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Dynamic Asymmetries in House Price Cycles: A Generalized Smooth Transition Model

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Abstract

This paper investigates dynamic asymmetries in house price cycles. We introduce an *ad-hoc* nonlinear model to capture real estate cycles. The suggested model involves a particular parametrization of the transition function used in the transition equation of a smooth transition autoregressive model which improves the fit in the non-central probability region. The dynamic symmetry in house price cycles is strongly rejected in a number countries taken into consideration. Further, our results show that the proposed model performs well in a out of sample forecasting exercise.

Keywords: house price cycles, dynamic asymmetries, non-linear models, forecasting.

JEL Classification: C10, C31, C33.

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1 Introduction

Academics are interested in modelling the housing market due to spillover effects on the whole economy. Accurate modelling of house prices is also crucial for policy makers interested in preventing unsustainable swings in the housing markets. However, modeling house price appreciation proved to be a challenging task due to the strong vulnerability of the housing markets to finance systems, regulatory structures, and market fundamentals.

In the literature there is growing consensus that economic series exhibit nonlinear behavior. Well know examples of such series are GDP, industrial production, and unemployment rates (see for example Neftci (1984), Sarantis (2001) among others). Common sense suggests that house prices also incorporate some nonlinear properties. Abelson *et al.* (2005) suggest that during house price increse, households exhibit forward looking behavior, while the equity constraint factor plays only a minor role. On the other hand, households are less willing to buy or sell properties during contraction phases due to loss aversion and more pronounced equity constraints causing stickiness on the downside of the housing market cycle.

The purpose of this paper is to seek empirical evidence of asymmetric behaviour in house prices cycles. In order to capture dynamic asymmetries in the housing market an *ad-hoc* regime-switching model is proposed. Almost all previous empirical work relating to house prices modelling is based on linear specifications (see for example Abraham and Hendershott (1996)). Yet, the well established literature on cyclical behavior of macroeconomic variables suggests that nonlinearity of house prices should stem from the asymmetric properties of house price determinants like GDP, interest rate or bank lending. Few available empirical works corroborate this hypothesis. For example, Kim and Bhattacharya (2009) use smooth transition models (STAR) to test for nonlinearity in the regional hosing market in the United States. It is found that West and Northeast regions (and to some extent also the South) are characterized by high speed transition between regimes. Balcilar, Gupta and Miller (2015) use a STAR-type model to forecast house price distributions. Nonlinear models are also used in Crawford and Fratantoni (2003) to forecast house price changes.

Regime-swiching models such as the STAR allow the dynamic of house price growth rates to evolve according to a smooth transition between regimes that depends on the sign and magnitude of past realization of house price growth rates (see Chan and Tong (1986)). The low speed of transition between different regimes in house price growth found in empirical studies validate the choice of smooth transition models. A possible shortcoming of these type of nonlinear models is that in the model specification a symmetric transition function is used to capture oscillations from the conditional mean of the changes in house price series. Although STAR-type models efficiently describe nonlinearity in house price growth rates, the most commonly used transition functions may not be suitable to capture dynamic asymmetries in real estate cycles.

In this paper we address the following question: Do contractions in the house prices are steeper than expansions? Or, does the amplitude of troughs in house prices exceeds that of picks? From the methodological point of view the question is as follows: Is the rate of change in the left tail of the transition function of the STAR-type model different with respect to that of the right tail? And if so, how much? We argue that being the transition function generally adopted in STAR-type models symmetric, by construction, the resulting model may not be suitable to address the issues above. We believe that what economic literature has called "asymmetry" may better be reflected by a statistical model that uses a more general parametrization of the transition function. Accordingly, in this work it is suggested that a class of models indexed by two shape parameters be used to model dynamic asymmetry in house price cycles. By using shape parameters that influence the symmetry and heaviness of the tails of the fitted transition equation the proposed model may be more suitable to fit the non-central probability regions and therefore better able to capture the asymmetries that have been often found in empirical research. The suggested transition function, which is in the spirit of Stukel (1988), has two parameters governing the two tails of the sigmoid function in the nonlinear component of the model. The advantage of the proposed parameterization is that the resulting model is able to preserve smoothness of the transition function without requiring any restriction in the parameters. This feature may be appealing in empirical applications. Asymmetric behaviour over the business cycle has long been object of interest in applied and theoretical research. It is therefore not surprising that several variations of the STAR model have been suggested in the literature. For example, Sollis, Leybourne and Newbold (1999) suggest to raise the transition function of the STAR to an exponential. Alternatively, Sollis, Leybourne and Newbold (2002) propose to add a parameter inside the transition function in order to control asymmetry of both tails of the transition function. The suggested procedures successfully address the issue of dynamic asymmetry in several classical macroeconomic series. A possible shortcoming, however, is that the transition function suggested by Sollis *et al.* (2002) may be non-smooth. Also, the effect of increasing the asymmetry of the parameter may resolve to a shift of the transition function (see Zanetti Chini (2014) and Lundbergh and Teräsvirta (2006)).

Our results reveal several insights on the patterns of the housing markets under consideration. In particular, it is found that house prices deviate above their mean at an exponential rate, whereas they return at the equilibrium level at a logarithmic rate. This implies that when improving economic conditions boost housing demand above the potential stock, prices raise rapidly above their expected level. On the other side, house prices fall slowly when economic condition worsen and changes in house prices are below the expect value. We then consider whether forecasting with the nonlinear proposed model leads to improvements in forecast performance over forecasting with an autoregressive linear model. Comparing several criteria it is found that, overall, the proposed nonlinear model performs better than its symmetric counterpart.

The reminder of this paper proceeds as follows. In Section 2 some of the literature relating house price cycles is reviewed. In Section 3 the specification, estimation and testing of the proposed nonlinear model is presented. In Section 4 the empirical results are described. An Appendix reports the results of a small Monte Carlo experiment and the derivations for the misspecification tests used in Section 4. Finally, some concluding remarks are given in Section 5.

2 Real Estate Price Cycles

A large body of literature shows that real estate cycles are closely related to the business cycle. From the theoretical point of view, in the literature models used to describe housing market cycles fall within the demand-supply framework, where supply is assumed to be rigid. From one side, improving of the economic conditions tend to increase income of households and therefore to boost housing demand. On the other side, when property prices raise above the replacement costs, property developers initiate the construction process based on current property prices. However, the supply of new properties is by definition a slow process. By the time new properties are delivered economic conditions may have changed for the worse and prices start to decline. This inertia of supply responsiveness causes asymmetries in the real estate cycles (Davies and Zhu (2005)).

Numerous studies have explained real estate cycles using demand and supply framework. For example, Abraham and Hendershott (1996) describe an equilibrium price level to which the housing market tend to adjust. The authors divide the determinant of house price appreciation in two groups: one that explains changes in the equilibrium price and another that accounts for the adjustment mechanism in the equilibrium process. Slow adjustment toward the equilibrium can be regarded as an indication of asymmetries in real estate cycles. Muellbauer and Murphy (1997) explore the behaviour of house prices in the UK. The authors suggest that presence of transaction costs associated with the housing market cause important nonlinearity in house price dynamics. Further, Seslen (2004) argues that households exhibit rational responses to returns on the upside of the market but do not respond symmetrically to downturns. On an upswing of the housing cycle households exhibit forward looking behaviour and are more likely to trade up, with equity constraint playing a minor role. On the other hand, households are less likely to trade when prices are on the decline causing stickiness on the downside of the housing market cycle.

Traditionally, in empirical literature house price dynamics have been analyzed using error correction mechanisms to investigate short-run deviations from the house fundamental value. For example, Hendershott and Abraham (1993) estimate a cointegrated model which includes lagged house price changes among other explanatory variables. They found evidence of slow adjustment toward the equilibrium which implies a cyclical adjustment path. Abelson, Joyeux and Milunovich (2005) estimate an asymmetric threshold cointegrated model to investigate nonlinearity in house prices in Australia. Malpezzi (1999) analyzes the impact of supply and demand factors on the path of house price adjustments. Most studies conclude that some locations may be more prone to house price cycles, in particular, to rigid supply conditions delaying response to demand-side shocks.

A recent strand of the literature has related asymmetric cyclical movements of housing markets to nonlinearity of change of credit cycles. Theoretical research has argued that endogenous developments in financial markets can greatly amplify the effect of small income shocks through the economy. In an influential paper Bernanke, Gertler and Gilchrist (1996) call this amplification mechanism the "financial accelerator" or "credit multiplier". The key idea behind the financial accelerator is that under the assumption of fixed leverage ratio positive or negative shocks to income have a procyclical effect on the borrowing capacity of household and firms. This in turn affects housing prices. Positive shocks to household income translate into larger house prices increase where prevailing leverage ratio are higher (e.g. U.S. and UK) and smaller in countries where such leverage ratios are lower (e.g. Italy). Following a similar argument, Kiyotaki and Moore (1997) show that rising asset prices may ignite a lending boom by increasing collateral values. Reversal in fundamentals then increase loan default rate. Rajan (1994) suggests that herd behaviour by bank managers can also feed a rapid growth spiral: shortsighted managers choose to behave like their peers as losing market share hurts their jobs prospect throw reputation effect, and mistakes are punished more lenient when they are common across banking sector. In a related work by Hott (2011) a theoretical framework that explains the relation between real estate prices and mortgage default is proposed. It is argued that on one hand banks contribute to the creation of real estate cycles by providing increasing level of financial resources for real estate purchases. On the other hand, banks suffer high losses when the tide changes. An attractive feature of the theoretical framework in Hott (2011) is that irrational expectations of banks play a crucial role in characterizing bank behaviour. Therefore, irrational behaviour of banks contributes to the creation of real estate cycles.

All in all, consensus literature suggests that asymmetric cycles are an important feature of house prices. Thus, if real estate markets feature asymmetric patterns of adjustment, models that take account of such nonlinearity may perform better than those which impose symmetric adjustment to rising and falling prices. Accordingly, below a nonlinear model which allows for dynamic asymmetric adjustments in housing price cycles is presented. We refer to the proposed model as generalized smooth transition model (GSTAR).

