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(Article begins on next page)

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The holographic supersymmetric Casimir energy

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We consider a general class of asymptotically locally AdS_5 solutions of minimal gauged supergravity, which are dual to superconformal field theories on curved backgrounds $S^1 \times M_3$ preserving two supercharges. We demonstrate that standard holographic renormalization corresponds to a scheme that breaks supersymmetry. We propose new boundary terms that restore supersymmetry, and show that for smooth solutions with topology $S^1 \times \mathbb{R}^4$ the improved on-shell action reproduces both the supersymmetric Casimir energy and the field theory BPS relation between charges.

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I. THE SUPERSYMMETRIC CASIMIR ENERGY

In [1, 2] a new observable of $d = 4$ superconformal field theories has been introduced: the supersymmetric Casimir energy. This is defined by putting the theory on certain curved backgrounds $M_4 = S^1_\beta \times M_3$, where S^1_β is a circle of length β and M_3 is a compact three-manifold. These are rigid supersymmetric backgrounds, and the supersymmetric Casimir energy is defined as

$$E_{\rm susy} = -\lim_{\beta\to\infty} \frac{\mathrm{d}}{\mathrm{d}\beta} \log Z^{\rm susy}_{S^1_\beta \times M_3} \ . \tag{1}$$

Here the partition function $Z^{\rm susy}$ is computed with periodic boundary conditions for the fermions around S^1_β. A key point is that, unlike the vacuum energy of general $d = 4$ conformal field theories (CFTs), $E_{\rm susy}$ is scheme-independent and thus an intrinsic observable.

The rigid supersymmetric backgrounds of interest comprise a metric on M_4 of the form

$$g_4 \,=\, \mathrm{d}\tau^2 + g_3 \,=\, \mathrm{d}\tau^2 + (\mathrm{d}\psi + a)^2 + 4\mathrm{e}^w \mathrm{d}z\mathrm{d}\bar{z} \ , \tag{2}$$

where $\tau \sim \tau + \beta$ is a coordinate on S^1_β. The vector ∂_ψ is Killing, and generates a transversely holomorphic foliation of M_3, with local transverse complex coordinate z. The local one-form a satisfies $\mathrm{d}a = \mathrm{i}u\,\mathrm{e}^w \mathrm{d}z \wedge \mathrm{d}\bar{z}$, where $w = w(z,\bar{z})$, $u = u(z,\bar{z})$. In addition there is a non-dynamical Abelian gauge field, which couples to the R-symmetry current and arises when the field theory is coupled to background conformal supergravity, given by

$$\begin{aligned} A = \tfrac{\mathrm{i}}{8} u\,\mathrm{d}\tau + \tfrac{1}{4} u(\mathrm{d}\psi + a) + \tfrac{\mathrm{i}}{4}(\partial_{\bar{z}} w\,\mathrm{d}\bar{z} - \partial_z w\,\mathrm{d}z) \\ + \gamma\,\mathrm{d}\psi + \mathrm{d}\lambda(z,\bar{z}) \ . \end{aligned} \tag{3}$$

Notice that the second line is locally pure gauge; however, the constant γ will play an important role.

The background geometry thus depends on the choice of the two functions $w(z,\bar{z})$, $u(z,\bar{z})$, and via (1) the supersymmetric Casimir energy also *a priori* depends on this choice. These backgrounds admit two supercharges of opposite R-charge, and associated to each of these is an integrable complex structure (*i.e.* they are ambi-Hermitian). In [3] it is argued that the supersymmetric partition function depends on the background only via the choice of complex structure(s). In the present set-up, this implies that $Z^{\rm susy}$ depends only on the transversely holomorphic foliation generated by ∂_ψ. In particular, deformations of $w(z,\bar{z})$ and $u(z,\bar{z})$ that leave this foliation fixed should not change $E_{\rm susy}$.

