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Abstract At the operational decision level, the problem arising in the Operating Room (OR) planning is also called "surgery process scheduling", which usually consists in selecting elective patients from a waiting list and assigning them to a specific operating room on a specific day, and determining the sequence of surgical procedures and the allocation of resources for each OR session. The Real Time Management (RTM) of operating rooms is the decision problem arising during the fulfillment of the surgery process scheduling, that is the problem of supervising the execution of such a schedule and, in case of delays, to take the more rational decision regarding the surgery cancellation or the overtime assignment. The RTM is characterized by the uncertainty of its main parameters, that is, for instance, the duration of a surgery and the arrivals of non-elective patients. In this chapter we propose online optimization approaches for the RTM capable to deal with (i) the elective and non-elective patient flows within a single surgical pathway (Non-Elective Worst Fit algorithm), and with (ii) the resource sharing among different surgical pathways of elective patients (Flexible Overtime Allocation and Flexible Scheduling policies). We assess the effectiveness of the proposed solutions on simulated surgical clinical pathways under several scenarios. From a methodological point of view, our analysis suggested that online optimization can be a suitable methodology to deal with the inherent stochastic aspects arising in the majority of the health care problems.

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1 Introduction

Operating Room (OR) planning and scheduling is a research topic widely discussed in the literature. Cardoen et al. (2010) and Guerriero and Guido (2011) provide an exhaustive review of the problems belonging to this topic, also analyzing in detail multiple fields related to the problem settings and summarizing significant trends in research areas of future interest. Such problems are usually classified into three phases corresponding to three decision levels (Testi et al., 2007), namely strategic (long term), tactical (medium term) and operational (short term).

At the operational decision level, the problem arising in the Operating Room (OR) management is also called "surgery process scheduling", which usually consists in (i) selecting elective patients from an usually long waiting list and assigning them to a specific OR time session (i.e., an operating room on a specific day) over a planning horizon, (ii) determining the precise sequence of surgical procedures and the allocation of resources for each OR time session, and (iii) dealing with the arrival of non-elective patients requiring a surgery within a given time threshold.

The Real Time Management (RTM) of operating rooms is the decision problem arising during the fulfilment of the surgery process scheduling of elective and non-elective patients, that is the problem of supervising the execution of such a schedule and, in case of delays, to take the more rational decision regarding the surgery cancellation or the overtime assignment. The RTM is characterized by the uncertainty of its main parameters, that is, the duration of a surgery and the arrival of non-elective patients. The RTM could deal with different objectives, that is to maximize the operating room utilization and to minimize the number of surgeries cancelled. Especially when considering the inherent stochasticity of the problem (Bruni et al., 2015; Addis et al., 2016; Landa et al., 2016; van Veen-Berkx et al., 2016), the two objectives are conflicting (Beaulieu et al., 2012).

The optimization literature reports few attempts to address the problem as reported in Hans and Vanberkel (2012). The problem of rescheduling the elective patients upon the arrival of emergency patients is addressed in Erdem et al. (2011, 2012). The authors proposed a MILP model which considers the overtime cost of the operating rooms and/or the post-anesthesia care units, the cost of postponing or preponing elective surgeries, and the cost of turning down the emergency patients. They proposed a genetic algorithm for its approximate and faster solution. The results of the case study suggest that, instead of shuffling the elective surgeries, it would be worthwhile to consider performing the elective surgeries using the overtime of the operating rooms. Note that the problem of rescheduling patients can be addressed as a particular job shop scheduling problem (see, e.g., Pham and Klinkert (2008); Stuart and Kozan (2012)) but these experiences can not be directly applied to the operating room context due to its peculiarity in the evaluation of a solution, as we will show along the chapter.

The current development of the health care systems is aimed to recognize the central role of the patient as opposed to the one of the health care providers. In this context, Clinical Pathways (CPs) shift the attention from a single health benefit to the health care chain that starts to resolve the illness episode. They can be

defined as "health-care structured multidisciplinary plans that describe spatial and temporal sequences of activities to be performed, based on the scientific and technical knowledge and the organizational, professional and technological available resources" (Campbell et al., 1998). A comprehensive approach to the surgery process scheduling of only elective patients has been discussed in Duma and Aringhieri (2015), in which the authors proposed also an online algorithm for the RTM. In that paper, the authors provided an extensive analysis of the impact of several optimization procedures on the performance of a surgical CP estimated exploiting a series of patient- and facility- centered indices as previously discussed in Aringhieri and Duma (2016).

Starting from the problem with only elective patients, it could be of great interest, both from a methodological and managerial point of view, to consider in our analysis two crucial aspects of the OR management, that is the management of non-elective patients and the resource sharing among different surgical pathways. These two aspects make the starting problem more challenging requiring the adoption of unconventional solution methodologies (Aringhieri et al., 2013).

While the list of the elective patients is known at the moment of the surgery process scheduling, the arrivals of the non-elective patients are unknown. Non-elective patients differs from elective ones by their request of a compulsory surgery within a tight time limit, usually ranging from "*as soon as possible*" to "*within 24 hours*". Thus the management of non-elective patients poses a challenge problem since delaying their surgery may increase the risk of postoperative complications and morbidity. As reported in literature (see, e.g., Van Riet and Demeulemeester (2015)), two main policies can be adopted to deal with non-elective patients, that is to have a certain number of dedicated ORs (see, e.g., Heng and Wright (2013)) or to share available ORs (see, e.g., Wullink et al. (2007)). From the RTM point of view, the sharing OR policy is more interesting to study because of the need to establish how to insert non-elective patients within OR sessions on which the elective patients have been already scheduled. Therefore, the need of making available an OR session within a tight time limit and without an ex-ante planning improves the complexity of supervising the execution of the surgery schedule affecting both the number of cancellations of elective surgery and the use of the overtime.

Among different surgical pathways, the critical resources that can be shared in order to improve the pathway management, are the overtime and the OR sessions. The master surgical schedule usually defines the specific assignment of OR sessions to different surgical pathways (Testi and Tànfani, 2009), and it should be updated whenever the total amount of the OR time or the requirements of some surgical pathways change. This can occur not only as a response to long term changes in the overall OR capacity or staffing fluctuations, but also in response to seasonal fluctuations in demand (see, e.g., Banditori et al. (2013); van Oostrum et al. (2008)). The objective of the resource sharing is therefore to have a fair assignment of both the overtime and the OR sessions to different surgical pathways.

In this chapter we propose online optimization approaches for the RTM capable to deal with (i) the elective and non-elective patient flows within a single surgical

pathway, and with (ii) the resource sharing among different surgical pathways of elective patients. The chapter is organized as follows.

Section 2 describes the problem of dealing with the elective and non-elective patient flows in a single surgical pathway. The online approach for the RTM for a joint flow of elective and non-elective patients is discussed in Section 3. In Section 4, we also provide a mixed-integer programming model to compute the optimal offline solution, that is the optimal solution assuming to know in advance all the information that are acquired over time by the online solution. Such a solution provides a significant contribution to evaluate the competitiveness of the online approach. The quantitative analysis to prove the effectiveness of the proposed approach is reported in Section 5.