3 The GSTAR Model

Let be y_t a realization of a the house price series observed at $t = 1-p,\ 1-(p-1), .., -1, 0, 1, T-1, T$. Then, the univariate process $\{y_t\}_t^T$ can be specified using the following model

$$y_t = \phi' z_t + \theta' z_t G(\gamma, h(c_k, s_t)) + \epsilon_t, \qquad \epsilon_t \sim I.I.D.(0, \sigma^2) \tag{1}$$

$$G(\gamma, h(c_k, s_t)) = \left(1 + \exp - \left\{ \prod_{k=1}^{K} h\left(c_k, s_t\right) \right\} \right)^{-1}, \tag{2}$$

$$h(c_k, s_t) = \left\{ \begin{array}{ll} \gamma_1^{-1} \exp(\gamma^{-1} |s_t - c_k| - 1, & \text{if } \gamma_1 > 0 \\ s_t - c_k, & \text{if } \gamma_1 = 0 \\ -\gamma_1^{-1} \log(1 - \gamma_1 |s_t - c_k|), & \text{if } \gamma_1 < 0 \end{array} \right\} \tag{3}$$

for $(s_t - c_k) > 0$ (or, equivalently, $h(c, s_t) > 1/2$) and

$$h(c_k, s_t) = \left\{ \begin{array}{ll} \gamma_2^{-1} \exp(\gamma^{-1} |s_t - c_k| - 1, & \text{if } \gamma_2 > 0 \\ s_t - c_k, & \text{if } \gamma_2 = 0 \\ -\gamma_2^{-1} \log(1 - \gamma_2 |s_t - c_k|), & \text{if } \gamma_2 < 0 \end{array} \right\} \tag{4}$$

for $(s_t - c_k) \leq 0$ (or, equivalently, $h(c_k, s_t) < 1/2$).

In equation (1)-(4) the vectors $z_t = (1, y_{t-1}, \ldots, y_{t-p})'$, $\phi = (\phi_0, \phi_1, \ldots, \phi_p)'$, $\theta = (\theta_0, \theta_1, \ldots, \theta_p)'$ are parameter vectors. The process $\{\epsilon_t\}_t^T$ in (1) is assumed to be a martingale difference sequence with respect to the history of the time series up to time $t-1$, denoted as $\Omega_{t-1} = [y_{1-(p-a)}, y_{t-p}]$, with $E[\epsilon_t | \Omega_{t-1}] = 0$ and $E[\epsilon_t^2 | \Omega_{t-1}] = \sigma^2$.

The expression $G(\tilde{\gamma}, h(c_k, s_t))$ defines the transition function, which is assumed to be continuously differentiable with respect to the scale parameters $\tilde{\gamma} \in (\gamma_1, \gamma_2)$ and bounded between 0 and 1. Also, $G(\tilde{\gamma}, h(c_k, s_t))$ is continuous in the function $h(c_k, s_t)$ and $h(c_k, s_t)$ is strictly increasing in the transition variable s_t. These assumptions on the transition function are required in order to circumvent the identification problem highlighted in Davies (1977) when testing for dynamic symmetry.

The transition variable s_t is assumed to be a lagged endogenous variable, that is, $s_t = y_{t-d}$ for certain integer $d > 0$. Note that s_t can also be an exogenous variable or possibly a function of lagged endogenous variables. The parameters $c_k \in \{1, 2\}$ are the location parameters.

To simplify the notation in (2)-(4) it is convenient to denote the kernel of the model corresponding to the k-th location as $\eta_{k,t} \equiv s_t - c_k$. Therefore, the transition function in (2)-(4) can be written as

$$G\left(\tilde{\gamma}, h\left(\eta_{k,t}\right)\right) = \left(1 + \exp\left\{\prod_{k=1}^{K}\left[\begin{array}{c} h\left(\eta_{k,t}\right) I_{(\gamma_1 \leq 0, \gamma_2 \leq 0)} + h\left(\eta_{k,t}\right) I_{(\gamma_1 \leq 0, \gamma_2 > 0)} + \\ +h\left(\eta_{k,t}\right) I_{(\gamma_1 > 0, \gamma_2 \leq 0)} + h\left(\eta_{k,t}\right) I_{(\gamma_1 > 0, \gamma_2 > 0)} \end{array}\right]\right\}\right)^{-1}. \tag{5}$$

Asymmetric behaviour in house price dynamics is introduced in the model by equations (3)-(4). In particular, equation (3) models the higher tail of the probability function, whereas equation (4) models the lower tail of the probability function. The velocity of the transition function is controlled by the slope parameters $\tilde{\gamma}$. In (5) if $\tilde{\gamma} > 0$, the function $h(\eta_{k,t})$ is an exponential rescaling which increases more quickly than a standard logistic function. On the other side, if $\tilde{\gamma} < 0$, the function $h(\eta_{k,t})$ is a logarithmic rescaling which increases more slowly than a standard logistic function. The case with $h(\eta_{k,t}) = \eta_{k,t}$ implies that the function nests a one-parameter symmetric logistic STAR model (see Teräsvirta (1994)) with slope $\gamma_1 = \gamma_2 = \gamma$.

Different choices of the transition function $G(\tilde{\gamma}, h(c_k, s_t))$ give rise to different types of regime-switching behaviour. If $k = 1$ in (2) the parameters in the right hand side of (1) change monotonically as a function of s_t from ϕ to $\phi + \theta$ and the corresponding transition function is given by

$$G\left(\tilde{\gamma}, h\left(\eta_{1,t}\right)\right) = \left(1 + \exp\left\{-h\left(\eta_{1,t}\right) I_{(\gamma_1 \leq 0, \gamma_2 \leq 0)} + h\left(\eta_{1,t}\right) I_{(\gamma_1 \leq 0, \gamma_2 > 0)} + h\left(\eta_{1,t}\right) I_{(\gamma_1 > 0, \gamma_2 \leq 0)} + h\left(\eta_{1,t}\right) I_{(\gamma_1 > 0, \gamma_2 > 0)}\right\}\right)^{-1} \tag{6}$$

with $h(\eta_{1,t})$ given in equation (3) and (4).

When $k = 2$ and $c_1 \neq c_2$, the model in (1) nests the following nonlinear model with second order generalized logistic function

$$G(\tilde{\gamma}, h(\eta_t)) = 1 - \exp\left\{-h(\eta_{2,t})\right\}, \tag{7}$$

where

$$h(\eta_{2,t}) = \left\{\begin{array}{ll} \gamma_1^{-1}\exp(\gamma_1\left|(s_t - c_1)(s_t - c_2)\right| - 1), & \text{if } \gamma_1 > 0 \\ (s_t - c_1)(s_t - c_2) & \text{if } \gamma_1 = 0 \\ -\gamma_1^{-1}\log(1 - \gamma_1\left|(s_t - c_1)(s_t - c_2)\right|), & \text{if } \gamma_1 < 0 \end{array}\right\}, \tag{8}$$

for $(s_t - c)^2 > 0$ (or, equivalently, $h(\eta_t) > 1/2$) and

$$h(\eta_{2,t}) = \left\{\begin{array}{ll} -\gamma_2^{-1}\exp(\gamma_2^{-1}\left|(s_t - c_1)(s_t - c_2)\right| - 1), & \text{if } \gamma_2 > 0 \\ (s_t - c_1)(s_t - c_2) & \text{if } \gamma_2 = 0 \\ -\gamma_2^{-1}\log(1 - \gamma_2\left|(s_t - c_1)(s_t - c_2)\right|), & \text{if } \gamma_2 < 0 \end{array}\right\}, \tag{9}$$

for $(s_t - c_1)(s_t - c_2) < 0$ (or, equivalently, $h(\eta_{2,t}) < 1/2$), whith $\eta_t \equiv \eta_t = (s_t - c_1)(s_t - c_2)$.

A particular case of GSTAR holds when $k = 2$ and $c_1 = c_2$, in which case the model (1) nests an exponential generalized exponential autoregressive model, which is defined as in (7)-(9), apart the fact that $h(\eta_{2,t}) = (s_t - c)^2$ when $\gamma_1 = 0$ for $(s_t - c)^2 > 0$ and $\gamma_2 = 0$ for $(s_t - c)^2 \leq 0$. In this case, the parameters $\phi + \theta G(\cdot)$ change asymmetrically at some (undefined) point where the function reaches its own minimum.

The GSTAR nests several well known linear and non-linear models. Before considering the estimation procedure of the GSTAR it is of interest at this point to relate the proposed model to other models available in the literature.

First, the model in (1) with the transition function in (5) implies that the GSTAR model reduces to a one-parameter symmetric logistic STAR model (see Teräsvirta (1994)) with slope $\gamma_1 = \gamma_2 = \gamma$. However, with respect to the STAR model a clear advantage of the indicator functions in (5) is that slope parameters are not constrained. Positiveness of the slope parameter is an identifying condition which was a crucial assumption in Teräsvirta (1994). Second, the transition function in (5) nests an indicator function $I_{(s_t > c)}$ when $\tilde{\gamma} \to +\infty$. Therefore, the GSTAR reduces to the model in Tong (1983) when $\tilde{\gamma} \to +\infty$ and it becomes a straight line around 1/2 for each s_t when $\tilde{\gamma} \to -\infty$. Finally, the GSTAR model nests a linear AR model when $\tilde{\gamma}$ is a null vector. In this case $h(\eta_t)$ in (5) reduces to a simpler function given by

$$\tilde{h}(\eta_t) = \left\{ \begin{array}{ll} \gamma_1^{-1} \exp\left(\gamma_1 \left|\eta_t\right| - 1\right) & \text{if } \gamma_1 > 0 \\ 0 & \text{if } \gamma_1 = 0 \\ \gamma_1^{-1} \log\left(1 - \gamma_1 \left|\eta_t\right|\right) & \text{if } \gamma_1 < 0 \end{array} \right\}, \tag{10}$$

for $\eta_t \geq 0$ $(\mu > 1/2)$ and

$$\tilde{h}(\eta_t) = \left\{ \begin{array}{ll} \gamma_2^{-1} \exp\left(\gamma_2 \left|\eta_t\right| - 1\right) & \text{if } \gamma_2 > 0 \\ 0 & \text{if } \gamma_2 = 0 \\ \gamma_2^{-1} \log\left(1 - \gamma_2 \left|\eta_t\right|\right) & \text{if } \gamma_2 < 0 \end{array} \right\},$$

for $\eta_t < 0$ $(\mu < 1/2)$. This special case is used below to construct a test for the null of linearity against the alternative hypothesis of dynamic asymmetry.

a) Modelling Strategy

Following standard practice related to nonlinear modelling literature, the first step in the model specification procedure is to test whether a linear $AR(p)$ representation is adequate for the data at hand. If the answer is negative, then the second step involves the selection of a nonlinear symmetric model. Crucially, the next step is to test for dynamic symmetry in the house price series. The resulting general-to-specific type of modelling strategy is summarized below:

Step 1. Estimate a suitable linear autoregressive model.