Later in this paper we will focus on the case that topologically $M_3 \cong S^3$. Here we may embed $S^3 \subset \mathbb{R}^4 = \mathbb{R}^2 \oplus \mathbb{R}^2$, and write[1] $\partial_\psi = b_1 \partial_{\varphi_1} + b_2 \partial_{\varphi_2}$, where φ_1, φ_2 are standard 2π periodic azimuthal angles. In this case the above statements imply that $E_{\rm susy}$ should depend only on b_1, b_2, and the explicit calculation in [1] gives

$$E_{\rm susy} = \frac{2(b_1+b_2)^3}{27 b_1 b_2}(3\mathtt{c} - 2\mathtt{a}) + \frac{2}{3}(b_1+b_2)(\mathtt{a} - \mathtt{c}) \, . \tag{4}$$

Here $\mathtt{a}$ and $\mathtt{c}$ are the usual trace anomaly coefficients for a $d = 4$ CFT. For field theories admitting a large N gravity dual in type IIB supergravity, to leading order in the $N \to \infty$ limit one has $\mathtt{a} = \mathtt{c} = \pi^2/\kappa_5^2$, where κ_5^2 is the five-dimensional effective gravity constant and we have set the AdS radius to 1. In this limit (4) reduces to

$$E_{\rm susy} = \frac{(b_1+b_2)^3}{b_1 b_2} \frac{2\pi^2}{27\kappa_5^2} \ . \tag{5}$$

In particular the conformally flat $S^1_\beta \times S^3$, where $M_3 \cong S^3$ is equipped with the standard round metric of radius r_3, has $b_1 = b_2 = 1/r_3$, leading to $E_{\rm susy} = 16\pi^2/27 r_3 \kappa_5^2$. We will reproduce (5) from a dual supergravity calculation.

II. DUAL SUPERGRAVITY SOLUTIONS

The gravity duals are constructed in $d = 5$ minimal gauged supergravity, whose solutions uplift to type IIB supergravity. In Euclidean signature, the bosonic part of the action reads

$$\begin{aligned} S_{\rm bulk} = -\frac{1}{2\kappa_5^2} \int_{M_5} \Big[\mathrm{d}^5x \sqrt{\det G}\, \left(R_G - \mathcal{F}_{\mu\nu}\mathcal{F}^{\mu\nu} + 12\right) \\ - \tfrac{8\mathrm{i}}{3\sqrt{3}} \mathcal{A} \wedge \mathcal{F} \wedge \mathcal{F} \Big] \ . \end{aligned} \tag{6}$$

[1] In this paper our conventions are such that $b_1, b_2 > 0$.

Here $G = (G_{\mu\nu})$ denotes the five-dimensional metric, R_G is its Ricci scalar, $\mathcal{A}$ is the graviphoton and $\mathcal{F} = \mathrm{d}\mathcal{A}$.

We are interested in supersymmetric solutions that are asymptotically locally Anti-de Sitter (AlAdS), with metric and graviphoton on the conformal boundary given by (2) and (3). Employing a coordinate system defined canonically by supersymmetry, we have solved the supersymmetry conditions and equations of motion in a series expansion near the boundary. We have then cast the solution in Fefferman-Graham coordinates [4], where the metric is $G = \mathrm{d}\rho^2/\rho^2 + h_{ij}(x,\rho)\mathrm{d}x^i\mathrm{d}x^j$, and

$$h = \frac{1}{\rho^2}\left[h^{(0)} + h^{(2)}\rho^2 + \left(h^{(4)} + \tilde{h}^{(4)}\log\rho^2\right)\rho^4 + \mathcal{O}(\rho^5)\right],$$
$$\mathcal{A} = A^{(0)} + \left(A^{(2)} + \tilde{A}^{(2)}\log\rho^2\right)\rho^2 + \mathcal{O}(\rho^3)\ . \tag{7}$$

Here the conformal boundary is at $\rho = 0$. The terms at leading order in the expansions, $h^{(0)} \equiv g_4$ and $A^{(0)} \equiv -A/\sqrt{3}$, coincide with the metric (2) and gauge field (3), respectively. These depend only on the functions $w(z,\bar{z})$ and $u(z,\bar{z})$, which we therefore refer to as *boundary functions*. $h^{(2)}$, $\tilde{h}^{(4)}$, and $\tilde{A}^{(2)}$ are uniquely fixed in terms of these, whereas $h^{(4)}$ and $A^{(2)}$ are not determined by the conformal boundary, and parametrize the one-point functions of the dual field theories. These depend on four new functions $k_1(z,\bar{z})$, $k_2(z,\bar{z})$, $k_3(z,\bar{z})$ and $k_4(z,\bar{z})$, that we refer to as *non-boundary functions*. The first three of these appear in the expansion of the gauge field:

