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# Diagrams and mathematical events: Encountering spatio-temporal relationships with graphing technology

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*This paper presents the diagrammatic activity of some secondary school students exploring motion through graphing technology, which captures a pair of space-time graphs on a single Cartesian plane. Focus is on a written task about the connections between two imaginary movements and (between) the corresponding graphs. Drawing on a vision that conceives mathematical thinking as a place of events instead of objects, we discuss three unexpected diagrams for how they bring forth inventive and speculative possibilities for learners to encounter and problematize spatio-temporal relationships, rather than seeing them as ways of being mistaken.*

*Keywords: Graphing technology, movement, diagram, event, problematic.*

## Introduction

In this paper, we deal with the issue of how students might learn about a representational system in which temporospatial relationships are the ground for the mathematical doing. Our interests are also in how visual, proprioceptive and kinaesthetic aspects of experiencing these relationships might move the learning of mathematics in unexpected and unconventional directions. We follow here de Freitas (2013) in rethinking mathematics as the place of events, instead of objects, where creativity and contingency prevail and the problematic—rather than the axiomatic—better capture the vitality of mathematical activity. The idea is that deduction “moves from the problem to the ideal accidents and events that condition the problem and form the cases that resolve it.” (Smith 2006, p. 145). Thus, mathematics is concerned with the occurrence of events more than with the existence of objects, and attention is on the material encounters with the mathematical.

In this perspective, we present an activity that was carried out with grade 9 students working with graphing motion technology to study function. In particular, the technology requires that two remote controllers of the Nintendo Wii game console (Wiimotes) are moved at the same time in front of a sensor bar, and it displays two space-time graphs on the same Cartesian plane. In the graphs, space is given by the distance of each controller from the bar. Thus, the software captures the movement of the Wiimotes over time. Our focus is on a written task that asks the students, divided into groups, to draw a space-time graph related via movement to a given graph. The task is called “Rob and Bob”. In it, Rob and Bob are the names of two little robots that are imagined to be moving the controllers in front of the sensor. The graph associated to Rob’s movement is given on paper, together with the instructions with which Bob is supposed to be moving with respect to Rob. The students are expected to complete the task adding Bob’s graph on the Cartesian plane. We will discuss how three different graphs are presented as solutions to the task from different groups, and we will develop how we think that these are significant in terms of the novel mathematical meanings that the students are articulating. In the meanwhile, we will also draw attention to aspects of the experience with the technology that might support this novelty, raising issues about the role of perception on the one side, and about the features of the technology on the other side.