Step 2. Test for the linearity of the model.

Step 3. If the linearity hypothesis is rejected, test the symmetry of the tails of the transition function.

Step 4. If the hypothesis of symmetry is rejected, estimate the GSTAR model.

In Step 1 the correct order autoregressive order p is selected using for example Bayesian Information Criterion and Portmanteau test for serial correlation. In Step 2 linearity can be tested using the inference procedure suggested in Luukkonen, Saikkonen and Teräsvirta (1988). As far as Step 3 is concerned, the dynamic symmetry hypothesis is tested using a LM-type test. The test statistic is described in detail below. Finally, in Step 4 the choice of the transition function can be guided by economic theory. Alternatively, a procedure suggested in Tsay (1989) and Teräsvirta (1994) can be used.

b) Testing for Dynamic Symmetry

Testing for linearity against dynamic symmetry is problematic as the GSTAR model is only identified under the alternative hypothesis. In order to overcome the identification problem, following an idea in Luukkonen, Saikkonen and Teräsvirta (1988), we linearize the transition function by taking the third order Taylor expansion of $G(\cdot)$. This approximation leads to an augmented artificial model that can be used to calculate an LM test statistic.

Consider (1) with $G(\tilde{\gamma}, \tilde{h}(\eta_t))|_{\gamma=0}$ and define $\tau = (\tau_1, \tau_2)'$, where $\tau_1 = (\phi_0, \phi')'$, $\tau_2 = \gamma$. Let $\hat{\tau}_1$ the LS estimator of τ_1 under $\mathcal{H}_0 : \tilde{\gamma} = 0$, $\hat{\tau} = (\tau_1', 0')'$. Moreover, let $\Xi = [\phi, \theta, \tilde{\gamma}, c]$, $z_t(\tau) = \frac{\partial \epsilon_t}{\partial \tau}$ and $\hat{z}_t = z_t(\hat{\tau}) = (\hat{z}_{1,t}, \hat{z}_{2,t})$, where the partition conforms to that of τ. Then the general form of LM statistic is

$$LM(\Xi) = \frac{1}{\hat{\sigma}^2} \hat{u}' \hat{Z}_2 (Z_2' \hat{Z}_2 - \hat{Z}_2' \hat{Z}_1 (\hat{Z}_1' \hat{Z}_1)^{-1} \hat{Z}_1' \hat{Z}_2)^{-1} \hat{Z}_2' \hat{u}, \tag{11}$$

where $\hat{u}_t = [\hat{u}_1, .., \hat{u}_T]'$ is as previously defined, $\hat{\sigma}^2 = \frac{1}{T} \sum_1^T \hat{u}_t^2$ and $\hat{u}_t = y_t - \hat{\tau}' z_t$, $\hat{Z}_i = (\hat{z}_{i1}, \ldots, \hat{z}_{it}, \ldots, \hat{z}_{iT})'$, $i = \{1, 2\}$, $t = 1, \ldots, T$.

The linearized GSTAR model is given by

$$y_t = \phi' z_t + \theta' z_t T_3 \left[h\left(\eta_{k,t}\right) I_{(\gamma_1 \leq 0, \gamma_2 \leq 0)} + h\left(\eta_{k,t}\right) I_{(\gamma_1 \leq 0, \gamma_2 > 0)} + h\left(\eta_{k,t}\right) I_{(\gamma_1 > 0, \gamma_2 \leq 0)} + h\left(\eta_{k,t}\right) I_{(\gamma_1 > 0, \gamma_2 > 0)} + \epsilon_t' \right]. \tag{12}$$

The formulation in (12) leads to the following auxiliary regression for testing linearity and symmetry

$$\hat{u}_t = \hat{z}_{1t}' \tilde{\beta}_1 + \sum_{j=1}^{p} \beta_{2j} y_{t-j} y_{t-d} + \sum_{j=1}^{p} \beta_{3j} y_{t-j} y_{t-d}^2 + \sum_{j=1}^{p} \beta_{4j} y_{t-j} y_{t-d}^3 + v_t \ , \tag{13}$$

where $v_t \sim I.I.D.(0, \sigma^2)$, $\tilde{\beta}_1 = (\beta_{10}, \beta_1')'$, $\beta_{10} = \phi_0 - (c/4)\theta_0$, $\beta_1 = \phi - (c/4)\theta + (1/4)\theta_0 e_d$, $e_d = (0, 0, \ldots, 0, 1, 0, \ldots, 0)'$ with the d-th element equal to unit and $T_3(G) = f_1 G + f_3 G^3$ is the third-order Taylor expansion of $G(\Xi)$ at $\tilde{\gamma} = 0$, $f_1 = \partial G(\Xi)/\partial \Xi|_{\gamma=0}$ and $f_3 = (1/6)\partial^3 G(\Xi)/\partial \Xi|_{\gamma=0}$, $G(\Xi)$ being defined in previous section. To test the null hypothesis

$$\mathcal{H}_0 : \beta_{2j} = \beta_{3j} = \beta_{4j} = 0 \qquad (j = 1, \ldots, p). \tag{14}$$

in (13) the following test statistic can be used

$$LM = (SSR_0 - SSR)/\hat{\sigma_v}^2 \ , \tag{15}$$

were SSR_0 and SSR denotes the sum of squared estimated residuals from the estimated auxiliary regression (12) and under the null and alternative, respectively, and $\sigma_v^2 = (1/T)SSR$. Under the null hypothesis the LM test in (15) is asymptotically distributed as χ_p^2 distribution.

c) Estimation

As far as the estimation of the GSTAR model is concerned, following standard practice (see for example Leybourne, Newbold and Vougas (1998)), estimation is done by concentrating the sum of square residuals function with respect to the vectors θ and ϕ, that is minimizing:

$$SSR = \sum_{t=1}^{T}(y_t - \hat{\psi}' x_t')^2, \tag{16}$$

where

$$\hat{\psi} = [\hat{\phi}, \hat{\theta}] = \left(\sum_{t=1}^{T} x_t'(\tilde{\gamma}, c) x_t(\gamma, c) \right)^{-1} \left(\sum_{t=1}^{T} x_t'(\tilde{\gamma}, c) y_t \right),$$

and

$$x_t(\hat{\gamma}, \hat{c}) = \left[z_t z_t' G\left(\hat{\gamma}, h\left(\hat{c}, s_t\right)\right) \right].$$

Note that under the assumption that the vectors $\tilde{\gamma}$ and c are known and fixed, the GSTAR model is linear in the vectors θ and ϕ. Therefore, the non-linear least square minimization problem reduces to a minimization on three (four) parameters and can be solved via a grid search over γ_1, γ_2, c. In our applications, both γ_1 and γ_2 are chosen between a minimum value of -10 and a maximum of 10 with rate 0.5; the grid for parameters c is the set of values computed between the 10^{th} and 90^{th} percentile of s_t with rate computed as the difference of the two and divided for an arbitrarily high number (here, 200).

Before the estimated GSTAR model can be accepted as adequate, it should be subjected to misspefication tests. Important hypotheses which might be tested are the hypothesis that there is no residual correlation, no remaining nonlinearity and parameter constancy. These inference procedures are derived in Appendix 1.

4 Empirical Results

In this section the proposed GSTAR model is used to model dynamic asymmetries in house prices cycles. The outline of this section is as follows. First the data are described in some details. Next, the estimation results of the GSTAR model are reported. The out of sample forecasting properties of the proposed model are also evaluated.

4.1 Data and Descriptive Statistics

The data under consideration are obtained from the Bank for International Settlement and relate to quarterly nominal residential properties prices over the period 1970:1 to 2014:1 for the U.S. and three European countries. Namely, Spain, UK and Ireland.

The U.S. housing market has experienced significant cyclical volatility over the last twenty-five years due to major structural changes in the banking sector. However, in 2007 entered a crisis of unprecedented proportions. The recent financial crisis was triggered by problems in the U.S. domestic subprime mortgage market, where cumulative loss rate of securitized subprime loan portfolios exceeded 20 percent by end-2010. In the wake of the crisis, the U.S. housing default rate increased sharply reaching the highest level since the 1930s. At the end of 2010, when the housing market started to recover, nearly a quarter of residential properties had negative equity mortgage.

The recent evolution of the housing market in Spain and Ireland, and to a lesser extent the UK, also provide examples of unsustainable asymmetric cycles where booms in the housing market were followed by prolonged recession and financial instability. Although, social and economic factors fueling housing market dynamic were peculiar to each country, cycles in the housing markets under consideration present many similarities. First, housing markets in these countries have been consistently more volatile than most other countries in Europe. Second, the supply of housing measured by new constructions, is relatively price inelastic compared to the rest of Europe. Third, starting from the 80ths, they experienced strong liberalization and deregulation of the financial system and the mortgage market in particular. Financial liberalization in these countries went further that it did in much of the rest of Europe. Excessive competition and aggressive lending policies that followed in the wake of financial deregulation caused a deterioration of lending standards which translated to a prolonged expansion phase of the house price cycle followed by deep busts.

Table 1 reports the descriptive statistics for the data under consideration. The p-values of the Jarque-Bera test (JB) test for normality, Engle's test for conditional heteroskedasticity (ARCH), and the augmented Dickey-Fuller test (ADF) are also reported. Note that the descriptive statistics refer to the values of the time series in levels, whereas the calculated test statistics in Table 1 refer to the series of house prices in differences, that is $\Delta y_t = y_t - y_{t-4}$.

From Table 1 it appears that Ireland had the highest house price volatility during the period under consideration. This is probably due to the extreme cycle that the Irish market experienced during the last 15 years. Comparing to the other three housing markets, the U.S. house prices present the lowest mean and standard deviation. This is probably due to the greater heterogeneity of the U.S. housing market. An analysis of the skewness and the kurtosis coefficients reveals that all the series under consideration are skewed and fat-tailed.