Section 6 introduces the problem of sharing resources among different surgical pathways, highlighting the differences with respect to the problem discussed in Section 3. Different and alternative policies for the management of the ORs and the overtime are proposed in Sections 6.1 and 6.2, respectively. Following the same framework of analysis introduced in Section 5.1, we determine the best policy combination to manage the shared resources in Section 6.3.

General remarks and conclusions are discussed in Section 7.

2 The management of elective and non-elective patients

In this section we would describe the operative context that we are considering in order to better motivate our approach. An ex-ante scheduling is usually performed before the starting of the planning time horizon, which is usually set to one week in the literature. Here we consider the following version of the surgery process scheduling: for each OR session available, it determines which elective patients should be operated on (*advanced scheduling*) selecting them from the pre-admission list, and the sequence of their surgical interventions (*allocation scheduling*). We consider the surgery process scheduling of a single specialty under the block scheduling or closed block planning paradigm (van Oostrum et al., 2010): for each planning period, a number of OR time blocks are assigned to the specialty, which schedules their surgical cases within these time blocks.

Let us consider the set J of the operating rooms and let K be the set of the days of the week. Then, let $S \subseteq J \times K$ be the set of the OR sessions (j,k), each one denoting an OR $j \in J$ available the day $k \in K$ and having duration equal to d_{jk} minutes. The overall number of OR sessions is $n = |S|$. Let I be the set of the elective patients in the pre-admission list and $L \subseteq I$ the set of scheduled patients to be operated on during the next planning time horizon. Note that L can be partitioned in n subsets $L_{jk} \subseteq L$ where L_{jk} is the set of the patient scheduled on the OR session (j,k).

For each patient $i \in I$ the Estimated Operating Time (EOT) e_i and the number of days elapsed in the waiting list t_i are known. After the surgery, also the Real Operating Time (ROT) r_i will be known. Another relevant information is the Diagnosis Related Group (DRG). A DRG defines a general time limit expressed in days before

which the patient should be operated on (i.e., days to surgery). In our context, a *Urgency Related Group* (URG) is assigned to each patient belonging to the same DRG: the URG states a more accurate time limit called *Maximum Time Before Treatment* (MTBT) and denoted by $t_i^{\max}$. In other words, URG allows to define a partition of the patients in the same DRG in order to prioritize their surgical operation.

Usually, the advanced scheduling has the objective to maximize the utilization of each OR session considering its overall duration d_{jk} promoting the selection from I of those patients whose t_i is closer to $t_i^{\max}$. Since the ROT r_i is unknown, such a scheduling is usually performed taking into account the EOT e_i determined by the physician during the outpatient visit. Note that the EOTs are also used in the allocation scheduling to determine the patient sequence within each OR session.

The RTM arises during the accomplishment of the surgery process schedule. Considering the OR session (j,k), it could happen that $r_i > e_i$ for the patient $i \in L_{jk}$. When this occurs, the schedule could be delayed and the overall delay could determine the exceeding of the OR session duration d_{jk}. In this case, two possible decisions can be considered, that is that of postponing a surgery or that of assigning a part of the available overtime Ω to allow the completion of the surgery plan. Such decisions have to take into account the need of guarantee the patient surgery before his/her MTBT but, at the same time, to avoid an over-allocation determining a possible overtime failure in the next planning days. The RTM requires an online approach because the overtime demand until the end of the planning horizon is unknown. Further, the RTM is challenged by the unforeseeable arrivals of non-elective patients that must be operated on within their tight time limit. Unlike the elective ones, the non-elective patients are characterized only by the values e_i and r_i. Table 1 summarizes the notation introduced in this section.

Table 1 Notation.

J: set of operating rooms	K: set of the days of the week
j: index of the operating room	k: index of the day
S: set of OR sessions	n: number of OR sessions
d_{jk}: duration of OR session (j,k)	Ω: weekly overtime available
I: set of patients in the pre-admission waiting list	L: set of scheduled patients
L_{jk}: set of patients scheduled into the OR session (j,k)	$t_i^{\max}$: MTBT of patient i
t_i: waiting days to surgery of the i-th patient	e_i: EOT of patient i
r_i: ROT of patient i	

3 *Ex-ante* approach: the online solution

In this section we introduce our online approach to deal with elective and non-elective patients.

3.1 The RTM with only elective patients

Let us suppose to consider the generic OR session $(j,k) \in S$. Let ρ_{jk}^{τ} be the time elapsed in the OR session (j,k) from the beginning of the session at the time τ. If the surgery of the m-th patient belonging to the schedule of L_{jk} ends at time τ, the effective time elapsed to operate on the first m patients is

$$\rho_{jk}^{\tau} = \sum_{i=i_1,\ldots,i_m} r_i. \tag{1}$$

Let us introduce the following parameter:

$$\beta_k^{\tau} = 1 + \frac{n_k}{n} - \frac{\Omega_k^{\tau}}{\Omega} \tag{2}$$

where Ω_k^{τ} is the remaining overtime at the time τ and n_k is the number of OR sessions from the day after k, that is

$$n_k = \left|\left\{(j',k') \in S : k' > k\right\}\right|.$$

The value β_k^{τ} would measure the overtime still available with respect to the number of OR sessions to be still performed. Actually, β_k^{τ} is close to 1 when the overtime has been used proportionally; it is between 0 and 1 or it is greater than 1 when it is underused or overused, respectively. Because of n_k is equal to 0 when k is the last day of the planning, we remark that β_k^{τ} is always less than or equal to 1 hence promoting the use of overtime.

In order to establish when the cancellation of a patient could lead to the exceeding of the MTBT, we define the parameter

$$\widetilde{w}_i = \frac{t_i + \delta}{t_i^{\max}} \tag{3}$$

where δ is the number of days until the beginning of the next planning horizon. It is worth noting that we would to avoid the cancellation of patients with $\widetilde{w}_i > 1$ because the rescheduling of such patients in the next planning horizon would exceed the MTBT.

Let $L_{jk}^{\tau} \subset L_{jk}$ be the set composed of the patients in L_{jk} still waiting for a surgery at the time τ, that is $L_{jk} \setminus \{i_1,\ldots,i_m\}$. The online algorithm for the RTM with only elective patients starts every time a surgery ends and

$$\rho_{jk}^{\tau} + \sum_{i \in L_{jk}^{\tau}} e_i > d_{jk}. \tag{4}$$

It consists of two procedures.

