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Abstract The recent literature has analyzed binary choices dynamics providing interesting results. Most of these contributions consider interactions within a single group. Nevertheless, in some situations the interaction takes place not only within a single group but also between different groups. In this paper we investigate the choice dynamics when considering two populations where one serves as a *reference group*. Considering this influence effect enriches the dynamics. Although the structurally stable resulting dynamics are attracting cycles only, with any positive integer period, the reference group makes the dynamics of the influenced population much more complex. We considered both the possibility that the reference group has the same or the opposite attitude towards the distribution over the choices. We show how the dynamics and the bifurcation structure are modified under the influence of the reference group. Our results illustrate how the propensity to switch choices in the reference groups may, indirectly, affect choices in the first group.

Keywords discontinuous 2-dim maps · border-collision bifurcations · periodicity tongues · influence · reference group · binary choices

1 Introduction

Most contributions which analyze binary choices dynamics consider interactions within the group (see e.g., [8,6]). Actually, several situations involve

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intergroup and not only intragroup interactions. In fact, according to [1, p.17], "everywhere on earth we find a condition of separateness among groups". Furthermore, individuals have preferences in relating with others belonging to their same group. However, social interaction is not limited among individuals belonging to the same group, rather interactions both in terms of cooperation and competition are quite common between groups (see e.g., [30]). As a matter of fact, interaction between groups is not limited to competition and cooperation as individuals may wish to be included in other groups and may relate themselves to the standards and expectations of another group, which in the social psychology literature [1] is called *reference group.* As defined in [17] a reference group is any group that individuals use for social comparison, i.e., they determine their own social worth by comparing their accomplishments to the achievements of members of the identified groups. According to [22], the term "reference group" was first introduced in [21] where some of its properties were explored. This construct has been elaborated by several scholars; in particular [26] related the reference group to the concept of anticipatory socialization, i.e., when individuals –led by the aspiration to become members– socialize themselves to other group norms. Furthermore, [23] made an important distinction between comparative and normative reference groups. This distinction corresponds to the two functions of reference groups: as standards of comparison for self-appraisal and as the source of the individual's norms, attitudes, and values. Actually, according to [2] the term can be used in three different situations. The first concerns the personal evaluation when comparing oneself on some characteristics with a referent which may be another person or category of persons. In the second situation the behavior of individuals in the reference group become a sort of "model". Finally, the last situation refers to how others' behavior is interpreted depending on the contest of some social groups. In this paper we consider the second kind of situations which is related to Newcomb's Bennington studies [27] and, according to [24], highlights how a group may influence individual attitudes and preferences. Evidence of the reference group influence is large; we can find examples in different disciplines such as economics, marketing and sociology. In economics for instance, [3] finds that individuals' contributions are affected by those in a reference group. Furthermore, according to [16] in the ultimatum game acceptance threshold is affected by the reference group with consequences on the offers which may be sustained in equilibrium. Seed money plays an important role in fund-rising (see [18]); in the study reported in [25], seed money serves also as a signal of quality, and it is shown that in donation campaigns increasing seed money may increase fund-raising for threshold public goods. In marketing, [5] investigates different types of reference group influence on brand and product decisions across several product categories. In sociology, [11] use reference group theory to study the relationships among religiosity, socioeconomic status, and sexual morality. According to this study when values and beliefs in a group and a reference group are opposed to one another, individuals are more likely to conform to their group's values and beliefs. Finally, [12] apply the reference group theory to study the connection between religion and drinking behavior, using people's religion as a reference group.

In this paper we consider two populations facing the same binary choices with externalities in a dynamic setting. Starting from the adopted dynamics in the literature analyzing a single population [8,6], we model the dynamics for a two-population system. Then we derive the map in the case in which one of the group serves as reference group for the other.

The paper is organized as follows. In Section 2 the influence of a reference group is modeled. The analysis and the results are presented in Section 3, and the last section is devoted to conclusions and further research.

2 Formalization

We consider a repeated game with two populations $\mathbb{X}$ and $\mathbb{Y}$ where a continuum of agents chooses actions from set $A = \{L, R\}$, i.e., they are facing a binary choice as in [29]. Within each population, agents update their choices at time $t = 0, 1, 2, \ldots$. The two sets of agents are normalized[1] to the interval $[0, 1]$. Individuals in both populations are affected by their payoff. We introduce the following notation, at any time t:

- $x_t^L \in [0, 1]$ is the fraction of agents in population $\mathbb{X}$ choosing action L;
- $x_t^R \in [0, 1]$ is the fraction of agents in population $\mathbb{X}$ choosing action R;
- $y_t^L \in [0, 1]$ is the fraction of agents in population $\mathbb{Y}$ choosing action L;
- $y_t^R \in [0, 1]$ is the fraction of agents in population $\mathbb{Y}$ choosing action R.

As we are considering binary choices, when at any time t a fraction x_t^R of population $\mathbb{X}$ chooses action R, a fraction $x_t^L = 1 - x_t^R$ chooses action L. The same reasoning applies to population $\mathbb{Y}$, where $y_t^L = 1 - y_t^R$. In this way it is possible to consider only one independent variable for each population and in the following we will omit both x_t^L and y_t^L (given as complement values). To further simplify the notation, from now on we will write $x_t := x_t^R$ and $y_t := y_t^R$.

Thus, the phase space –that is, the set of feasible vectors (x, y)– is the unitary square $U := [0, 1] \times [0, 1]$, with corners $P_{LL}(0, 0)$, $P_{LR}(0, 1)$, $P_{RL}(1, 0)$, $P_{RR}(1, 1)$. Obviously if:

- $(x_t, y_t) = (0, 0)$ then in both populations agents choose action L;
- $(x_t, y_t) = (0, 1)$ then the whole population $\mathbb{X}$ chooses action L and the whole population $\mathbb{Y}$ chooses action R;
- $(x_t, y_t) = (1, 0)$ then the whole population $\mathbb{X}$ chooses action R and the whole population $\mathbb{Y}$ chooses action L;
- $(x_t, y_t) = (1, 1)$ then in both populations agents choose action R.

Corners can be named *population's local unanimity* vertices since in each of them the two populations have agents choosing the same action. In particular,

[1] This assumption can be easily dropped when considering percentages.

corners P_{LL} and P_{RR} are *global unanimity* corners, as both groups wholly agree on the same choice.

For both populations, the payoff functions are common knowledge and are assumed to be linear functions depending on how the agents distribute over actions. Respectively, with subscript $\zeta \in \{X, Y\}$:

- $L_\zeta : [0,1] \to \mathbb{R}$ are the payoffs associated to action L:
  - $L_X(x) = a_{XL}x + b_{XL}$ for population $\mathbb{X}$
  - $L_Y(y) = a_{YL}y + b_{YL}$ for population $\mathbb{Y}$
- $R_\zeta : [0,1] \to \mathbb{R}$ are the payoffs associated to action R:
  - $R_X(x) = a_{XR}x + b_{XR}$ for population $\mathbb{X}$
  - $R_Y(y) = a_{YR}y + b_{YR}$ for population $\mathbb{Y}$

for all $a_{XL}, b_{XL}, a_{XR}, b_{XR}, a_{YL}, b_{YL}, a_{YR}, b_{YR} \in \mathbb{R}$.

