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A Target-Oriented Approach: A "One-Size" Model to Suit Humans and Econs Behaviors

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Abstract

Thaler and Sunstein (2008) introduce two stereotypical decision makers: the Econs, imaginary people who always behave as strictly rational expected utility maximizers, and the Humans, real people subject to ordinary behavioral biases. This note sheds light on how the axiomatic *target-oriented approach* introduced by Castagnoli and Li Calzi (1996) may fit well the behavior of both of them. We show that although Econs and Humans use a different language, they maximize the same functional, *e.g.* the probability of meeting the goal. So declaring the probability distribution of the goal permits to elicit the agent utility function. A number of different distributions for goals are discussed and the family of the skew normal ones is proposed for its user-friendly flexibility. We show how moving the skewness parameter along its range every stereotypical decision maker's profile may be modelled.

Keywords: Goal-oriented decision making, Utility function assessment, Skew normal uncertain target

1 Introduction

Thaler and Sunstein (2008) introduce the notions of perceptive architecture (which influences what people perceive), prospective architecture (which influences what people consider) and choice architecture (which influences what people choose). Based on these architectures, they introduce two stereotypical decision makers: the Econs, imaginary people who always behave as strictly rational expected utility maximizers, and the Humans, real people subject to ordinary behavioral biases. The former group accomplishes the Savage (1954) "rationality" axioms on the basis of the von Neumann-Morgenstern (1947) (vNM) theory that normatively states that an agent should choose the action which maximizes her expected utility. For a long time that has been the standard precept to model the behavior of agents in financial markets. Yet, much experimental evidence has confirmed that in real life individuals do not always act according to these “rational” assumptions. Many paradoxes have challenged across the centuries, from St. Petersburg’s (Bernoulli, 1738) to Allais (1953). Since 1979 Amos Tversky and Daniel Kahneman developed a non-expected utility theory, the so called Prospect Theory (PT) (see Kahneman and Tversky, 1979), according to which the agents may be influenced by “heuristics and bias” and rules of thumb. Agents acting in this way are called Humans by Thaler and Sunstein (2008).
This note aims to shed light on how the *target-oriented approach* introduced by Castagnoli and Li Calzi (1996) may fit well both the Econs and Humans behaviors.
The paper is laid out as follows. In the next section, the target-oriented model is discussed. Then in Sec. 3 In Sec. 4, a number of different functionals for different agent utility functions is examined. Specifically, we highlight the potentiality of the skew-normally distributed ones. Finally, in the last section, concludes the note.

2 The normative target-oriented model and simple statistics

The target-oriented decision-making model has been formally set up by Castagnoli and Li Calzi (1996) and subsequently extended by Bordley and Li Calzi (2000). To show how it can be applied in real problems we reword a decision example discussed by Goldstein et al. (2008) and Donkers et al. (2013).
Suppose the agent is planning her risky investments she should commit to in view of saving for her retirement. She does not know which investment she likes best but does know where she wants to retire and the current cost of living (and cost of housing) for retirees in that area. She forecasts that the total wealth required for this retirement is t. If she were absolutely certain of this forecast, the agent would make decisions to deliver a final wealth X that maximizes the probability of exceeding t. But because she knows that the cost of living may change, she recognizes that the wealth required for retirement must be described by a *random variable* T. As a result, she makes decisions which maximize the probability of X exceeding her target T.

Let model the desired target by the random variable $T = t + \varepsilon$, where t is a constant that may be interpreted as the expected value of the uncertain goal T and ε is a zero-mean error term that is *stochastically independent* of the risky options at the disposal to the agent. So the optimization guideline is to pick the risky option that maximizes the probability $P(X \geq T)$. Suppose that the (stochastically independent) target T has distribution function $u(x)$ and that the final wealth X associated with an investment has distribution function $F(x)$. Then

$$P(X \geq T) = \int P(x \geq T) dF(x) = \int u(x) dF(x) \qquad (1)$$

By Equation (1) the following statement follows (see Castagnoli and LiCalzi, 1996): to maximizing the probability of meeting a random target $T = t + \varepsilon$ with distribution function $u(x)$ is equivalent to maximizing the expected utility function $u(x)$. It is worthwhile noting that Equation (1) was already pinpointed by Borch (1968) who using a different terminology states that ranking among several possible uncertain insurance prospects to choosing the prospect with highest survival probability *is equivalent* to maximizing the expected utility.

In conclusion, although Econs and the Humans use a different language, they converge in maximizing the same functional (1), *e.g.* the probability of meeting the goal.