Resequencing. The objective of the resequencing is twofold, that is to ensure the surgery of those patients close to their MTBT, and to maximize the OR utiliza-

tion. To this end, patients are ordered in such a way to put first the patients having $\widetilde{w}_i > 1$; then, if such patients does not run out the available operating time d_{jk}, a subset of the remaining patients are selected to maximize the OR utilization trough a dynamic programming approach, which is an adaptation of that discussed in Section 3.4.1 of Martello and Toth (1990); in any case, the unselected patients are inserted at the end of the schedule since they could be operated on using the overtime;

Overtime allocation. Let i_{m+1} be the next patient in the schedule. Then, if

$$\rho^{\tau}_{jk} + e_{i_{m+1}} > d_{jk}\,,$$

the patient i_{m+1} could incur in a cancellation. Therefore, the required overtime is assigned to patient i_{m+1} if the overtime available is sufficient and at least one of the two following conditions is satisfied:

$$\widetilde{w}_i > 1\,, \tag{5}$$

$$\beta^{\tau}_{k}\left(\frac{e_{i_{m+1}} + \rho^{\tau}_{jk}}{d_{jk}}\right) \leq 1\,. \tag{6}$$

Otherwise the surgery of the patient i_{m+1} is postponed to the next week. This assumption is justified by the high level of utilization imposed by the advanced scheduling, which is reported in Section 5.1.

3.2 The RTM with elective and non-elective patients

Non-elective patients should require to be operated on within different but usually tight time limits depending on their urgency. Such time limits can range from "*as soon as possible*" to "*within 24 hours*" (see Table 4 in Van Riet and Demeulemeester (2015)). When the time limit is very short, that is few hours, the most reasonable decision is to schedule the intervention as soon as possible. This situation can be handled by the algorithm for the RTM reported in Section 3.1 by properly increasing the available overtime (Duma and Aringhieri, 2015).

Here we consider the arrivals of non-elective patients with a time limit of 24 hours, that is they must be treated within the end of the current day k. In this context, the online approach has to decide in which OR session the non-elective patient has to be scheduled. Such a decision can determine a different need of overtime or the cancellation of the elective patients previously scheduled.

Let $S_k \subseteq S$ be the set of the OR sessions planned on the day k. Let us also to introduce the set Q^{τ} composed of the non-elective patients still waiting for a surgery at the time τ, respectively. Therefore the next surgery should be selected from the set $L^{\tau}_{jk} \cup Q^{\tau}$.

To deal with this online decision we develop the Non-Elective Worst Fit (NEWF) algorithm, which is a greedy construction of an alternative schedule of the patients

in which we try to insert the non-elective patients in Q^τ, only when $Q^\tau \neq \emptyset$. On the basis of this alternative schedule, the online algorithm NEWF establishes the next patient to be operated on during the OR session (j,k), that is to continue with the planned schedule or to insert a non-elective patient in (j,k).

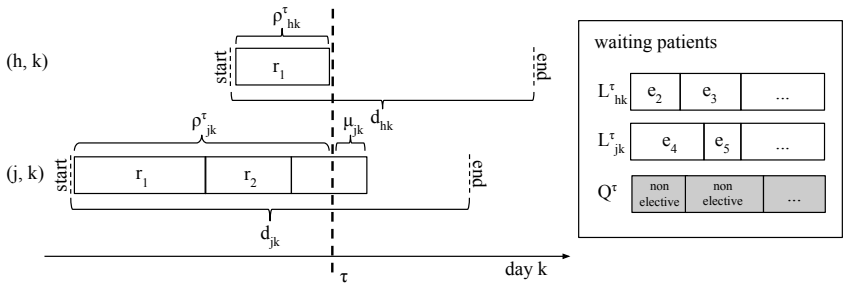


Fig. 1 Example of the context in which the NEWF operates.

The NEWF operates in the following online context: on the day k, we consider the instant during which the surgery of the patient $i_m \in L_{hk}$ ends; at that moment, let μ_{jk} be the estimated time remaining to the end of the ongoing surgery in the OR session (j,k) (note that $\mu_{hk} = 0$). An example of the context in which the NEWF operates is reported in Figure 1.

Algorithm 1 Non-Elective Worst Fit

procedure NON-ELECTIVE WORST FIT
initialization:
  $p^e \leftarrow$ next patient in L^τ_{hk}; $Q' \leftarrow Q^m$;
  for each OR session (j,k) **do** $L'_j \leftarrow L^\tau_{jk}$;
loop:
  $p^{ne} \leftarrow$ patient in Q' with the maximum waiting time;
  $j^\star \leftarrow \arg\min_{(j,k)} \left(\rho^\tau_{jk} + \mu_{jk} + \sum_{i \in L'_j} e_i - d_{jk}\right)$;
  if $j^\star = h$ **then return** p^{ne};
  $L'_{j^\star} \leftarrow L'_{j^\star} \cup \{p^{ne}\}$; $Q' \leftarrow Q' \setminus \{p^{ne}\}$;
  if $Q' \neq \emptyset$ **then goto** *loop*;
end of the procedure:
  return p^e;

The pseudocode reported in Algorithm 1 describes the algorithm NEWF. After the initialization of the auxiliary data structures, the algorithm starts a loop to determine the alternative schedule. At each iteration, the current non-elective patient p^{ne} is scheduled on the OR session (j,k) which minimizes the difference between the estimated total duration of the operated and non-operated patients in L_{jk}

$$\rho_{jk}^{\tau} + \mu_{jk} + \sum_{i \in L'_j} e_i \tag{7}$$

and its duration d_{jk}. Such a rule corresponds to insert p^{ne} in the OR session with the maximum unused OR time in such a way to minimize the overtime demand when d_{jk} is greater than (7). The aim is to balance the workload among the OR sessions. If one non-elective patient is scheduled in the OR session (h,k) then the NEWF returns p^{ne}, otherwise it returns p^{e}.

The online algorithm for the RTM with elective and non-elective patients starts every time a surgery ends in an OR session (j,k) and the value of (7) is greater than d_{jk} and/or at least one non-elective patient is waiting to be operated on. Accordingly, when $Q^{\tau} = \emptyset$ only the former condition has to be checked and it is equivalent to the condition (4) defined for the RTM with only elective patients because $\mu_{jk} = 0$ and $L'_j = L^{\tau}_{jk}$. The algorithm for the RTM performs three steps, that is the resequencing, the NEWF and the overtime allocation.

Finally, we would like to remark that the amount of effective used overtime could slightly exceed the maximum overtime available Ω. It depends on whether the overtime is assigned basing the decision on the EOT since ROT is not available. Under special circumstances, extra overtime can be required for the surgery completion but all the available overtime has been previously assigned. In this case, we assume to allow the surgery completion setting the parameter Ω equal to the effectively used overtime.

4 *Ex-post* approach: the offline solution

The online solution is characterized by the lack of knowledge about what might happen in the remaining of the planning horizon. This is due to the difference between estimated and real duration of a surgery, and to the unforeseeable arrivals of non-elective patients.