Table 1. Descriptive statistics.

Country	Mean	Std. Dev.	Skewness	Kurtosis	JB	ARCH	ADF
U.S.	105.95	66.40	0.54	-0.78	0.003	0.000	0.004
UK	131.04	108.87	0.69	-0.86	0.001	0.000	0.005
IRELAND	143.27	125.24	0.88	-0.97	0.005	0.000	0.000
SPAIN	109.58	97.87	0.69	-0.44	0.041	0.000	0.003

Looking now at the calculated test statistics in Table 1, the JB test rejects the joint hypothesis of the skewness being zero and the excess kurtosis being zero for all the house price series in consideration. The ARCH-test also rejects the null hypothesis the there is no autoregressive conditional heteroskedasticity. Finally, the ADF test rejects the null hypothesis of unit root in the house price series.

4.2 Estimation Results

The modelling procedure adopted follows the steps described in Section 3. First the maximal lag order of the AR model is chosen by Bayesian information criterion and Portmanteau test for serial correlation. Then, the linearity tests are conducted. To test for linearity the procedure suggested by Luukkonen, Saikkonen, and Teräsvirta (1988) is used. This involves testing for the fact that the nonlinear function $G(\cdot)$ is zero in (6). Hence, under the null hypothesis that $\mathcal{H}_0 : \tilde{\gamma} = 0$ against $\mathcal{H}_1 : \tilde{\gamma} \neq 0$. Under the null hypothesis the LM-type test is asymptotically distributed as a $\chi^2(3p)$ distribution.

The second and third columns of Table 2 report the p-values of test for linearity and the test for dynamic symmetry, respectively. In Table 2 the null hypothesis is rejected at 5% significance level for U.S., UK and Ireland and 10% for Spain. Coming to the test for dynamic symmetry, the LM-type test statistic has under the null hypothesis $\mathcal{H}_0 : \tilde{\gamma} = 0$, $\hat{\tau} = (\tau_1', 0')'$ which, as seen in the previous section, corresponds to testing the null hypothesis

$$\mathcal{H}_0 : \beta_{2j} = \beta_{3j} = \beta_{4j} = 0 \;\; j = 1, \ldots, p.$$

in the auxiliary equation (12). From Table 2 on the base of the empirical p-values reported in column three the null hypothesis of dynamic symmetry can be rejected.

Table 2. Empirical p-values for the tests for nonlinearity and dynamic symmetry.

Country	Linearity Test	Dynamic Symmetry Test
U.S.	0.031	0.022
UK	0.023	0.032
IRELAND	0.002	0.000
SPAIN	0.072	0.001

As far as the choice of the transition function is concerned, as discussed in Section 3, the choice of the number of location parameters k affects the type of asymmetric behaviour characterized by the model. The model in equation (6) with $k = 1$ is suitable to describe housing markets with rapid change in house prices during expansions but slow rate of change during recessions. On the other end, when $k = 2$ in (6), the resulting exponential form of the transition function suggests that recessions and recovery phases have similar dynamic but different duration and intensity. This feature is referred to as "deepness" of the cycle in Sichel (1993). The GSTAR model introduced in Section 3 is capable of describing both types of asymmetric behavior in house price changes. However, in this context we are more interested in modelling situations where expansions and recessions have different dynamic rather than midrange regimes. For this reason, the model in equation (6) with $k = 1$ is considered.

In Table 3 the estimated parameters are reported. From Table 3 it appears that most estimated θ coefficients for the house prices under consideration are significantly different from zero. The parameters γ_1 and γ_2 indicate the speed of the transition between regimes. These coefficients are also significantly different from zero. Looking at the signs of these coefficients we note that, no matter the country taken into consideration, the parameter γ_1 is always positive, whereas γ_2 is negative. This indicates that the speed of the transition from one regime to the second regime is increasing during periods of house price booms at a rate that is greater than the one which would be consistent with a standard logistic curve, but it is decreasing during the periods of house price recessions at a rate which is slower than the one that would be consistent with a standard logistic function. The signs of the estimated γ_1 and γ_2 are consistent with asymmetric price adjustment models found in applied economic literature. The fact that house price changes deviate above their mean at an exponential rate, whereas they return at a logarithmic rate implies that when improving economic conditions boost housing demand above the potential stock, prices raise rapidly above their expected level, but they fall slowly when economic conditions worsen and price changes are below the expect value. This implies that, for a given housing stock, a positive demand shock will pushes up prices, but it will have a relatively small effect on housing supply. In contrast, a negative demand shock will have a relatively larger impact on housing supply.

Looking at the magnitude of the estimated γ_1 and γ_2 coefficients,[1] the parameters take value of 9.0 for the U.S., approximately 5.0 for the UK and Spain and 3.6 for Ireland. The higher magnitude of the speed parameters γ_1 and γ_2 indicates a sharper transition for the U.S. from one regime to another in this country with respect to the European countries. This is probably due to more conservative housing policy in Europe. Note that the relatively small estimates of γ_1 and γ_2 indicate that other types of nonlinear models in the class of regime switching, such as for example the Markov switching or the TAR models are no suitable to capture housing market dynamics since these models assume a sudden transition between one regime and the other (i.e. in these models $\gamma_1 = \gamma_2 \longrightarrow \infty$ by assumption). Coming now to the parameter c, this indicates the halfway point between the expansion and contraction phases of the housing markets. In all cases, the values of c is statistically significant at 5%. The estimated parameters c are positive for all countries, but Ireland.

[1]Note that the magnitude of the two slope should not be confused with symmetry, in what it simply state that a strong departure from the conditional mean is followed with a slow recovery.

Once that the model has been estimated the goodness of fit of the model can be evaluated using misspecification tests. The diagnostic statistics considered here are: *i*) the LM test for serial independence for the hypothesis that there is no serial correlation against the q-order autoregression (for $q \in \{1, ..., 4\}$), *ii*) the test for no remaining asymmetry, *iii*) the test for parameter constancy (see Appendix 1 for details on these inference procedures).

The p-values of the tests are reported at the bottom panel of Table 3. Looking at the misspecification tests it emerges that the autocorrelation test does not reject the null hypothesis of no autocorrelation against q-order autoregression for all estimated models. There is also no evidence of remaining asymmetry given that the test does not reject the null hypothesis for the estimated models. Similarly, the test of parameter constancy also does not reject the null hypothesis at the 5% significant level for all the estimated models. Overall, the results in Table 3 suggest that the estimated models do not suffer from misspecification problems.

Table 3. Estimated Model and diagnostic tests.

Parameters	U.S.	UK	Ireland	Spain
ϕ_0	-0.463	-0.191	-6.756	-4.417
	(0.708)	(2.103)	(4.203)	(10.078)
ϕ_1	2.012^{**}	1.321^{*}	1.872^{*}	-1.433^{*}
	(-1.041)	(0.268)	(0.359)	(0.205)
ϕ_2	-1.041^{*}	-0.405	-1.288^{*}	-0.381^{**}
	(0.335)	(0.307)	(0.5213)	(0.218)
θ_{10}	1.823^{*}	3.433^{*}	7.997^{**}	17.57^{**}
	(0.730)	(0.214)	(4.209)	(10.108)
θ_{11}	-0.974^{*}	0.349	-0.973^{*}	-0.488^{*}
	(0.366)	(0.272)	(0.360)	(0.220)
θ_{12}	0.869^{*}	-0.457	1.273^{*}	0.324
	(0.335)	(0.309)	(0.522)	(0.246)
γ_1	9.000^{*}	4.800^{*}	3.600	5.000^{*}
	(0.491)	(2.214)	(4.209)	(0.417)
γ_2	-9.000^{*}	-4.800^{*}	-3.600^{*}	-5.000^{*}
	(0.480)	(0.272)	(0.360)	(0.388)
c	6.659^{*}	4.268^{*}	-4.330^{*}	19.895^{*}
	(0.094)	(0.309)	(0.553)	(0.358)
LogLikelihood	-430.70	-464.545	-990.492	-537.787
R^2	0.951	0.920	0.904	0.933
Diagnostic Tests (p-values)				
No Error Correlation				
$q = 1$	0.799	0.858	0.962	0.381
$q = 2$	0.968	0.984	0.998	0.682
$q = 4$	0.999	0.999	0.999	0.944
No remaining asymmetry	0.999	0.998	0.999	0.998
Parameter constancy				
H1	0.999	0.999	0.999	0.999
H2	0.999	0.995	0.999	0.998
H3	0.999	0.989	0.999	0.999

Note: *) and **) indicates significance level at 5% and 10%, respectively.

Figure 2 reports the estimated transition functions for the housing markets under consideration plotted against the transition variable (s_t) with one dot for every observation (note that a single dot may represent more than one observation). The plot of the estimated transition functions shed further light on the degree of asymmetry of the sigmoid function. For the UK housing market almost two thirds of the observations lie in the lower part of the graph, corresponding to the segment between 0.2 and 0.4 of the vertical axis, while the remaining are in the extreme regime, corresponding to 1. Similar story holds for Spain. The case of U.S. is however more extreme, as confirmed by the higher magnitude of the nonlinear parameters. On the other hand, for Ireland these proportions are inverted. In Figure 2, the shape of the transition functions reflect the different signs of the estimated slope parameters in Table

3: the negative sign of γ_2 corresponds to a logarithmic rescaling of the segment 0 - 0.5 of the vertical axis, whereas the positive γ_1 produces an exponential transformation of the upper part.