This way, in each population the payoffs are equal (and the agents are indifferent) when $L_X(x) = R_X(x)$ and $L_Y(y) = R_Y(y)$ respectively, for some $x, y \in [0,1]$. That is, when $x = x^*$ and $y = y^*$, where

$$x^* = \frac{b_{XR} - b_{XL}}{a_{XL} - a_{XR}}, \qquad y^* = \frac{b_{YR} - b_{YL}}{a_{YL} - a_{YR}} \quad \text{with } a_{XL} \neq a_{XR} \text{ and } a_{YL} \neq a_{YR}. \tag{1}$$

We say that the two indifference points are feasible if $x^*, y^* \in [0,1]$.

The agents are homogeneous and maximize their next period utility using impulsive choices as in [7]. At time $t+1$ variables x_t, y_t become common knowledge, and each agent in both populations can observe payoffs $L_X(x_t), L_Y(y_t), R_X(x_t)$ and $R_Y(y_t)$. If at time t a fraction $x_t^L = 1 - x_t$ of population $\mathbb{X}$ chooses action L, a fraction x_t chooses action R, and the payoffs are such that $R_X(x_t) > L_X(x_t)$, then a fraction of the $1 - x_t$ agents who chose action L will switch to action R at next time period $t+1$. This is the same for population $\mathbb{Y}$ and for all actions which give the larger payoff. In other words, at any time t all the agents decide their future action at time $t+1$ comparing payoffs $L_X(x_t)$ to $R_X(x_t)$ and $L_Y(y_t)$ to $R_Y(y_t)$ according to the following rules:

$x_{t+1} = F_1(x_t)$ and $y_{t+1} = G_1(y_t)$ with $F_1, G_1 : [0,1] \to [0,1]$ and the maps

$$x_{t+1} = F_1(x_t) = \begin{cases} x_t - \delta_{XL}x_t & \text{if} \quad L_X(x_t) > R_X(x_t) \\ x_t + \delta_{XR}(1 - x_t) & \text{if} \quad L_X(x_t) < R_X(x_t) \end{cases} \tag{2}$$

$$y_{t+1} = G_1(y_t) = \begin{cases} y_t - \delta_{YL}y_t & \text{if} \quad L_Y(y_t) > R_Y(y_t) \\ y_t + \delta_{YR}(1 - y_t) & \text{if} \quad L_Y(y_t) < R_Y(y_t) \end{cases} \tag{3}$$

Parameters $\delta_{XL}, \delta_{XR} \in [0,1]$ represent the proportion of population $\mathbb{X}$ agents who may switch to action L and R, respectively; similarly, parameters $\delta_{YL}, \delta_{YR} \in [0,1]$ represent the proportion of population $\mathbb{Y}$ agents who may switch to action L and R respectively. When within a given population these two parameters are equal, there are no differences in the propensity to switch to any of the actions involved. The switching rate to a different action just depends on the sign of the difference between the payoffs, without foresight (see [15]). This setting formalizes the dynamics of two isolated groups.

As the main goal of the paper is to describe the effect of a reference group influence we assume that the reference group distribution over the two choices influences the other population choice mechanism. Following the second interpretation of reference group provided in [2], we assume that population $\mathbb{Y}$ choice distribution is considered the "model" to follow; in other words population $\mathbb{X}$ agents aim to obtain the same choice distribution as in $\mathbb{Y}$. Therefore, as it concerns the population $\mathbb{X}$, a further evaluation is based on population $\mathbb{Y}$ behavior, as population $\mathbb{Y}$ is assumed to be the reference group for population $\mathbb{X}$. If at time t a fraction x_t of population $\mathbb{X}$ chooses action R and a fraction y_t of population $\mathbb{Y}$ chose action R, and fractions are such that $y_t > x_t$, then a fraction of the $1-x_t$ agents of population $\mathbb{X}$ who chose action L will switch to action R at next time period $t+1$. This is not the same for population $\mathbb{Y}$ which takes the role of reference group and is assumed not to consider population $\mathbb{X}$ choice. Therefore, at any time t all the population $\mathbb{X}$ agents decide their future action at time $t+1$ also comparing x_t to y_t according to the following rule: $x_{t+1} = F_2(x_t, y_t)$ with $F_2 : [0,1]^2 \to [0,1]$ and the map is

$$x_{t+1} = F_2(x_t, y_t) = \begin{cases} x_t - c_X x_t & \text{if} \quad x_t > y_t \\ x_t + c_X(1-x_t) & \text{if} \quad x_t < y_t \end{cases} \tag{4}$$

Parameter $c_X \in [0,1]$ is the proportion of population $\mathbb{X}$ agents who may switch action as a consequence of the comparison between the fraction of agents choosing R in their own group and the same fraction in the reference group. When considering both the decision rules, the joint comparisons can be illustrated geometrically. From the population $\mathbb{X}$ perspective, the phase space U can be partitioned in different regions denoted as follows: the first lower index is related to the action with the largest payoff; the second lower index indicates the choice suggested by population $\mathbb{Y}$ distribution. The regions illustrated in Figure 1a are defined as:

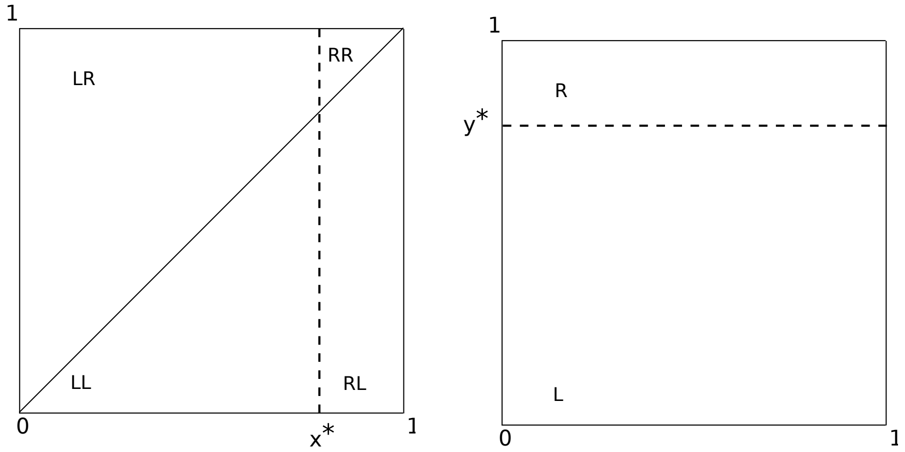
- $R_{LL} = \{(x,y) \in U : \quad L_X(x) > R_X(x) \quad \text{and} \quad x > y\}$
- $R_{LR} = \{(x,y) \in U : \quad L_X(x) > R_X(x) \quad \text{and} \quad x < y\}$
- $R_{RL} = \{(x,y) \in U : \quad L_X(x) < R_X(x) \quad \text{and} \quad x > y\}$
- $R_{RR} = \{(x,y) \in U : \quad L_X(x) < R_X(x) \quad \text{and} \quad x < y\}$

From population $\mathbb{Y}$ perspective, the phase space U is simply partitioned as (see Figure 1b):

- $R_L = \{(x,y) \in U : \quad L_Y(y) > R_Y(y)\}$
- $R_R = \{(x,y) \in U : \quad L_Y(y) < R_Y(y)\}$

For example, in region R_{LR} action L is dominant for population $\mathbb{X}$ (first lower index), and action R (second lower index) is more popular in population $\mathbb{Y}$ than in population $\mathbb{X}$. As it concerns population $\mathbb{Y}$ regions the lower index simply denotes the dominant choice.