On the contrary, at the end of the planning horizon we have a complete information about what is happened. Thus is possible to evaluate what would be the optimal decisions to be taken assuming to know in advance all the information that are acquired over time by the online solution. In our case, such information includes the ROTs of the elective patients and the surgery demand of the non-elective patients, that is their amount, the ROTs and the day in which they must be operated on.

We denote this set of decisions as *offline solution*. Such a solution provides a significant contribution to evaluate the effectiveness of the online approach. In this section, first we provide a linear programming model to compute the optimal offline solution in the case of only elective patients. Then, we extend this model to take into account also the non-elective patients.

Figure 2 reports an example in which the difference between EOTs and ROTs caused the request of an amount of overtime. In the OR session $(1,k)$ the overtime

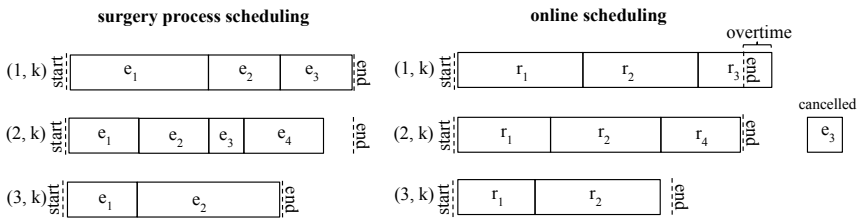


Fig. 2 Surgery process scheduling vs. online scheduling: elective patients.

has been allocated in order to operate on the last patient, while in the OR session $(2,k)$ the surgery of the patient with index 3 has been postponed.

To determine an offline solution in the case of dealing with only the elective patients, the only relevant decision is that of postponing the surgery interventions. Since the ROTs are known, we remark that any sequencing of the surgery planned into an OR session determines the same outcome. Thus the sequencing is not relevant for the offline solution. Let us introduce the following decision variables

$$x_i = \begin{cases} 1 & \text{if the surgery intervention of the patient } i \in L \text{ is postponed} \\ 0 & \text{otherwise} \end{cases}$$

and the non-negative integer $v_{jk} \in \mathbb{Z}_+$ measuring the overtime assigned to the OR session (j,k).

To model the offline solution we introduce the following constraints:

$$\sum_{i \in L_{jk}} (1 - x_i) r_i \leq d_{jk} + v_{jk} \qquad \forall (j,k) \in S \tag{8}$$

$$\sum_{(j,k) \in S} v_{jk} \leq \Omega \tag{9}$$

$$x_i = 0 \qquad \forall i \in L_{\text{first}} \tag{10}$$

Constraints (8) ensure that the overall duration of the surgery performed during the OR session (j,k) can not exceed the duration of the OR session plus the additional overtime assigned. Constraint (9) limits the use of the overtime to the maximum overtime available. Finally, we remark that the first patient scheduled in each OR session (j,k) is not the subject of an online decision, that is he/she will be always operated on. Therefore, we are required to model this fact in our offline solution introducing the constraints (10) where $L_{\text{first}} \subset L$ is the set of all the patients sequenced as the first of an OR session.

We recall that our online solution would maximize the utilization of the OR sessions and to minimize the number of postponed patients whose $\widetilde{w} > 1$, that is those patients for which the MTBT will be exceeded. Thus our objective function should take into account these requirements.

We define the overall utilization of the OR sessions as the ratio between the total duration of the operated patients and the sum of the duration of all the OR sessions, limited to 1 to avoid greater values, that is when using the overtime

$$u = \min \left\{ \frac{\sum_{i \in L}(1-x_i)r_i}{\sum_{(j,k)\in S} d_{jk}}, 1 \right\}.$$

To promote a solution with higher utilization, we introduce an auxiliary continuous variable $u \in [0,1]$ and the constraint

$$u \sum_{(j,k)\in S} d_{jk} \leq \sum_{i \in L} r_i(1-x_i). \tag{11}$$

Our aim is to maximize the objective function defined as follows

$$z \equiv (1-\alpha)u + \alpha \frac{\sum_{i\in L}(1-x_i) - \sum_{i \in L_{\widetilde{w}>1}} x_i p_i}{|L|}, \tag{12}$$

which is the convex combination of two terms in $\alpha \in [0,1]$. The former is the utilization defined by the constraint (11). The latter is the number of the patients operated on minus a sum of the penalties associated to those patient whose surgery is postponed and their $\widetilde{w} > 1$. Since the utilization ranges in $[0,1]$, the latter term is normalized on the overall number of scheduled patient $|L|$. The penalties are defined as

$$p_i = \widetilde{w}_i^2. \tag{13}$$

in order to limit the impact of the symmetries (see, e.g., Ghoniem and Sherali (2011)).

Finally, the offline solution in the case of only elective patients can be computed by finding the optimal solution of the following mixed-integer linear program

$$\begin{aligned} M^e: \qquad \max z \quad & \text{s.t. (8)–(11)} \\ & x_i \in \{0,1\} \quad \forall i \in L \\ & v_{jk} \in \mathbb{Z}_+ \quad \forall (j,k) \in S \\ & u \in [0,1] \end{aligned}$$

Figure 3 reports an example of solution of the problem in which two non-elective patients have been scheduled, determining the request of an amount of overtime for some of the OR sessions.

Model M^e can be modified to address also the management of the non-elective patients. Let Q_k be the set of the non-elective patient arrived the day k. We introduce the following decision variable

$$y_{ijk} = \begin{cases} 1 & \text{if the patient } i \in Q_k \text{ is inserted in the session } (j,k) \\ 0 & \text{otherwise} \end{cases}.$$

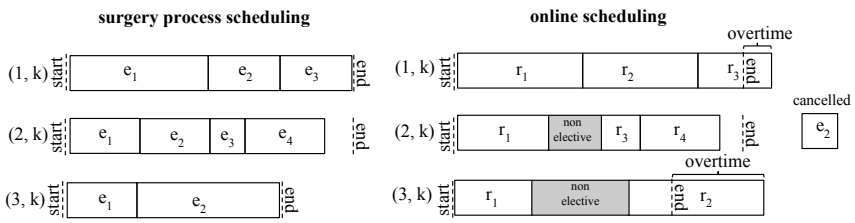


Fig. 3 Surgery process scheduling vs. online scheduling: elective and non-elective patients.

The constraints

$$\sum_{(j,k')\in S:k'=k} y_{ijk'} = 1 \quad \forall i \in Q_k, \forall k \in K \tag{14}$$

ensure that each non-elective patients in Q_k is operated on during only one OR session (j,k). We remark that we have to modify the constraints (8) and (11) to take into account the insertion of the non-elective patients. By consequence, the new constraints are

$$\sum_{i\in L_{jk}} (1-x_i)r_i + \sum_{i\in Q_k} y_{ijk} r_i \leq d_{jk} + v_{jk} \qquad \forall (j,k) \in S \tag{15}$$

$$u \sum_{(j,k)\in S} d_{jk} \leq \sum_{i\in L} (1-x_i) r_i + \sum_{i\in Q} r_i \tag{16}$$

where $Q = \bigcup_{k\in K} Q_k$.