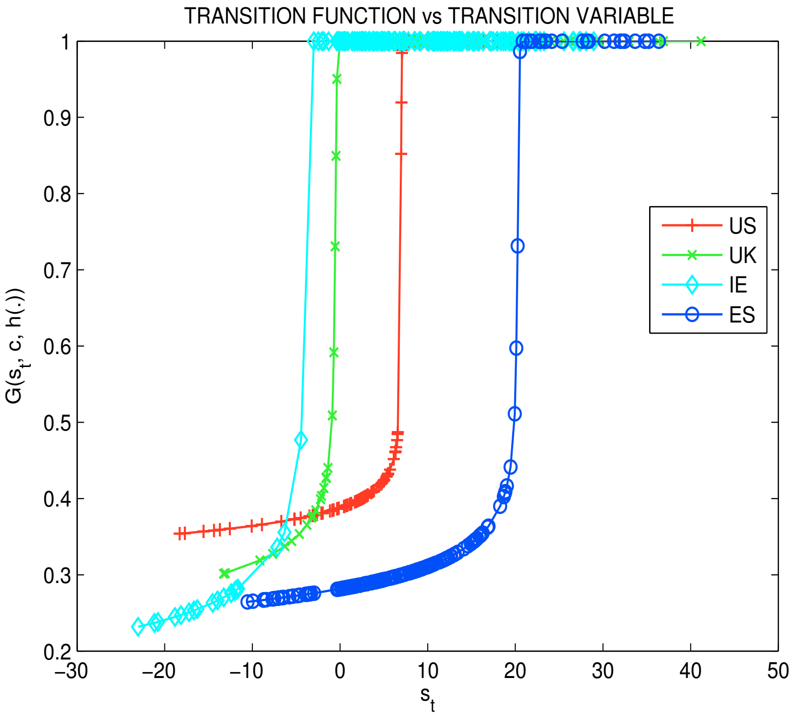

Figure 1. Estimated transition function and the transition variable. Note: each dot represents at least one observation.

The estimated transition functions are shown over time in Figure 2 a)-d) along with house price changes, Δy_t. In Figure 2 a)-d) the transition functions for the estimated models take values at or close to one during periods of price booms and values close to zero during periods of housing market busts. Overall, looking at the plot of the two regimes it emerges that the GSTAR model captures reasonably well contractions and expansions in the housing markets under consideration.

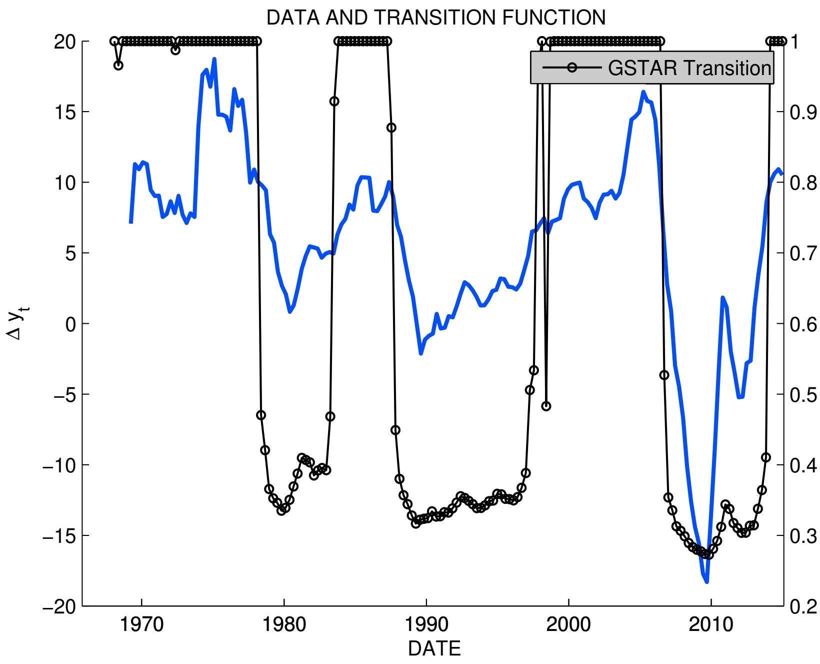

Figure 2a. U.S.: Observed values and estimated transition functions over time.

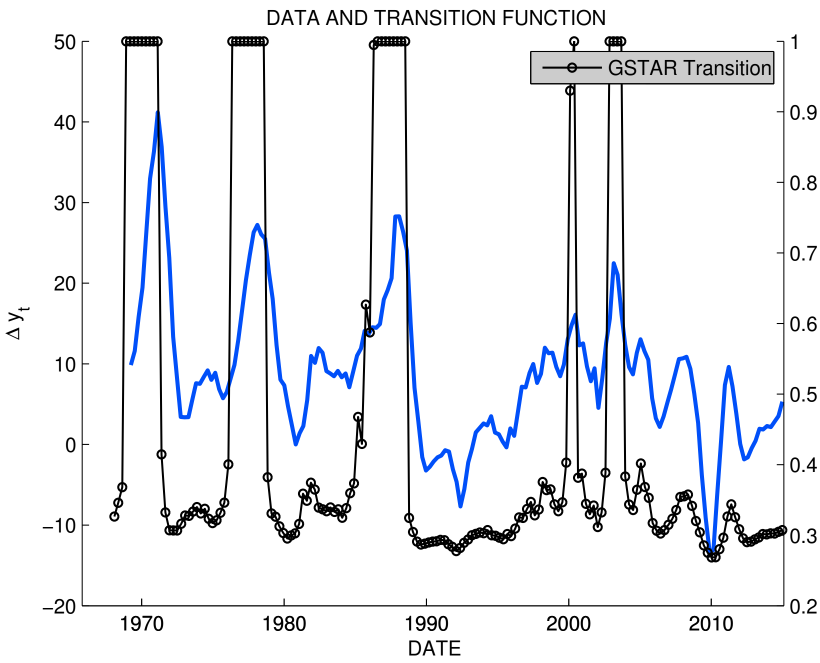

Figure 2b. UK: Observed values and estimated transition functions over time.

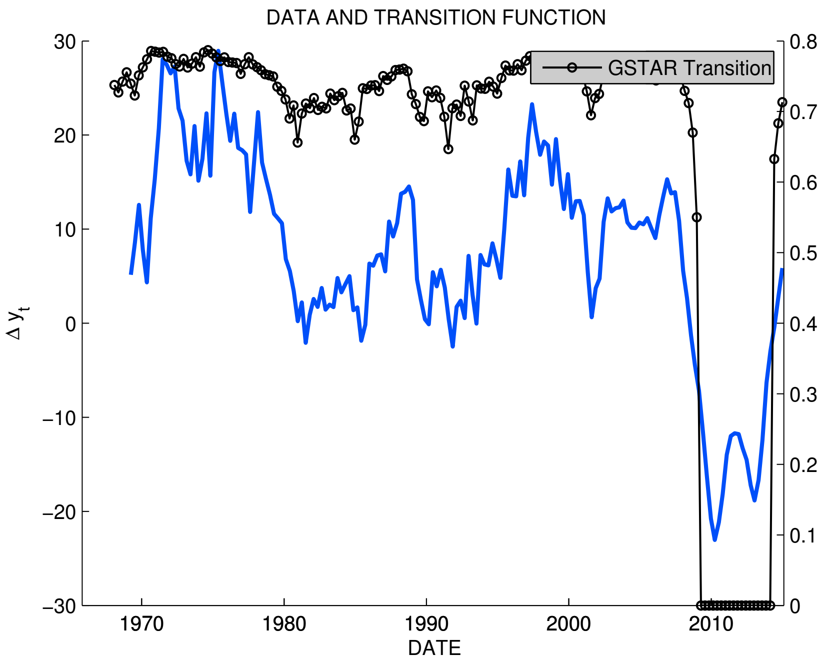

Figure 2c. Ireland: Observed values and estimated transition functions over time.

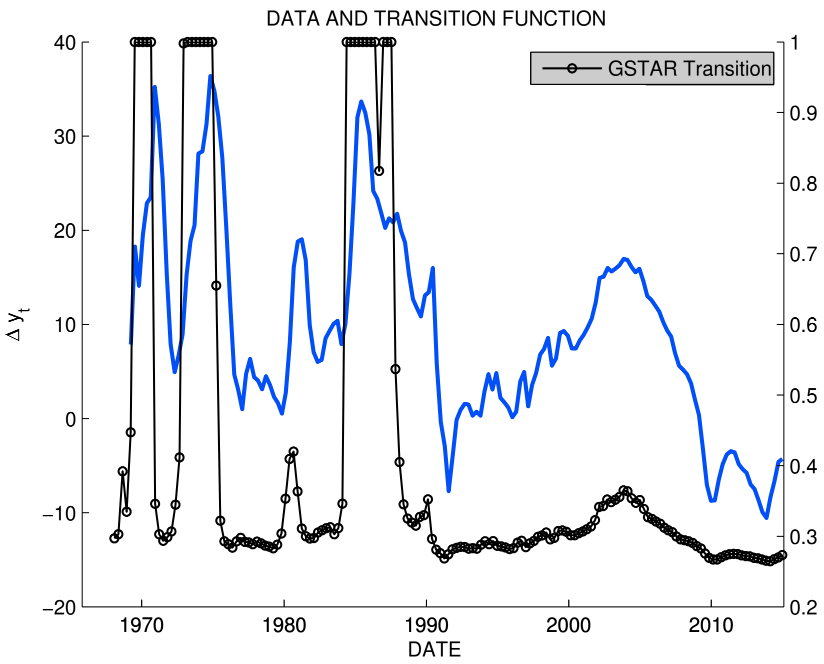

Figure 2d. Spain: Observed values and estimated transition functions over time.

4.3 Forecasting House Prices

In order to investigate the forecasting ability of the GSTAR model a rolling forecast experiment is implemented.

With this target in mind, for each country, the house price series is split two subsamples: a pre-forecast period (for $t = 1, ..., T_{s-1}$) from which the model is estimated and a forecast period $t = T^s, \ldots, T$ with $T^s = t + h$. Then h-step-ahead forecasts are computed and compared with the pre-forecast period. The forecast period under consideration is $h = \{1, 3, 6, 12\}$.