## Theoretical highlights

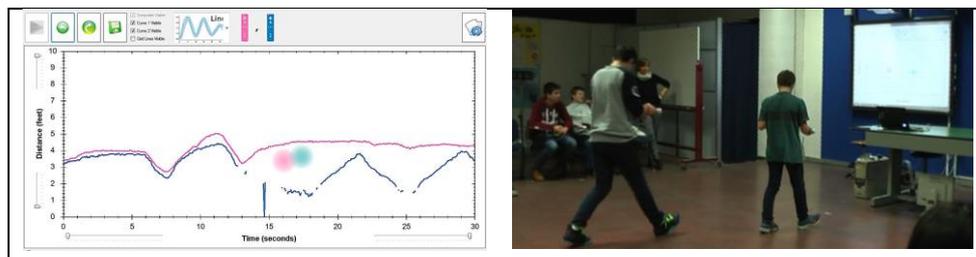
The representational system we refer to in this paper is the (space-time) Cartesian plane, which our students encounter through activity with the technology. However, we want to trouble traditional ideational assumptions that conceive such system as inhabited by mathematical figures or functions that, in their essence, are representations (particular instantiations or attributes, concrete instances) of some form, inert, transcendent, abstract and disembodied. In fact, claims de Freitas (2013), “the process of instantiation fails to capture the creative and material act of individuation that is entailed when we do mathematics.” (p. 586). We instead embrace an animate vision of the mathematical drawing/creation or act of drawing/creation of a figure or a function as event-structured, full of potentiality, temporality and movement, immanent, contingent to material circumstances, and incidentally subject to transformation. This positions us in the broad discussion on the theorising about the embodied nature of mathematics thinking and learning, which attempts to look at knowledge in non-representational ways and to overcome body/mind Cartesianisms (see e.g. Nemirovsky et al., 2013; Sinclair, 2014; Ferrara, 2015; Roth, 2016). According to this view, learning is much more about encountering concepts than about recognizing concepts. Cutler & MacKenzie (2011) might argue that thus the challenge is to treat learning as an ontological rather than an epistemological problem, staying away, we would add, from opposing the mathematical and the physical. The issue of representations is crucial here. As Sinclair (2014) points out, it is not that “symbols, diagrams, programming languages and even gestures” (and any other system, we would add) “do not at times *function* to re-present mathematical concepts and relations”, rather “they are inevitably bound up with bodies and discourses and thus potentially poised to open up new meanings.” (p. 174, *emphasis in the original*). Our own reading of this makes sense as regards our commitment to a mobile view of mathematics and mathematical doing that tries to escape concrete versus abstract and matter versus thought divides (Ferrara & Ferrari, 2016). In Ferrara (2015), these divides are challenged through a vision of perception and creation in/of mathematics for which perceiving is conceiving, thinking is acting and creating is learning. The work of philosopher Gilles Châtelet (1987, 1993/2000) on inventive diagramming was provocative to us in considering the centrality of mobility or virtuality to bridging the physical and the mathematical. The virtual is the necessary link between the two realms. Roth (2016) also draws on Châtelet to underline how one way of thinking about dynamic systems is just in terms of the virtual. We can better understand this if we take the examples that Châtelet (1987) makes about historical contributions of new ideas by Leibniz and Abel. Leibniz theorised differential calculus thinking of points as if they were alive, as powers of explosion (“*puissances d’explosion*”), while Abel saw the curve not as fixed but in terms of its power of receiving intersections (“*comme puissance à recevoir des intersections*”). The virtual restores concepts to mobility, granting them inventive force and power. For de Freitas (2014), Châtelet shows us “how we might study a particular practice for how its lines of flight flourish and act generatively in unfolding new *intensive* dimensions.” (p. 290, *emphasis in the original*). The virtual is that which nourishes encounters with mathematics, linking the concrete and the abstract and allowing recoding the indeterminate contours of the sensible and the intelligible. This has to do with the potentiality or virtuality that is always entailed in perception: “We never just register visual information from that which is in front of our eyes: we see potentiality, relationality, mobility, occurrence. Students are not seeing an object; they are seeing an event” (de Freitas, 2014, p. 298).

In this paper, we take this perspective to look at the students' mathematical encounters with spatio-temporal relationships, focussing on the material and virtual dimensions of these encounters.

## Method and activity

The activity, which is the focus of this paper, is part of a classroom-based intervention (Stylianides & Stylianides, 2013) aimed at introducing the concept of function through the use of graphing technology. The wider research had the main purpose of investigating how learners might articulate meanings for functional relationships through modelling motion, and how their embodied activity with the technology might affect these meanings. A class of 30 grade 9 students and their regular mathematics teacher participated in the study, which lasted for a period of about three months with weekly sessions. During this time, the students worked on individual tasks, in groups of three people and in pairs of groups, taking part in class discussions. The authors were both present in the classroom and two cameras were used to film the mathematical activity of the students during all the sessions. Data for the research analyses are based on the films and students' written productions and diagrammatic activity. A microethnographic methodology (e.g. Streeck & Mehus, 2005) is essentially chosen for studying interactions and discourse in the classroom through strands of semiotic and representational activity over short periods of time, drawing attention to the material circumstances of the mathematical events.

The technology the students used in the case we consider here is WiiGraph, an interactive software application, which leverages two Wiimotes to display the space-time graphs of two users moving the remote controllers in front of a sensor bar. WiiGraph provides several challenges and composite operations, including shape tracing, maze traversal and ratio resolution. Choosing the plain visualisation (Line), the software captures the distances of the controllers over time and two graphs appear, in real time and with different colours, on a single Cartesian plane. Figure 1 shows a case of this type of visualisation for a 30-second default time and two students who move the controllers.



**Figure 1: The graphical system in Line and two students moving**

Visual and bodily (especially proprioceptive and kinaesthetic) interactions partake in the students' encounters with the graphical system in relation to experiencing spatial and temporal aspects with the technology. We will not refer to other types of graphical activity, since this is the one of interest in the case of Rob and Bob.