3 How to choose the suitable utility function?

A standard assumption in neoclassical economics is that agents display risk aversion at *all* levels of their wealth. That implies that an agent acting according to the vNM axioms be endowed with a concave utility function. That fits the case of an Econ agent. Kahneman and Tversky (1979) with the famous adage "losses loom larger than gains" formalize the popular belief that people in real life impute greater value to a given item when they give it up than when they acquire, *e.g.* they display loss aversion. This "endowment effect" clearly violates the vNM-risk aversion axiom. As a substitute to a concave utility function, Kahneman and Tversky (1979) suggest an S-shaped utility function, concave over gains and convex over losses, able to grasp the risk aversion and the loss aversion of the agent for wealth levels respectively below and above the inflection point. As mentioned by Heath et al. (1999), the inflection point represents the goal to hit for an S-shaped utility function.

4 From the target distribution vs. the utility function

In Equation (1) $u(x)$ is assumed a non-decreasing function to interpret the basic rational axiom stating that "agents prefer more to less", but no restrictions are imposed on the concavity. Specifically, according to the Kahneman and Tversky PT: *(i)* agents evaluate outcomes, not according to final wealth levels, but according to their perception of gains and losses relative to a reference point, typically the desired goal (see Heath et al., 1999). That phenomenon is called *framing effect*; *(ii)* agents are risk-averse for gains and risk-seeking for losses;

and *(iii)* agents are more sensitive to losses than to gains of the same magnitude and display *loss aversion*. It is worthwhile noting that this behavior *(i)-(ii)-(iii)* has been documented in several experimental works (see Thaler and Sunstein, 2008). Condition *(ii)* means that the utility function is concave over gains and convex over losses, *e.g.* it is an S-shaped function concave for gains above the reference point *t* and convex for losses below the reference point *t*. That means that differences between small gains or losses close to the reference point are assigned a high value, whereas differences further away from the reference point are assigned smaller values, whereas *(iii)* implies that the S-shaped utility function is steeper for losses than for gains (i.e. tails are not symmetrical respect to the reference point). Above can be reworded using the target-based language. Attitude *(ii)* means that the distribution function of the target *T* is S -shaped (*e.g.* for a Gaussian target). A sufficient condition for S-shaping is that *T* is unimodal with the inflexion point at the mode. However, the two tails of *T* may not have the same curvature. Hence, this kind of utility function is consistent with *(iii)* as *u* has a steeper slope over losses than over gain. That is the case of a *unimodal and positively skewed distribution* (*e.g.* for a lognormal target), this implies that the probability density for the target has fatter right tail, that implies a behavior that is less risk averse over gains than it is risk seeking over losses.
For an explanatory purpose, let discuss a number of common distributions that may fit *T*. At the end, we propose that of skew-normal that thanks the proper modulation of the skewness parameter permits to fit any agent's risk preferences.

4.1 Exponentially distributed targets for fitting rational Econs

Let suppose the agent evaluates the stochastic target T is distributed as an exponential random variable with parameter $\lambda > 0$ and cumulative distribution $F(x;\lambda) = 1 - e^{-\lambda x}$. By (3) that means that the agent is endowed with the concave vNM-utility function $U(x;\lambda) = 1 - e^{-\lambda x}$ with constant absolute risk aversion coefficient λ. It follows that the risk aversion of the agent is not influenced neither by her wealth level nor the probability of hitting the target *T*. So she acts as a Econ.

4.2 Normally distributed targets for fitting behavioral Humans

Let now the target $T \sim N(t,\sigma)$ be a normal random variable. Since we can write $T = t + \varepsilon$, $\varepsilon \sim N(0,\sigma)$ is interpretable as a zero-mean white noise about the mean *t*. The distribution function $F(x;t,\sigma) = \frac{1}{2}\left[1 + erf\left(\frac{x-t}{\sqrt{2\sigma^2}}\right)\right]$ corresponds to a prospect theory S-shaped utility function with inflection point at $x = t$ in correspondence of the mean value *t* of *T* (see Bordley et al., 2014).

4.3 Uniformly distributed targets for fitting risk neutral agents

Let now the target $T \sim Unif(a,b)$ be an uniform random variable, with distribution function $F(x) = \frac{x-a}{b-a}$ for $a \leq x < b$, $F(x) = 0$ for $x < a$ and $F(x) = 1$ for $x \geq b$. It follows that the agent is endowed with the linear utility function $U(x) = F(x)$ and she is risk neutral. In such case the expected utility of wealth is simply given by a linear function of expected wealth. So, maximizing the expected wealth is equivalent to maximizing the probability to exceeding the target.