For each country we compare a linear AR(3) model and the GSTAR model in their out-of-sample point forecast. The out-of-sample forecast comparisons do not rely on a single criterion, for robustness we compare the results of four different measures. Namely, the mean forecast error (MFE), the root mean square forecast error (RMSFE), the symmetric mean absolute percentage error (sMAPE) and the median relative absolute error (mRAE). The four performance measures are calculted as follows:

$$
\begin{aligned}
MFE_h &= \frac{1}{T-h-T^s+1}\sum_{t=T^s}^{T-h}\left(\Delta y_{t+h} - \Delta\hat{y}_{t+h|t}\right), && (16)\\
sMAPE_h &= \frac{100|\Delta y_{t+h} - \Delta\hat{y}_{t+h}|}{0.5(\Delta y_{t+h} - \Delta\hat{y}^{j}_{t+h|t})}, && (17)\\
mRAE_h &= \frac{|\Delta y_{t+h} - \Delta\hat{y}_{t+h}|}{|\Delta y_{t+h} - \Delta\hat{y}^{(1)}_{t+h}|}, \text{ with (1) indexing the benchmark model;} && (18)\\
RMSFE_h &= \frac{1}{T-h-T^s+1}\sum_{t=T^s}^{T-h}\left(\Delta y_{t+h} - \Delta\hat{y}_{t+h|t}\right)^2. && (19)
\end{aligned}
$$

Table 4 reports the results of the forecasting results. In columns 2 and 3 the forecasting horizon and the forecast error measures are reported, respectively, whereas in columns 4-7 the forecasting results for each housing market are reported. From the top panel of Table 4 it is clear that according to the MFE and sMAE criteria the GSTAR model performs better than its symmetric counterpart. Similarly, in the bottom panel, according to the RMSFE the GSTAR has superior forecasting properties . However, looking at the mRAE measure the results are mixed.

Table 4. Forecasting house prices: Point predictive performances.

Model	Forecast Horizon	Forecast Error Measure	U.S.	UK	Ireland	Spain
AR	1	MFE	0.0137	-0.177	-0.022	-0.536
	2		0.2537	-0.195	-0.050	-0.535
	4		0.4541	-0.206	-0.077	-0.571
	8		0.5757	-0211	-0.223	-0.594
GSTAR	1	MFE	-0.073	-0.130	-0.148	-0.166
	2		-0.069	-0.195	-0.163	-0.178
	4		-0.078	-0.207	-0.173	-0.143
	8		-0.113	-0.208	-0.246	-0.138
AR	1	sMAE	0.004	0.003	0.006	0.004
	2		0.005	0.003	0.007	0.004
	4		0.006	0.004	0.007	0.004
	8		0.008	0.004	0.006	0.004
GSTAR	1	sMAE	0.003	0.004	0.003	0.004
	2		0.003	0.005	0.005	0.004
	4		0.003	0.007	0.005	0.004
	8		0.004	0.003	0.005	0.004
AR	1	mRAE	1.000	1.000	1.000	1.000
	2		1.000	1.000	1.000	1.000
	4		1.000	1.000	1.000	1.000
	8		1.000	1.000	1.000	1.000
GSTAR	1	mRAE	1.257	0.414	1.071	1.046
	2		1.481	0.424	1.199	1.075
	3		1.924	0.406	1.480	1.032
	4		2.174	0.733	1.334	1.950
AR	1	RMSFE	0.197	0.315	0.329	0.226
	2		0.202	0.317	0.331	0.228
	3		0.220	0.421	0.339	0.233
	4		0.254	0.932	0.346	0.242
GSTAR	1	RMSFE	0.140	0.286	0.280	0.226
	2		0.142	0.305	0.284	0.228
	3		0.145	0.406	0.290	0.231
	4		0.150	0.920	0.302	0.242

5 Conclusion

This paper investigates dynamic asymmetries in house price cycles. We introduce an *ad-hoc* nonlinear model to capture real estate cycles. The suggested model is capable of parametrizing the asymmetry in the transition equation by using a particular generalization of the logistic function. Although applied in this paper specifically to the analysis of housing markets, the proposed model can be used to test for dynamic asymmetries in a more general context.

The application to house prices reveals several insights on the patterns of the housing markets under consideration. It is found that house price changes increase at exponential rate during expansion periods, whereas contractions in home prices follow a logarithmic rate. Therefore, contractions occur for a more prolonged period than expansions. An interesting conclusion from our analysis is that the direction of the estimated asymmetry is the same for all countries considered.

Our results have important consequences for policy makers. In the aftermath of the global financial crisis, there is a growing consensus that the housing sector plays a key role in financial stability. The recent financial crisis was born in the housing sector and grew up in the financial sector. Then, a dip recession hit the real economy causing a high level of unemployment and spread misery throughout the world. Good prudential policy requires action before the overheating of the housing sector goes too far and requires policy intervention early in the boom period. For this reason, our findings on the asymmetric behaviour of house prices over the business cycle may be useful to policy makers and perhaps help to fulfill their willingness to lean against the wind.

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Appendix 1: Misspecification Tests

Like any other model, the GSTAR needs to be evaluated for possible misspecification before it can be used for forecasting purposes. To investigate the quality of the estimated model tests for serial independence, no remaining asymmetry and parameter constancy are derived below. The suggested inference procedures are derived as a generalization of the misspecification tests in Eitrheim and Teräsvirta (1996).

a) Test for Serial Independence

Consider the general additive model in equation (1), where:

$$\epsilon_t = a'v_t + u_t = \sum_{j=1}^{q} a_j L^j \epsilon_t + u_t, \qquad u_t \sim I.I.D.(0, \sigma^2), \tag{1A}$$

with L^j denoting the lag operator, $v_t = (u_{t-1}, \ldots, u_{t-q})'$, $a = (a_1, \ldots, a_q)'$, $a_q \neq 0$. Under the assumption of stationarity and ergodicity, the null hypothesis of serial independence is $\mathcal{H}_0 : a = 0$. By pre-multiplying equation (2) by $1 - \sum_{j=1}^{q} a_j L^j$ we obtain

$$y_t = \sum_j a_j L^j y_t + \phi' z_{\mathbf{t}} - \sum_j a_j L^j \phi' z_t + \theta' z_t G(\cdot) - \sum_j a_j \theta' G(\cdot) + \epsilon_t \ . \tag{2A}$$

Hence, assuming that the initial values $y_0, y_{-1}, \ldots, y_{-(p+q)+1}$ are fixed, the pseudo normal likelihood for $t = 1, \ldots, T$ is

$$\begin{aligned}
\mathcal{L}_t &= constant + \frac{1}{2}\ln\sigma^2 - \frac{\epsilon_t^2}{2\sigma^2}, \\
\epsilon_t &= y_t - \sum_j a_j L^j y_t - \phi' z_t + \sum_j a_j L^j \phi' z_t - \theta' G(z_{t-j}, \Xi) + \sum_j a_j \theta' G(z_{t-j}, \Xi).
\end{aligned}$$

Consistent with the model initial assumptions, the information matrix is block diagonal and σ^2 is fixed. Therefore, the partial derivatives with respect to a_j and $\mathbf{\Xi}$ are given by

$$\frac{\partial \mathcal{L}_t}{\partial a_j} = \frac{\epsilon_t}{\sigma^2}[y_{t-j} - \phi' z_{t-j} - \theta' G(z_{t-j}, \Xi)] \tag{3A}$$

$$\frac{\partial \mathcal{L}_t}{\partial \mathbf{\Xi}} = \frac{\epsilon_t}{\sigma^2}\left[\theta' z_t \frac{\partial G(z_{t-j}, \Xi)}{\partial \Xi} - \sum_j a_j \theta' \frac{\partial G(z_{t-j}, \Xi)}{\partial \Xi}\right]. \tag{4A}$$

Under the null hypothesis, consistent estimators of $(3A)$ - $(4A)$ are given by

$$\frac{\partial \hat{\mathcal{L}}_t}{\partial a_t}|_{\mathcal{H}_0} = \frac{1}{\sigma^2}\hat{u}_t \hat{v}_t \ \text{ and } \ \frac{\partial \hat{\mathcal{L}}_t}{\partial \mathbf{\Xi}}|_{\mathcal{H}_0} = -\frac{1}{\sigma^2}\hat{u}_t \hat{z}_t, \tag{5A}$$

where the vector $\hat{u}_t = (\hat{v}_{t-1}, \ldots, \hat{v}_{t-q})'$, $\hat{v}_{t-j} = y_{t-j} - \phi' z_{t-j} - \theta' G(z_{t-j}, \hat{\Xi})$, $j = 1, \ldots, q$, $\hat{\Xi}$ is the QMLE estimator of Ξ and

$$\hat{z}_t = \frac{\partial G(z_t, \hat{\Xi})}{\partial \hat{\Xi}} = k_t^G = [\theta' z_t G_{\gamma 1}, \theta' z_t G_{\gamma 2}, \theta' z_t G_{\gamma c}].$$

The resulting LM test statistic is given by

$$LM = \frac{1}{\hat{\sigma}}\left(\hat{u}_t'\hat{v}_t\right)\left\{\hat{v}_t'\hat{v}_t - \hat{v}_t'\hat{z}_t\left(\hat{z}_t\hat{z}_t'\right)^{-1}\hat{z}_t'\hat{v}_t\right\}^{-1}\left(\hat{v}'\hat{u}_t\right), \tag{6A}$$

with $\hat{\sigma}^2 = \frac{1}{T}\sum_t u_t^2$. Under the null hypothesis the statistic $(6A)$ is asymptotically χ_q^2 distributed.

Note that in finite and moderate samples the LM statistic can be severely size-distorted. In general, the size distortion of the LM test is sensitive to the number of nuisance parameters. It may also be the case that the estimated residual vector could be non-orthogonal to the gradient vector $\hat{z}_t$. A small sample corrected F-type test is therefore proposed below. To calculate the test one may proceed as follows:

Step 1. Estimate the GSTAR model under the assumption of uncorrelated errors and compute the residual sum of squares $SSR_0 = \sum_{t=1}^{T}\hat{u}_t^2$.