$$A^{(2)} = \tfrac{1}{64\sqrt{3}}\Big[\left(-96k_1 - 32uk_2 + 4u\Box w + \tfrac{3}{2}u^3\right)\mathrm{i}\mathrm{d}\tau + \frac{1}{u}\Big(128k_3 - 32uk_1 - \tfrac{64}{3}k_2^2 + 16\Box k_2 - \tfrac{32}{3}k_2\Box w - 16u^2k_2$$
$$+ 3\Box(\Box w + u^2) - 2(\Box w)^2 - \tfrac{5}{3}u^2\Box w - 3\mathrm{e}^{-w}\partial_z u\partial_{\bar{z}}u - \tfrac{5}{12}u^4\Big)\left(-\mathrm{i}\mathrm{d}\tau + \mathrm{d}\psi + a\right) - *_2\mathrm{d}\left(32k_2 + u^2\right)\Big], \tag{8}$$
$$\tilde{A}^{(2)} = \tfrac{1}{32\sqrt{3}}\left[\Box u\,\mathrm{i}\mathrm{d}\tau + \left(2\Box u - u\Box w - \tfrac{1}{2}u^3\right)(\mathrm{d}\psi + a) + *_2\mathrm{d}\left(2\Box w + u^2\right)\right]\ ,$$

where $\Box \equiv \mathrm{e}^{-w}\partial_z\partial_{\bar{z}}$ and $*_2\mathrm{d} \equiv \mathrm{i}(\mathrm{d}\bar{z}\,\partial_{\bar{z}} - \mathrm{d}z\,\partial_z)$. A more exhaustive discussion will be presented in [5].

The bulk action evaluated on a solution is divergent and must be renormalized by the addition of counterterms. As usual, we include the Gibbons-Hawking term

$$S_{\mathrm{GH}} = -\frac{1}{\kappa_5^2}\int_{\partial M_\epsilon}\mathrm{d}^4x\sqrt{\det h}\,K\ , \tag{9}$$

to have a well-defined variational principle. Here h is the metric (7) induced on a four-dimensional hypersurface $\partial M_\epsilon = \{\rho = \epsilon = \text{constant}\}$, and K the trace of its second fundamental form. The counterterms

$$S_{\mathrm{ct}} = \frac{1}{\kappa_5^2}\int_{\partial M_\epsilon}\mathrm{d}^4x\sqrt{\det h}\left(3 + \tfrac{1}{4}R_h\right)\ , \tag{10}$$

cancel all divergences as $\epsilon \to 0$. In general there is also a $\log\epsilon$ divergence in the action, related to the field theory Weyl anomaly; but in the limit $\epsilon \to 0$ for this class of backgrounds we have $\mathcal{E} \equiv 0$, $C_{ijkl}C^{ijkl} \equiv 8\mathcal{F}_{ij}\mathcal{F}^{ij}$, and this term vanishes identically [6]. Here R_h, C_{ijkl} and $\mathcal{E}$ are the Ricci scalar, Weyl tensor and Euler density of the metric h, respectively. We also include a linear combination of the standard finite counterterms

$$\Delta S_{\mathrm{st}} = -\frac{1}{\kappa_5^2}\int_{\partial M_\epsilon}\mathrm{d}^4x\sqrt{\det h}\left(\varsigma\,R_h^2 - \varsigma'\,\mathcal{F}_{ij}\mathcal{F}^{ij}\right), \tag{11}$$

where ς and ς' are arbitrary constants. These affect the holographic one-point functions, as well as the on-shell action. The ordinary renormalized action is obtained as

$$S \equiv \lim_{\epsilon\to 0}\left(S_{\mathrm{bulk}} + S_{\mathrm{GH}} + S_{\mathrm{ct}} + \Delta S_{\mathrm{st}}\right)\ . \tag{12}$$