The task was given in a written form to the class during the second session and did not imply direct use of the technological devices. In the first session, the students explored Line and its graphical potential, became acquainted with the devices and started discussing about pairs of functions (for example, horizontal or slanted straight lines), with graphs originating in real time and projected on an interactive whiteboard. The activity of Rob and Bob was designed with the purpose of unfolding the

slope/speed relation (early insights emerged out of class discussion in the first session), and how it may reveal relationships between two space-time graphs (functions).

### Rob and Bob

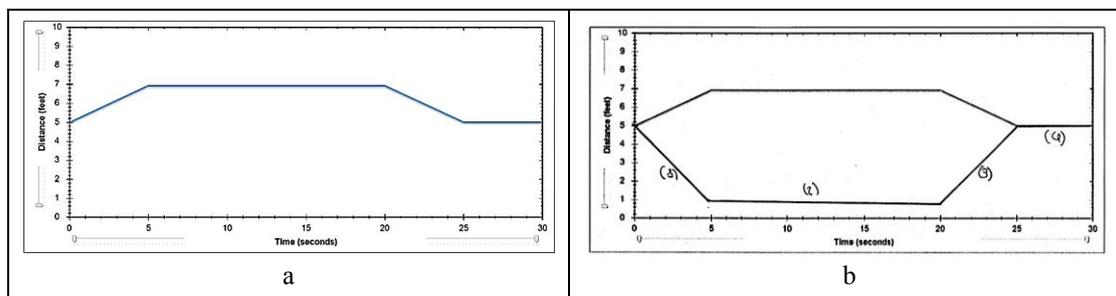
The task was faced by the students divided into groups of three people, and followed by a class discussion led by one of the authors. It focuses on an imaginary experience with WiiGraph in which two little robots move (the controllers) in front of the sensor bar, but only the graph associated to one robot's movement is given (Figure 2a). The text of the task is the following:

*Rob and Bob are two little robots, which can be taught to move in front of the sensor very precisely. Suppose that, in response to Rob's movement, WiiGraph produces the line below (Figure 2a). Imagine that Bob also moved: it started together with Rob, at the same distance from the sensor, but moved at a double speed and in the opposite direction.*

- Which graph would WiiGraph show for Bob's movement?
- Did Rob and Bob meet again after the start?

*Justify your answers.*

The task has an unconventional nature with respect to the representational system offered by the technology, because it does not ask the students to merely reason on the model to motion, or motion to model, shift. Instead, information about the missing graph is given in terms of the relationships between the two robots' movements ("double speed", "opposite direction"), so that the students are moved to think about the relationships between the two graphs (double slope with opposite sign), through their perceptual and bodily experience with the tool. In addition, the simultaneity of the two movements, which by the way recalls the usual way of using the tool, is embedded in information about the starting instant/point ("it started together with Rob", "at the same distance").



**Figure 2: (a) The given graph, (b) The expected solution to the task given by one group**

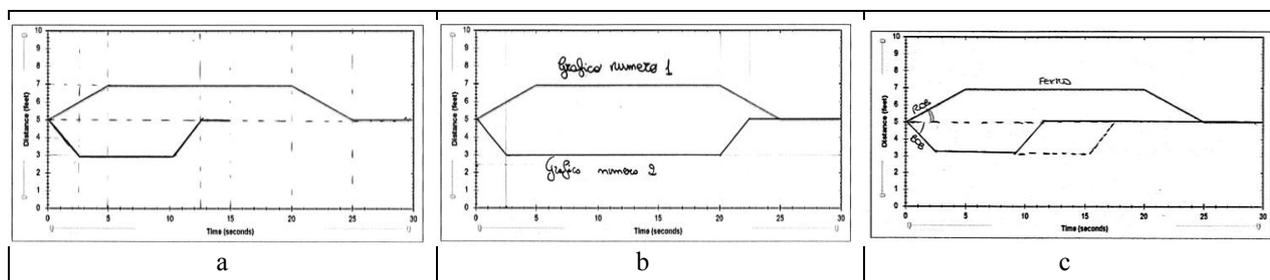
The given graph is that of a piecewise function made up of four pieces, which capture alternate ways of moving by Rob: stepping further from the sensor for the first five seconds, stopping for the next fifteen seconds, returning to the starting position in other five seconds, and stopping for the last five seconds (Rob keeps constant speed in each time interval). We expected the students to complete the Cartesian plane drawing a graph like the one in Figure 2b. It is the graph of a piecewise function again made up of four pieces, defined on the same sequence of time intervals as the given graph. These pieces correspond to four ways of moving by Bob: getting close to the sensor for the first five seconds, stopping for the next fifteen seconds, returning to the starting position in five more seconds, and stopping for the last five seconds (however, Bob is supposed to cover double space with respect to