Finally, the offline solution in the case of elective and non-elective patients can be computed by finding the optimal solution of the following mixed-integer linear program

$$\begin{aligned} M^{ne}: \quad & \max z \quad \text{s.t. (9)–(10), (14)–(16)} \\ & x_i \in \{0,1\} \quad \forall i \in L \\ & v_{jk} \in \mathbb{Z}_+ \quad \forall (j,k) \in S \\ & y_{ijk} \in \{0,1\} \quad \forall i \in Q, \forall (j,k) \in S \\ & u \in [0,1] \end{aligned}$$

5 Quantitative analysis

This section reports the quantitative analysis performed under several scenarios to evaluate the effectiveness of the proposed online methods providing two different but complementary analysis. The first one is to embed our online approaches on a simulated surgical clinical pathway in such a way to evaluate their impact on the

RTM week by week, that is how the previous decisions (e.g., determining less or more cancellations) can impact on the current decisions. The second one exploits the computation of the corresponding offline solutions in such a way to assess the competitiveness of the proposed online solutions. The section is organized as follows. We describe the simulated surgical clinical pathway in Section 5.1 while we describe the the performance indices and the different evaluation scenarios in Section 5.2. We report and discuss the results of our computational tests in Section 5.3 while some computational remarks are discussed in Section 5.4.

5.1 The simulated surgical clinical pathway

In a surgical context, the main activities composing the clinical pathway are summarized in Figure 4, which depicts also the moment on which the optimization problems arises. Our simulated surgical clinical pathway is a straightforward implementation of the surgical pathway described in Figure 4. The simulation framework is based on a Discrete Event Simulation (DES) since it is the most suitable methodology to analyze a discrete and stochastic workflow. Further, DES is the only approach capable to represent the single entity within a CP, which is a necessary condition to apply our online algorithms.

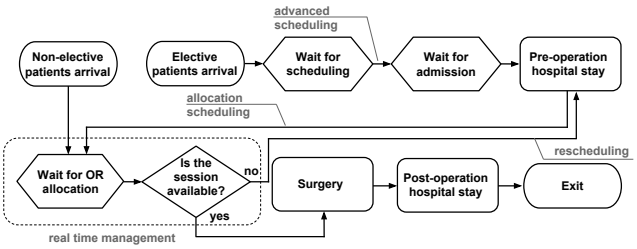


Fig. 4 The surgical CP and the optimization modules.

The optimization modules embedded in the hybrid model are the RTM algorithms presented in Section 3 and the following:

Advanced scheduling: a metaheuristic based on a greedy construction of an initial solution and then a local search to improve that solution as reported in Duma and Aringhieri (2015);

Allocation scheduling: patients are sequenced in decreasing order of $\tilde{w}_i$.

Furthermore, the canceled surgeries are rescheduled in one of the OR sessions of the first day of the next week. We recall that the advanced scheduling is aimed at maximizing the OR utilization. This fact directly influences the number of possible cancellations during the scheduling posing a challenge for the RTM. Furthermore,

we recall that it makes really difficult to insert a patient whose surgery has been postponed by the RTM, as reported in Duma and Aringhieri (2015). This justify our choice to schedule on the next week all the postponed patients.

The resulting hybrid model is implemented using AnyLogic 7.1 (Borshchev, 2013): its Enterprise Library is exploited for the implementation of the DES simulation framework whilst the optimization modules are implemented from scratch in Java, which is the native programming language of AnyLogic.

5.2 Scenarios and indices

The model briefly discussed in Section 5.1 is inspired to the case study reported in Ozcan et al. (2011) for the thyroid surgical treatment of elective patients. Since our model is capable to represent such a setting, in our previous work Duma and Aringhieri (2015), we validated our model through a comparison with the one discussed in Ozcan et al. (2011). To avoid to get trapped on a single case study, which could be a limitation from our point of view, we introduce four scenarios in such a way to provide more accurate insights from our quantitative analysis. To this end, we will consider four scenarios (E, NE1, NE2, NE2b) obtained by varying the non-elective arrival ratio and available overtime while the other parameters characterizing them are fixed. All the parameters are reported in Table 2.

Table 2 Parameters characterizing the four scenarios.

Varying parameters					
scenario	non-elective	Ω	scenario	non-elective	Ω
(E)	—	10 hours	(NE2)	30 per week	50 hours
(NE1)	15 per week	15 hours	(NE2b)	30 per week	40 hours
Common parameters to all scenarios					
parameter	unit	value	parameter	unit	value
elect. arrival rate	patients/day	25	initial $\|I\|$	patients	500
avg. EOT	minutes	140	s. dev. EOTs	minutes	75
s. dev. $r_i - e_i$	minutes	30	max ROT	minutes	480
n	sessions/week	45	d_{jk}	minutes	480

The flow of elective patients is described in terms of urgency class, frequency and MTBT in Table 3. Finally, all the simulation model parameters are the same of those reported in Duma and Aringhieri (2015).

Table 4 reports the two types of indices adopted to evaluate the impact of the optimization modules. We define a set of patient-centered indices in such a way to evaluate the performance from a patient point of view. We also define a set of facility-centered indices in such a way to evaluate them against to the patient-centered ones.

Table 3 Urgency classes and MTBTs of the elective patients.

URG class	frequency	MTBT (days)	URG class	frequency	MTBT (days)
A	3%	8	B	5%	15
C	7%	30	D	10%	60
E	15%	90	F	25%	120
G	35%	180			

Table 4 Patient-centered and facility-centered indices.

Index	Definition
	Patient-centered
p	number of surgeries performed
c	number of cancellations
f_{MTBT}	percentage of patients operated within the MTBT
ℓ_{avg}	average length (number of patients) of the waiting list
t_{avg}	average waiting time (days) spent in the waiting list
w_{avg}	average value of patient's $w_i = t_i/t_i^{\max}$ at the time of their surgery
	Facility-centered
u_{OR}	OR utilization
u_{over}	overtime utilization

The reason of considering both patient- and facility- centered indices relies in the more general idea of using a CP to enhance the quality of care by improving patient outcomes, promoting patient safety, increasing patient satisfaction, and optimizing the use of resources.

5.3 Results

In this section we report the results of our quantitative analysis obtained by running our methods on the four different scenarios.

In order to provide a term of comparison, we introduce a baseline configuration in which the algorithms for the advanced scheduling and the allocation scheduling are those reported in Section 5.1 while the solution for the RTM is simpler than those proposed in Section 3. In the baseline configuration, the resequencing is not performed, the overtime is a priori uniformly distributed among the OR sessions and the non-elective patients are assigned to the first free OR session. The baseline configuration does not claim to fit perfectly the clinical practice (since we are not dealing with a specific case study) but it would represent a more general operative context in which some optimization approaches are performed on the planning side

but without taking into account the inherent uncertainty arising in the management of a surgical pathway. The introduction of the baseline configuration allows us to evaluate the actual impact of the RTM on the management of the surgical pathway.