A variation of the total on-shell action with respect to boundary data takes the form

$$\delta S = \int_{M_4}\mathrm{d}^4x\sqrt{\det g_4}\left(-\tfrac{1}{2}T_{ij}\delta g_4^{ij} + j^i\delta A_i\right)\ , \tag{13}$$

where g_4 is the ρ-independent metric (2) on the conformal boundary. The holographic energy-momentum tensor T_{ij} and R-symmetry current j^i may be computed with standard formulas (see *e.g.* [7]). The former is particularly unwieldy, but we have verified that these satisfy the expected Ward identities. In particular, the R-symmetry current is conserved, $\nabla_i j^i = 0$, and the energy-momentum tensor obeys the correct conservation equation

$$\nabla^i T_{ij} = j^i F_{ji}\ , \tag{14}$$

with $F = \mathrm{d}A$. However, we will show next that imposing supersymmetric Ward identities requires a non-standard modification of the holographic renormalization scheme.

III. SUPERSYMMETRIC HOLOGRAPHIC RENORMALIZATION

According to the gauge/gravity duality, the renormalized on-shell gravitational action is identified with minus the logarithm of the partition function of the dual field theory, in the large N limit. Namely

$$Z^{\mathrm{susy}}_{S^1_\beta\times M_3} = \mathrm{e}^{-S[M_5]}\ , \tag{15}$$

where $S[M_5]$ is evaluated on an appropriate supergravity solution, as described in the previous section. Assuming (15), the field theory results summarized in the first section imply that S should be invariant under deformations of the boundary geometry that leave fixed the transversely holomorphic foliation generated by ∂_ψ. Concretely, this implies that S should be invariant under $w \to w + \delta w$, $u \to u + \delta u$, where $\delta w(z,\bar{z})$, $\delta u(z,\bar{z})$ are arbitrary smooth global functions on M_3, invariant under ∂_ψ. The corresponding variation of S may be computed explicitly using the general formula (13). We find

$$\begin{aligned}\delta_w S = \int_{M_4} \frac{\mathrm{d}^4x\sqrt{\det g_4}}{2^6 3\kappa_5^2}\,\delta w\,\big[&(1-96\varsigma+16\varsigma')u^2R_{2d} + \tfrac{1}{2}(1-96\varsigma+28\varsigma')\Box u^2 + 12\varsigma' u\Box u \\ &-\tfrac{1}{32}(19-288\varsigma+192\varsigma')u^4 - 8(-24\varsigma+\varsigma')(R_{2d}^2+2\Box R_{2d}) + \tfrac{8}{9}\gamma(2uR_{2d}+2\Box u - u^3)\big]\,, \\ \delta_u S = \int_{M_4} \frac{\mathrm{d}^4x\sqrt{\det g_4}}{2^9 3^2\kappa_5^2}\,\delta u\big[&-24(1-96\varsigma+16\varsigma')uR_{2d} - 288\varsigma'\Box u + (19-288\varsigma+192\varsigma')u^3 + \tfrac{32}{3}\gamma(3u^2-4R_{2d})\big]\,.\end{aligned} \tag{16}$$

Here $R_{2d} \equiv -\Box w$ is the Ricci scalar of the transverse two-dimensional metric $4\mathrm{e}^w \mathrm{d}z\mathrm{d}\bar{z}$. We emphasize that this is locally, but not globally, a total derivative. Notice the dependence on the constant γ, which appears in the boundary gauge field A in (3). In the first variation in (16) we hold $\mathrm{d}a$ fixed, meaning that $\delta(u\,\mathrm{e}^w) = 0$ and hence $\delta u = -u\,\delta w$; while the second variation in (16) is the change in S under an arbitrary variation δu. In obtaining these expressions we have used Stokes' theorem to discard total derivative terms. In particular, we find that all dependence on the non-boundary functions drops out of these integrals, as does $\mathrm{d}\lambda(z,\bar{z})$ in (3).

Crucially we see that there is no choice of ς, ς' for which these variations are zero for an arbitrary background. The standard holographic renormalization of the previous section hence does not correspond to the supersymmetric renormalization scheme used in field theory. This result explains why previous attempts to obtain the holographic supersymmetric Casimir energy have failed.