Step 2. Regress $\hat{u}_t$ on $\hat{v}_t$, z_t, $z_t\hat{G}(z_{t-d})$, $\hat{G}_{\gamma 1}$, $\hat{G}_{\gamma 2}$, $\hat{G}_c$ and compute SSR;

Step 3. Compute the test F-type statistic

$$F_{LM} = \frac{q^{-1}\left(SSR_0 - SSR\right)}{(T-n-q)^{-1}SSR}, \qquad \text{where } n = \dim(\hat{\mathbf{z}}_{\mathbf{t}}).$$

A variation of the suggested procedure is proposed in Eitrheim and Teräsvirta (1996) where an extra-step to is added to Step 2. Eitrheim and Teräsvirta (1996) suggest to regress the estimated errors on z_t, z_t, $z_t\hat{G}(z_{t-j})$, $\hat{G}_{\gamma 1}$, $\hat{G}_{\gamma 2}$, $\hat{G}_c$. Then, to use the resulting errors $\tilde{u}_t$ to calculate $SSR_1 = \sum_{t=1}^{T}\tilde{u}_t^2$.

b) No Remaining Asymmetry

Consider the additive GSTAR model:

$$y_t = \phi' z_t + \theta' z_t G_1(\tilde{\gamma}, h(\eta_t)^{(1)}) + \pi' z_t G_2(\tilde{\gamma}, \tilde{h}(\eta_t)^{(2)}) + u_t, \tag{7A}$$

with $u_t \sim I.I.D.\ (0, \sigma^2)$. The null of neglected asymmetry is:

$$\mathcal{H}_0 : \tilde{h}(\eta_t)^{(2)} = 0 \quad \text{versus} \quad \mathcal{H}_1 : \tilde{h}(\eta_t)^{(2)} \neq 0. \tag{8A}$$

If the vector $\tilde{\gamma}$ is found being not null, the investigator can easily check if the additive nonlinear part is significant. We assume that, under the null hypothesis, Ξ can be consistently estimated by QML. However, the model in $(7A)$ is not identified under the null hypothesis. Therefore, once again a Taylor expansion of the transition function $G(\cdot)$ can be used in order to circumvent the problem. In this case, assume that $G_2(\cdot)$ is a generalized logistic function and replace it with its third-order Taylor expansion about $h(\gamma)^{(2)} = 0$. This implies

$$T_2 = g_{20} + g_{21}y_{t-l} + g_{22}y_{t-l}^2 + g_{23}y_{t-l}^3\ , \tag{9A}$$

where g_{2j}, $j = 0, 1, 2, 3, 4$ are functions of the vector $\gamma^{(2)}$ such that $g_{20} = g_{21} = g_{22} = g_{23} = 0$ for $\gamma^{(2)} = 0$, consistently with the definition of $h_{\gamma}(s_t)$. By re-parametrizing, the model $(7A)$ becomes:

$$y_t = \beta_0' z_t + \theta' z_t G_1(\cdot) + \beta_1' \tilde{\mathbf{z}}_{\mathbf{t}} y_{t-l} + \beta_2' \tilde{z}_{\mathbf{t}} y_{t-l}^2 + \beta_3' \tilde{z}_{\mathbf{t}} y_{t-l}^3 + r_t \ , \tag{10A}$$

where $\tilde{z}_t = (y_{t-1}, \ldots, y_{t-p})'$. The null hypothesis of no additive nonlinearity is $\mathcal{H}_0 : \beta_1 = \beta_2 = \beta_3 = 0$ (note also that under the null hypothesis $r_t = u_t$). The LM statistics is asymptotically distributes as a $\chi^2(3p)$. Since there are no modifications in the statistical assumptions concerning the errors distribution, the asymptotic distribution of the test can be derived as in Eitrheim and Teräsvirta (1996).

c) Parameter constancy

Consider the model:

$$y_t = \phi(t)' \bar{z}_t + \theta(t)' \tilde{z}_t G(\gamma, h(\eta_t)) + u_t, \quad u_t \sim I.I.D.\ (0, \sigma^2) \ , \tag{11A}$$

with $\bar{z}_t$ denoting the $k \leq p+1$ element of z_t for which the corresponding element of ϕ is not assumed zero a priori, $\tilde{z}_{\mathbf{t}}$ is the same $(l \times 1)'$ for the element of θ. Let $\tilde{\phi}$ and $\tilde{\theta}$ denote the equivalent $(k+1)$ and $(l+1)$ parameter vectors, $\phi(t) = \tilde{\phi} + \lambda_1 G_j(t; \tilde{\gamma}, h(\eta_t)^{(1)})$, and $\theta(t) = \tilde{\theta} + \lambda_2 G_j(t; \tilde{\gamma}, h(\eta_t)^{(2)})$ with λ_1 and λ_2 being a $(k \times 1)$ and $(l \times 1)$ vectors respectively. Then the null hypothesis of parameter constancy in $(11A)$ is

$$\mathcal{H}_0 : G(t; \gamma, h(\eta_t)) \equiv 0. \tag{12A}$$

Three forms for $G(\cdot)$ can be considered

$$\begin{aligned}
G_1(t; \gamma, h(c, s_t)) &= (1 + \exp\{-h(\eta_t^{GL})\})^{-1} \quad \text{with} \\
\eta_t^{GL} &\equiv t - c \ , \\
G_2(t; \gamma, h(c, s_t)) &= (1 + \exp\{-h(\eta_t^{GE})\})^{-1} \ \text{with} \\
\eta_t^{GL} &\equiv (t - c\ )^2 \ , \\
G_3(t; \gamma, h(c, s_t)) &= (1 + \exp\{-h(\eta_t^{C})\}) \qquad \text{with} \\
\eta_t^{C} &\equiv \left(t^3 - c_{12} t^2 + c_{11} t + c_{10}\ \right).
\end{aligned} \tag{13A}$$

The null of parameter constancy is $\mathcal{H}_0 : \tilde{\gamma} = 0$. Note that in this case the model is identified also in case of $\tilde{\gamma} < 0$, so that the only identifying restriction is that $\tilde{\gamma} \neq 0$. G_1 and G_2 are the generalized logistic and exponential smooth transition of the change in parameters, while G_3 is a cubic function which allows for both monotonically and non-monotonically changing parameters and can be seen as a general case of G_1 and G_2 when building up a test. As suggested by the literature, we use a third-order Taylor expansion of G_3 about $\tilde{\gamma} = 0$

$$T_3(t; \tilde{\gamma}, h(\eta_t)) = \frac{1}{4} h(\tilde{\gamma})(t^3 + c_{12}t^2 + c_{11}t + c_{10}) + R(t, \tilde{\gamma}, h(\eta_t)). \tag{14A}$$

in order to approximate $\phi(t)$ and $\theta(t)$ in $(11A)$ using $(14A)$. This yields

$$\begin{aligned} y_t &= \beta_0^{'}(\bar{z}_t) + \beta_1^{'}(t\bar{\mathbf{z}}_\mathbf{t}) + \beta_2^{'}(t^2\bar{\mathbf{z}}_\mathbf{t}) + \beta_3^{'}(t^3\bar{\mathbf{z}}_\mathbf{t}) + y_t \\ &+\{\beta_4^{'}(\tilde{\mathbf{z}}_\mathbf{t}) + \beta_5^{'}(t\tilde{\mathbf{z}}_\mathbf{t}) + \beta_6^{'}(t^2\tilde{\mathbf{z}}_\mathbf{t}) + \beta_7^{'}(t^3\tilde{\mathbf{z}}_\mathbf{t})\}G(t; \tilde{\gamma}, h(\eta_t)) + r_t^* \ , \end{aligned} \tag{15A}$$

where $r_t^* = u_t + R(t; \tilde{\gamma}, h(\eta_t))$. Under $\mathcal{H}_0$, $r_t^* = u_t$. In $(15A)$, $\beta_j = h(\eta_t)\bar{\beta}$, $j = 1, \ldots, 7$, hence the null hypothesis in terms of $(15A)$ becomes $\mathcal{H}_0 : \beta_j = 0$, $j = 1, \ldots, 7$. Consequently, the locally approximated pseudo normal log-likelihood under $\mathcal{H}_0$ (ignoring R) is

$$\begin{aligned} \mathcal{L}_t &= const - \frac{1}{2}\ln\sigma^2 - \frac{1}{2}\sigma^2[y_t - \beta_0^{'}w_t - \beta_1^{'}(t\bar{w}_t) - \beta_2^{'}(t^2\bar{w}_t) - \beta_3^{'}(t^3\bar{w}_t) - \\ &-\{\beta_4^{'}(\tilde{w}_t) + \beta_5^{'}(t\tilde{w}_t) + \beta_6^{'}(t^2\tilde{w}_t) + \beta_7^{'}(t^3\tilde{w}_t)\}G(y_{t-d}; \tilde{\gamma}, h(\eta_t)]^2. \end{aligned}$$

The partial derivatives are:

$$\frac{\partial \mathcal{L}_t}{\partial \beta_j} = \frac{1}{\sigma^2} u_t(t^j \bar{w}_\mathbf{t}), \qquad j = 0, \ldots, 3, \tag{16A}$$

$$\frac{\partial \mathcal{L}_t}{\partial \beta_j} = \frac{1}{\sigma^2} u_t(t^j \tilde{w}_t) G(y_{t-d}; \tilde{\gamma}, h(\eta_t)), \qquad j = 4, \ldots, 7, \tag{17A}$$

$$\frac{\partial \mathcal{L}_t}{\partial \gamma_1} = \frac{1}{\sigma^2} u_t\{\beta_4^{'}(\tilde{w}_t) + \beta_5^{'}(t\tilde{w}_t) + \beta_6^{'}(t^2\tilde{w}_t) + \beta_7^{'}(t^3\tilde{w}_t)\}G_{\gamma_1} \ , \tag{18A}$$

$$\frac{\partial \mathcal{L}_t}{\partial \gamma_2} = \frac{1}{\sigma^2} u_t\{\beta_4^{'}(\tilde{w}_t) + \beta_5^{'}(t\tilde{w}_t) + \beta_6^{'}(t^2\tilde{w}_t) + \beta_7^{'}(t^3\tilde{w}_t)\}G_{\gamma_2} \ , \tag{19A}$$

$$\frac{\partial \mathcal{L}_t}{\partial c} = \frac{1}{\sigma^2} u_t\{\beta_4^{'}(\tilde{w}_t) + \beta_5^{'}(t\tilde{w}_t) + \beta_6^{'}(t^2\tilde{w}_t) + \beta_7^{'}(t^3\tilde{w}_t)\}G_c \ , \tag{20A}$$