However to fit the attitudes *(i)-(ii)-(iii)* we suggest the use of a skewness parameter-dependent family, the skew-normal distributions.

4.4 Skew-normally distributed targets for fitting rational Econs and behavioral Humans

The original skew-normal definition is due to Azzalini (1985), and in recent years it is becoming in financial and insurance modelling (see Eling, 2012) for a recent review see Azzalini (2013). We use an alternative definition as the original Azzalini's one that appears more suitable in this context. A continuous random variable *X* is skew-normally distributed if and only if the following representation holds:

$$X = \xi + \omega\left(\delta|Z_1| + \sqrt{1-\delta^2}Z_2\right), \tag{2}$$

Where $\delta \in [-1,1]$; Z_1 and Z_2 are independent standard Normal, and $|\cdot|$ stands for Half-Normal; ξ and ω (with $\omega \geq 0$) are the location and the scale parameters, respectively. Skew-normal distributions are an extension of normal ones, in fact they reduce to the standard normal random variable for $\delta = 0$ and to the Half-Normal when $\delta = \pm 1$. Calibrating the parameter δ, called the *Azzalini skewness parameter*, the presence of the Half–Normal $|Z_1|$ can be weighted on one-side tail of *X*. The more δ is positive (negative), the more the probability mass is pronounced on the right (on the left) tail, so moving δ the final distribution shape changes. Since $|Z_1|$ has a concave distribution and Z_2 a S-shaped one, their combination permits to model the tails as desired. The skew-normal is unimodal and it is diminishing in sensitivity if $\delta < 0$.

In conclusion, modelling the stochastic target T with the skewness-dependent Skew Normal distribution permits to suit well both Econs and Humans, in fact:

- If $\delta = \pm 1$, then T coincides with a Half-Normal variable with strictly concave distribution function $F(x;\sigma) = \left[erf\left(\frac{x}{\sqrt{2}\sigma}\right)\right]$, this means that F is the utility function of a rational vNM Econ;
- If $\delta = 0$ then *T* coincides with a normal variable with S-shaped distribution function *F*, so the utility function although is s-shaped is symmetrical to the reference point *t* and represents a behavioral PT Human acting;
- If $-1 < \delta < 1$ and $\delta \neq 0$ then T has an S-shaped distribution function with asymmetrically steeped tails. Specifically, if $\delta < 0$ the probability mass is more on the left tail than on the right one. That perfectly captures the loss-adverse Human feeling that losses hurt more than gains.

In conclusion, it is sufficient to moving the Azzalini's δ skewness parameter to fit the risk profile of any agent. The advantage in using this class of distributions is that it is intuitively understandable the impact of δ on the shape of the probability distribution (see the distribution plotting program developed by Adelchi Azzalini available on The Skew-Normal Probability Distribution homepage). Moreover, the possible asymmetrical tails of T permit to capture the agent loss aversion summarized by the adage "losses loom larger than gains".

Conclusion

In this short note we have shown that the target-oriented model may represent the decision making of the both rational Econs and behavioral Humans. Due to the Equation (1) (see Castagnoli and LiCalzi, 1996; Bordley and LiCalzi, 2000) they are maximizers of the probability to exceeding the subjective stochastic target or equivalently wording, maximizers of the expected utility/utility function.

The key difference between the two decision maker groups is the shape of their utility/utility function: the Econs accomplishing the von Neumann-Morgenstern (1947) axioms, so display risk aversion and are endowed with concave utility functions, whereas the Humans acting according to the Kahneman and Tversky (1991) prospect theory, may display risk aversion and risk seeking at given wealth levels. So, their preferences are fit by S-shape utility functions. Moreover if they display loss aversion their S-shaped utility functions have different steeped tails.

Summarizing, the target-oriented approach for decision making:

- undergoes the optimizing principle according to which agents should choose the action which maximizes the probability of meeting the target (Manski, 1988);
- accomplishes the rational expected utility maximization criterion (see Bordley and Li Calzi, 2000);
- is compatible with the prospect theory (Kahneman and Tversky, 1979, 1991);
- if the target is assumed skew normally distributed, the steepness of the left and right tail of the S-shape utility function can be intuitively regulated by choosing the skewness-parameter δ.

And last but not least, the target-oriented approach has an *immediate linkage with practice.* In fact, using a computer program designed to gaining insight into agent's preferences as the Distribution Builder (DB) developed by Sharpe et al. (2000) it is possible to elicit the target probability distribution in an user-friendly way.

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