Remarkably, we have found that if we define the new "finite counterterms"

$$\Delta S_{\mathrm{new}} = -\frac{1}{\kappa_5^2}\int_{M_4}(\mathrm{i}A\wedge\Phi + \Psi)\,, \tag{17}$$

where

$$\begin{aligned}\Phi &\equiv \tfrac{1}{2^3 3^3}\left(u^3 - 4uR_{2d}\right)\mathrm{i}\,\mathrm{e}^w \mathrm{d}z\wedge\mathrm{d}\bar{z}\wedge(2\,\mathrm{d}\psi + \mathrm{i}\,\mathrm{d}\tau)\,, \\ \Psi &\equiv \tfrac{1}{2^{11}3^2}\left(19u^4 - 48u^2R_{2d}\right)\mathrm{d}^4x\sqrt{\det g_4}\,,\end{aligned} \tag{18}$$

then (16) implies that

$$S_{\mathrm{susy}} \equiv \lim_{\epsilon\to 0}(S_{\mathrm{bulk}} + S_{\mathrm{GH}} + S_{\mathrm{ct}}) + \Delta S_{\mathrm{new}} \tag{19}$$

is invariant under $w \to w+\delta w$, $u \to u+\delta u$. We claim that (19) is the correct renormalized supergravity action for the class of backgrounds introduced in the first section, in the sense that this corresponds to the unique supersymmetric renormalization scheme used in field theory. In particular, this result should be valid for arbitrary topology of M_3. Specializing to the case $M_3 \cong S^3$, in the next section we shall not only show that (19) correctly reproduces (5), but moreover we are able to determine the holographic charges in this scheme, and prove that these satisfy the correct BPS relation in field theory.

IV. ON-SHELL ACTION AND HOLOGRAPHIC CHARGES

In general to evaluate the bulk action one needs to know the full solution. However, with some additional topological assumptions, and assuming that a bulk filling exists, one can compute S_{susy} in (19) explicitly.

We henceforth take $M_3 \cong S^3$. In this case the boundary supercharges are sections of a trivial bundle, and correspondingly A in (3) is a *global* one-form. As shown in [1] this fixes the constant $\gamma = (b_1+b_2)/2$, which physically is the charge of the spinors under ∂_ψ. As in the solution of [7], we assume the bulk filling is smooth with topology $S^1\times\mathbb{R}^4$, with the bulk graviphoton $\mathcal{A}$ smoothly extending A on the boundary. These assumptions, together with supersymmetry, allow one to write the bulk action as a total derivative, and hence express S_{susy} as the limit of a term evaluated near the conformal boundary. However, this expression still depends on non-boundary functions, which are only determined by regularity in the deep interior of the solution. Fortunately, we may bypass this problem using another idea from [7]. If $\mathcal{C} \cong \mathbb{R}^4$ is a regular hypersurface at $\tau =$ constant, with boundary $M_3 \cong S^3$ at infinity, then combining the Maxwell equation and Stokes' theorem on $\mathcal{C}$ one can show that

$$\int_{M_3}\left(*_5\mathcal{F} + \tfrac{2\mathrm{i}}{\sqrt{3}}\mathcal{A}\wedge\mathcal{F}\right) = 0\,. \tag{20}$$

Substituting (7) and (8) in, this identity may be used to eliminate *all* dependence of the on-shell action on non-boundary functions. Also discarding terms which are total derivatives on M_4, and noting that (17) leads to extensive cancellations, (19) evaluates to the remarkable

formula

$$S_{\rm susy} \,=\, \frac{\gamma^2}{27\kappa_5^2}\int_{M_4} {\rm d}^4x\sqrt{\det g_4}\, R_{2d} \ . \qquad (21)$$

We reiterate that this has been derived here for $M_4 \cong S^1_\beta \times S^3$, although as we shall explain in [5] this formula has larger validity. As remarked earlier, R_{2d} is locally but not globally a total derivative. Its integral is a topological invariant of the foliation, proportional to the transverse first Chern class. Using the explicit formulas in [1] for the metric functions and coordinate ranges for $M_3 \cong S^3$ with $\partial_\psi = b_1\partial_{\varphi_1} + b_2\partial_{\varphi_2}$, we find

$$\int_{M_3} {\rm d}^3x\sqrt{\det g_3}\, R_{2d} = 2(2\pi)^2\frac{b_1+b_2}{b_1 b_2} \ . \qquad (22)$$

Substituting this into (21), using $\gamma = (b_1+b_2)/2$ and that τ has period β, we find that $S_{\rm susy} = \beta E_{\rm susy}$, where $E_{\rm susy}$ is the field theory result (5)!