Modeling the joint comparisons in population $\mathbb{X}$ is non-trivial even when considering impulsive agents. Assuming the payoff and reference group as antecedents of agents' behavior, would require to consider at least three kinds of



(a) Population $\mathbb{X}$ (b) Population $\mathbb{Y}$

Fig. 1: Partition of set U from the perspective of each population.

agents: those for which either a difference in payoff or the reference group are sufficient to switch choice and those for which both a difference in payoff and the reference group are necessary to switch. For the sake of analytic tractabilty we consider homogeneous agents. Therefore, when the switching decision based on payoff comparison is coherent with the reference group norm, we assume that a fraction c_X of the agents who are not switching to the larger payoff choice will be following the reference group norm. It is interesting to observe that if we assume that the decision is made first with respect to the reference group norm and then with respect of the payoff difference the result is the same; in fact, when considering R_{LL}, we have:

$$x_t - \delta_{XL} x_t - c_X \left(x_t - \delta_{XL} x_t\right) = x_t - c_X x_t - \delta_{XL} \left(x_t - c_X x_t\right). \tag{5}$$

By contrast, when the payoff comparison switch is opposite to the reference group norm the switching fractions will be determined respectively by parameters $\delta_{X\sigma}$ and c_X, with $\sigma \in \{L, R\}$. For example, assuming choice L provides a larger payoff than R and in the reference group the fraction of agents choosing R is larger than in population $\mathbb{X}$, then in the latter population, a fraction δ_{XL} of the agents choosing R will switch to L and a fraction c_X of those choosing L will switch to R.

As a result of their own perspectives, the two populations follow the respective decision rules derived from (2), (4) and (3):

$$x_{t+1} = \begin{cases} (1 - \delta_{XL} - c_X + \delta_{XL} c_X) x_t & \text{if} \quad (x_t, y_t) \in R_{LL} \\ (1 - \delta_{XL} - c_X)\, x_t + c_X & \text{if} \quad (x_t, y_t) \in R_{LR} \\ (1 - \delta_{XR} - c_X)\, x_t + \delta_{XR} & \text{if} \quad (x_t, y_t) \in R_{RL} \\ (1 - \delta_{XR} - c_X + \delta_{XR} c_X)\, x_t + \delta_{XR} + c_X - \delta_{XR} c_X & \text{if} \quad (x_t, y_t) \in R_{RR} \end{cases} \tag{6}$$

$$y_{t+1} = \begin{cases} (1-\delta_{YL})\, y_t & \text{if} \quad (x_t, y_t) \in R_L \\ (1-\delta_{YR})\, y_t + \delta_{YR} & \text{if} \quad (x_t, y_t) \in R_R \end{cases} \tag{7}$$

Several contributions show the existence of two forces acting together in presence of a reference group. For example, [11] in their study on sexual morality provide evidence that people are "torn" between two reference points: their socioeconomic status collectivity and their religious group. However, as we consider the influence of a reference group the two population dynamics need to be considered jointly as $(x_{t+1}, y_{t+1}) = \mathbf{F}(x_t, y_t)$ such that $\mathbf{F} : U \to U$, where

$$\mathbf{F}(x_t, y_t) = \begin{cases} \begin{cases} x_{t+1} = (1-\delta_{XL} - c_X + \delta_{XL}c_X)\, x_t \\ y_{t+1} = (1-\delta_{YL})\, y_t \end{cases} & \text{if } (x_t, y_t) \in R_{LLL} \\ \begin{cases} x_{t+1} = (1-\delta_{XL} - c_X + \delta_{XL}c_X)\, x_t \\ y_{t+1} = (1-\delta_{YR})\, y_t + \delta_{YR} \end{cases} & \text{if } (x_t, y_t) \in R_{LLR} \\ \begin{cases} x_{t+1} = (1-\delta_{XL} - c_X)\, x_t + c_X \\ y_{t+1} = (1-\delta_{YL})\, y_t \end{cases} & \text{if } (x_t, y_t) \in R_{LRL} \\ \begin{cases} x_{t+1} = (1-\delta_{XL} - c_X)\, x_t + c_X \\ y_{t+1} = (1-\delta_{YR})\, y_t + \delta_{YR} \end{cases} & \text{if } (x_t, y_t) \in R_{LRR} \\ \begin{cases} x_{t+1} = (1-\delta_{XR} - c_X)\, x_t + \delta_{XR} \\ y_{t+1} = (1-\delta_{YL})\, y_t \end{cases} & \text{if } (x_t, y_t) \in R_{RLL} \\ \begin{cases} x_{t+1} = (1-\delta_{XR} - c_X)\, x_t + \delta_{XR} \\ y_{t+1} = (1-\delta_{YR})\, y_t + \delta_{YR} \end{cases} & \text{if } (x_t, y_t) \in R_{RLR} \\ \begin{cases} x_{t+1} = (1-\delta_{XR} - c_X + \delta_{XR}c_X)\, x_t + \\ \qquad +\delta_{XR} + c_X - \delta_{XR}c_X \\ y_{t+1} = (1-\delta_{YL})\, y_t \end{cases} & \text{if } (x_t, y_t) \in R_{RRL} \\ \begin{cases} x_{t+1} = (1-\delta_{XR} - c_X + \delta_{XR}c_X)\, x_t + \\ \qquad +\delta_{XR} + c_X - \delta_{XR}c_X \\ y_{t+1} = (1-\delta_{YR})\, y_t + \delta_{YR} \end{cases} & \text{if } (x_t, y_t) \in R_{RRR} \end{cases} \tag{8}$$

and $R_{\sigma\tau\upsilon} = R_{\sigma\tau} \cap R_\upsilon$ with $\sigma, \tau, \upsilon \in \{L, R\}$.

Each expression defining x_{t+1}^R and y_{t+1}^R linearly depends on the same state variable only, and not on the other one. However, depending on the payoff values, the state variable may change the region to which it belongs to, leading to a change in the dynamics.

Depending on the payoff functions, the feasible discontinuity points x^*, y^* defined in (1) partition set U in regions where, case by case, a different strategy is dominant, given the distribution (x, y) of the agents of each population on the different strategies. To make the reading of the figures clearer, each region is labeled according to the regions of map (8). In Figure 2, the first letter indicates the dominant strategy for population $\mathbb{X}$. When map (2) has the discontinuity with an increasing jump, for $x < x^*$ the dominant strategy is L while it is R for $x > x^*$. Vice versa, if the discontinuity has a decreasing jump, for $x < x^*$ the dominant strategy is R while it is L for $x > x^*$. With the same rule, the last letter indicates the dominant strategy for population $\mathbb{Y}$, while the middle one indicates the choice suggested by the reference group. In this case, as illustrated in Figure 2, up to seven different regions can coexist.