where G_{γ_1}, G_{γ_2}, G_c are the derivatives of $G(y_{t-d}, \tilde{\gamma}, h(\eta_t))$ with respect to γ_1, γ_2 and c. With this notation, the estimators of $\frac{\partial \mathcal{L}_t}{\partial \gamma_1}$, $\frac{\partial \mathcal{L}_t}{\partial \gamma_2}$ and $\frac{\partial \mathcal{L}_t}{\partial c}$ are $\frac{\partial \hat{\mathcal{L}}_t}{\partial \gamma_1} = \frac{1}{\hat{\sigma}^2} u_t \hat{G}_{\gamma_1}$, $\frac{\partial \hat{\mathcal{L}}_t}{\partial \gamma_2} = \frac{1}{\hat{\sigma}^2} u_t \hat{G}_{\gamma_2}$, $\frac{\partial \hat{\mathcal{L}}_t}{\partial c} = \frac{1}{\hat{\sigma}^2} u_t \hat{G}_c$ respectively, so that: $\hat{z}_t = (1, \bar{z}_t', \tilde{z}_t'\hat{G}(y_{t-d}; \cdot), \hat{G}_{\gamma_1}, \hat{G}_{\gamma_2}, \hat{G}_{\gamma_c})'$ and $\hat{u}_t = (t\bar{z}_t', t^2\bar{z}_t', t^3\bar{z}_t', t\tilde{z}_t'\hat{G}(y_{t-d}, \cdot), t^2\tilde{z}_t'\hat{G}(y_{t-d}, \cdot), t^3\tilde{z}_t'\hat{G}(y_{t-d}, \cdot))$. Like in the symmetric case, under $\mathcal{H}_0$, the statistic $(6A)$ has a χ^2 distribution with $3(k + l)$ degrees of freedom and the equivalent F-distribution has $3(k + l)$ and $T - 4(k + l) - 2$ degrees of freedom (the statistic is denoted LM_3). The following rule is used: if $\mathcal{H}_1$ is $(11A)$ with transition function G_3, then $(6A)$ is based on $(15A)$ assuming $\beta_3 = 0$ and $\beta_7 = 0$ (statistic LM_2) and, if the same alternative hypothesis has the transition function G_2, the test is based on $(15A)$, assuming $\beta_2 = \beta_3 = 0$ and $\beta_6 = \beta_7 = 0$ (statistic LM_2).

Appendix 2: Small Sample Properties of the Test for Dynamic Symmetry: A Monte Carlo Experiment.

In this section we investigate the finite sample properties of the test for dynamic symmetry presented in Section 3 by undertaking a small Monte Carlo simulation experiment. The aim of the experiment is to investigate the sensitivity of the test statistic to variations of the parameter space. In particular, a specific numerical problem exists in the estimation of the GSTAR when the parameters γ_1 and γ_2 are large so that there are not many observations in the neighbourhood of the location parameter c. The problem is more likely to occur in small samples where such a cluster of observations is more difficult to find. This lack on information due to the estimated value of the asymmetry parameter may affect the power of the test statistic. For this reason in the experiment design the parameters of the vector $\tilde{\gamma}$ in the data generating process (DGP) are chosen replicate the effect of different cases of null, medium and extreme asymmetry. Moreover, in Section 3 the innovation term of the GSTAR model is considered $\varepsilon_t \sim I.I.D.\left(0, \sigma^2\right)$. However, non-normality of innovations is often found in empirical work (see for example Crawford and Fratantoni (2003)). As Clements and Krolzig (2003) point out asymmetries in the observed variable can arise either from asymmetries in the model's propagation mechanism or from asymmetries in the innovations. Dynamic asymmetry translates in an heavy tail behaviour of the estimated density of the innovation process. Non-normality of the innovation may affect the the finite sample properties of the inference procedure under consideration.

In designing the simulation experiment the findings of Section 4 are exploited by using the autoregressive parameters of the UK data to generate the DGP. In particular, the following DGP is used:

$$y_t = 0.19 + 1.32 y_{1,t-1} - 0.40 y_{1,t-2} + (3.43 - 0.35 y_{1,t-1} + 0.45 y_{1,t-2})^{(s)} G(\Xi) + \epsilon_t \tag{21A}$$

and

$$G(\Xi) = (1 + \exp\{-h(\eta_t) I_{(\gamma_1<0,\gamma_2<0)} + h(\eta_t) I_{(\gamma_1>0,\gamma_2<0)} + h(\eta_t) I_{(\gamma_1<0,\gamma_2>0)} + \\ + h(\eta_t) I_{(\gamma_1>0,\gamma_2>0)}]\})^{-1}.$$

In equation $(21A)$ the process $\{y_t\}_{t=1}^{T}$ is simulated with $s = y_{t-1}$, $c = \frac{1}{T} y_t$ and a number of $N = 1,000$ Monte Carlo replications. Two sets of experiments are carried out. In the first innovations are drawn from a normal distribution, so that $\epsilon_t \sim N(0,1)$. In the second set of experiments the innovations follow a mixture of normal distribution where $\epsilon_t \sim N(a(\mu_1, \sigma_1^2) + (1-a)(\mu_2, \sigma_2^2))$ with $\mu_1 = -1.5$, $\mu_2 = 1.5$, $\sigma_1^2 = 1$, $\sigma_1^2 = 2$, $a = 0.3$ denoting the mixing parameter. In this case the realization of y_t conveys an additive non-linear model with accentuated (observed) dynamic asymmetry. This last is captured by the mixing parameter a which governs the propagation of the shock from $(\mu_1,\, \sigma_1^2)$ to $(\mu_2,\, \sigma_2^2)$.

For illustration purposes Figure 3 shows several transition functions simulated using the parameters γ_1 and γ_2 taken from Table 3. Consistently with the findings reported in Table 3 the two tails of the transition function are treated asymmetrically. The parameter γ_1 relates the speed of adjustment during expansion faces, whereas the parameter γ_2 relates to contraction periods. The greater the magnitude of the γ_1 and γ_2 in modulus, the steaper the slope of the transition function and therefore the faster is the transition from one regime to the other.

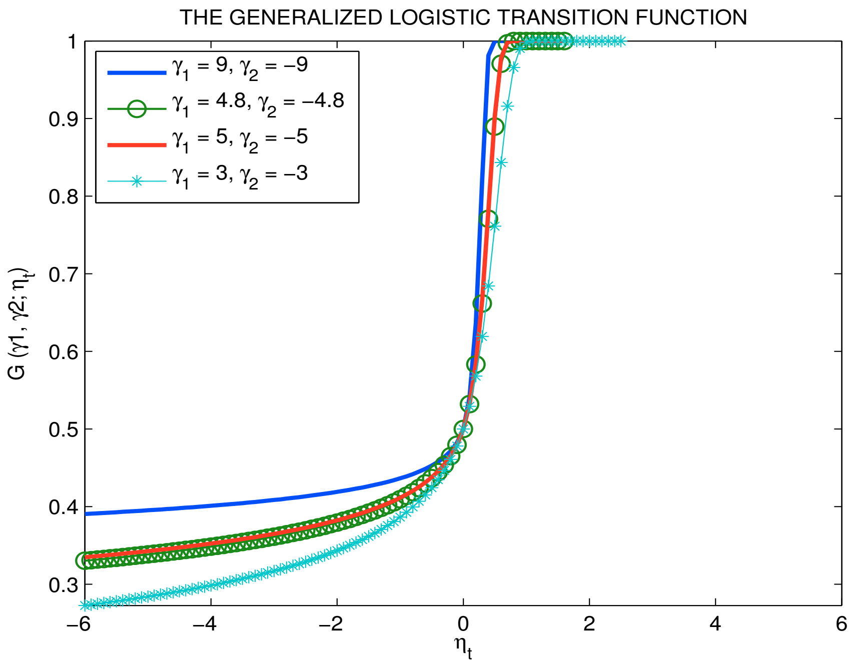

Figure 3. Simulated transition functions for different values of γ_1 and γ_2.

In Table 1A the simulation results for the empirical size and power of the test are reported. In the third column the simulation results for the case with the DGP in (21A) and $\epsilon_t \sim N(0,1)$ are reported, whereas the fourth column relates to the results where innovations follows a mixture of normal distribution. The top panel of Table 1A reports the empirical sizes against a nominal significance level of 5%, whereas the rejection frequencies relating the power are reported in the bottom panel. In order to evaluate the power properties of the test we use a set of values of vector $\tilde{\gamma}$ to simulate the function $h(\eta_t)$. The slope parameters in the DGP are chosen replicate the effect of different cases of null, medium and extreme asymmetry. The effect of having different kind of asymmetry is controlled by the different signs in the two parameters γ_1 and γ_2.

Table 1A. Empirical size and power of the test for dynamic symmetry.

Empirical Size			
T	γ_1, γ_2	$\epsilon_t \sim N(0,1)$	$\epsilon_t \sim N(0.3(-1.5,1)+0.7(1.5,2))$
100	(-4.8, 4.8)	0.004	0.003
300	(-4.8, 4.8)	0.040	0.024
1000	(-4.8, 4.8)	0.080	0.067

Empirical Power			
T	γ_1, γ_2	$\epsilon_t \sim N(0,1)$	$\epsilon_t \sim N(0.3(-1.5,1)+0.7(1.5,2))$
100	(2,1)	0.718	0.666
	(2,-1)	0.440	0.678
	(-2,1)	0.736	0.654
	(20,10)	0.731	0.678
	(20,-10)	0.473	0.675
	(-20,10)	0.755	0.656
	(200,100)	0.999	0.995
	(200,-100)	0.523	0.999
	(-200,100)	0.477	0.999
300	(2,1)	0.704	0.898
	(2,-1)	0.705	0.704
	(-2,1)	0.713	0.885
	(20,10)	0.707	0.905
	(20,-10)	0.761	0.857
	(-20,10)	0.737	0.885
	(200,100)	1.000	1.000
	(200,-100)	1.000	1.000
	(-200,100)	0.852	1.000
1000	(2,1)	0.996	1.000
	(2,-1)	0.994	1.000
	(-2,1)	0.995	1.000
	(20,10)	0.991	0.991
	(20,-10)	0.993	0.993
	(-20,10)	0.995	0.995
	(200,100)	1.000	1.000
	(200,-100)	1.000	1.000
	(-200,100)	1.000	1.000

From Table $1A$ it appears that the inference procedure has good size properties for $T > 100$, thus revealing that the test suffers finite from sample size distortion. As far as the power is concerned, In Table $1A$ power estimates show that the sample size and the distance between the null and the alternative hypothesis play an important role in determining the power of the test statistics under consideration. Comparing, the power properties of the test it appears that the power of the test is not much affected by the choice of the distribution of the innovations: the test is relatively well behaved when the innovations are fat-tailed but i.i.d.