Two further configurations are introduced to properly evaluate the online approach in the case of non-elective patients, that is one configuration with the NEWF algorithm (conf. 2) and one without (conf. 1). When NEWF is not considered, the non-elective patients are scheduled as soon as possible.

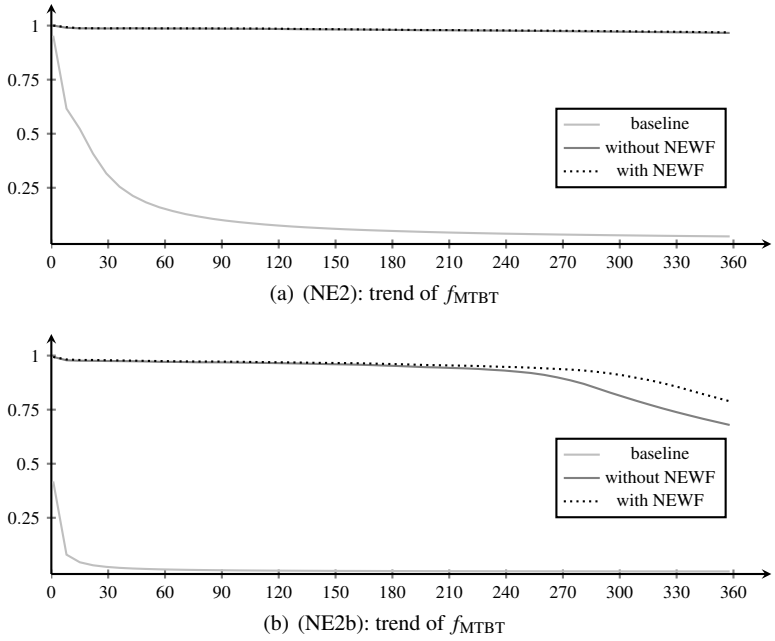
Table 5 Performance indices for each scenario and RTM configuration.

Scenario id	RTM config.	**Performance indices**							
		p	c	f_{MTBT}	ℓ_{avg}	t_{avg}	w_{avg}	u_{OR}	u_{over}
(E)	baseline	7532	506	44.0%	2879	111	1.05	83.4%	13.3%
	1	7991	399	91.9%	2168	86	0.81	88.5%	56.9%
(NE1)	baseline	7050	999	17.3%	3400	130	1.25	86.5%	17.5%
	1	7657	693	65.6%	2465	98	0.92	93.1%	100.0%
	2	7796	595	76.2%	2317	93	0.87	94.6%	100.0%
(NE2)	baseline	6866	1120	2.5%	3855	147	1.42	93.9%	21.2%
	1	8131	366	96.6%	1952	81	0.75	100.0%	95.8%
	2	8147	356	96.8%	1921	80	0.74	100.0%	96.2%
(NE2b)	baseline	6717	1244	0.2%	4064	154	1.50	92.3%	20,2%
	1	7780	572	66.7%	2440	99	0.92	100.0%	98.0%
	2	7878	540	77.4%	2347	96	0.89	100.0%	98.4%

Table 5 reports the value of the performance indices, which are obtained by taking the corresponding average value running the simulation model (depicted in Section 5.1) 30 times on a given configuration and, each time, starting from a different initial condition. Each run replicates two years of operating room management. Data are collected only on the second year. Remarks on running time are reported in Section 5.4.

With respect to the baseline configuration, the reported results showed that the adoption of the online approach for the RTM – both in the case of only elective or non-elective patients – can largely improves the patient-centered performance indices while maintaining the facility-centered ones.

The most relevant improvement is that regarding the percentage of patients operated within the MTBT (f_{MTBT}), which measure the capability of the hospital to respect their deadlines ensuring to deliver a surgery in a proper way. The results prove the positive impact of the NEWF algorithm. For instance, an improvement of more than the 10% of the f_{MTBT} can be observed on the scenarios (NE1) and (NE2b) while this improvement is limited in the scenario (NE2).



(a) (NE2): trend of f_{MTBT}

(b) (NE2b): trend of f_{MTBT}

Fig. 5 Trend of f_{MTBT} (data referred to the 2nd year, days on x-axis, percentage of patients on y-axis).

Figure 5 reports the trend of the f_{MTBT} value during the simulation in order to compare the behavior in the scenario (NE2) (Figure 5(a)) and (NE2b) (Figure 5(b)). As reported in Table 2. the difference between the two scenarios is the amount of available overtime, that is 50 hours for (NE2) and 40 for (NE2b). While in Figure 5(a) the f_{MTBT} is quite stable along the time, we observe that in Figure 5(b) a drop is reported after about 240 days. This highlights the fundamental role of the overtime as a flexible resources, when the required amount is correctly evaluated. We also remark that the NEWF is able to limit the negative impact of an overtime underestimation (Figure 5(b)).

Table 6 reports the competitive analysis, that is the comparison between online and offline solutions. The results are obtained as follows. Among the 30 runs reported before, we selected the run whose performance indices are closest to the average values of the performance indices. From this run, we extracted the information required to generate the 52 instances corresponding to the second year of simulated time. Finally, we computed the optimal solution for each of the 52 instances by solving the corresponding mixed-integer linear problem.

Therefore, Table 6 reports the average values over 52 instances of the following quantities: π and π' are respectively the number of elective patients scheduled and the number of elective patients scheduled whose $\tilde{w}_i > 1$; γ and γ' are respectively the number of cancellation and the number of cancellation whose corresponding

patients have $\tilde{w}_i > 1$; z_{avg} is the value of the objective function (12). Finally, the columns regarding the competitive ratio report both the experimental average and worst ratio.

The competitive analysis confirms the quality of the online solutions. In particular, the analysis of the z_{avg} values and the average and the worst competitive ratio values validates the remark about the positive impact of the NEWF algorithm.

Table 6 Comparison between online and offline solutions.