The conditions on the parameters determining the slopes of the linear functions, and thus the map eigenvalues in the linear pieces, all lead to contractions. A point $(x, y) \in R_{\sigma\tau\upsilon}$ has Jacobian matrix either

$$J(x,y) = \begin{pmatrix} 1-\delta_{X\sigma}-c_X+\delta_{X\sigma}c_X & 0 \\ 0 & 1-\delta_{Y\upsilon} \end{pmatrix} \text{ if } \quad \sigma = \tau,$$

or

$$J(x,y) = \begin{pmatrix} 1-\delta_{X\sigma}-c_X & 0 \\ 0 & 1-\delta_{Y\upsilon} \end{pmatrix} \text{ if } \quad \sigma \neq \tau.$$

The first matrix eigenvalues, $\lambda_1 = (1-\delta_{X\sigma}-c_X+\delta_{X\sigma}c_X)$ and $\lambda_2 = (1-\delta_{Y\upsilon})$, are real and belong to $[0, 1)$, except for $\delta_{X\sigma}+c_X-\delta_{X\sigma}c_X = 0$ and $\delta_{Y\upsilon} = 0$, in which case both the eigenvalues equal 1. However, as it would be $\delta_{X\sigma} = c_X = \delta_{Y\upsilon} = 0$, this would correspond to the case of two isolated groups who never switch choice internally, and which is not of interest in this paper. The same applies for the second matrix eigenvalues.

Under such conditions only stable cycles can exist, as all the eigenvalues of the components are non-negative and less then one, or at most equal to 1. In fact, any possible cycle of period $k \geq 1$ has eigenvalues which are necessarily non-negative and not greater than one. Furthermore, we recall that the eigenvalues of a cycle are given by the eigenvalues of the Jacobian matrices product in the periodic points. Thus, no chaotic behavior can occur, neither divergence, as the map is defined from U onto U, and the following proposition is proved.

Proposition 1 *Map* $\mathbf{F}$ *in* (8) *can only have* k*-cycles for any* $k \geq 1$.

We are interested in the effect of the reference group influence both in the occurrence of positive and negative externalities, when considering binary choices in a population with impulsive agents [6, 7]. By externalities we mean the advantageous/adverse effects of one's choices on others' payoffs as illustrated by the several examples provided in [29]. When there is no reference group influence, i.e. $c_X = 0$, we have two independent populations whose dynamics are determined by iterated maps and where each map is characterized either by an *increasing* or a *decreasing* jump at the discontinuity point d. We recall that when such a linear map has an increasing jump, the dynamics are very simple (see Figure 3a). There are two stable fixed points, the boundary steady states $x = 0$ and $x = 1$, with basins of attraction separated by the discontinuity point d: any initial condition $x_0 \in [0, d)$ will converge to the fixed

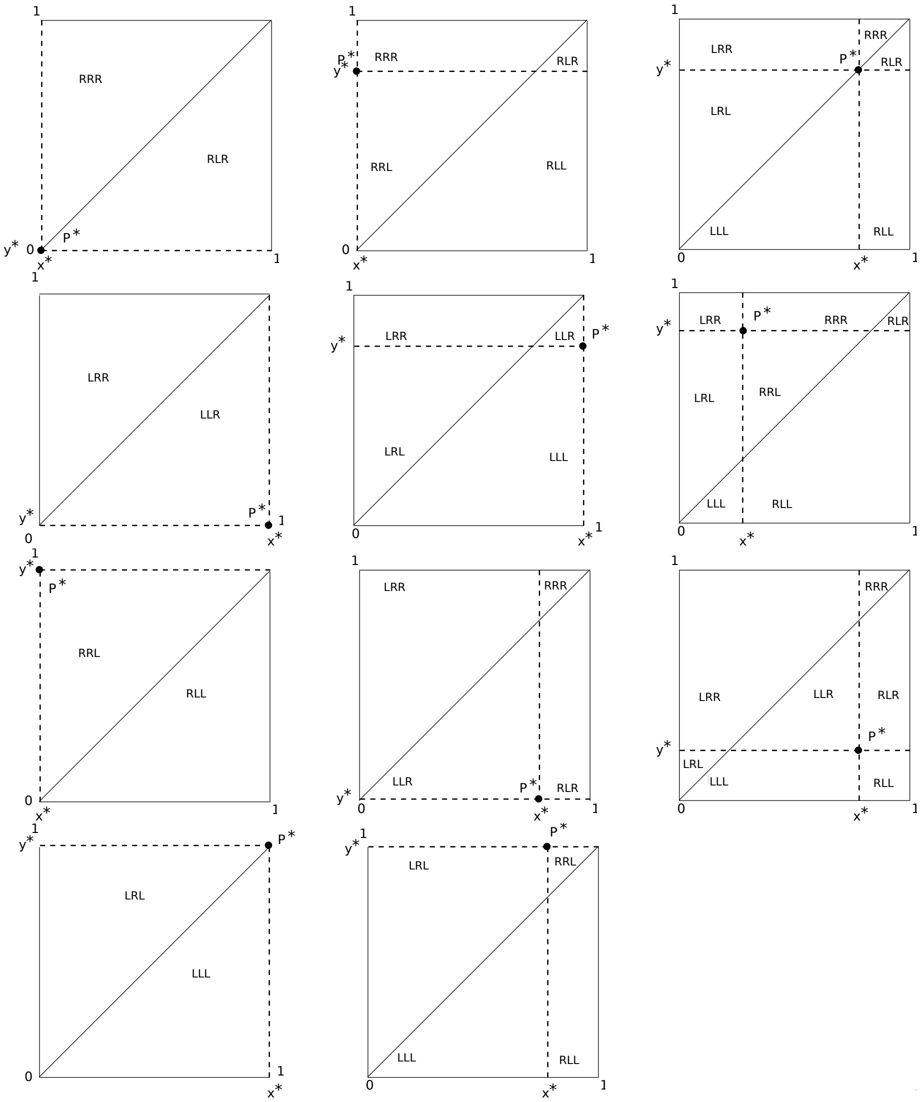
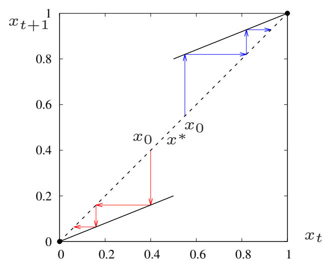
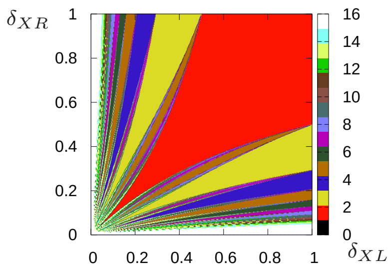


Fig. 2: The possible partitions of the set U when the maps of the two populations have both a discontinuity with an increasing jump. The areas are labeled according to the rule described in Section 2.

point $x = 0$ while any initial condition $x_0 \in (d, 1]$ will generate a trajectory that converges to the fixed point $x = 1$. By contrast, when considering a decreasing jump, as illustrated in [6], periodic cycles of any period may occur. The two maps (2) and (3) are piecewise linear maps with one discontinuity point, x^* and y^* respectively. As we have already said that for both maps the slopes on the left and right sides of the discontinuity points are between zero and one, in the parameter space of each map the period adding structure can be observed [6], as illustrated in the bifurcation diagram in Figure 3b.