The above argument applies to any solution with topology $S^1 \times \mathbb{R}^4$, but it is worth emphasizing that there are explicit examples. The new counterterms (17) are non-zero even for AdS_5 in global coordinates, whose boundary is the conformally flat $S^1_\beta \times S^3$ geometry with $b_1 = b_2 = 1/r_3$ mentioned at the end of the first section. The solution of [7] has a squashed $S^1_\beta \times S^3_v$ boundary, with the bulk solution depending non-trivially on the squashing parameter v. However, $b_1 = b_2 = 1/vr_3$, and we find that $S_{\rm susy}$ is a simple rescaling of the action of AdS_5.

Finally, we turn to the holographic charges. Let us start from the standard charges, which may be obtained from T_{ij} and j^i (defined through (12), (13)). Due to the Ward identity (14) the canonical Hamiltonian H and angular momentum J associated to translations along ∂_τ and $-\partial_\psi$ are defined as

$$\begin{aligned} H \,&\equiv\, \int_{M_3} {\rm d}^3x\sqrt{\det g_3}\,(T_{\tau\tau} + j_\tau A_\tau) \ , \\ J \,&\equiv\, {\rm i}\int_{M_3} {\rm d}^3x\sqrt{\det g_3}\,(T_{\tau\psi} + j_\tau A_\psi) \ , \end{aligned} \qquad (23)$$

respectively. On the other hand, the holographic R-charge is defined as

$$Q \,\equiv\, -{\rm i}\int_{M_3} {\rm d}^3x\sqrt{\det g_3}\, j^\tau \ . \qquad (24)$$

In the dual field theory, these are identified with the vev of the corresponding operators $\langle H\rangle$, $\langle J\rangle$, and $\langle Q\rangle$.

Utilizing a trick introduced in [7] and elaborated in [5], one can then show that

$$\beta H \,=\, S \qquad \text{and} \qquad J \,=\, 0 \ . \qquad (25)$$

Recall that the supersymmetry algebra implies that in the field theory vacuum the BPS relation

$$\langle H\rangle + \langle J\rangle + \gamma\langle Q\rangle \,=\, 0 \ , \qquad (26)$$

should hold, with $\langle H\rangle = E_{\rm susy}$ [2]. However, for the Euclidean AdS_5 solution, which is expected to correspond to the vacuum of theories in conformally flat space, one finds that $J\,|_{\mathrm{EAdS}_5} = Q\,|_{\mathrm{EAdS}_5} = 0$, implying that (26) is violated.

Assuming that the identity (13) holds replacing S with $S_{\rm susy}$, and correspondingly $T_{ij} \to T_{ij}^{\rm susy}$, $j_i \to j_i^{\rm susy}$, we can define "supersymmetric" versions of the holographic charges, via formulas analogous to (23) and (24). In particular, the improved electric charge may be defined as

$$Q_{\rm susy} \equiv -{\rm i}\int_{M_3}\frac{\delta S_{\rm susy}}{\delta A_\tau} = Q - \frac{1}{\kappa_5^2}\int_{M_3}\Phi \ , \qquad (27)$$

and by direct computation we find

$$\gamma Q_{\rm susy} = -\frac{1}{\beta}S_{\rm susy} \ . \qquad (28)$$

Moreover, using the relations (25) applied to the improved Hamiltonian and angular momentum, we deduce that $\beta H_{\rm susy} = S_{\rm susy}$ and $J_{\rm susy} = 0$, thus showing that these obey the BPS relation (26).

V. CONCLUDING REMARKS

We have constructed new boundary terms of five-dimensional minimal gauged supergravity that we argued are necessary to restore supersymmetry of the gravitational action in a large class of AlAdS_5 solutions. Including these counterterms, we have reproduced the supersymmetric Casimir energy and the field theory BPS relation between charges [1, 2]. More details, as well as a number of generalizations, will be presented in [5]. For example, we will perform an analogous computation in four-dimensional gauged supergravity, finding that no new counterterms are needed. In five dimensions we will consider M_3 with more general topology, making contact with [8], as well as a twisting of S^1_β over M_3.

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