = \frac{1}{\ln \frac{\tau_{2}}{\tau_{1}}} \left( 1 - \frac{\frac{1 - e^{-\alpha H}}{1 + e^{-\alpha H}}}{\frac{1 - e^{-\beta}}{1 - e^{-\beta}}} \right)$$

When 
$$\beta >>1$$
,  $S_V(T,H) \cong \frac{1}{\ln \frac{T_2}{\tau_1}} \left(1 - \frac{1 - e^{-\alpha H}}{1 + e^{-\alpha H}}\right) = \frac{2}{\ln \frac{\tau_2}{\tau_1}} e^{-\alpha H}$ .

b) Magnetic field taking an arbitrary direction  $\phi_n$  with respect to the easy axis  $(-\pi/2 \le \phi_n \le \pi/2)$ 

The field's components parallel and perpendicular to the nanoparticle's easy axis are:  $H_{//} = H$   $\cos \phi_n$ ,  $H_{\perp} = H \sin \phi_n$ . The distribution of population in the two wells is determined by  $H_{//}$ , the component  $H_{\perp}$  not contributing to the asymmetry of the double well; therefore:

$$N_{\phi_n 1} = N_{\phi_n} \frac{1}{1 + e^{-\beta \cos \phi_n}}$$

$$N_{\phi_n 2} = N_{\phi_n} \frac{e^{-\beta \cos \phi_n}}{1 + e^{-\beta \cos \phi_n}}$$

 $N_{\phi n}$  being the total number of double-well systems with easy axis taking an angle  $\phi_n$  with respect to the field direction. Usually  $\beta \cos \phi_n >> 1$  and  $N_{\phi n1} >> N_{\phi n2}$ ; however  $N_{\phi n1} = N_{\phi n2} = N_{\phi n} / 2$  when  $\phi_n \rightarrow \pi/2$ .

When the temperature is lowered to  $T \ll T_A$  and H is decreased to  $H \ll H_A$ , the starting magnetization in the direction of the field is:

$$M_{\varphi_n i} = (N_{\varphi_n 1} - N_{\varphi_n 2})_i \mu \cos \varphi_n + \varepsilon_n \sin \varphi_n$$

where  $\varepsilon_n = \frac{N_{\phi_n} \mu^2 H}{2U} \ll 1$  is the contribution to the magnetization in the field direction, U being the anisotropy barrier of the double well. The final (equilibrium) magnetization is:

$$M_{\varphi_n f} = (N_{\varphi_n 1} - N_{\varphi_n 2})_f \mu \cos \varphi_n + \varepsilon_n \sin \varphi_n$$

## c) System of nanoparticles with randomly oriented anisotropy axes

Let us consider a system of nanoparticles with randomly oriented anisotropy axes; it is assumed that the number of nanoparticles is evenly distributed ( $N_{\phi n} = \text{const.}$ ). Dividing the interval ( $-\pi/2$ ,  $+\pi/2$ ) into N-1 intervals (N>>1), the initial value of the overall magnetization along H is:

$$M_i = \frac{1}{\mathcal{N}} \left[ \mu \sum_{n=1}^{\mathcal{N}} \frac{1 - e^{-\beta cos\phi_n}}{1 + e^{-\beta cos\phi_n}} \cos\phi_n + \epsilon \sum_{n=1}^{\mathcal{N}} sin\phi_n \right] = \frac{1}{\mathcal{N}} \left[ \mu \sum_{n=1}^{\mathcal{N}} \frac{1 - e^{-\beta cos\phi_n}}{1 + e^{-\beta cos\phi_n}} \cos\phi_n \right]$$

In fact, the second sum gives zero by symmetry reasons,  $\epsilon$  being a constant quantity if  $N_{\phi n}$  = const. By analogy, the final value of the overall magnetization is:

$$M_{f} = \frac{1}{N} \left[ \mu \sum_{n=1}^{N} \frac{1 - e^{-\alpha H \cos \phi_{n}}}{1 + e^{-\alpha H \cos \phi_{n}}} \cos \phi_{n} \right]$$

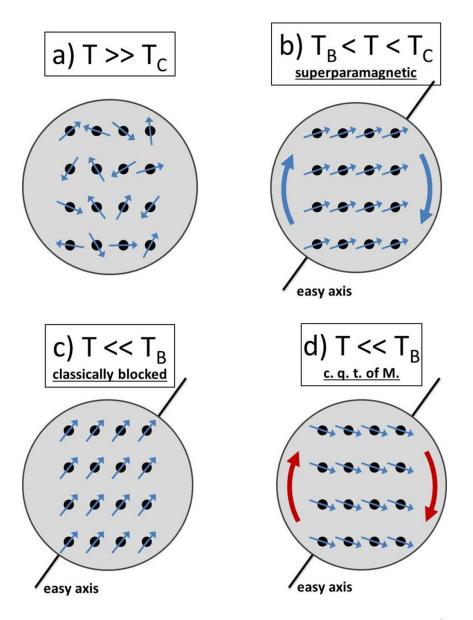
and the magnetic viscosity takes the form:

$$S_{V}(T,H) = \frac{1}{\ln \frac{\tau_{2}}{\tau_{1}}} \left( 1 - \frac{M_{f}}{M_{i}} \right) = \frac{1}{\ln \frac{\tau_{2}}{\tau_{1}}} \left( 1 - \frac{\sum_{n=1}^{N} \frac{1 - e^{-\alpha H \cos \phi_{n}}}{1 + e^{-\alpha H \cos \phi_{n}}} \cos \phi_{n}}{\sum_{n=1}^{N} \frac{1 - e^{-\beta \cos \phi_{n}}}{1 + e^{-\beta \cos \phi_{n}}} \cos \phi_{n}} \right)$$
(1)

In particular, when H = 0 one has  $S_V(T, 0) = \frac{1}{ln\frac{\tau_2}{\tau_1}}$  because M<sub>f</sub>=0.

## 3. A pictorial representation of the different magnetic states in magnetic sub-nanoparticles

The magnetic states of the investigated systems are summarized in Fig. S3. Classical blocking (case c) is actually overcome by QTM (case d).



**Figure S4.** Overall picture of arrangement and behavior of Ni<sup>2+</sup> spins in samples 1 and 2 at different temperatures.