(a)



(b)

Fig. 3: Single population dynamics. (a) One dimensional map with an increasing jump at the discontinuity point x^*. (b) Bifurcation diagrams in the parameters plane $(\delta_{XL}, \delta_{XR})$; the regions of periodicity are represented by different colors.

In the next section we examine the increasing/decreasing jump combinations for the two populations and analyze how the reference group affects population $\mathbb{X}$ dynamics.

3 The influence of the reference group

When jointly considering the two populations we have four possible cases depending on the payoff functions. Although, when considering a reference group, one would expect the two populations to exhibit the same dynamics, there are examples in which the two populations may have different dynamics. One comes from how public transportation is used in different countries. Indeed, there are some differences in automobile use between European countries and the US [19]. Nevertheless, according to [20, p.14] "the roots of today's motor industry can be traced back to Henry Ford"; as a matter of fact the structure of FIAT's main plants in Turin were inspired by Fordist mass production [28]. Therefore, as it concerns transportation, we can assume that the US can be considered as a reference group and that in the US using private transportation can be considered a dominant choice while in Europe it could be more similar to a minority game [4]: if the majority of agents uses public transportation then it would be preferable to use the car as roads are empty; vice versa, when everybody is using private transportation it would be better to use public transportation because of congestion [29,6].

In general, while with independent populations set U is partitioned in four parts as illustrated[2] in Figure 4a, the reference group induces a comparison between the decisions taken in the two groups and is modeled by means of the 45-degree line, as shown in Figure 4b. This line allows to distinguish the areas of the set U that are above this line $(y > x)$ and those that are below $(y < x)$. If the proportion of agents in the reference group who have chosen strategy R exceeds that of population $\mathbb{X}$, then strategy R becomes the reference strategy and this letter is inserted between the previous two.

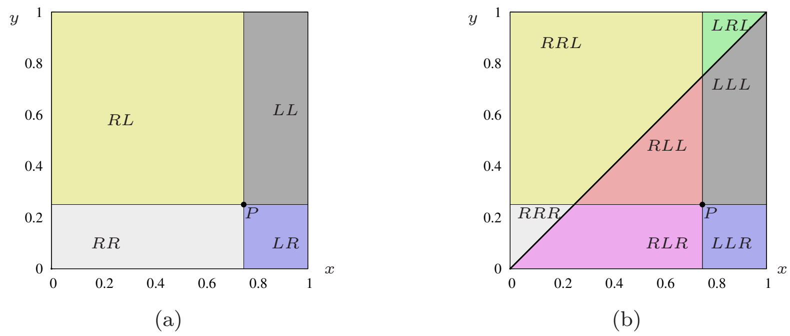


Fig. 4: Feasible set U for a two-population system, both with increasing jump (a) without reference group influence and (b) with reference group influence.

In the rest of the section we present the four possible cases depending on the jump at the discontinuity point. In order to make a proper comparison among them and for the sake of simplicity, we assume that population $\mathbb{Y}$ converges either to a fixed point or to a 2-period cycle. In the examples we provide, all the parameter values are kept fixed, unless otherwise stated: $\delta_{XL} = 0.1$, $\delta_{XR} = 0.3$, $\delta_{YL} = 0.6$ and $\delta_{YR} = 0.7$. Finally, we consider the following values of reference group influence: $c_X = 0.01$, 0.1 and 0.3.

3.1 Two maps with an increasing jump at the discontinuity point: $F_1(x^{*-}) < F_1(x^{*+})$ and $G_1(y^{*-}) < G_1(y^{*+})$

This is the simplest of the four cases and therefore it can be analyzed theoretically. When the jump is increasing for both populations, the resulting dynamics (with $c_X = 0$) has four stable fixed points, the boundary steady states $P_{LL}(0,0)$, $P_{LR}(0,1)$, $P_{RL}(1,0)$, and $P_{RR}(1,1)$, with basins of attraction separated by the discontinuity lines defined as the set $\{(x,y) \in U : x = x^* \vee y = y^*\}$. Any initial condition $(x_0, y_0) \in R_{LL} = [0, x^*) \times [0, y^*)$ will converge to $P_{LL}(0,0)$;

[2] For the sake of brevity in Figure 4 the case of an increasing jump for both populations is considered. The other cases are similar albeit with different labeling of the regions.

$(x_0, y_0) \in R_{LR} = [0, x^*) \times (y^*, 1]$ to point $P_{LR}(0,1)$; $(x_0, y_0) \in R_{RL} = (x^*, 1] \times [0, y^*)$ to point $P_{RL}(1,0)$; $(x_0, y_0) \in R_{RR} = (x^*, 1] \times (y^*, 1]$ to point $P_{RR}(1,1)$; that is, each population will converge to the local unanimity equilibrium depending on the initial condition. When assuming that for each population the indifference point is internal, we have four regions as illustrated in Figure 5. In this case, two local and two global unanimity equilibria occur.

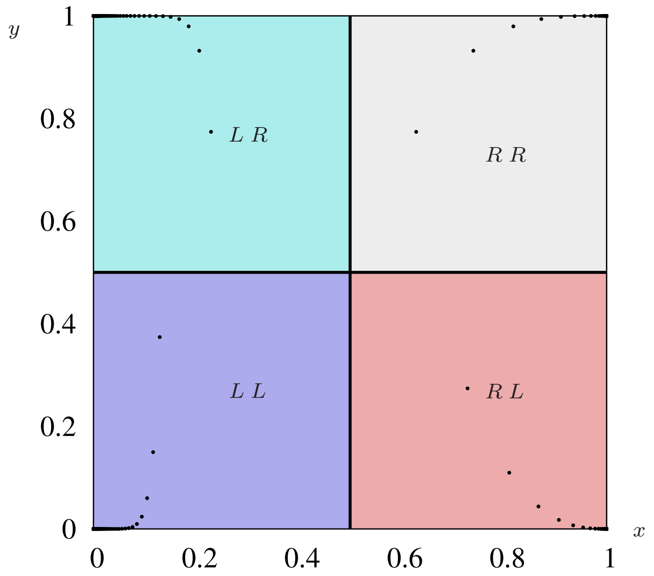


Fig. 5: Trajectories for the case increasing-increasing jumps at the discontinuity point with no reference group influence: $c_X = 0$. Payoff functions: $L_X(x) = -x + 2$ $R_X(x) = x + 1$ $L_Y(y) = -y + 2$ $R_Y(y) = y + 1$.

However, when parameter $c_X > 0$, the off-diagonal points P_{LR}, P_{RL} –that is, the local unanimity points– undergo the influence of the reference group modeled by parameter c_X and are moved from the corners of set U. The new points are respectively $A\left(\frac{\delta_{XR}}{\delta_{XR}+c_X}, 0\right)$ and $B\left(\frac{c_X}{\delta_{XL}+c_X}, 1\right)$, as illustrated in Figure 6. It is immediate to see the influence of the reference group on these two fixed points, by letting c_X take values in $(0,1]$. The observer of population $\mathbb{X}$ can correctly predict the existence of two equilibria, yet these equilibria are not the expected distribution of the agents over the two choices, as there is no unanimity.