Scenario id	RTM config.	input $\pi\,(\pi')$	online sol $\gamma\,(\gamma')$	online sol z_{avg}	offline sol $\gamma\,(\gamma')$	offline sol z_{avg}	comp. ratio avg.	comp. ratio worst	time secs.
(E)	baseline	157 (78)	12 (5)	0.8607	3 (1)	0.9347	1.09	1.15	0.12
	1	161 (8)	7 (0)	0.9243	4 (0)	0.9467	1.02	1.09	0.11
(NE1)	baseline	156 (109)	20 (13)	0.8022	2 (1)	0.9830	1.23	1.45	0.96
	1	161 (42)	13 (2)	0.9188	2 (0)	0.9883	1.08	1.26	35.31
	2	161 (24)	12 (0)	0.9353	2 (0)	0.9894	1.06	1.13	3.98
(NE2)	baseline	154 (138)	22 (20)	0.7708	0 (0)	0.9978	1.30	1.63	0.20
	1	163 (6)	8 (0)	0.9768	1 (0)	0.9971	1.02	1.06	68.75
	2	163 (5)	7 (0)	0.9786	1 (0)	0.9972	1.02	1.06	0.53
(NE2b)	baseline	152 (147)	23 (22)	0.7317	1 (1)	0.9929	1.37	1.78	0.49
	1	161 (37)	12 (1)	0.9616	2 (0)	0.9932	1.03	1.12	117.31
	2	161 (31)	10 (0)	0.9681	2 (0)	0.9939	1.03	1.07	110.16

The analysis of the average competitive ratio proves the challenging of the problem of dealing with the management of a flow of elective and non-elective patients. Actually, the competitive ratio of the baseline solution largely increases as soon as the arrival rate of the non-elective increases or the available overtime is tight. On the contrary, the competitive ratio of the configurations 1 and 2 is quite stable. Furthermore, the gap between the two competitive ratios (baseline vs. configuration 1 or 2) is quite acceptable for the scenario (E) while increases for the other non-elective scenarios demonstrating the need of an online solution to cope in a effective way the management of non-elective patients.

5.4 Computational remarks

The results reported in Section 5.3 are obtained running our computational tests on a 64 bit Intel Core i5 CPU with 3.33GHz and 3.7GB of main memory.

On average, one single run of the simulation model requires from 7.0 to 20.5 seconds when running with scenario (E) and baseline configuration or with scenario (NE2b) and configuration 2, respectively. This means that no more than 615 seconds are needed to simulate two years of operating room management. Finally, we

remark that the algorithm for the advanced scheduling is the most time consuming component while the running time required by the other components is negligible.

The mixed-integer linear programs are solved using IBM ILOG CPLEX Optimization Studio 12.3. The CPLEX running time are reported in the last column of Table 6. Note that usually few seconds are enough to solve an instance of the offline problem. The high average values are determined by few instances requiring a lot of time to close the optimality gap. For example, the number of instances requiring more than 5 seconds in the scenario (NE2b) are 10 and 4 for configuration 1 and 2, respectively. This is probably due to the large number of symmetries determined by the decision variables y_{ijk} for a given day k.

6 Sharing resources among surgical pathways

The specific assignment of OR sessions to be shared among different surgical CPs, is defined by the Master Surgical Schedule (MSS). The MSS must be updated whenever the total amount of OR time changes or when the requirements of some surgical CPs change. This can occur not only as a response to long term changes in the overall OR capacity or staffing fluctuations, but also in response to seasonal fluctuations in demand. As already discussed in the previous sections, RTM deals with the overtime management in the case of a single surgical CP. However, when the overtime is a shared resources, the online decision of using the overtime, or to cancel a surgery, should take into account a fairness criterion.

The objective of the management of the shared resources is therefore to have a fair assignment of the overtime and the OR sessions to surgical CPs.

In the following, we will consider two or more different surgical CPs corresponding to different specialties. Each CP is essentially the same surgical pathway described in Figure 4 but without the source of non-elective patients. We will consider only elective patients since we would like to have a more clear idea of the impact of the proposed sharing policies: actually, in the current context, a flow of non-elective patients would correspond to a higher workload, that we are able to manage as shown in Section 5. For the sake of simplicity, we refer hereafter to those surgical CPs as specialties.

6.1 Policies for sharing ORs

We would define how to assign the available OR sessions among different specialties in such a way to ensure enough and balanced OR sessions to each specialty. In our operative context, the MSS is updated every time period (usually one month or one week). To this end, we define three alternative policies.

The first policy is *Based on Lengths* (BL), that is, every four weeks, the OR sessions are reassigned so that they are proportional to the number of patients in

the waiting list of each specialty. On the contrary, the second policy is *Based on the EOTs* (BE) in which every four weeks, the OR sessions are reassigned so that they are proportional to the sum of the EOTs of the patients in the waiting list of each specialty.

The last policy consists in a *Flexible Scheduling* (FS) in which MSS and Advanced Scheduling are solved at the same time every week. The algorithm implementing the FS policy is an adaptation of that proposed in Aringhieri et al. (2015). It consists of a greedy construction of the initial solution and an improvement phase performed by a local search engine: (i) at the beginning of the greedy construction, the patients are ordered by decreasing value of $\widetilde{w}_i$, and each OR session is not assigned to any specialty, except for the OR sessions used to reschedule the patients postponed during the last week; (ii) during the greedy construction, patients can be inserted only into OR sessions assigned to their specialty, or into OR sessions not already assigned (that is empty OR sessions); in the latter case, such an OR session is assigned to the specialty of the patient; (iii) during the local search, only swaps between patients belonging to the same specialty are allowed.

6.2 Policies for sharing overtime

When sharing overtime, we are interested in guaranteeing a fair access to the available overtime from the different specialties. We propose two alternative policies.

The first policy is called *Dedicated Overtime Allocation* (DOA). in which a dedicated amount Ω^s of weekly overtime is allocated to the specialty s, so that it is proportional to the number of OR sessions assigned by the MSS. By consequence, the RTM will take into account Ω^s as the overtime available when applying the criterion (6).

The second policy is called *Flexible Overtime Allocation* (FOA), in which all the specialties share the total available overtime Ω; in order to foster a balanced use of the overtime, we adapt the criterion (6) as follow:

$$\beta^{\tau}_{jk}\left(\frac{e_i+\rho^{\tau}_{jk}}{d_{jk}}\right)\left(1+\frac{v^s}{\Omega}-\frac{n^s_k}{n}\right)\leq 1 \tag{17}$$

where v^s is the amount of weekly overtime used by the specialty s until that time and n^s_k is the number of OR sessions of that specialty from the day after k.

The policy FOA introduces a new factor which measure the overtime still available with respect to the number of OR sessions to be still performed by the specialty. This value is closed to 1 when the overtime has been used proportionally with respect to the assigned and completed sessions. On the contrary, it is between 0 and 1 or it is greater than 1 when it is underused or overused, respectively.

6.3 Quantitative analysis

Following the same analysis framework proposed in Section 5, we report a quantitative analysis to evaluate the impact of the policies for the resource sharing.

In the current analysis, we consider as "baseline" the configuration 1 introduced in Section 5.3, that is the best one when dealing with only elective patients. The only difference relies on the allocation scheduling in which the patients having $\tilde{w}_i \leq 1$ are sequenced with the Longest Processing Time (LPT) rule. This is due to the slightly more sophisticated policy used in Duma and Aringhieri (2015) and Aringhieri and Duma (2016), that would make it difficult to implement a linear programming model for the competitive analysis.

In the baseline configuration, the overtime is allocated using the DOA rule while the number of OR sessions are proportional to the arrival rate and it does not change over time.

Table 7 Parameters of the two scenario.