Rob, according to the constraint of moving at a double speed. Of course, this is true when he moves, and trivially when he does not, since the distance covered is null).

Instead of looking at the expected graph as the correct one and speaking of difference in terms of being mistaken, we dwell on different unexpected solutions emerged from the groups about their potential to bring forth new relational possibilities for the two robots' movements as well as for the pair of graphs. In the next section, we take these solutions as the problematic actualizations of the mathematical events that the groups encounter in solving the task. It is this idea of novelty that speaks directly to inventive mathematics and makes students alive to their engaging with the task.

## Graphs and discussion

The groups worked on the task for half of the time, then they took part in a collective discussion in which their graphical solutions were compared. Only one group drew the expected solution (Figure 2b), while eight out of ten created one of the three unexpected lines shown in Figure 3 (For the sake of ease, we refer these lines to three graphs labelled with numbers 1, 2 and 3).



**Figure 3: Unexpected solutions — (a) graph 1, (b) graph 2 and (c) graph 3**

The three graphs added for Bob's movement have some similarity. They all show that taking into account information about opposite direction and capture it visually in the diagram is not an issue for the students. Each added graph is made up of four pieces, which embed the opposite way of moving with respect to Rob: first getting close, then returning to the start (first a decreasing piece, then an increasing piece). Not even slope is an issue: the double speed of movement is double slope in the three diagrams. However, the duration of Bob's movement is problematic for the students. In fact, while there is correspondence between ways of moving there is no embodiment of duration: there is no correspondence between time intervals in which both robots either move or stand still. The lengths of the horizontal pieces are different from each graph to the other, and the constraint for Bob to move at a double speed with respect to Rob is no longer preserved. Thus, the problematics of duration evolved along various accidental threads for the students, driven by their perceptual and bodily engagement with the task. These broke with causal connections and direct determination, opening up to speculative and inventive investments and to a generative movement, implicating the perturbation of spatio-temporal relationships. For example, in the case of graphs 1 and 3 (Figures 3a and 3c), some encountered the event for which Bob already stands still while Rob is still moving and, later, Bob moves towards the starting position while Rob is still standing still. Some groups introduced the new event in which the second robot stops just after fifteen seconds, in the very middle of the experience with WiiGraph, and ideally disappears from the view of the sensor, so that the second graph might accidentally stop in the middle of the diagram (Figure 3a). Almost all the students engaged with the kinaesthetic question of Rob and Bob always covering the same space, no matter the time spent, as

shown in the three diagrams. These threads are actualized through the groups' written explanations, then during class discussion. Types of explanation are the following:

Graph 1: *"The line we represented is half of Rob's line. The lines are steeper because speed is doubled and Bob moved faster than Rob and in the opposite way. The graph ends at 15 because Bob, moving at a double speed, stopped at half of 30."*

Graph 2: *"The two configurations are different from each other, indeed slopes also change since times change: Rob is slower. So, covering the same space in different time, there will be a higher steepness."*

Graph 3: *"Bob goes at a double speed with respect to Rob, so it finishes "the lap" before Rob. The rest of the way it stood still and at the end it met Rob. Speed changes between the ways of Bob and Rob, indeed Bob has to cover the same space backwards using half of the time."*