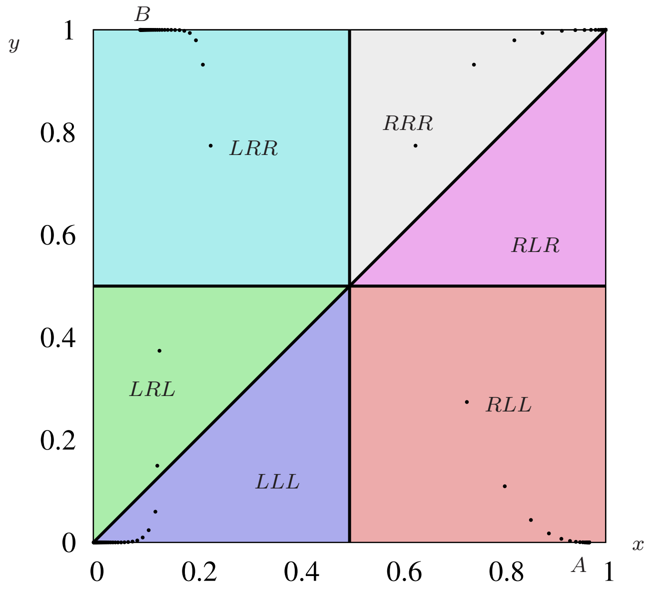


Fig. 6: Trajectories for the case increasing-increasing jumps at the discontinuity point with reference group influence $c_X = 0.01$. Payoff functions: $L_X(x) = -x + 2$ $R_X(x) = x + 1$ $L_Y(y) = -y + 2$ $R_Y(y) = y + 1$.

3.2 The case of increasing-decreasing jumps at the discontinuity point: $F_1(x^{*-}) < F_1(x^{*+})$ and $G_1(y^{*-}) > G_1(y^{*+})$

Without reference group influence, when the jump is increasing for population $\mathbb{X}$ and decreasing for population $\mathbb{Y}$, the first component dynamics has two stable fixed points while for the second component dynamics, period cycles of any period may occur. The resulting dynamics has two attractors, both stable cycles of period $k = k_Y$ depending on parameter values δ_{YL}, δ_{YR}: one on boundary $x = 0$; the other one on boundary $x = 1$. Any initial condition (x_0, y_0) s.t. $x_0 \in [0, x^*)$ will generate a trajectory convergent to the attractor on $x = 0$; any initial condition (x_0, y_0) s.t. $x_0 \in (x^*, 1]$ to the attractor on $x = 1$. In any case, observing just population $\mathbb{X}$, only the two unanimity equilibrium points are possible. In other words the dynamics of population $\mathbb{X}$ is the same as the one described in Figure 3a and population $\mathbb{Y}$ dynamics is characterized by a bifurcation diagram as the one illustrated in Figure 3b.

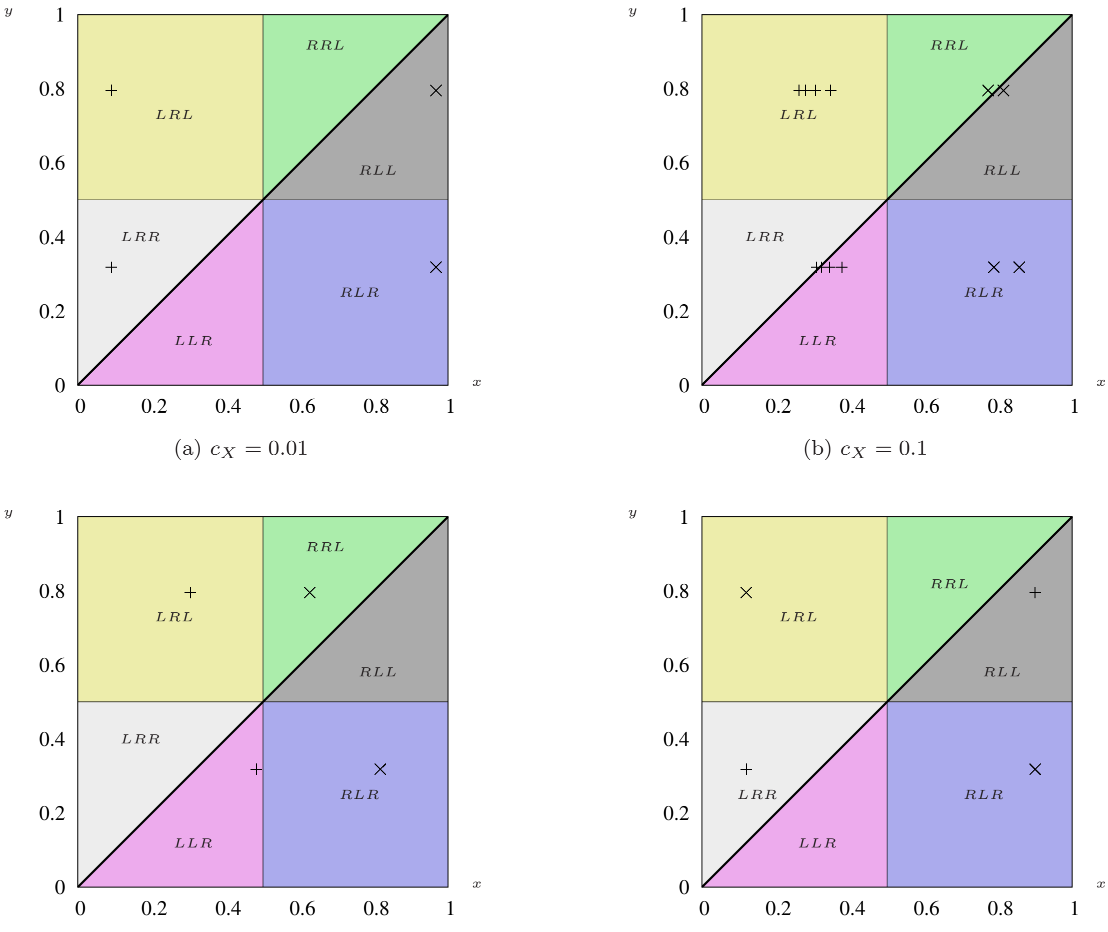
If we assume the reference group has an influence on population $\mathbb{X}$, that is, when $c_X > 0$, the dynamics of population $\mathbb{Y}$ affects the dynamics of population $\mathbb{X}$. In order to investigate this case we consider the following example.

Example 1 Considering payoff functions

$$L_X(x) = -x + 2, \quad R_X(x) = x + 1, \quad L_Y(y) = y + 1, \quad R_Y(y) = -y + 2$$

then map (8) is defined in the regions illustrated in Figure 7.