Parameters	unit of measure	Scenario S_1	Scenario S_2
arrival rate			
pathways 1, 2, 3	patients/day	$12.5, 12.5, -$	$24.0, 12.0, 4.0$
initial waiting list	patients	1000	1500
MTBT URG $A, \cdots, G$	days	$8, 15, 30, 60, 90, 120, 180$	$8, 15, 30, 60, 90, 120, 180$
frequency URG $A, \cdots, G$			
pathway 1		$5\%, 15\%, 40\%, 15\%, 10\%, 10\%, 5\%$	$5\%, 15\%, 40\%, 15\%, 10\%, 10\%, 5\%$
pathway 2		$14\%, 14\%, 14\%, 14\%, 15\%, 15\%, 15\%$	$14\%, 14\%, 14\%, 14\%, 15\%, 15\%, 15\%$
pathway 3			$8\%, 9\%, 11\%, 12\%, 15\%, 15\%, 30\%$
EOT average			
pathways 1,2,3	minutes	$120, 180, -$	$150, 120, 180$
EOT deviation			
pathways 1,2,3	minutes	$75, 75, -$	$75, 75, 75$
n	ORs	50 (10 a day)	75 (15 a day)
d_{jk}	minutes	480	480
Ω	minutes	600	900

Table 7 describes the two different scenarios in which we evaluate our sharing policies. The two scenarios differ from (i) the number of specialties, (ii) the amount of available resources (number of operating rooms and weekly overtime), and (iii) the patient features, namely the arrival rates, the EOT distributions and the urgency distributions.

Tables 8 and 9 report the analysis of the proposed policies for the resource sharing. We denote with "B" the baseline configuration while those obtained by combining the different policies are denoted with an integer from 1 to 6. The combination of the different policies is described in the second and the third columns: the column "MSS" denotes the policy used for OR sharing while the column "FOA" indicates when the FOA policy has been adopted in alternative to the DOA one. Finally, for each configuration, the performance indices are reported.

Considering the scenario S_1 in Table 8, we can remark that all the configurations 1–6 indicates a general improvement of the performance indices with respect to the

baseline configuration, except for the number of cancellations. This justify the need of *ad hoc* solutions to deal with the resource sharing. Furthermore, the DOA rule seems to be more effective than the FOA, especially when considering the patient-centered indices. These considerations are confirmed also by the analysis reported in Table 9 for the Scenario S_2, in which the effectiveness of the DOA with respect to the FOA is more evident.

Considering both scenarios S_1 and S_2, the configuration 3 – that is, that adopting a flexible scheduling policy for the MSS and a dedicated allocation for the overtime – leads to a general and robust improvement of
all the indices resulting as the best configuration. Table 10 reports in more detailed way the results for that configuration reporting also the performance indices for the two pathways analyzed in the scenario S_1.

From the results for each pathway reported in Table 10, it is evident the effectiveness of the flexible scheduling policy: actually, the results for configuration 3 demonstrate a good balance of the performance indices relative to the two pathways, especially that regarding the percentage of surgeries performed within their MTBT time threshold.

7 Conclusions

The RTM of operating rooms is the decision problem arising during the fulfillment of the surgery process scheduling, that is the problem of supervising the execution of such a schedule and, in case of delays, to take the more rational decision regarding the surgery cancellation or the overtime assignment. The RTM is characterized by the uncertainty of its main parameters. In this chapter, we considered two challenging extensions of the original problem with only elective patients. The resulting quantitative analysis showed the crucial role of the RTM of the operating rooms.

The first extension is that of considering a joint flow of elective and non-elective patients. We evaluated the effectiveness of the RTM on a simulated surgical clinical

Table 8 Scenario S_1: performance indices (B is the baseline configuration).

Policies			**Performance indices**							
id	MSS	FOA	p	c	f_{MTBT}	ℓ_{avg}	t_{avg}	w_{avg}	u_{OR}	u_{over}
B			8380	352	56%	1634	56	0.99	88%	44%
1	BL		9058	387	91%	984	43	0.75	97%	77%
2	BE		9040	380	87%	973	44	0.79	97%	74%
3	FS		9050	380	93%	918	42	0.74	97%	79%
4	BL	$\surd$	8903	539	81%	1134	50	0.87	95%	77%
5	BE	$\surd$	8895	493	61%	1215	53	0.94	95%	59%
6	FS	$\surd$	8850	628	67%	1229	53	0.94	95%	69%

Table 9 Scenario S_2: performance indices (B is the baseline configuration).

Policies			Performance indices							
id	MSS	FOA	p	c	f_{MTBT}	ℓ_{avg}	t_{avg}	w_{avg}	u_{OR}	u_{over}
B			13547	609	33%	2410	56	1.09	92%	26%
1	BL		14189	626	58%	1853	51	0.91	97%	52%
2	BE		14198	605	60%	1877	51	0.92	97%	54%
3	FS		14238	577	83%	1864	51	0.88	97%	74%
4	BL	√	14031	725	40%	2145	57	1.03	96%	46%
5	BE	√	14009	718	38%	2179	58	1.04	96%	45%
6	FS	√	13858	870	39%	2289	60	1.05	95%	46%

pathway under several scenarios and also reporting a competitive analysis with respect to an offline solution obtained by solving a mixed-integer programming model. The quantitative analysis showed the capability of the online solutions to address the inherent uncertainty of the RTM determining a general improvement of the patient-centered indices without deteriorating the facility-centered ones. Further, the analysis of the competitive ratios confirmed the challenging of the problem of dealing with a flow of non-elective patients sharing the ORs with a flow of elective patients.

The second extension dealt with the management of the shared resources among different surgical pathways. The shared resources considered in our analysis are the ORs and the overtime. Our analysis demonstrated that different pathways can benefit from sharing the resources when adequate policies are adopted.

From a methodological point of view, our analysis suggested that online optimization can be a suitable methodology to deal with the inherent stochastic aspects arising in the majority of the health care problems. Although online optimization does not exploit sophisticated mathematical approaches, the competitive analysis reported in Table 6 suggested its capability to deal with the stochastic aspects of a

Table 10 Configuration 3, scenario S_1: detailed analysis.

id	pathways	Performance indices							
		p	c	f_{MTBT}	ℓ_{avg}	t_{avg}	w_{avg}	u_{OR}	u_{over}
	both	8380	352	56%	1634	56	0.99	88%	44%
B	1	4587	251	99%	40	8	0.25	80%	77%
	2	3793	101	5%	1595	114	1.89	96%	11%
	both	9050	380	93%	918	42	0.74	97%	79%
3	1	4544	289	94%	392	36	0.75	98%	92%
	2	4507	91	91%	526	47	0.73	97%	70%

problem whenever such aspects are embedded into a well-structured optimization problem, such as those arising in the health care management.

Future research avenues could consider a more systematic analysis of the optimization solutions provided in literature to deal with non-elective patients under the sharing OR policy, and to compare the results with those obtained by the dedicated OR policy.

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