The logical equivalence between double speed as double distance in the same time and as the same distance in half of the time is lost, and the problematic of covering a fixed space in less time drives students' perception and visualisation in the diagramming of the missing graph. Graph 1 (Figure 3a) is the most coherent in respect to the axiomatic way of reasoning about double speed but at once the most incoherent in relation to kinaesthetic actions with the technology. Briefly speaking, it is nothing but a temporal shrinking of the given graph. Instead, graph 3 (Figure 3c) is in line with the usage of WiiGraph, because it embraces all the thirty seconds of the modelling process. The same occurs in the case of graph 2 (Figure 3b), which is particular though, since it struggles to depict the simultaneity of the two robots' movements. In the discussion, different students actualize in different ways the problematics that sustain the mathematical events that occurred in solving the task. Below, Lorenzo, Luigi, Giulio and Oliver bring forth in the discourse the issues of duration and simultaneity of movements, of moving at a double speed and of covering the same space, issues that are entangled in their diagrammatic and written activity. Lorenzo speaks about graph 1 (Figure 3a), Luigi and Giulio refer to graph 2 (Figure 3b), while Oliver argues about graph 3 (Figure 3c).

Lorenzo: 'Cause, moving at a double speed, distance remained constant, even though it was the opposite, but maybe, if Rob performed a movement in 10 seconds, Bob performed it in 5 seconds because speed was double.

Luigi: For me, hem, the graph took up the same time because, moving simultaneously, maybe, at the time they were moving, it took less time for one than for the other one to cover the same space, to move in the same space, but then one stood still until the other one also did move again, so both graphs last for 30 seconds. (...)

Giulio: For me, it [the graph] finished at 30, 'cause it's not that Rob [Bob] could know Bob's [Rob's] movement in advance, so it [the graph] cannot finish at 15, it [Bob] has to wait for it [Rob] to perform the same but opposite movement, 'cause we did see Bob's graph (*miming it in the air*) but if they move simultaneously, it means that one cannot anticipate the movements, so it cannot finish at 15 seconds. (...)

Giulio: We depicted slope at 2,5 seconds but then we stood still until 20 seconds, 'cause anyway it's true that time is halved, but Bob doesn't know what Rob will do later, so it has to wait for it.

- Researcher: Did you say that time is halved?
- Giulio: Yes, 'cause it [Bob] does things in half of the time, it's true, however it's not that he can know what it [Rob] will do later, hem, 'cause we do know it, but if they move simultaneously...
- Oliver: Bob does our graph with the movements with which it's been set up, and then, in the end, it's not that it waits for Rob, it goes on just as it likes and wants, then at a certain point, when movements are finished, it stops and the line keeps straight for the rest of the time.
- Researcher: Are you saying that time is halved?
- Oliver: Yes, because of the double speed. (...)
- Lorenzo: Indeed, for explaining a little the graph, I said that if there's a distance to cover, and that distance is 50 kilometres and you go at a speed of 50 kilometres per hour, it takes you 1 hour to cover that distance. Instead, if you go at 100 kilometres per hour it takes you 30 [minutes].
- Giulio: Yes, for me he's right about the first piece, but then if you stand still, so speed is null, zero times two is always zero and so speed has to be equal in the positions in which it stands still, for me.

Discourse with the researcher unfolds the event-nature of unexpected threads traversed in solving the task. We see how the students inscribe themselves into the temporality of imaginary situations with the robots. The ways of perceiving this temporality are different for different (groups of) students: some imagine that one robot has to "wait for" the other to know what to do (Giulio, Luigi); for others, coordination is not needed (Oliver) or considered (Lorenzo). Time is duration and simultaneity of movements: both aspects become problematic for learners. Both are crucial in making sense with WiiGraph of time as the independent variable in space-time functions.

### **Conclusive remarks**

In this paper we have discussed some unexpected graphical solutions to a given diagramming task based on modelling motion through the use of Wii graphing technology. We have focused not on how these solutions were incorrect with respect to the expected diagram, but on ways in which they brought forth new possibilities for the students to encounter spatio-temporal relationships. In so doing, we looked at visual, proprioceptive and kinaesthetic aspects of experiencing the technology as that which sustained the occurrence of new mathematical events in the classroom, bringing into being problematic perturbations of the given situation, like shrunk graphs as well as not coordinated movements and fixed paths, which break with the conventional visualization and activity of the representational system in use. The written, the diagrammatic, the discursive and the bodily, as the groups attempted to grapple with the task (to make sense of it), have to be seen as that which animated the task without ever exhausting it or fully determining it. The temporality of the events speaks directly to the material contingency of learning: the students are dynamically affected by the diagrammatic activity while telling stories of motion related to graphing technology.

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