- $R_{LLR} = \{(x, y) \in U : x < 1/2, y < 1/2, y < x\}$
- $R_{LRL} = \{(x, y) \in U : x < 1/2, y > 1/2, y > x\}$
- $R_{LRR} = \{(x, y) \in U : x < 1/2, y < 1/2, y > x\}$
- $R_{RLL} = \{(x, y) \in U : x > 1/2, y > 1/2, y < x\}$
- $R_{RLR} = \{(x, y) \in U : x > 1/2, y < 1/2, y < x\}$
- $R_{RRL} = \{(x, y) \in U : x > 1/2, y > 1/2, y > x\}$



(a) $c_X = 0.01$ (b) $c_X = 0.1$

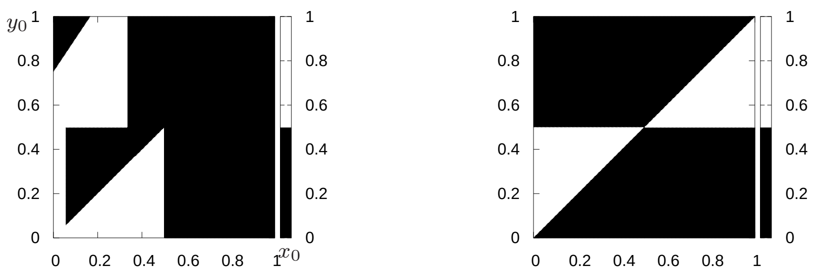
(c) $c_X = 0.3$ (d) $c_X = 0.9$

Fig. 7: Cycles for the case increasing-decreasing jumps at the discontinuity point. Payoff functions as in Example 1; initial condition $x_0 = 0.129$, $y_0 = 0.374$.

The two-population system attracting sets do not any longer belong to the two boundaries $x = 0$ and $x = 1$. As a result of the reference group influence, the trajectories converge to a 2-cycle in the interior of either region $R_{LRL} \cup R_{LRR}$ if $x_0 \in [0, x^*)$, or $R_{RLL} \cup R_{RLR}$ if $x_0 \in (x^*, 1]$. This holds for small values of c_X as illustrated in Figure 7a. In the limit case of $c_X = 0$ the cycles collapse respectively on the boundaries $x = 0$ and $x = 1$. The basins of attraction remain connected sets. The observer who ignores the reference group still see that population $\mathbb{X}$ dynamics has two equilibria, albeit different from those with no reference group influence.

Increasing the reference group influence a new periodic k-cycle for population $\mathbb{X}$ appears. For example, in Figure 7b, with $c_X = 0.1$, a cycle of period eight (i.c. (x_0, y_0) with $x_0 < x^*$) coexists with a cycle of period four (i.c. (x_0, y_0) with $x_0 > x^*$). The basins of attraction are still connected sets. However, by ignoring the reference group influence the occurrence of cycles for population $\mathbb{X}$ might be hard to explain.

When $c_X = 0.3$ as in Figure 7c the two-population system has two coexisting 2-cycles. However, neither all the trajectories with initial conditions in $R_{LRL} \cup R_{LRR} \cup R_{LLR}$ are converging to the cycle to the left, nor all those in $R_{RRL} \cup R_{RLL} \cup R_{RLR}$ are converging to the cycle to the right. In Figure 8a we show in white the set of points whose trajectory is convergent to the 2-cycle with orbits to the left and in black to the 2-cycle to the right. In this case one of the basins is disconnected. When $c_X = 0.9$ both basins are disconnected as illustrated in Figure 8b as $(0.5, 0.5)$ is the unstable fixed point.



(a) $c_X = 0.3$ (b) $c_X = 0.9$

Fig. 8: Basins of attraction for map attractors as in Example 1. In white, basins of attraction of the cycles denoted with $+$, in black, those denoted with $\times$ in Figures 7c and 7d.

As a last remark, as parameter c_X increases, that is, when the reference group influence is very strong, we observe that the periodic points of the two coexisting 2-cycles switch from $\mathbf{p_1} = (x_1, y_1)$, $\mathbf{p_2} = (x_2, y_2)$ to $\mathbf{q_1} =$

(x_2, y_1), $\mathbf{q_2} = (x_1, y_2)$. In fact, as illustrated in Figure 7d when $c_X = 0.9$, with initial conditions $(x_0, y_0) \in R_{LRR} \cup R_{RLL}$, the 2-cycle is $\mathbf{p_1} = (0.12, 0.318)$, $\mathbf{p_2} = (0.9, 0.795)$, otherwise the cycle is $\mathbf{q_1} = (0.9, 0.318)$, $\mathbf{q_2} = (0.12, 0.795)$. Therefore, in this case the reference group influence reduces local unanimity in population $\mathbb{X}$ which, dependently on the initial conditions, exhibits a behavior either conforming or opposite to the reference group one.

3.3 The case of decreasing-increasing jumps at the discontinuity point: $F_1(x^{*-}) > F_1(x^{*+})$ and $G_1(y^{*-}) < G_1(y^{*+})$

When the jump is decreasing for population $\mathbb{X}$ and increasing for population $\mathbb{Y}$, and the reference group has no influence ($c_X = 0$), the dynamics is similar to the case discussed in Section 3.2 by switching populations. The two attractors are stable cycles of period $k = k_X$ depending on parameter values δ_{XL}, δ_{XR}; the trajectories will asymptotically converge either to the attractor on the boundary $y = 0$ if $y_0 \in [0, y^*)$, or to the boundary $y = 1$ if $y_0 \in (y^*, 1]$. When the influence of the reference group is positive, the two-population system still exhibits cycles of any period $k \geq 1$: depending on the initial condition y_0 the trajectory will converge either to the cycle on the boundary $y = 0$ or $y = 1$, as illustrated in the following example.

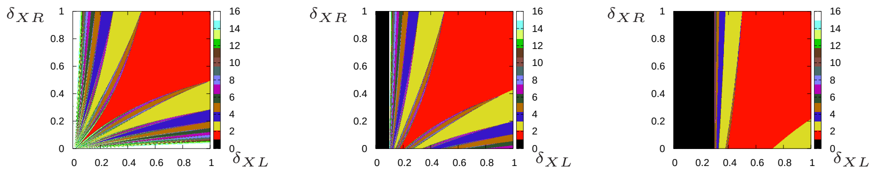
Example 2 Considering payoff functions

$$L_X(x) = x+1, \quad R_X(x) = -x+2, \quad L_Y(y) = -y+2, \quad R_Y(y) = y+1 \tag{9}$$

then map (8) is defined in the regions:

- $R_{LLL} = \{(x, y) \in U : x > 1/2, y < 1/2, y < x\}$
- $R_{LLR} = \{(x, y) \in U : x > 1/2, y > 1/2, y < x\}$
- $R_{LRR} = \{(x, y) \in U : x > 1/2, y > 1/2, y > x\}$
- $R_{RLL} = \{(x, y) \in U : x < 1/2, y < 1/2, y < x\}$
- $R_{RRL} = \{(x, y) \in U : x < 1/2, y < 1/2, y > x\}$
- $R_{RRR} = \{(x, y) \in U : x < 1/2, y > 1/2, y > x\}$

For small values of c_X, just by observing population $\mathbb{X}$ only, any k-cycle can occur as expected. However, when c_X is large enough, the reference group influence may lead the dynamics to converge to a fixed point rather than to a cycle. This can be inferred by observing the modifications of the bifurcation diagram of population $\mathbb{X}$ in Figure 9, as large period tongues disappear and for some choices of parameters the dynamics converges to a fixed point. Furthermore, the symmetry is lost and the period adding structure is destroyed. It is worth observing that for $c_X \geq 0.3$ fixed points become more and more influenced by the reference group fixed points, and population $\mathbb{X}$ dynamics tends to conform to either one of the reference group, as the fixed point region on the plane $(\delta_{XL}, \delta_{XR})$ widens.



(a) $c_X = 0.01$ (b) $c_X = 0.1$ (c) $c_X = 0.3$

Fig. 9: Bifurcation diagrams for population $\mathbb{X}$ in the parameters plane $(\delta_{XL}, \delta_{XR})$ for the case decreasing-increasing jumps at the discontinuity point; the regions of periodicity are represented by different colors.

3.4 The case of decreasing-decreasing jumps at the discontinuity point: $F_1(x^{*-}) > F_1(x^{*+})$ and $G_1(y^{*-}) > G_1(y^{*+})$

When the jump is decreasing for both populations and $c_X = 0$, given the parameter values $\delta_{XL}, \delta_{XR}, \delta_{YL}, \delta_{YR}$ the resulting dynamics obviously has a unique attractor, a stable cycle of period $k = l.c.m.(k_X, k_Y)$, for any initial condition $(x_0, y_0) \in U$ as it can be derived from the single population dynamics studied in [6]. We recall that k_X and k_Y are the periods of stable cycles of populations $\mathbb{X}$ and $\mathbb{Y}$ respectively, when $c_X = 0$.

With the influence of the reference group the dynamics of the system may be dramatically affected and the analysis is too complex to be analyzed here, rather it deserves a study of its own. However, for the sake of completeness, a numerical analysis is conducted and illustrated by the following example. Also in this case, for the sake of comparison, we consider the same parameter values as in the previous ones.

Example 3 Considering payoff functions

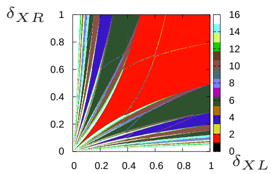
$$L_X(x) = x + 1, \quad R_X(x) = -x + 2, \quad L_Y(y) = y + 1, \quad R_Y(y) = -y + 2$$

then map (8) is defined in the regions:

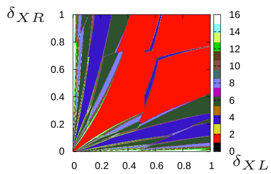
- $R_{LLL} = \{(x, y) \in U : x > 1/2, y > 1/2, y < x\}$
- $R_{LLR} = \{(x, y) \in U : x > 1/2, y < 1/2, y < x\}$
- $R_{LRL} = \{(x, y) \in U : x > 1/2, y > 1/2, y > x\}$
- $R_{RLR} = \{(x, y) \in U : x < 1/2, y < 1/2, y < x\}$
- $R_{RRL} = \{(x, y) \in U : x < 1/2, y > 1/2, y > x\}$
- $R_{RRR} = \{(x, y) \in U : x < 1/2, y < 1/2, y > x\}$

When the two populations are independent, with the same switching parameters values fixed as in Section 3, population $\mathbb{Y}$ dynamics has a 2-cycle as before and population $\mathbb{X}$ a 19-cycle. Thus, the two-population system dynamics has a 38-cycle.

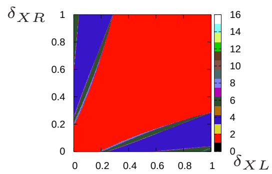
With these parameter values and $c_X = 0.01$, the stable attractor is a cycle of period 30 and there is no coexistence with other attractors for all possible initial conditions. However, as parameter c_X increases, with the same initial condition the period of the attractor decreases: a 4-cycle when $c_X = 0.1$ and a 2-cycle when $c_X = 0.3$. The bifurcation diagrams in the parameters plane $(\delta_{XL}, \delta_{XR})$ for these different values of parameter c_X are reported in Figure 10 and show how higher periodicity cycles vanish and population $\mathbb{X}$ dynamics conforms to a 2-cycle as the reference group. In particular, we see that the bifurcation diagram is not symmetric in Figure 10b: this is due to the asymmetry of population $\mathbb{Y}$ cycle periodic points. However, this is not the unique peculiarity of this case. As a matter of fact, by considering different values of the payoff parameters, we would see that not only symmetry is lost, but also the bifurcation structure modifies into a more complex structure and, as it can be observed in Figures 10a and 10b, we can have coexistence. For example, with $\delta_{XL} = 0.58517$ and $\delta_{XR} = 0.811623$, when the initial condition is $x_0 = 0.6$ and $y_0 = 0$ we obtain a 2-period cycle, while with initial condition $x_0 = 0$ and $y_0 = 0$ the period of the cycle is 4. This is quite surprising as, without the influence of the reference group, coexistence would not be possible for any of the two populations.



(a) $c_X = 0.01$



(b) $c_X = 0.1$



(c) $c_X = 0.3$

Fig. 10: Bifurcation diagrams for population $\mathbb{X}$ in the parameters plane $(\delta_{XL}, \delta_{XR})$ for the case decreasing-decreasing jumps at the discontinuity point; the regions of periodicity are represented by different colors.

4 Conclusions

We modeled the dynamical effect – in discrete time – of a reference group in a two-population system with binary choices and externalities. We considered the reference group as a "model" in the sense of [2] where it is not important what the majority of the group does, rather how the group distributes over the choices.

Our contribution to the literature of population games with binary choices and externalities in discrete time (e.g., [8,6,7,14,13]) is two-fold: firstly, we extended such a dynamics to two populations; secondly, we introduced in these kind of models the influence of a reference group which, with its behavior, affects the other population's dynamics. It is known in this literature, as proven in [6], that the dynamics is piecewise linear with one discontinuity point, and that at such a point either an increasing or a decreasing jump can be observed. In the former case, the trajectories converge to one of two fixed points located at the extremes of the feasible set, depending on the initial condition; in the latter, the unique attractor is a stable cycle of any period $k > 1$. Following this literature, we examined all the possible combinations when considering the two-population system.

The model allowed us to make comparisons of the population behavior with and without the reference group influence and also to calibrate the influence strength. Some characteristics of the reference group dynamics affect the other population. For example, in some cases the dynamics we found are similar to those with one population. In fact, we showed both examples with the coexistence of unanimity equilibria as in [29] and examples with cyclic behaviors as those analyzed in [6]. Furthermore, by introducing the influence of the reference group we described how these equilibria are modified, depending on the switching propensities of the reference group. Further analysis showed dynamics which are different from those observed when the reference group has no influence. In Example 1 large values of reference group influence may modify unanimity across populations. In other cases (Example 3) the complexity which can arise leads to the expected result that the influence of the reference group cannot be ignored when studying the behavior of a population.

The analysis of the examples we proposed evidenced bifurcation structures impossible to be observed in binary choices with only one population. On one hand, we could find also with the reference group influence the same border collision bifurcation structure (adding scheme, see [6]) as in an isolated population; on the other hand, in the two last examples we found more complicated bifurcation diagrams. In particular, we found in Example 2 that the period adding scheme is destroyed, and in Example 3 that the bifurcation diagram becomes asymmetric because of the asymmetry in the orbits describing the reference group behavior and we can have coexistence of attractors which is an effect of the influence of the reference group. In particular, for Example 1, we also described regions in the parameter space associated with overlapping periodicity regions, leading to bistability between two cycles of different periods, none of which is a fixed point.

Future research will analyze the mathematical properties of the more complex bifurcation structures which arose in some of the reported examples; also, it would be interesting to introduce other kinds of behavior that are familiar in the dynamic game theory literature, such as proportional [14] and replication dynamics [9, 10]. Finally, following the discussion about choices reported in Section 2, it would be interesting to consider the heterogeneity of agents when considering switching choices.

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